ABSTRACT

IVES, SARAH ELIZABETH. Learning to Teach Probability: Relationships among Preservice Teachers' Beliefs and Orientations, Content Knowledge, and Pedagogical Content Knowledge of Probability. (Under the direction of Hollylynne Stohl Lee.)

The purposes of this study were to investigate preservice mathematics teachers' orientations, content knowledge, and pedagogical content knowledge of probability; the relationships among these three aspects; and the usefulness of tasks with respect to examining these aspects of knowledge. The design of the study was a multi-case study of five secondary mathematics education preservice teachers with a focus on their knowledge as well as tasks that were used in this study. Data from individual interviews and test items were collected and analyzed under a conceptual framework based on the work of Hill, Ball, and Schilling (2008); Kvatinsky and Even (2002); and Garuti, Orlandoni, and Ricci (2008). The researcher found that the preservice teachers held multiple orientations towards probability yet tended to be mostly objective (mathematical and statistical) with little evidence of subjective orientations. Relationships existed between the preservice teachers' orientations and their content knowledge, as well as their pedagogical content knowledge. These relationships were found more in tasks where they were required to make a claim about a probability within some sort of real-world context. The researcher also found that tasks involving pedagogical situations tended to be more effective at eliciting knowledge than tasks involving only questions.

Learning to Teach Probability: Relationships among Preservice Teachers' Beliefs and Orientations, Content Knowledge, and Pedagogical Content Knowledge of Probability

by
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BIOGRAPHY

Sarah Ives was born in Clinton, Iowa on March 10, 1977. She is the youngest of three children. She graduated from East Grand Rapids High School in 1995 and went on to pursue a Bachelor's degree in mathematics from the College of Wooster. She graduated in May, 1999 and from there went on to the University of North Carolina at Wilmington to complete a Master's in mathematics. During her studies at UNCW Sarah was a teaching assistant and gained valuable experience assisting several different professors. After graduating in 2001 Sarah continued her education at North Carolina State University to complete a Doctorate degree in mathematics.

While studying in the mathematics department Sarah had a teaching assistant position. Within this capacity Sarah taught Calculus I, Introduction to Mathematics, and Mathematics Finance. In fall 2003 Sarah decided to change her major from mathematics to mathematics education. Knowing since high school that she wanted to teach mathematics at the college level Sarah realized she would be a more effective teacher with a degree in mathematics education.

Starting in the spring of 2004 Sarah began taking course work in mathematics education. During the calendar year of 2004 she had the opportunity to work with Dr. Hollylynne Lee on a research project focused on technology and collaboration between mathematics and engineering education. Since that time Sarah has worked within the Wake

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CHAPTER 1: INTRODUCTION

Probability can be considered one of the most important areas of mathematics for students to study due to the fact that it applies to many areas of daily life, including health decisions, sports, and weather. In the United States over the past 20 years, there has been increased importance of probability within the recommendations given by the National Council of Teachers of Mathematics (NCTM, 1989, 2000) for standards pertaining to K-12 mathematics. This is not a nationally adopted curriculum but a suggested set of standards that most states use to guide their state-level curriculum. Due to this increase, students are expected to learn, and teachers are expected to teach, probability concepts as early as primary school. Yet recent research has shown that many teachers, at primary, middle, secondary, as well as post-secondary levels, often struggle to teach probability concepts (e.g., Batanero, Godino, & Roa, 2004; Begg & Edwards, 1999; Haller, 1997; Liu, 2005; Mojica, 2006).

Little research has been conducted specifically on preservice teachers' understanding of both the learning and teaching of probability. This study, therefore, examined preservice teachers' orientations and beliefs about probability, content knowledge, and pedagogical content knowledge of probability. Additionally this study examined relationships that exist among these three aspects.

Significance of Study

A strong conceptual understanding of probability is important for a variety of reasons: it is required for critically reading newspaper reports, interpreting statistics, and to be an informed citizen; it helps one to understand and evaluate information in the world around us; and it is a prerequisite to entering many other fields of study (Ahlgren & Garfield, 1991). Reasons to include probability and statistics in the K-12 curriculum include "the usefulness of statistics and probability for daily life, its instrumental role in other disciplines, the need for a basic stochastic knowledge in many professions and its role in developing critical reasoning" (Batanero, et al., 2004, p. 1).

The NCTM's (2000) *Principles and Standards for School Mathematics* (PSSM) is intended as a guide for improving the teaching and learning of mathematics in the US. Within the PSSM, NCTM stresses the importance of probability by including 'Data analysis and Probability' as one of the five content standards recommended. In addition, Shaughnessy (1993) stated "… perhaps no other branch of mathematical sciences is as important for all students, college-bound or not, as probability and statistics" (p. 244). Therefore, it is important to teach probability in schools which requires a demand for preparing future teachers in this area.

Extensive literature has addressed the complexity of understanding probability for students at grade levels from elementary up to college level (e.g., Batanero & Serrano, 1999; Drier, 2000; Glencross, 1998; Jones, Langrall, Thornton, & Mogill, 1999; Lee,

Rider, & Tarr, 2006; Maher, Speiser, Friel, & Konold, 1998; Pfannkuch & Brown, 1996; Pratt, 2000; Shaughnessy, 2003; Vahey, 2000; Watson & Moritz, 2003). In light of this evidence of problems with understanding probability, there has been a call for studies on the understanding of the teaching of probability (Jones, Langrall, & Mooney, 2007; Pfannkuch, 2005; Pratt, 2005; Shaughnessy, 1992, 2003). Little research on probability and statistics education has attended to teacher's understanding of probability and statistics or to their thinking on how to teach these subjects (Liu, 2005). However, a few studies have been done specifically on preservice mathematics teachers' understanding of teaching probability (Carnell, 1997; Carter, Zientek, & Capraro, 2005; Koirala, 2003; Lane, 2002; Peard, 2005). More research in this area is needed if we are to improve the teaching of probability.

The emergence of the importance of better understanding issues related to teaching probability and statistics is evidenced in the joint International Commission on Mathematical Instruction (ICMI) and International Association for Statistical Education (IASE) conference in 2008 that focused on these issues. The title of this conference was "Statistics Education in School Mathematics: Challenges for Teaching and Teaching Education," and one of the topic working groups in which the researcher participated was: "Teachers' Attitudes, Knowledge, Conceptions and Beliefs in Relation to Statistics Education." Within this topic, one of the guiding questions, "How do teachers' attitudes and beliefs about statistics and teaching statistics affect their pedagogical approaches?"

was posed as a significant research question (Batanero, 2006, p. 6). However, at the completion of the conference, the facilitators of that working group concluded that this question had yet to be adequately addressed in the current research (Batanero, & Burrill, personal communication, July 3, 2008). Additionally, research is needed to address how teacher educators can influence change in practicing and preservice teachers' beliefs about learning and teaching probability. The increased importance of probability in the classroom, the evidence of students' difficulties learning probability, and the lack of research on preservice teachers' beliefs and pedagogical content knowledge of probability, all lead to a need for the research in this study.

Teachers' Knowledge and Beliefs as Influences on Pedagogy

Students at all grade levels have difficulty understanding probability concepts. One reason for this difficulty is because of commonly held intuitive biases that are contradictory to probabilistic reasoning (Aspinwall & Shaw, 2000; Fischbein & Gazit, 1984; Konold, 1995; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Pfannkuch & Brown, 1996; Sedlmeier & Gigerenzer, 1997). Another reason for students' difficulty learning probability is that teachers are often unprepared, ill-equipped, or unmotivated to teach probability (Batanero, et al., 2004; Garfield & Ahlgren, 1988; Greer & Mukhopadhyay, 2005; Konold, et al., 1993; Stohl, 2005; Watson, 2001). Additionally, "many teachers have not studied probability in their own K-12 mathematics courses and sometimes need convincing as to why they need to learn and teach probability concepts"

(Papaieronymou, 2008, p. 2). Teachers that do not possess a strong understanding of the concepts within probability will likely not be able to teach probability effectively. While the extent of teachers' subject matter knowledge of mathematical concepts has an effect on their ability to teach mathematics, their content knowledge alone is not sufficient (Ball, 1990, 2000; Ball, Lubienski, & Mewborn, 2001; Thompson, 1984).

Teachers' content knowledge, or lack thereof, has an influence on their pedagogical decisions, such as the selection and creation of mathematical tasks, instructional orientations, and types of assessments (Ball, 2000; Shulman, 1986). These decisions are based on a teacher's knowledge that is specific to the content – a knowledge referred to as pedagogical content knowledge (PCK). This interplay of content knowledge and pedagogical content knowledge plays an important role in how well students understand mathematics in instructional settings (Hill, Rowan, & Ball, 2005; Thompson, 1984).

To address the demand for probability and statistics in the classroom, future teachers should have the content knowledge needed to make informed pedagogical decisions. Careful analysis of preservice teachers' content and pedagogical content knowledge can inform teacher educators' practice. Furthermore, there are many orientations and perspectives unique to probability that may have an impact on content and pedagogical content knowledge. Thus, there is a need to research the relationships among these orientations, content knowledge, and pedagogical content knowledge.

One way to improve the teaching of probability at the primary and secondary levels is through teacher education programs. In this respect, it is necessary for teacher educators to first understand what preservice teachers know about probability and the teaching of probability. Therefore, the purpose of this study is to address the need for research focused on preservice teachers' pedagogical content knowledge of probability, as well as relationships among beliefs, content knowledge, and pedagogical content knowledge. Through the use of task based interviews and test item analysis, this study explores the nature of preservice teachers' beliefs, content and pedagogical content knowledge of probability. There are three goals of this study: (a) to investigate preservice teachers' orientations towards probability, content knowledge, and pedagogical content knowledge of probability; (b) to better understand relationships among preservice teachers' orientations about probability, content knowledge, and pedagogical content knowledge of probability; and (c) to investigate the usefulness of pedagogical tasks designed to confront and improve orientations, content knowledge, and pedagogical content knowledge of probability among preservice teachers. A better understanding of preservice teachers' understanding (this includes their orientations, content knowledge, and pedagogical content knowledge as well as relationships among these) can assist mathematics teacher educators in designing useful teaching methods, or intervention strategies, that may help preservice teachers gain a firmer understanding of the teaching and learning of probability.

Definitions of Terms

Some of the terms used for this study are defined in this section. It is important to clarify what is meant by *beliefs, orientations, content knowledge,* and *pedagogical content knowledge.* Additionally the definition for *preservice teachers* is given as well.

For the purpose of this study, *beliefs* refer to how a person views the meaning of probability. Philipp (2007) offered this definition, "beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). These beliefs can be about mathematics content and also pedagogy. How a person believes students learn and what the role of a teacher is are some examples of beliefs.

O*rientations* refer specifically to how a person views the meaning of probability and how to calculate probability. There are four orientations considered in this study: statistical, mathematical, subjective, and personal (Garuti, Orlandoni, & Ricci, 2008); also, a person can be categorized as having more than one orientation. For example, someone may interpret probability as the number of favorable outcomes out of the number of total outcomes, using a mathematical orientation, or a classical objective understanding of probability. However the same person may also take the view that probability is more subjective and is a degree of belief that is based on past experiences or personal beliefs.

Content knowledge of probability is defined as the general knowledge one has of probability concepts that are not specific to the teaching of probability. Some of the

concepts related to probability addressed in this study are: randomness, empirical probability, theoretical probability, sample space, sample size, variability, and the law of large numbers.

Pedagogical Content Knowledge was first defined by Shulman (1986) as the knowledge teachers need to have of the content they teach. Ball (2000) expanded on this notion of bridging the two areas of knowledge, pedagogy and content: "It is not just what mathematics teachers know, but how they know it and what they are able to mobilize mathematically in the course of teaching" (p. 243). Pedagogical content knowledge involves blending ideas into an understanding of how problems and tasks are organized, represented, and adapted to meet the diverse needs of learners with different abilities.

Preservice teacher is defined as a college student of mathematics education preparing to be a teacher but who has only had limited experiences in classrooms. In this study *preservice teacher* refers to the participants because they have not started their teaching careers yet and are currently studying mathematics education.

In summary, the goals of this study are: to investigate preservice teachers' orientations, content knowledge, and pedagogical content knowledge of probability; identify relationships that exist among these aspects; and to offer suggestions of tasks for teacher educators to use when preparing preservice teachers to teach probability. A thorough literature review and the framework are presented in Chapter 2. The specific research questions are then posed at the end of Chapter 2.

CHAPTER 2: LITERATURE REVIEW

Over the past two decades there has been increased attention on the teaching and learning of probability and statistics (Borovcnik & Peard, 1996; Garfield & Ahlgren, 1988; Jones, 2005; Kapadia & Borovcnik, 1991; NCTM, 1989, 2000; Shaughnessy, 1992). NCTM (2000) proposed that ideas about probability should start as early as pre-kindergarten where the treatment of probability ideas is informal. However, most teachers at all levels- elementary, middle, and secondary- are inadequately prepared to teach probability concepts (Haller, 1997; Lopes & de Moura, 2002; Stohl, 2005; Watson, 2001). Teachers' content knowledge is closely tied to their beliefs; thus in order to improve future teachers' knowledge of probability, we must first understand their beliefs, current knowledge *and* how they use their beliefs and knowledge to teach probability (Philipp, 2007; Thompson, 1992).

There are several aspects from the literature that are pertinent in laying a foundation for this study: (a) the nature of probability and randomness, (b) students' and teachers' beliefs and content knowledge of probability, and (c) teachers' pedagogical content knowledge of probability. Each topic will be presented in depth. The chapter concludes with a presentation of the framework developed to guide the research and the specific research questions.

Beliefs and Content Knowledge of Probability

"Although many articles in the education literature recommend how to teach statistics better, there is little published research on how students actually learn statistics concepts" (Garfield & Ahlgren, 1988). In their 1988 review of the literature on students' understanding of probability and statistics concepts, Garfield and Ahlgren found that more cross-disciplinary research on how students think about probability and statistical concepts is needed.

Since that time, research has been conducted on students' learning of probability at many levels: elementary (Drier, 2000; Jones, Langrall, et al., 1999; Jones, Thornton, Langrall, & Tarr, 1999), middle (Aspinwall & Shaw, 2000; Stohl & Tarr, 2002; Vahey, 2000), secondary (Batanero & Serrano, 1999; Castro, 1998; Konold, et al., 1993; Maher, et al., 1998; Shaughnessy, 1999), and tertiary (Kahneman & Tversky, 1972; Konold, 1989, 1995; Shaughnessy, 1977). Among the tertiary studies, a few have been dedicated to preservice mathematics education teachers and their understanding of probability (Begg & Edwards, 1999; Canada, 2006; Capraro, Capraro, Parker, Kulm, & Raulerson, 2005; Carter & Capraro, 2005; Fabrizio, Lopez, & Plencovich, 2007; Fast, 1997; Lane, 2002).

This section of the literature review is focused on beliefs and content knowledge of probability and is organized into two sections: students' beliefs and content knowledge of probability, and teachers' content knowledge of probability. Some of the topics within

this first section include: the mathematical knowledge needed, effects of intuitions and strategies on probabilistic reasoning, and interests and beliefs.

Students' Beliefs and Content Knowledge of Probability

The most prevalent finding among the research in this area is that students of all ages have difficulties understanding probability. Garfield and Ahlgren (1988) offer three main reasons for these difficulties: "1) students do not have a strong enough understanding of basic rational numbers and proportional reasoning; 2) probability ideas often contradict how students view the world; and 3) students may already dislike probability because they have mainly been taught in an abstract formal way" (p. 47).

In a more recent review of the literature on the learning and teaching of probability, Jones, Langrall, and Mooney (2007) focused on research related to students' understanding of three key themes: 1) randomness and chance, 2) sample space, and 3) probability measurement. These themes were chosen because they reflected the themes that were prevalent in their review of curricula from the United States, United Kingdom, and Australia.

A majority of the research found that difficulties in understanding probability and misconceptions of probability are due to students' intuitions and how these affect their understanding (Aspinwall & Shaw, 2000; Borovcnik, Bentz, & Kapadia, 1991; Fischbein, 1987; Fischbein & Gazit, 1984; Konold, 1995; Pfannkuch & Brown, 1996; Sedlmeier & Gigerenzer, 1997; Vahey, 1997, 2000). Three main contributions to students'

understanding of probability are discussed in this section: 1) the mathematical knowledge needed to understand probability, 2) effects of intuitions on understanding probability, and 3) beliefs about and interest in probability.

Mathematical Knowledge Needed

In the book *Navigating through Probability in Grades 6-8,* published by NCTM, it is the expectation given by Bright, Frierson, Tarr, and Thomas (2003) that all students, age-appropriately, will learn to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data; and
- understand and apply basic concepts of probability (p. 2).

These expectations reveal NCTM's position that probability should be taught using statistical data and students should be able to reason probabilistically about chance situations.

One aspect of probabilistic reasoning is that it is associated with a context of uncertainty (DelMas, 2004; Jones, Thornton, et al., 1999; Scheaffer, 2006). By observing the thinking of elementary and middle school students, Jones Thornton, et al. (1999) identified six key concepts within probabilistic reasoning: sample space, experimental

probability, theoretical probability, probability comparisons, conditional probability and independence. In order for students to be able to reason probabilistically, they also need to have an understanding of such mathematical topics as proportions, multiplicative relationships, and the connections between fractions, percents, and decimals.

Many students have difficulty with probability due to their lack of understanding of proportions, part-part, and part-whole relationships. For a review on the difficulties students have when reasoning about proportions see Tourniaire and Pulos (1985). Also for further detail on the role of proportional reasoning in the contexts of sampling and comparing data sets see Watson and Shaughnessy (2004).

Because probabilities can be expressed in a classical approach as a ratio of frequencies – desired outcomes divided by total outcomes – it can be inferred that students must have an understanding of fraction and proportional reasoning. In their study on the improvement of probabilistic reasoning with age, Fischbein and Gazit (1984) argued that probabilistic reasoning and proportional reasoning- while connected- are based on two distinct mental schema. They found that although a student might improve their understanding of proportions, this did not necessarily mean they improved their probabilistic reasoning. This finding supports the argument that probabilistic reasoning is complex and is based on many, sometimes conflicting, constructs.

The literature on students' understanding of probabilistic concepts that has been outlined above has implications for teacher education. Preservice teachers need to be

aware of these conceptions and misconceptions related to probability. The underlying importance of randomness, proportional reasoning, and the equivalence between fractions, decimals, and percents, should be addressed when preparing future teachers to teach probability.

Effects of Intuitions and Strategies on Probability Understanding

Students as well as teachers have intuitive biases that are often contradictory to appropriate probabilistic reasoning (Fischbein & Gazit, 1984; Konold, et al., 1993; Shaughnessy, 1977). The epistemological nature of probability requires one to make inferences, reason critically, and draw upon intuitive notions within the context of the specific problem. Freudenthal (as cited in Fischbein, 1975) stated, "in no mathematical domain is blind faith in techniques more often denounced than in probability; in no domain is critical thought more often required" (p. 3). The area of probability is particularly appropriate for the study of intuition because probabilities are closely tied to action (Fischbein, 1975). The construction of the concept of probability starts from specific experiences, which are stochastic in character. "Intuition plays an essential part in the domain of probability, perhaps more conspicuously and strikingly than it does in other domains of mathematics" (p. 5). Fischbein (1999) discussed how intuitive tendencies can lead to misconceptions that contradict the formal knowledge taught in school.

One study explored this issue by looking at middle school students' judgments on the fairness of games and the influence of intuition on these judgments (Aspinwall & Shaw, 2000). Their intuitions were evident in their responses when asked to explain or justify their reasoning, "It's just luck. In any game, anybody could win" (p 219). Aspinwall and Shaw found that the students tended to believe their intuitions despite multiple samples and the graphic example of a tree diagram.

These intuitive misconceptions are one of the reasons that learning probability, as well as learning to teach probability, is difficult. In order to address the misconceptions, teachers need to first be aware of what these misconceptions are and second, have rich pedagogical content knowledge in order to change them effectively. Teachers must also be aware of their own intuitive biases (Ball, 2000; Garfield & Ahlgren, 1988). Another reason learning and teaching probability is so difficult is due to students' interest and beliefs.

Interest and Beliefs

Konold (1989) studied college students and found that many tend to focus on outcomes of single events. These students interpreted probability as related to predicting rain, into three discrete categories, rather than a continuous spectrum of belief. If the probability was greater than 50% the situation is certain to happen, if the probability is equal to 50% this doesn't tell you whether it will or will not happen, and if the probability is less than 50% the situation will not happen.

Another insight into students' beliefs about probability can be seen in the following quote from a student, "I don't believe in probability. Just because the probability of something is 40% doesn't mean that it can't happen. Even if the probability is only 2%, it can still happen. So I don't believe in probability" (Konold, as cited in Ahlgren & Garfield, 1991, p. 120). This quote shows that students' beliefs can have an impact on their understanding of how probability can be a measure for uncertainty. Since this student "doesn't believe in probability" they may not have a grasp on the difference between a 40% chance and a 2% chance.

Another area of the literature that received much attention over the last 30 years is related to students' beliefs about probability using judgmental heuristics. Tversky and Kahneman (1974) defined a judgmental heuristic as a strategy that relies on a natural assessment to produce estimation or a prediction. These natural assessments are based on the student's perception of events. Shaughnessy (1992, 1993) expounded on this definition that judgmental heuristics are used by people to make estimates of the likelihood of events. Such judgments can lead to biases that cause misconceptions with regard to probability. These judgmental heuristics provide a theoretical framework for the learning of probability and statistics. They can be characterized as general features underlying intuitive reasoning.

Examples of judgmental heuristics are representativeness, negative and positive recency effects, availability, adjustment and anchoring, and the conjunction fallacy

(Tversky & Kahneman, 1974). The representativeness heuristic is used to estimate likelihoods for events based on how well an outcome represents some aspect of its parent population. There are situations in which individuals use an availability heuristic to assess the frequency of a class or the probability of an event based on how easily one can think of instances or occurrences of that event (Tversky & Kahneman, 1974). The conjunction fallacy is employed when "people rate certain types of conjunctive events as much more likely to occur as their parent stem events" (Shaughnessy, 1992, p. 472).

In this section on students' beliefs and content knowledge of probability, the research has shown that in addition to mathematical knowledge, beliefs and intuitions impact understanding. For teachers to effectively teach probability they will need to be aware of how these factors impact students' learning. In addition, teachers themselves need to have a strong content knowledge of probability concepts. The next section looks at the research that is related to teachers' understanding of probability.

Teachers' Content Knowledge of Probability

There have been a few research studies aimed at gaining a better understanding of teachers' and preservice teachers' content knowledge of probability. These studies focused on different levels of teachers; elementary, middle, as well as secondary. Carter and Capraro (2005), and Capraro et al. (2005) both looked at preservice teachers at the elementary and middle school levels. Carter and Capraro conducted a study involving 108 preservice teachers' that were enrolled in an introductory statistics class. In an on-

line assessment instrument, preservice teachers' exhibited misconceptions regarding sample space, law of large numbers, representativeness, and the conjunction fallacy. They found that the majority of participants "do not have a sufficient intuitive understanding of chance" (Carter & Capraro, 2005, p. 110). From their findings they suggest that preservice teachers' should have an introduction to probability that explicitly and directly addresses these misconceptions.

Liu (2005) focused on secondary teachers' content knowledge of probability and statistics. Within her dissertation she created a theoretical framework to explain teachers' understanding of hypothesis testing, margin of error, and probability. According to her framework, which developed through the analysis of the data, Liu proposed that there are two main perspectives teachers can have – a stochastic conception, and a non-stochastic conception. This relates to the deterministic and probabilistic viewpoints discussed by Schaeffer (2006). Through her study Liu found that teachers can have one perspective or the other, or both depending on the situation.

Mojica (2006) and Lee and Mojica (2008) also studied teachers' understanding of probability. One interesting finding from Liu's study that relates to Mojica's (2006) and Lee and Mojica's (2008) studies on lessons implemented by practicing middle school teachers is teachers' strongly held belief that there is one true answer for each situation (a non-stochastic conception). Mojica (2006) found this result in her analysis, "Although participants rely heavily on a theoretical estimation as the true probability, some have

difficulty calculating the likelihood of uncertain events using a theoretical approach" (p. 2). And Lee and Mojica found that, "Teachers overwhelmingly privileged the theoretical probability as the best or most reliable estimator of probability, often using empirical differences across groups to justify this preference" (p. 5). Also, Stohl (2005) in her discussion of the preparation of teachers, pointed out that this attitude, or belief, in one true answer stems from a deterministic (or non-stochastic) view of probability.

As has been shown by these few studies, the current state of our understanding of teachers' content knowledge is wanting. To meet the demands of the curriculum, which calls for an increased emphasis on probability and statistics (NCTM, 2000, 2004), teachers need to have a strong content knowledge base as well as instruction on how to use that knowledge pedagogically.

The next section of the literature review is focused on research regarding pedagogical content knowledge of probability. This section is organized into six sections: 1) a general discussion of the connection between content knowledge and pedagogical content knowledge, 2) the role of beliefs in mathematics education, 3) connections between content knowledge and pedagogical content knowledge of probability, 4) approaches to probability instruction, 5) technology use in teaching probability, and 6) teacher education practices to develop pedagogical content knowledge.

Pedagogical Content Knowledge of Probability

In order to teach probability concepts with meaning, teachers must be able to draw upon a strong knowledge base. Not only do teachers need to know *what* to teach (the mathematical content), but they also need to know *how* to teach within the context of mathematics (the pedagogical content of the mathematics). Shulman (1986) introduced the construct of 'pedagogical content knowledge' to refer to knowledge associated with the teaching of a specific topic. Since that time several researchers have worked to more closely define this type of knowledge and also find ways to measure it (see Ball, 2000; Ball, et al., 2001; Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008; Hill, et al., 2005; Silverman, 2005; Van Dooren, Verschaffel, & Onghena, 2002).

Connection Between Content Knowledge and Pedagogical Knowledge

Shulman saw a missing paradigm which he referred to as "a 'blind spot' with respect to content that now characterizes most research on teaching and, as a consequence, most of our state-level programs of teacher evaluation and certification" (1986, pp. 7-8). Within the research literature on teaching what was missing were questions about understanding of the content needed for teaching. Thus the term 'pedagogical content knowledge' was coined to refer to the pedagogical knowledge that is related to teaching a specific content.

Included in this knowledge is one of the toughest challenges for novice teachers: how to develop the capacity to deconstruct their own knowledge into a form that is

accessible to the new learner. One way to address this challenge is to have opportunities in methods courses that would enable preservice teachers to use what they know in a variety of contexts.

Ball, Lubienski, and Mewborn (2001) further developed what they called Mathematical Knowledge for Teaching (MKT). In their review of the research literature on teaching and learning they found that 15% of those articles focused on teachers' knowledge and beliefs but only 5% focused on how teachers' mathematical understanding affected their practice and only 2% examined how it affected students' learning. Previous research has taken two approaches, one on researching characteristics of *teachers*, and the other on teachers' *knowledge*. What Ball, Lubienski, and Mewborn argued is that there needs to be an alternative approach that shifts the focus to *teaching* and on *teachers' use of mathematical knowledge.*

Several researchers have since built on this research agenda (Ball, et al., 2008; Hill, et al., 2008; Hill, et al., 2005; Hill, Schilling, & Ball, 2004). To better define what is needed within MKT, Ball, Thames, and Phelps (2008) as well as Hill, Ball, and Schilling (2008) offered a domain map (Figure 1) delineating the specific constructs that make up MKT.

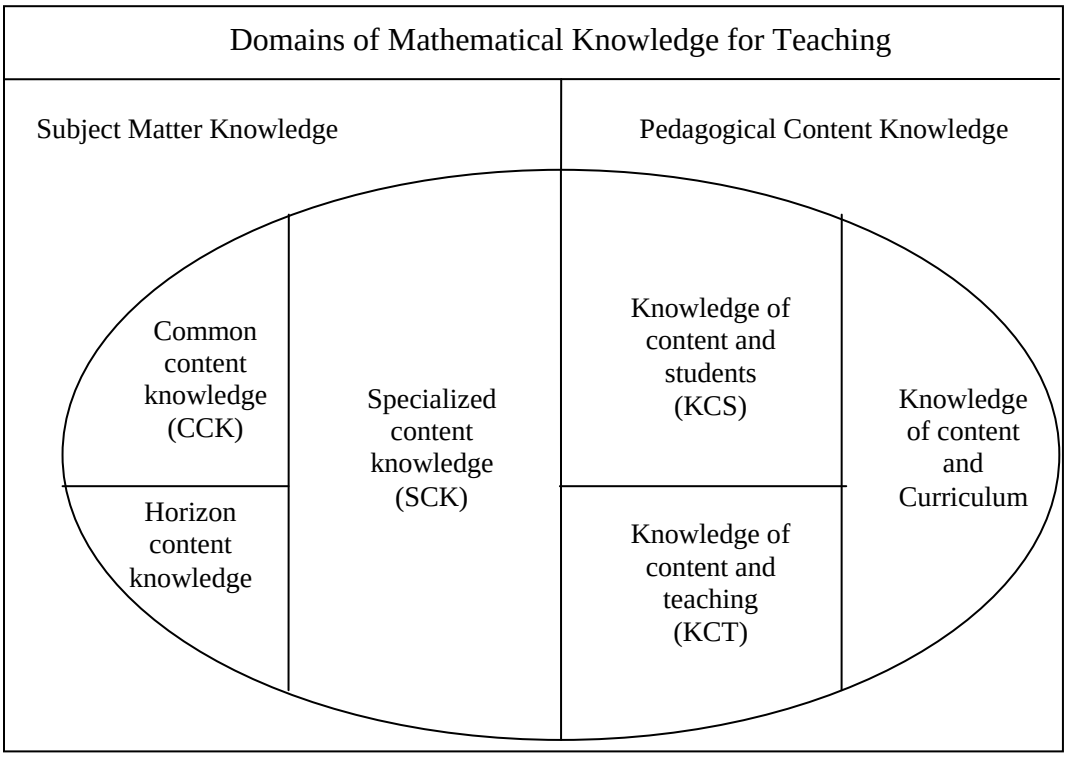


Figure 1. Domain Map for Mathematical Knowledge for Teaching (Ball, et al., 2008, p. 403)

Two constructs within this domain are in addition to what Shulman defined as PCK, namely *common content knowledge* (CCK) and *specialized content knowledge* (SCK). CCK is defined as the knowledge of mathematics that is commonly used in other professions that use mathematics; whereas SCK is the knowledge of mathematics that is specifically used within teaching. Some examples of this include ways to accurately represent mathematical ideas, being able to provide explanations for rules and procedures, and understanding unusual solution methods to problems. Both of these fall

under *subject matter knowledge* because they do not require understanding of students or teaching. Under PCK, Hill, Ball, and Schilling (2008) went into further depth of the construct *knowledge of content and students* (KCS). They argued that KCS is distinct from subject matter knowledge and is focused on teachers' understanding of how students learn particular content. For example, a teacher might have a strong mathematical understanding of the content but weak knowledge of how students learn that content.

As for *knowledge of content and teaching* (KCT), Ball, Thames, and Phelps (2008) further explained this construct. They focused their research not on what teachers need to know, but more on how teachers need to know the content in order to teach effectively. They sought to define what KCT is needed within the practice of teaching, what activities are required to develop a classroom "in which mathematics is treated with integrity, students' ideas are taken seriously, and mathematical work is a collective as well as an individual endeavor" (2008, p. 385). Some of the activities that are involved in teaching include: planning, evaluating students' work, writing and grading assessments, explaining classwork to parents, managing homework, as well as dealing with issues of equity and working with colleagues and administration.

While Ball, Thames, and Phelps (2008) focused on the *teaching* of mathematics, Capraro, Capraro, Parker, Kulm, and Raulerson (2005) focused their research on knowledge of *teachers*. Capraro, et al. conducted a study to investigate the relationship

between teachers' content knowledge and teachers' pedagogical content knowledge using grades on math courses taken, state teacher certification test, observations, reflection journals, and a methods course. They found that a lack of mathematical content knowledge leads to ineffective mathematics instruction. The qualitative findings indicated that those preservice teachers with stronger mathematical knowledge exhibited progressively more pedagogical content knowledge as they proceeded through their mathematics methods course. Similar to Ball (2000), the case studies provided some evidence that mathematics course-taking success does not guarantee that preservice teachers apply their knowledge correctly in the classroom. For this reason, Capraro et. al. (2005) argue that preservice teachers need experience in a school setting to build teaching confidence, allow preservice teachers to work on questioning strategies, and increase understanding of strategies used by students at various grade levels.

The Role of Beliefs in Mathematics Education

Beliefs and attitudes that teachers have about mathematics and teaching affect their practice and thus need to be considered when discussing teacher knowledge (Correa, Perry, Sims, Miller, & Fang, 2008; Philipp, 2007; Philipp, et al., 2007; Thompson, 1992). While the term 'beliefs' is often popular in education research, it is also undefined by those who study it (Philipp, 2007; Thompson, 1992). Philipp (2007) offered this definition, "beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). In differentiating knowledge and

belief some argue that knowledge has the added characteristic of truth (Clement, 1999; Pajares, 1992; Thompson, 1992). For example, Scheffler (1965) described knowledge as having three conditions: one believes Q, one has the right to believe Q, and Q is true.

Philipp (2007) in his literature review of research on teachers' beliefs about students' mathematical thinking found these beliefs to be correlated to teachers' instruction and students' learning. Research has demonstrated a relationship between teacher beliefs, instructional practices, and student learning (e.g., Richardson, Anders, Tidwell, & Lloyd, 1991; Staub & Stern, 2002, as cited in Correa, et al., 2008). Furthermore, when preservice elementary school teachers were provided with opportunities to learn about children's mathematical thinking, their beliefs changed. A reason for this change was attributed to the intensity of the experience associated with focusing upon students' thinking (Ambrose, 2004; Vacc & Bright, 1999). This provides an argument for why it may be a useful practice to engage preservice teachers with opportunities to learn about students' mathematical thinking.

So far in this discussion on pedagogical content knowledge the focus has been in general. The next four sections are focused on what literature has been written about the teaching of probability in particular. The first section is focused on the connections between content and pedagogical content knowledge for teaching probability. The second part is focused on several guidelines that have been given for the teaching of probability and statistics. This area within the literature has been given more attention especially

within the past decade. The next part is focused on approaches to teaching probability, and the last part for this section on pedagogical content knowledge is focused on the use of technology in the teaching of probability.

Connection Between Content and Pedagogical Content Knowledge of Probability

Haller (1997), Mojica (2006), and Lee and Mojica (2008) conducted studies that examined the content knowledge and pedagogical content knowledge of middle grades teachers. Haller (1997) studied 35 middle grades teachers as they participated in a summer institute in an effort to document changes in teachers' knowledge of probability as well as describe the relationship between teachers' instructional practice and their knowledge of probability and teaching experience. Mojica (2006) examined 4 middle grades teachers' understanding and teaching of empirical and theoretical probability as the teachers participated in a professional development project. And Lee and Mojica (2008) studied 9 middle grades teachers that were enrolled in a graduate level course that was focused on probability and statistics for middle grades teachers. All three of these studies found that teachers' knowledge of the teaching and learning of probability is lacking.

Haller (1997) found that teachers' probability knowledge had more of an impact on teachers' effectiveness in the classroom than did teaching experience. Prior to instruction (participation in the summer institute) only 17% of the 35 middle school teachers were able to correctly answer 10 or more of the 11 test items. After instruction

that percentage rose to 67%. One area assessed was common misconceptions. With regards to the effect of sample size and the law of large numbers, only 11 teachers prior to instruction and 20 teachers post instruction understood this concept. The idea of sample size and its relation to variance is not intuitive (Kahneman & Tversky, 1972), requiring explicit instruction of this concept within teacher education.

Within Haller's study, those teachers with a stronger content knowledge of probability were better able to capitalize on students' questions and responses, had fewer errors or misconceptions within their lessons, and seemed more confident in their teaching. Haller's study adds support to the claim that the extent of a teacher's content knowledge directly affects their pedagogical content knowledge.

Mojica (2006) studied teachers' understanding of probability by looking at their pedagogical decisions during teaching episodes, which include- use of models and representations, examples, and approaches. She found that although the teachers attempted to connect empirical and theoretical probability, they did not have a thorough understanding of the relationship between the two concepts. In addition some participants were unable to easily move from empirical to a theoretical model and most participants' use of representations and examples were limited.

Lee and Mojica (2008) collected teaching episodes from 9 teachers and analyzed teacher selected video clips from these episodes. What they found was that the teachers, while they did engage students in statistical investigations, often missed opportunities to

further students' statistical reasoning. Also the teachers' conception of empirical probability did not foster an understanding of probability as a limit of stabilized relative frequency. As for sample size choices, the teachers almost exclusively chose to use small samples in their lessons and also rarely pooled class data. And while some teachers did compare theoretical and experimental probabilities they "fail to address the heart of the issue: when should estimates of probability, using an experimental or theoretical approach, be similar? What variability should be expected in results from repeated trials within a sample and across a collection of samples?" (p. 6). The teachers did not go in depth and explore these questions during instruction. Thus Lee and Mojica concluded that more work is needed with teachers to develop connections between statistics and probability.

Approaches to Probability Instruction

Several teacher educators and researchers have offered guidelines and frameworks for the teaching of probability (Ahlgren & Garfield, 1991; Franklin, et al., 2005; Kvatinsky & Even, 2002; Shaughnessy, 2003; Stohl, 2005). Common themes across these guidelines and frameworks include: using data and experimentation, teaching probability within a context, making connections between statistics and probability, and the importance of connecting sample space and variability.

Ahlgren and Garfield (1991) looked at the issues surrounding the curriculum of probability. "There is the danger, however, of including topics in the curriculum without

first determining what we want students to know, why they should know these things, how they will use them, and what is known about the difficulties of designing and implementing new curriculum" (p. 107). These things need to be taken into consideration as we continue to add more probability into younger and younger grade levels. Depending on the level, what is it we want students to know? In addition, we need to address why students should know this and how we can teach it effectively for conceptual understanding.

To address these curricular concerns, the American Statistical Association (ASA) funded the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Project (Franklin, et al., 2005). The project published two reports, one which was focused on K-12 education and one which was focused on college courses. The K-12 report suggests a framework with four process components and three levels of understanding. The four process components are: 1) formulate question, 2) collect data, 3) analyze data, and 4) interpret results. Therefore the process of asking questions of data and using exploratory data analysis are key to the process of learning probability and statistics. The three levels A, B, and C, are not necessarily elementary, middle, and high, respectively. They are levels of depth of understanding and sophistication within the processes. In addition they argue that variability is a key concept students should understand and explain how the focus on variability should change from level A to B to C. For example,

at level A the focus is on variability within a group, level B adds variability between groups and covariability, and level C adds variability in model fitting.

In particular to probability, the GAISE report argued that it should be seen as a tool for statistics. Within experimental design there is randomization – random sampling, as well as randomly assigning individuals to different treatments. This randomization leads to chance variability and probability is a tool we can use to describe the variation we expect to see in samples when sampling is repeated a large number of times. Therefore, they suggest that probability in K-12 should be taught within an experimental context and students should explore data.

Shaughnessy (2003) in his article concerning research on students' understanding of probability suggested similar guidelines as those in the GAISE report. He agreed that probability should be taught using a problem-solving approach where students investigate probability problems and conduct their own experiments. Also, he suggested that probability be introduced through data and argued for the importance of making connections between variability in statistics with the notion of sample space in probability.

Further evidence for the importance of teaching probability from an experimental approach is given by Ainley and Pratt (2002). Ainley and Pratt (2002) discussed what they called the Planning Paradox. "If teachers plan from objectives, the tasks they set are likely to be unrewarding for the children and mathematically impoverished. But if

teachers plan from tasks, the children's activity is likely to be unfocused and learning difficult to assess" (Ainley & Pratt, 2002, p. 18). To remedy this paradox, Ainley and Pratt propose using tasks with utility and purpose. Several other researchers have found this to be an effective method to engage students in meaningful learning of probability and statistics (Aspinwall & Shaw, 2000; Batanero, et al., 2004; Lee, et al., 2006; Pratt, 2005). Tasks that require students to predict outcomes and then defend their predictions can have a motivating effect and subsequently increase students' interest.

Additionally, Bramald (1994) stated that "transferability is enhanced by work which emphasizes probabilities based upon the interpretation of statistical evidence" (p. 85). One of the reasons he gives for this stance is that few practical applications of probability are based on outcomes which are equally likely; thus students should be exposed to data and opportunities to interpret and discuss concepts of probability related to statistical evidence. Yet many mathematics teachers have the belief that mathematics is either right or wrong, that mathematics is rule-driven (Thompson, 1992). For this reason, many use a computational approach to teaching mathematics. This approach focuses on procedures and skill-based activities. As mentioned previously, there is a difference between mathematical reasoning and statistical reasoning which needs to be taken into consideration when discussing approaches to instruction in probability (DelMas, 2004; Rossman, Chance, & Medina, 2006; Scheaffer, 2006). Some of these differences include

dealing with uncertainty, the importance of context, and understanding processes that produce data.

Mathematics as a discipline can be described as the study of patterns and deals with abstract patterns and notation systems (DelMas, 2004). Thus mathematical reasoning tends to be more abstract and devoid of context. Although some would disagree with delMas, especially applied mathematicians, he claims that "ultimately, the purpose of mathematical inquiry is to develop an understanding of mathematical objects that is independent of real-world contexts" (p. 84). Conversely, statistics is grounded in context and the context is essential to statistical reasoning. "Unlike mathematical reasoning, statistical inquiry is dependent on data and typically is grounded within a context" (DelMas, 2004, p. 84). Implications for instruction are that students should have experiences that go beyond computations and procedures but are related to a physical or real-world context.

Within probability instruction, there are two approaches that are discussed in the literature: subjective and objective. Kvatinsky and Even (2002) in their framework for teachers' subject matter knowledge of probability include these alternative ways of approaching as one of seven aspects: 1) essential features, 2) strength of probability, 3) different representations and models, 4) alternative ways of approaching, 5) basic repertoire, 6) different forms of knowledge and understanding, and 7) knowledge about mathematics. The essential features of probability are what make it different from other

fields in mathematics, namely that probabilistic thinking is non-deterministic. For this reason, teachers should be aware of the different approaches to probability: objective-including classical and frequentist, and subjective. Teachers should also have an appreciation for the strength of probability and how it is used in dealing with uncertainty and random situations in almost every facet of life. Over thirty years ago, Freudenthal remarked that, "Probability applies in everyday situations, in games, in data processing, in insurance, in economics, in natural sciences. There is no part of mathematics that is as universally applied, except, of course, elementary arithmetic" (as cited in Kvatinsky & Even, 2002, p. 2).

Different representations and models, refers to the teachers' need to be able to use and interpret different representations such as tables, Venn diagrams, tree diagrams, pipe diagrams, area models, etc. In addition teachers need to be able to work between and among different representations in order to allow for deeper and more powerful understanding of probability. Basic repertoire includes specific examples and tasks that illustrate important ideas, properties, and concepts. Some concepts that Kvatinsky and Even feel should be included in a teacher's basic repertoire include independence, sample space, and conditional probability. The basic repertoire for teachers should include examples from their specific probability curriculum.

Steinbring (1991b) refers to the two approaches of probability as structural (objective) and intuitive (subjective). Within the structural approach the teacher believes

the character of mathematical knowledge is predetermined and structured hierarchically. From this perspective they organize their teaching activities according to the structure of defining the basic concepts first and then developing the theory. This approach could also be referred to as procedural and stems from the axiomatic approach, which focuses on formulas and rules. This is problematic within probability because the definitions of the concepts depend on the theory, that is, what is defined must already be assumed for a universal definition. For example, "the definition of equiprobability is based on equal likelihood of possibilities" (Steinbring, 1991b, p. 142). This structural approach is most often tied to the classical and frequentist definition of probability.

In the intuitive-concrete approach, the teacher may ask questions of intuitiveness and illustrate the concept through examples. This approach focuses on experiments, ideal games of chance, and real situations together with their conditions (Steinbring, 1991). Frequencies are used to define probability in these experiments and thus this approach relates to the frequentist approach. The experimental approach to teaching probability focuses on the process of estimating probabilities by creating situations for students to explore and build on their own intuitive notions. Researchers have found that the use of simulations to teach such concepts as sample space, and fairness can lead to a deeper understanding of variability of data and the law of large numbers. (Mojica, 2006; Stohl, 2005; Stohl & Tarr, 2002; Vahey, 2000; Watson & Moritz, 2003). There will be more discussion of simulations within technology later in this section.

Researchers agree that these different approaches need to be integrated within the teaching of probability (Batanero, Henry, & Parzysz, 2005; Kvatinsky & Even, 2002; Shaughnessy, 1992; Steinbring, 1991b; Stohl, 2005). That is, there is not a preference of one approach over the other; in fact, both approaches need to be used in an integrated way in order to gain a thorough understanding of probability. Both the structural and intuitive approaches should be used with respect to the student's current knowledge.

In summary, the research regarding teachers' pedagogical content knowledge of probability suggests that approaches for instruction should include a classical or axiomatic approach as well as a frequentist or experimental approach. Also that statistical thinking is different from mathematical thinking and is non-deterministic. Thus, the teaching of probability should include contexts with data and opportunities for students to make predictions, ask questions of data, and carry out experiments with data in order to learn key probabilistic concepts such as variability and sample space.

Technology Use in Teaching Probability

One way to address the issue of using real data and experimentation within the teaching of probability and statistics is through the inclusion of technology in the learning environment. By including such technology as microworlds and simulation software, researchers have found that students are able to test out their conjectures and influence their original intuitions of probability. Wilensky, (1993) asserted that intuitions can be constructed and that, "both the lack of good learning environments for probability and the

cultural and epistemological confusion surrounding probability do not support the construction of good probabilistic intuitions" (p. 100). The question becomes, 'what constitutes a good learning environment?'

Over 20 years ago Pea (1987) explained how technologies can be used within mathematics education as both a cognitive reorganizer as well as an amplifier. Technology can be used as an amplifier in the sense that it allows us to perform tasks and calculations faster and easier. Pea asserted that technology also has the capacity to act as a cognitive reorganizer and defined a cognitive technology as "any medium that helps transcend the limitations of the mind" (1987, p. 91). One way this can be done is through the use of "manipulable, dynamically linked, and simultaneously displayed representations" (p. 110). Ben-Zvi (2000) elaborated on this notion and applied it to statistical education; he explained how a powerful tool brings about the reorganization of physical or mental work in several ways, some of these are: by shifting the activity to a higher cognitive level; focusing the activity on transforming and analyzing representations; and accessing statistical conceptions by the use of graphics.

Jones et al. (2007) addressed this issue of using technology to teach probability in their review of research in probability. The studies of Pratt and Noss (2002) and Stohl (Drier, 2000; Stohl & Tarr, 2002) were reviewed and are discussed in more detail below. The remainder of this section is organized into two parts that address themes that are

common throughout the literature on the use of technology: design issues and representations.

Important Factors of Design

Children are curious and using an investigative approach to teaching probability concepts builds on this innate curiosity (Konold, 1995; Pratt, 2005). Not only the technology itself, but the tasks as well need to be open-ended and exploratory. For example, Pratt (2000, 2005) introduced a micro-world called *Chance-Maker* designed to develop students' understandings of chance and randomness. In the microworld, students are given gadgets that simulate random generators (coin, dice, and spinner) and a variety of tools that control the gadgets. The objective is for the students to determine which of the gadgets are fair (in their mind) and which are 'broken.' Within this design there are two constructs that Pratt feels are essential to quality task design: purpose and utility. A purposeful task is one that "has a meaningful outcome for the learner, in terms of an actual or virtual product, or the solution of an engaging problem" (Pratt, 2005, p. 182). A good task is one that affords the student to, "appreciate the utility of mathematical concepts and techniques in the sense that they learn how and why that idea is useful by applying it in a purposeful context" (p. 182). Through the use of *Chance-Maker* the students are engaged in the problem of 'fixing' the 'broken' gadgets and in the process come to appreciate the how and why random generators.

Stohl and Tarr (2002) addressed these same issues with the use of the software, *Probability Explorer*. *Probability Explorer* was designed as an open ended learning environment that enables learners to design, simulate, and analyze results of probability experiments. This in turn allows for understanding to evolve as a continuous and dynamic process through observation, reflection and experimentation (Stohl & Tarr, 2002; Tarr, Lee, & Rider, 2006; Weber, Maher, Powell, & Lee, 2008). In their research, one task the students were given was to determine which die company would be the best for their hypothetical school to buy as part of a game. Through simulations the students were to decide which die appeared to be unfair. This activity addresses Pratt's (2005) constructs of purpose and utility; the purpose is to find the 'broken' dice and the utility comes in through their interpretations of the data from simulations.

Along the lines of purpose is the idea of motivation. The medium of technology by itself can motivate students to be engaged more so than without technology. If the task is designed in such a way that it is game like, students can feel it is more play than work. Another additional element than can increase motivation is competition, 'which group (gadget, dice) will win.' These design issues need to be a part of teachers' pedagogical content knowledge in order to make the most effective use of technology.

Representations

In order for students to be able to grasp the complexities of probability, they should have multiple representations to choose from (Konold, 1995; Peard, 2005; Stohl &

Tarr, 2002). For example, *Probability Explorer* includes a pie-graph, pictograph, bar graph, and data tables. These different representations can support the learning of relative frequency and theoretical probability. One advantage within this design is that the representations update simultaneously as the experiment runs.

Abrahamson and Wilensky (2005) also address this issue of representation within their micro-world *ProbLab*. One of the components, Sample Stalagmite, looks at combinatorial sample space in a game like setting. "The histogramed combinatorial sample space is designed as a visualization bridge for students to ground a sense of the likelihood of an event in combinatorial analysis and proportional judgment" (p. 5). The model simulates the random generation of blocks of red and green squares. The squares accumulate into columns according to the number of red squares in each. This pictorial representation can help students to understand distribution and spread.

While representations are an important aspect of the technology, the teacher needs to be aware of how to direct students' attention to certain details that can deepen their understanding. Teachers also need to know how moving between the representations assist students in making connections and meaningful inferences. The use of these micro-worlds and software programs can greatly enhance students' learning of probability concepts; however, this can only happen if the teacher is well prepared to facilitate this learning.

Teacher Education Practices to Develop Pedagogical Content Knowledge

Recent efforts in mathematics teacher education have capitalized on research on classroom practices and students' mathematical learning to create opportunities for preservice teachers to engage in the practices of teaching through a variety of contexts that provide opportunities for situational learning. Many teacher educators and researchers have created and used artifacts from the practice of teaching mathematics (e. g., Ball & Even, 2009; Darling-Hammond & Snyder, 2000; Doerr & Thompson, 2004). Examples of artifacts of practice are sample student work, lesson plans, mathematical tasks, video cases, teacher reflections, and case studies of classroom episodes. Several uses of artifacts of practice were used in this study and will be briefly discussed: examining mathematical tasks, analyzing sample student work, and video cases.

Mathematical tasks can be used in the preparation of teachers in several effective ways (Hiebert, Morris, Berk, & Jansen, 2007; Hiebert, Morris, & Glass, 2003; Smith & Stein, 1998; Stein, Engle, Smith, & Hughes, 2008). Through analyzing mathematical tasks, preservice teachers can focus on several different aspects: the appropriateness of the task for students, the learning goals, what Smith and Stein (1998) refer to as "levels of cognitive demand," as well as reflecting on the usefulness of the task. According to Smith and Stein (1998), there are four levels of cognitive demand: memorization, procedures without connections, procedures with connections, and doing mathematics. One activity Smith and Stein used was to have teachers sort different tasks into the four categories;

this can be helpful for preservice teachers in determining the usefulness and effectiveness of tasks they may want to use in their teaching.

Stein et al. (2008) discussed a framework related to orchestrating productive mathematical discussions in the classroom. Cognitively challenging mathematical tasks used in conjunction with students' work on those tasks can provide a means for teaching preservice teachers how to conduct productive classroom discussions. Stein et al. (2008) felt that "these practices help make it more likely that teachers will be able to use students' responses to advance the mathematical understanding of the class as a whole" (p. 322).

Lee, Hollebrands, and Wilson (2007) developed curriculum materials to prepare teachers to teach mathematics with technology. One module within these materials is focused on engaging preservice teachers in data analysis and probability tasks using technology tools such as *TinkerPlots* (Key Curriculum Technologies, 2005), *Fathom* (Key Curriculum Technologies, 2007), *Probability Explorer*, spreadsheets, and graphing calculators. Within this module Lee, Hollebrands, and Wilson are also explicit in pointing out the pedagogical aspects within these tools. For example, within *Fathom* the user has the possibility to move objects and have the representations simultaneously coordinate the data in the tables and in the graphics. Another added feature of *Fathom* is that color gradation can represent another variable within the data. This can be pedagogically powerful because it allows students to focus on relationships among three variables at the

same time. In addition to recognizing the pedagogical aspects of the tools, these curriculum materials also engage preservice teachers in reflection on their own learning and to consider pedagogical issues such as anticipating student difficulties.

Another artifact of practice that can be used in teacher preparation is the use of student work. Student work can be used to help preservice teachers gain a better understanding of students' mathematical thinking and also conjecture possible misunderstandings (Crespo, 2000; Hiebert, et al., 2007; Kazemi & Franke, 2004). Crespo (2000) conducted a study where preservice teachers exchanged letters with students in a local elementary school (grade 4) as a way to carry out mathematical discussions, while Kazemi and Franke (2004) used student work with practicing elementary teachers as a way to facilitate discussion about students' mathematical thinking.

The use of video cases within teacher education can offer opportunities for productive discussions (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Darling-Hammond & Snyder, 2000; Hollebrands, Wilson, & Lee, 2007). Various ways that video cases can be used as a tool are focusing on students, focusing on teachers, and/or teachers' self evaluation. Videos can be useful because they are able to capture classroom dynamics that can offer an opportunity for discussion. Teacher educators can use videos to target specific areas they wish to examine, such as teacher practices and examples of student understanding (Borko, et al., 2008). An example of this can be seen in the materials by Lee et al (2007) where preservice teachers are asked to make conjectures about students'

understanding by watching a video case of students' work on comparing distributions using *TinkerPlots*. Hollebrands, Wilson, and Lee (2007) found that preservice teachers' analysis of student work using this video case could be categorized in four different ways: description, comparison, analysis, and restructuring. Preservice teachers often needed to first describe what the students were doing with the technology, often compared the students' work to their own work on the task (often in a judgmental way). However some of the preservice teachers were able to use the evidence in the video case to hypothesize what students' may be thinking when engaged in the task, and some were able to engage in restructuring activities where their analysis of student's thinking informed their own thinking about the task, the mathematical content, and implications for instruction.

This section on pedagogical content knowledge for probability has focused on guidelines for teaching probability, approaches to instruction of probability, the use of technology in teaching probability, and teacher education practices that promote development of pedagogical content knowledge. Some key aspects from this review are that probability is best taught using real data, integration of both an experimental and classical approach, and with the appropriate use of technological tools. In addition, the use of artifacts from classroom practice may help preservice teachers deepen their content understanding and develop their pedagogical content knowledge for teaching probability.

Summary of Literature

The teaching and learning of probability is complex and many students as well as teachers have difficulties. Part of this is due to the nature of probability and the different approaches, objective and subjective, one can take to understanding the meaning of probability. Another factor that contributes to this difficulty is that a person's beliefs about randomness and probability can be incomplete or incorrect and resistant to change. Teaching probability should include a classical, experimental, and subjective approach involving opportunities for students to explore data. Connections between statistics and probability can be made through the use of simulations and experimentation.

In light of these issues and suggestions for the learning and teaching of probability, the review of the literature has illuminated some areas of research that are still needed. It has been shown that the teaching of probability needs to be improved and in order to improve teaching probability, there needs to be more research within teacher education and among preservice teachers. This study intends to add to the literature by examining preservice teachers' understanding of probability as they are engaged in various tasks, including tasks that utilize artifacts from the practice of teaching. This study will also add to the literature by studying relationships that exist among preservice teachers' orientations, content knowledge, and pedagogical content knowledge. The following section describes the framework that guides the data collection and analysis for this study.

Framing the Research Study

From the review of the literature it is clear that there is a need for research on preservice teachers' beliefs and knowledge of the nature of probability concepts, their pedagogical content knowledge of probability, and the relationships that exist between these types of knowledge. To gain an understanding of this subgroup and their knowledge, several perspectives are used as a way to frame this investigation. For this study, several constructs are developed and defined, one for each aspect (orientations, content knowledge, pedagogical content knowledge), to build an overall conceptual framework which focuses on the relationships among these aspects. I will first give support as to why a framework that inter-relates the three aspects of interest in my study is warranted and appropriate, and then further explain how I am drawing upon literature to develop the individual constructs used to describe each aspect of interest in the study.

Importance in Examining Relationships Between Constructs

Several researchers have argued that a teacher's beliefs and content knowledge have an influence on and impact their pedagogical content knowledge (Hill, et al., 2008; Philipp, 2007; Thompson, 1984, 1992; Thompson, Philipp, Thompson, & Boyd, 1994). They also argue that it is important to define these types of knowledge and how they relate. As has been discussed in the literature review, Hill, Ball, and Schilling (2008) have a framework that relates a teacher's Mathematical Knowledge for Teaching (MKT)

to the subcategories of Subject Matter Knowledge (SK) and Pedagogical Content Knowledge (PCK). One area that is missing from this framework is that of orientations.

Thompson (1984) researched three junior high school teachers and found that, "the teachers' beliefs, views, and preferences about mathematics and its teaching played a significant, albeit subtle, role in shaping their instructional behavior" (p. 105). Thompson (1992) also argued for the position that researchers shouldn't separate the study of teachers' beliefs from teachers' knowledge, but rather study "whether and how, if at all, teachers' beliefs – or what they may take to be knowledge – affect their experience" (p. 129). As described in his 2007 synthesis, Philipp further supports the importance of studying the relationships between teachers' beliefs and knowledge and that further research is evidently needed. However the current research on teaching probability has yet to carefully consider the interactions among a teacher's beliefs about probability, content understandings, and pedagogical content knowledge, especially among preservice teachers. Once these relationships are more defined and better understood, they can be taken into account in designing and improving mathematics teacher education curriculum.

Specifically within probability a subset of a person's belief structures about teaching, students, and content is the notion of orientations. Orientations that one can have towards probability – statistical, mathematical, subjective, and/or personal – connect well with the different approaches to probability: frequentist, classical, subjective.

Therefore studying orientations towards probability in particular can provide a better defined construct than trying to study beliefs in general.

One important reason for studying the relationships among content knowledge, pedagogical content knowledge, and orientations is because it might be the case that the more narrow a teachers' orientation towards probability could imply their content knowledge is more narrow, and the more robust or varied their orientations, the more robust their content knowledge. For example, a teacher that demonstrates an orientation that is both statistical and mathematical may have a deeper understanding of how probabilities are estimated – both theoretically and empirically. Also a teacher that can view probabilities from both of these orientations may be better able to question students and direct classroom conversations; they may have more representations to draw from, and can design more cognitive demanding tasks than a teacher with more narrow orientations.

Components of the Three Aspects in the Framework

The various aspects of this study require various perspectives from studies done in mathematics teacher education in general, as well those specific to the domain of probability. By tying together frameworks used to describe the different aspects in isolation, the researcher is attempting to understand how the different aspects relate to each other. To examine orientations this study builds on the work of Batanero, et al. (2005) and Garuti, Orlandoni, and Ricci (2008). With respect to content knowledge of

probability the research of Kvatinsky and Evens (2002) and the work of Jones, Thornton, et al. (1999) are used. And the work of Hill, Ball, and Schilling (2008) is used to frame two aspects of pedagogical content knowledge, with Kvatinsky and Even's work being used to make explicit the constructs as they relate to probability. The following Figure 2, displays these three aspects and the constructs of interest within each. The entire figure represents the collection of the aspects, with orientations as foundational to the content knowledge and pedagogical content knowledge.

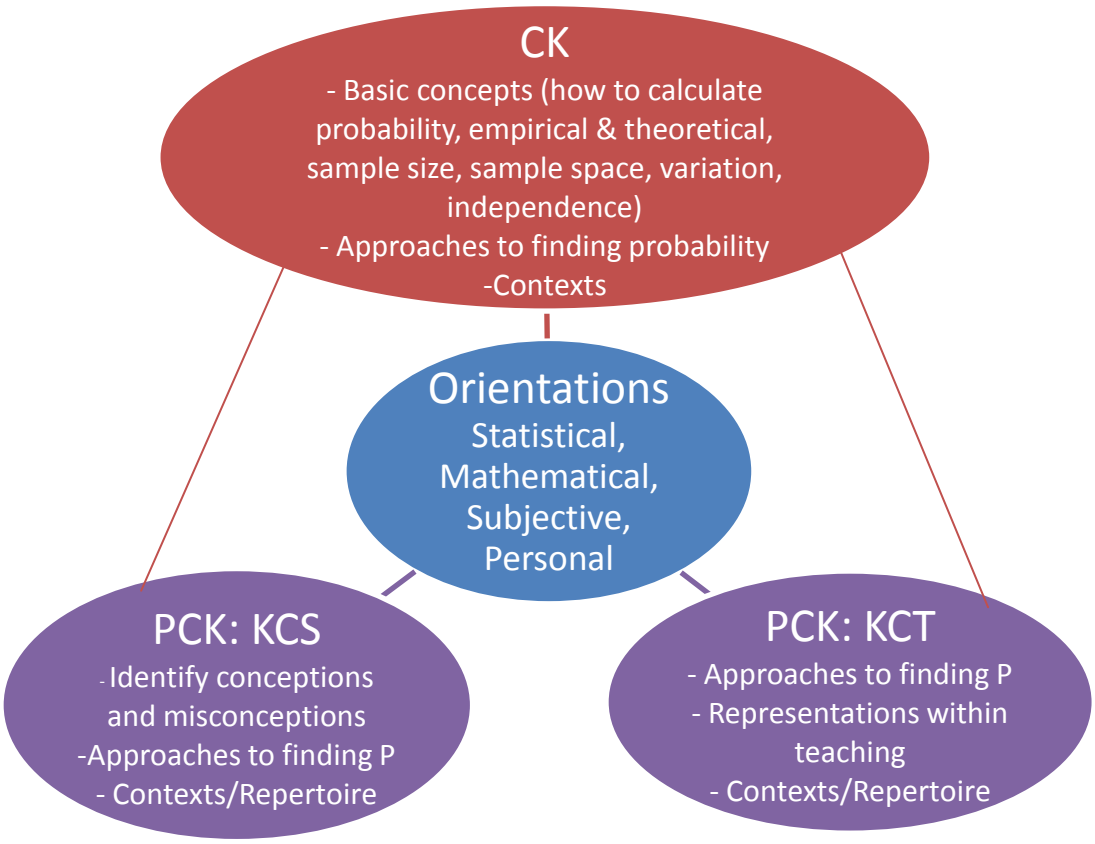


Figure 2. Framework for Knowledge for Teaching Probability

Orientations

For the purpose of describing each participant's orientations towards probability and randomness, I will define the characteristics of each perspective that can be used to identify a person's orientation(s). These definitions build on the work of Batanero, et al. (2005) and Garuti, Orlandoni, and Ricci (2008). As was discussed in the literature review there are three different commonly used approaches to understanding probability – classical, frequentist, and subjective. Garuti et al. described someone using a classical approach as having a *mathematical* orientation, someone with a frequentist approach as having a *statistical* orientation. They also defined someone as a sympathetic student as one that bases judgments only on personal feelings and impressions. For this study I will refer to the sympathetic student with the term *personal* and a *subjective* student as one that bases judgments on *both* the data from their own personal experiences and personal preferences.

Thus, when analyzing orientations, responses will be characterized as statistical (St), mathematical (M), subjective (S), and/or personal (P) personal. In addition to looking at the data and classifying responses in these four ways, these perspectives will be looked at across different contexts and compared – i.e. determining how well, or if, one's perspective in a content specific situation matches up to one's perspective in a pedagogical situation.

Content Knowledge

For the analysis of the preservice teachers' content knowledge this study does not use the three subsections given in Hill et al.'s (2008) characterization of subject matter knowledge. This is due to the fact that this study is not trying to measure subject matter knowledge in such fine grained detail. Instead, using the work of Kvatinsky and Even (2002) the analysis of preservice teachers' content knowledge will draw upon three of the seven aspects of their framework for teacher knowledge of probability. One aspect looks at alternative ways of approaching probability. When asked to estimate or interpret a probability of an event, it is of interest to know whether a preservice teacher uses a classical approach, frequency approach, or a subjective approach. Regardless of which approach or interpretation they use, we also want to record whether they recognize or show an awareness that the other approaches are possible to use. This is obviously influenced by, and can be characterized using the four orientations described in the previous section.

Another aspect within content knowledge is an understanding of basic concepts in probability. Within this aspect, this study examines what the preservice teachers understand of basic concepts within probability, such as, empirical probability, theoretical probability, sample size, law of large numbers, sample space, independence, and variability; these concepts are drawn from the research done by Jones Thornton, et al. (1999).

The third aspect from Kvatinsky and Even's framework is related to the repertoire of examples one brings to bear in dealing with probability and the contexts (e.g., weather, games of chance, lottery, etc) in which one situates their reasoning. Thus, it is of interest to know which examples and contexts the preservice teachers use in their own thinking about probability tasks, which may be similar or different to the examples or contexts they draw upon pedagogically. Some possible "real world" examples may include the weather and games of chance like the lottery. Kvatinsky and Even (2002) give an example of a basic number cube being used to illustrate an understanding of each outcome being independent or tossing a coin to illustrate one's understanding of the concept of sample space, in this case 'head' and 'tail'.

Pedagogical Content Knowledge

To study the preservice teachers' pedagogical content knowledge, the work of Hill Ball and Schilling (2008) is used. The two constructs from their research that are used in this study are Knowledge of Content and Students and Knowledge of Content and Teaching. Hill et al. include a third aspect, Knowledge of Curriculum, which for this study is subsumed under Knowledge of Content and Teaching due to the small scope of this study.

Knowledge of Content and Students is defined as "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (Hill, et al., 2008, p. 375). An example of this intertwined knowledge that Hill et al. give is

having the knowledge of how students add fractions and knowing common misconceptions that arise during the process. In this study, Knowledge of Content and Students can be ascertained as preservice teachers analyze student work with particular focus on what content knowledge the preservice teachers may draw upon. As can be seen in Figure 2 aspects of Knowledge of Content and Students that are included are based on the Kvatinsky and Even (2002) framework: identify conceptions and misconceptions, approaches to finding probability with students work that they consider valid, and contexts/ repertoire they draw upon to teach probability concepts as they relate to students interests.

The domain Knowledge of Content and Teaching "combines knowing about teaching and knowing about mathematics" (Ball, et al., 2008, p. 401). Some examples of Knowledge of Content and Teaching are choosing appropriate instructional strategies for a particular concept, sequencing tasks to build knowledge, types of questions to ask, knowing how different examples will illustrate different aspects of a concept, and knowing what different types of assessments will reveal about student understanding. For the study of the preservice teachers' Knowledge of Content and Teaching, the aspects included were taken from Kvatisnsky and Even's (2002) framework, namely, the approaches to finding probability in the context of teaching, representations within teaching, and the contexts/ repertoire they draw upon when referring to teaching situations.

With the use of this framework this study will examine not only the nature of preservice teachers' knowledge within these three aspects – orientations, content knowledge, and pedagogical content knowledge – but also look at possible relationships that may exist amongst them. The framework as outlined above will guide how this study answers the following research questions.

Research Questions

1. What is the nature of preservice teachers' orientations, content knowledge, and pedagogical content knowledge for teaching probability?
2. What are the relationships among preservice teachers' orientations, content knowledge, and pedagogical content knowledge for teaching probability?
3. How do different types of pedagogical tasks (open-ended questions, problem solving with and without a context, analyzing student work, analyzing teaching videos, multiple choice questions) elicit different aspects of preservice teachers' orientations and knowledge for teaching probability?

CHAPTER 3: METHODOLOGY

Overview

As has been shown in the literature, there is a gap within the research concerning preservice teachers' knowledge (orientations, content knowledge, and pedagogical content knowledge) of probability. Therefore the goals of this study are to contribute to our understanding of preservice teachers' orientations, content knowledge, and pedagogical content knowledge. Because this study is concerned with the nature of this knowledge, as well as the relationships among the aspects, an in-depth examination of teachers is needed, and thus, qualitative methods were used.

Qualitative research has several characteristics. The research 1) takes place in the natural world, 2) uses multiple methods that are interactive and humanistic, 3) is emergent rather than tightly prefigured, and 4) is fundamentally interpretive (Marshall & Rossman, 1999). Various methods, which include task-based interviews and pre/post-test analysis, were used to explore the nature of the participants' knowledge of teaching probability. This chapter presents the strategy of inquiry; participants and setting; data collection and analysis; and validity issues.

Case-Study Research

The strategy of inquiry that was used in this study was case-study research. This is research in which "the researcher explores in depth a program, an event, an activity, a process, or one or more individuals" (Creswell, 2003, p. 15). According to Marshall and

Rossman (1999), case study has a more complex design. A case study may require multiple sources of data, such as interviews and document analysis. Another characteristic of case study research is that it is bounded by time and activity (Stake, 1995). The cases in this study were bounded within a single university, a specific 400 level mathematics methods course, and a single semester.

This study focused on a particular group of individuals: preservice secondary mathematics teachers. As was shown in the literature review, this group is missing from the literature on the teaching of probability. Thus, a case in this study is defined as preservice secondary mathematics teacher. Within each case, the researcher attempted to understand their orientations, content knowledge, and pedagogical content knowledge related to the teaching of probability. Additionally, the preservice teachers were engaged in a variety of tasks that had various purposes and were given in different contexts – individual interview, written reflection, multiple choice test items. These tasks were used as the unit of analysis during the cross-case analysis. These tasks were bound by the interview setting and classroom setting. Also, the time of the interviews and class sessions were constrained.

Site and Participant Selection

This study took place in a university in the southeast US. The results from a variety of contexts and cultures suggest that the difficulty of understanding how to teach probability is not dependent on the location of the subjects (Fischbein & Schnarch, 1997;

Haller, 1997; Shaughnessy, 1977; Steinbring, 1991a). Therefore this site was chosen based on access to the participants.

The participants for this study were purposefully sampled from two sections of a junior level mathematics education methods course called Teaching Mathematics with Technology. Patton (1990) defines stratified purposeful samples as those samples that represent a particular subgroup of interest. The course from which the preservice teachers were sampled was purposefully chosen for several reasons: 1) there was a unit devoted to teaching probability and statistics with technology using the Lee, Hollebrands, & Wilson (2007) materials, 2) the preservice teachers were required to do a course project where they chose a topic (algebra, geometry, or probability and statistics) from which to design and teach a mini-lesson, and 3) a pre-post test was given at the beginning and end of the unit on probability and statistics. The pre-post test was being given as part of a larger NSF funded research study, *Preparing Teachers to Teach Mathematics with Technology* (PTMT) (Grant No. DUE 04-42319).

Five preservice teachers were sampled from this course, four were junior level undergraduate students – two female, Pam and Yasmin, and two male, Brad and Jeff – and one was a male graduate student, Sam, pursuing initial licensure. They were all studying mathematics education with a focus on high school. The reason for choosing this level of preservice teacher is due to the nature of the study. At this level, the participants have had at least one previous mathematics methods course and all were

enrolled in their second methods course from which they were sampled. Also none of them had completed student teaching and had about the same amount of field experiences in classrooms from their prior methods course. In addition, most of them should have completed at least one collegiate level statistics or probability course. These characteristics set them apart as a population from general undergraduate students as well as high school students. These preservice teachers chose to do their course project on designing a technology lesson on probability concepts and volunteered to take part in this study. Thus, they had shown an interest in learning to teach probability and were willing participants in the outside class activities.

Institutional Review Board permission was obtained for the protection of the participants (IRB# 291-06-9). Participation was voluntary and participants were compensated for their time. They were offered the choice of either a site license for the *Probability Explorer* software or a student membership to NCTM. The interviews and reflection took place outside of class time, the pre-post tests were administered during class time, and the participants did not receive any extra-credit in their course for their participation. All participants signed the informed consent letter found in Appendix A.

Instruments and Data Collection

The design of this study included the use of multiple data sources – individual and group interviews, a reflection, and pre and post tests. At the time the data was collected, the probability and statistics unit ,(Lee, et al., 2007) was being taught in the course from

which they were sampled. The data collection took place over an eight week period towards the end of the 2007 Spring semester. The next few sections detail when and how the different data sources were collected: pre-post test, initial interview, second interview, reflection, and final interview. How these sources were analyzed is explained in the following section on data analysis.

Pre-Post Test

The pre-post test was given to the entire class from which the participants were sampled. This test was developed as part of another research study and was adapted from Garfield's (2003) online test questions *Assessing Statistical Reasoning* (https://app.gen.umn.edu/artist/index.html). The test was administered in their course right before the teaching of the unit on probability and statistics and at the end of the semester. For this study, only test items that pertained to probability were used. These questions addressed probability content knowledge, with one pedagogical content knowledge question. The pre-post test questions in their entirety are given in Appendix B. The reason for the use of this instrument was to gain insight into the preservice teachers' content knowledge and pedagogical content knowledge within a setting other than an interview, as well as to compare their knowledge from the beginning of the study to the end. While the idea of growth of knowledge was not a focus of this study, it was a contributing finding.

Initial Interview

The initial interview was done individually, videotaped, and consisted of seven main questions. The following table gives a brief description of each question and the associated aspects of the framework it was designed to assess.

Table 1 *Brief description of initial interview questions related to aspects of framework*

1st Interview focus of question:	**Aspects of Framework:**
1. meaning of randomness; example of random; random in P&S class	Orientations Content Knowledge
2. probability of heads; 70% rain; meaning of probability	Orientations Content Knowledge
3. Why is important to teach probability	Orientations Pedagogical Content Knowledge
4. probability concepts important to teach (PST's topics); rate importance of topics (provided topics)	Orientations Content Knowledge Pedagogical Content Knowledge
5. Percent coverage for five mathematical strands in MS and HS	Pedagogical Content Knowledge
6. cars task: their answer; classroom discussion and (possible) analysis of students answers	Orientations Content Knowledge Pedagogical Content Knowledge
7. candy task: their answer; classroom use – task analysis(pros cons)	Orientations Content Knowledge Pedagogical Content Knowledge

The first five were questions designed to assess the preservice teachers' orientations and content knowledge. The first two of these questions addressed the preservice teachers' orientations and content knowledge of randomness. As was discussed in the literature, the concept of randomness is necessary for understanding

probability and people can have different conceptions of randomness (Batanero, et al., 2005; Batanero & Serrano, 1999; Borovcnik, et al., 1991; Green, 1991).

Questions 3, 4, and 5 were designed to assess the preservice teachers' orientations and content knowledge of probability. They were first asked to define probability in their own words and then asked to explain the meaning of two different types of probability situations, coin toss and weather. These two types were chosen because they represent different contexts, the first a situation involving equiprobable outcomes with a known sample space and the second a context where the theoretical probability is unknown and does not have a set sample space.

Additionally questions were asked that related to teaching probability. The researcher wanted to know what reasons preservice teachers' would give as to why probability is important for students to learn as well as what topics within probability that the preservice teachers' thought were important to teach. These questions were asked to gain insight into their pedagogical content knowledge of curriculum related to probability.

The last two questions from the initial interview were task-based questions that were presented in a teaching context. Hill, Ball, and Schilling (2008) argued for the need to research the knowledge of *teaching*; while the researcher could not assess these preservice teachers in the practice of teaching, these questions placed them in a teaching context where they had to address hypothetical student responses, assess tasks, and offer

suggestions for instruction. The first question was adapted from an item on Garfield's (2006) Assessment Resource Tools for Improving Statistical Thinking. This task asks preservice teachers to make a claim based on data they are given from a Consumer Reports study and three individuals' comments. They then were given three different hypothetical student responses that they had to evaluate. The second task-based question involved making a claim about the probability of selecting a red candy from a bowl of different colored candies. The distribution of the colors was not given to the preservice teachers in order to assess how they would make a claim about probability in this context. They were then asked if they would use this type of a task in their own teaching and if so how they would use it with students.

Some of the basic concepts within probability that were assessed in this initial interview include: how to calculate probability, effects of sample size, strategies to find probabilities, interpretation of meanings of probability, and knowledge of basic terms (randomness, chance, theoretical probability, empirical probability, law of large numbers, sample space, sample size, variation, independence, proportional reasoning, and permutations/combinations). The complete interview protocol is given in Appendix C.

Second Interview

The second interview took place within one to two weeks after the initial interview. Two of the preservice teachers did this interview individually and three participated in a group interview due to scheduling difficulties. This interview was

mainly designed to assess the preservice teacher's pedagogical content knowledge and indirectly their content knowledge.

The interview focused on data taken from a research study (Weber et al, 2008) looking at middle school aged students use of the Schoolopoly Task (see Figure xx). There were three main parts to this interview: 1) task analysis, 2) assessing student work in the forms of poster analysis, and 3) critiquing a video clip of student discourse and another video clip of a teacher intervention. See Appendix D for complete protocol and student posters.

The Schoolopoly Task (described in detail in the section on Organizing the Tasks) was originally designed and implemented in the research by Stohl & Tarr (2002) and in Tarr, Lee, and Rider (2006). The task was again used by Weber et al. (2008) and was chosen for this study because it focuses on making inferences from empirical data. By having preservice teachers examine this task, the researcher could get a sense of how they make sense of students' understanding of the connection between empirical and theoretical probability.

This task was designed in four parts to study different aspects of the preservice teachers' pedagogical content knowledge, and indirectly their content knowledge and orientations as well. The first part was to analyze the Schoolopoly Task; this is knowledge that is included in the knowledge of content and teaching. By first being presented with the task and asked to analyze the objectives, strengths and weakness, and

possible student difficulties, the researcher could gain an understanding of their initial thoughts about this task before seeing any student work or classroom discussions. These questions were also asked at the end of the interview in some cases.

Assessing student work is another aspect of pedagogical content knowledge that is related to knowledge of content and students. The researcher wanted to assess how the preservice teachers' could evaluate student work that was presented in the format of a poster created by students. These posters (see Appendix D) included screen shots from the students' work with the software. The content of the posters was what the students thought was important to help argue their conclusion as to whether the company they investigated made fair or unfair dice. The students also had to explain why they made their conclusions as well as what they thought the probabilities for each number on the die were.

The particular four posters (see Appendix D) that were used in this study were purposefully chosen because they present conflicting conclusions. Two posters were of students work on the Dice-R-Us company in which one group of students claimed this company produced fair dice while the other group presented evidence that the company's dice were not fair. The other two posters were of students work on the Calibrated Cubes company in which one group claimed the company's dice were fair while the other group claimed the dice were unfair. In addition to having different conclusions, the work

presented in these four posters showed different strategies the students used to make their inferences from data.

Two video clips were taken from the research done by Maher, Powell, Weber, and Lee (2006) and Weber, et al. (2008) and focused on part of a classroom discussion of the posters as well as a teaching intervention conducted by a teacher/researcher. In the class discussion clip the students are talking about the need for using a large sample size. One student in particular is arguing that a large sample is needed because he found the results to be different when he used large versus small samples. Other students disagree and at the end of this clip one student says "the percentages will be the same regardless (of the sample size)." This comment is applauded by other students. At this point in the interview with preservice teachers, the video is stopped and the preservice teachers are asked how they would direct the conversation. This was designed to assess their pedagogical knowledge of teaching as well as students to see how they would react to conflicting student beliefs.

The second video clip, also taken from Maher, et al.(2006) and Weber, et al. (2008), showed the teacher using the *Probability Explorer* software to illustrate the law of large numbers. The students directed the teacher as to how many trials to run and at different points in the simulation the teacher stopped the experiment and asked, "What do you think now?" This teaching intervention was meant to confront the students' misconception that the probabilities would be the same regardless of the sample size.

After watching this video clip the preservice teachers were asked to critique the teachers' actions as well as comment on what they perceived to be the students' understanding after the teaching. This was designed to assess the preservice teachers' pedagogical content knowledge of teaching and what they believed was the effectiveness of the teaching demonstration.

Reflection on Second Interview

After completing the second interview the preservice teachers were given a written reflection to answer and return. They completed this outside the interview setting one to two days after the interview. The purpose for this reflection was to gain further insight into the individuals' thoughts with regards to the task, student work, teaching episode and conversation. In addition, this reflection gave the preservice teachers a chance to possibly internalize some of the information that was learned through their experience in the interview and to express their thoughts after time for reflection. This became especially important since three of the preservice teachers participated in the second interview as a group, due to scheduling difficulties. The written reflection allowed the researcher to have an individual data source concerning the analysis of students' work on this task. See Appendix E for the complete prompt given for the reflection assignment.

Final Interview

The final interview was done individually and videotaped; it took place one to two weeks after the second interview. This interview was in two parts – the first part was

dependent on the individual and consisted of questions for further clarification from the first interview. Thus, these questions varied across participants. The second part of this interview consisted of seven questions. The focus of each question and it's relation to aspects of the framework are shown in Table 2.

Table 2 *Brief description of final interview questions related to aspects of framework*

Final Interview focus of question:	**Aspects of Framework:**
1. clarification questions, varied by case,	Orientations Content Knowledge Pedagogical content knowledge
2. statements calculating probability-thoughts, identify	Orientations
3. statements about defining probability-thoughts, identify	Orientations
4. what concepts would they want students to learn	Content Knowledge Pedagogical content knowledge
5. how would they teach those concepts	Pedagogical content knowledge
6. most meaningful for them [in this research process]	Orientations Content Knowledge Pedagogical content knowledge
7. what are some things they learned about the T&L of probability	Content Knowledge Pedagogical content knowledge
8. what have they learned about their *own* learning	Orientations Content Knowledge Pedagogical content knowledge

The first two were designed to address the preservice teachers' orientations; they were given four statements about interpreting probability and then four statements about the meaning of probability. They had to speak about each one and then chose which statement they identified with. These statements were purposefully designed to determine

the different orientations the preservice teachers might have. One of the statements was taken from Batanero et al. (2005) where they defined probability as the hypothetical number towards which the relative frequency tends when stabilizing.

The next two questions were related to pedagogical content knowledge and asked them to again list concepts they thought were important for students to learn and what strategies they would use to teach those concepts. And the final three were questions about what the preservice teacher thought they had learned since the beginning of the study. The protocol for the final interview is given in Appendix F.

Data analysis

To analyze the data, the framework for orientations and knowledge for teaching probability (see Figure 2), as described in Chapter 2, was used. In this section the analysis process is described, starting with coding the data and then describing the organization of tasks, within case analysis, and across case analysis.

Coding the Data

After the interviews were conducted they were transcribed verbatim and spreadsheets were used to organize, chunk, and code the data. The researcher then went through each data source (all three interviews, the reflection, and the pre-post tests) and coded the data with respect to orientations, content knowledge, and pedagogical content knowledge. For orientations, indications were coded as M, St, Su, and P for mathematical, statistical, subjective, and personal, respectively. For their content

knowledge the responses were analyzed as to evidence of awareness as well as understanding. For example, if they listed concepts to be taught in probability this showed an awareness of the topics but not necessarily understanding. And when analyzing student work they may have displayed an understanding of a topic and thus was coded under content knowledge. For pedagogical content knowledge, responses were coded as pertaining to knowledge of content and students or knowledge of content and teaching. Comments were made next to their responses as to the analysis of their responses.

Organizing the Tasks

As the analysis was underway it became clear that the data would be better analyzed by organizing each case according to their responses on different types of pedagogical tasks. There were several different types of tasks that addressed different aspects of orientations, content knowledge, and pedagogical content knowledge. The following Table 3 shows how the eight tasks were selected and organized from the instruments and data sources. As the analysis process progressed, certain questions from the first and final interviews were not deemed relevant to the research questions and were not used.

Table 3 *Description of tasks and data sources*

Description of 8 Tasks	**Data Source**
1: Interpretation of Randomness and Probability	Interview 1(Q1&2)
2: Curricular Issues	Interview 1(Q3&4a)
3: Real-world Context and Teaching Situation	Interview 1 (Q6)
4: Experimental Context and Critique/Plan	Interview 1 (Q7)
5: Schoolopoly Task and Reflection	Interview 2
5a: critique task, curriculum, planning	Interview 2(Q1a-d)
5b: analyze student work	Interview 2(Q2a-c)
5c: analyzing student discourse and teacher decisions	Interview 2(Q3 & 4)
5d: interview 2 reflection	Interview 2 (Q1-4)
6: Interpretation of Meaning of Probability	Interview 3 (Q2 & 3)
7: Curricular Issues Revisited	Interview 3(Q4, 5, & 7)
8: Multiple Choice and Free Responses from Pre/Post Tests	Pre/Post test (Q2, 3, 7-10, 15-17)

The first task came from the initial interview and consisted of the first and second questions. These included defining randomness and probability in the preservice teachers' terms, giving examples of things that happen in a random way, and interpreting meaning of probability.

1. What does random mean to you? Can you think of an example of something that happens in a random way? Does random mean something different in a probability and statistics class?
2. What does it mean to say the probability of getting a head on a coin toss is 50%? What does 70% chance of rain mean? If you had to explain what probability means what would you say?

Figure 3. Task 1 – Interpretation of Randomness and Probability

The second task consisted of two questions pertaining to curricular issues. The first question asked the preservice teachers to explain why they think it is important for students to learn probability, and the second question asked them to brainstorm topics they thought should be included in teaching probability.

1. Why do you think it is important for students to learn probability?
2. Take a few minutes to brainstorm what topics or concepts related to probability you think are important to teach at the MS/HS level.

Figure 4. Task 2 – Curricular Issues

The third task was an activity that was set in a real world context where the preservice teachers needed to make a decision based on information they were given about the mechanical problems of Toyotas versus Hondas. This task also asked them to comment on hypothetical student answers and discuss how they would direct the conversation in a classroom setting.

a. [adapted from Garfield: ARTIST] You are trying to decide between two types of cars. You first consult an issue of Consumer Reports, which compared rates of repairs for various cars. Records of repairs done on 400 cars of each type showed somewhat fewer mechanical problems with Hondas than with Toyotas. You have two friends that own Toyotas and one friend that owns a Honda.

   Both Toyota owners reported having a few mechanical problems, but nothing major.

   The Honda owner, however, exploded when asked how he liked his car: "First, the fuel injection went out, that cost $250 bucks. Next, I started having trouble with the rear end and had to replace it. I finally decided to sell it after the transmission went. I'd never buy another Honda."

   Given what you currently know, which car would you buy? Justify your response.

b. You are teaching a class and have posed this problem. The following are three different groups' responses. **How would you proceed with the class discussion?**
   a. We would recommend you buy the Toyota, primarily because of all the trouble your friend had with his Honda. Since you haven't heard similar horror stories about the Toyota, you should go with it.
   b. We would recommend you buy the Honda in spite of your friend's bad experience. That is just one case, while the information reported in Consumer Reports is based on many cases. According to that data, the Honda is somewhat less likely to require repairs.
   c. We would tell you that it does not matter which car you buy. Even though one of the models might be more likely than the other to require repairs, they could still, just by chance, get stuck with a particular car that would need a lot of

Figure 5. Task 3 – Real world Context and Teaching Situation

For the fourth task the preservice teachers were given a bowl of candy and asked to make a claim about the probability of choosing a red candy. They were then asked questions regarding the task: would you use this task in teaching, and how would you use this task?

1. [A bowl on the table has 500 wrapped hard candies: 20% are purple, 50% are red – watermelon and cherry, and 30% are green; interviewee does not know these percentages.]
   Given this bowl of candies, explain how you could make a claim about the probability of selecting a red candy.
   a. Is there any way to do experimentation to support the claim about the probability of selecting a red candy? What would you do with that data?
   b. Would you use this task with MS or HS students? Why, why not?

Figure 6. Task 4 – Experimental Context and Critique/Plan

The fifth task was built off of the Schoolopoly task, originally posed by Stohl and Tarr (2002), that was used in the research by Maher, et al. (2006).

Plainfield Middle Schoolopoly

Background
Plainfield Middle Schools are planning to create a board game modeled on the classic game of Monopoly. The game is to be called "Schoolopoly" and, like Monopoly, will be played with dice. Because many copies of the game will be sold as a fundraiser, several companies are competing for the contract to supply dice for Schoolopoly. Several companies, however, have been accused of making poor quality dice. These companies are to be avoided since players of Schoolopoly need to know that the dice they are using are actually "fair."
Each dice company has provided a sample die for analysis. You will be assigned one company to investigate:

Perfect Polyhedra	Calibrated Cubes
Dice R' Us	Dazzling Dice
Delta's Dice	Dice Depot

Your Assignment
Investigate whether or not the die sent to you by the company is fair. That is, are all six outcomes equally likely to occur?

You will need to create a poster to present to the School Board. The following three questions should be answered on your poster:

1. Would you recommend that dice be purchased from the company you investigated?
2. What evidence do you have that the die you tested is "fair" or "unfair"?
3. What do you think the chances are for rolling each of the six numbers?

Use Probability Explorer to collect data from simulated rolls of the die. Copy any graphs and screen shots you want to use as evidence and paste them in a Word document. Later, you will be able to print these to use on a poster. You will give a brief presentation pointing out the highlights of your group's poster.

Figure 7. Schoolopoly Task (Maher, et al., 2006)

This task was given in four parts, analysis of: the task, student work, a teaching episode, and a written reflection. In the first part the preservice teachers were asked to analyze the task and discuss the objectives, anticipated student difficulties, strengths and weaknesses, and if it is a task they would want to use.

5. a) **Discuss activity**: Begin with an open discussion (2-3mins) "tell me your thoughts on this activity." Then ask more direct questions:
    a. What do you see as the learning objectives associated with this task? What is the purpose of the task, what is it students should learn by doing this task?
    b. What difficulties might students have with this task?
    c. [If not already discussed:] In general what are the strengths and weaknesses of the task? Pros and cons?
    d. Is this a task you would use if you were teaching probability? Why or why not?

Figure 8. Task 5a – Schoolopoly Task Analysis

The second part was focused on students' work. The preservice teachers were shown four posters of students' work on the Schoolopoly Task (see Appendix D). They were asked to give their analysis of students' understanding and what they thought the students considered compelling evidence. They then were to compare the first two posters to each other, and the second two posters to each other, and then tell whether or not they were convinced by any of the posters.

5. b) **Analyze Posters**: Posters A & B (Calibrated Cubes), posters C & D (Dice R Us)

a. Based on the posters, what can you say about the students' understanding of probability?
b. What do you think they consider to be compelling evidence?
c. Compare posters A & B, and C & D. Are you convinced by any of these posters? Why or why not?

Figure 9. Task 5b – Analyze Schoolopoly Student Work

For the third part of Task 5 the preservice teachers were shown two video clips – the first one was of a class discussion and they were asked how they would direct the conversation at the end (when a student said that the percentages would be the same regardless and the other students agreed). The second clip showed the teacher demonstrating with *Probability Explorer* and different sample sizes. They were asked to critique the demonstration and say whether they thought it was effective and what the teacher could have done differently.

5. c) **View whole class discussion**: The class discussion is about sample size and Calibrated Cubes, some comments from the students were: "80 seems reasonable" "are you going to quit or are you going to keep playing to try to win your money back?" "It doesn't matter how many times the percentages will be the same regardless"

a. How would you direct the conversation at this point?
b. What is her misconception?
c. What ways could you intervene to address this misconception?

View intervention and discuss: The teacher uses the software to demonstrate the effect of large and small sample sizes starting with a sample size of 10.

d. Pros and cons – do you think this demonstration was effective? How can you tell?
e. What could the teacher have done differently?

Figure 10. Task 5c – Analyze Schoolopoly Teaching Episode

The fourth part of the Schoolopoly Task was a written reflection where the preservice teachers answered four questions as seen in Figure 11.

5. d) **Reflection**: During the focus group you participated in analyzing a task that was used to teach probability to middle school students. You looked at the task, students' work, as well as two video clips of class discussion related to the task.
   a. What do you see as the benefits and drawbacks if you were to use this task to teach probability?
   b. Based on the students work and the class discussion, if you were the teacher of this class, what would you do the next day?
   c. What did you take away from the experience? Write a few sentences about what you learned by participating in these activities.
   d. What did you learn about your own understanding of probability and of teaching probability?

Figure 11. Task 5d – Reflection on Second Interview

Task 6 was related to interpreting the meaning of probability and was in two parts. In the first part the preservice teachers were given four statements about calculating probability that they were to comment on and then identify the statement they agreed with the most (Figure 12). In the second part of this task the preservice teachers were given four statements about the meaning of probability to comment on and then identify the statement they agreed with the most (Figure 13). The purpose of the two parts was to see if the different contexts – calculating versus meaning – elicited different responses.

a) Read the following statements and say a little bit about each one.

1. I want to figure out the probability of rolling a 3 on a die. I simulated it and out of 1,000,000 rolls and 3 showed up 166,549 times. So the probability of getting a 3 is about 166,549/1,000,000 = .166549.
2. I think the probability of rolling a 3 is not very good. Based on my experience playing games with dice 3 doesn't come up very often. I'd say the probability of rolling a 3 is around 10%.
3. I want to know what the probability of rolling a 3 on a six sided die. I know there are six faces possible and 3 is one of those faces, therefore the probability of rolling a three is 1/6.
4. I want to find the probability of rolling a 3 on a die. I asked five of my friends as well as my brother and parents. Of the 8 people I talked to 5 of them told me it was 1/6. The other three told me it depends on the die, the probability could be small or high, you never know. So I am not sure, but based on what most of the people I asked said, I would say its 1/6.

b) Of these four statements, which one do you identify with, or agree with the most?

Figure 12. Task 6a – Interpretations of Calculating Probability

a) Read the following statements and say a little about each one.

1. Probability is a degree of belief based on personal judgment and information about experiences related to a given outcome.
2. Probability is the fraction of favorable outcomes over the total outcomes.
3. Probability is the hypothetical number towards which relative frequency tends when stabilizing.
4. Probability is the measured randomness of an occurrence.

b) Which of the four do you identify, or agree, with the most?

Figure 13. Task 6b – Interpretations of Meaning of Probability

In Task 7 the preservice teachers were again asked what major concepts they thought should be taught in probability. They were also asked what strategies they would use to teach those concepts. And finally what they had learned about the teaching and learning of probability.

1. As a teacher, if you want students to develop a deep understanding of probability, one that is applicable to life, what are the major concepts you would want them to learn?
2. What strategies would you use to teach these concepts?
3. What are some of the things you have learned about the teaching and learning of probability?

Figure 14. Task 7 – Curricular Issues Revisited

The final task was composed of questions from the pre-post test and was separated into three parts: a) multiple choice test questions about interpretations of probability and understanding of independence; b) multiple choice test questions assessing sample size, proportional reasoning, recognizing random distribution, and expected variability; and c) open-ended pedagogical questions critiquing student strategies.

There were four questions in the first part of the task and they were about interpreting a simple probability, interpreting a range of probabilities, and a sequence of coin tosses.

1) The following message is printed on a bottle of prescription medication.
   Warning: For applications to skin areas there is a 15% chance of developing a rash. If a rash develops, consult your physician.
   Which of the following is the best interpretation of this warning?
   a. Don't use the medication on your skin, there's a good chance of developing a rash.
   b. For applications to the skin, apply only 15% of the recommended dose.
   c. If a rash develops, it will probably involve 15% of the skin.
   d. About 15 out of 100 people who use this medication develop a rash.
   e. There is hardly a chance of getting as rash using this medication.
2) The Springfield Meteorological Center wanted to determine the accuracy of their weather forecasts. They searched the records for those days when the forecaster had reported a 70% chance of rain. They compared these forecasts to records of whether or not it actually rained on those particular days.
   The forecast of 70% chance of rain can be considered *very* accurate if it rained on:
   a. 95% - 100% of those days.
   b. 85% - 94% of those days.
   c. 75% - 84% of those days.
   d. 65% - 74% of those days.
   e. 55% – 64% of those days.
3) The following sequences represent the results from tossing a fair coin 5 times. Each sequence displays the results in the order in which they occurred. Which sequence is *most* likely?
   a. H H H T T
   b. T H H T H
   c. T H T T T
   d. H T H T H
   e. All four sequences are equally likely
4) Select the one explanation that best describes your reason for the answer you gave in item 3.
   a. Since the coin is fair, you ought to get roughly equal numbers of heads and tails.
   b. Since coin flipping is random, the coin ought to alternate frequently between landing heads and tails.
   c. If you repeatedly flipped a coin five times, each of these sequences would occur about as often as any other sequence.
   d. If you get a couple of heads in a row, the probability of a tails on the next flip increases.

Figure 15. Task 8a – Multiple Choice Test Questions Assessing Interpretations of Probability and Understanding of Independence

In the second part of Task 8 the preservice teachers had 3 questions that were related to sample size, proportional reasoning, and expected variability.

1) Two containers, labeled A and B, are filled with red and blue marbles in the following quantities:

Container	Red	Blue
A	6	4
B	60	40

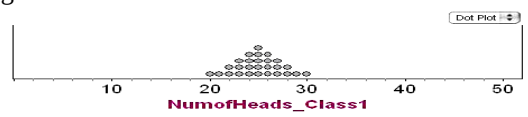
Each container is mixed thoroughly. After choosing one of the containers, you will reach in and, without looking, draw out a marble. If the marble is blue, you win $50. Which container gives you the best chance of drawing a blue marble?

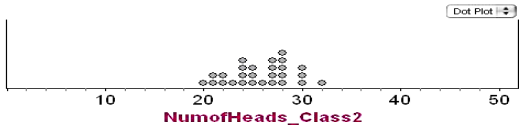
a. Container A (with 6 red and 4 blue)
b. Container B (with 60 red and 40 blue)
c. Equal chances from each container

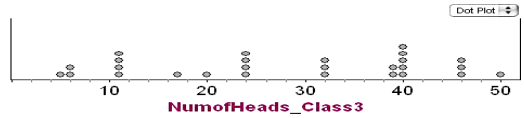
2) Half of all newborns are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?

a. Hospital A (with 50 births a day)
b. Hospital B (with 10 births a day)
c. The two hospitals are equally likely to record such an event.

3) Each student in a class tossed a penny 50 times and counted the number of heads. Suppose four different classes produce graphs for the results of their experiment. There is a rumor that in some classes, the students just made up the results of tossing a coin 50 times without actually doing the experiment. **Please select each** of the following graphs you believe represents data from actual experiments of flipping a coin 50 times.

a. 

b. **correct** 

c. 

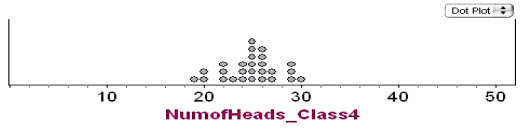
d. **correct** 

Figure 16. Task 8b – Multiple Choice Test Questions Assessing Sample Size, Proportional Reasoning, Recognizing Random Distribution, Expected Variability

The last part of Task 8 was a free response question where the preservice teachers were asked to critique two groups' strategies for determining if a die is fair or unfair.

1. Students are presented with a six-sided die and asked to experiment with it to make a claim as to whether the die is fair or unfair. You observe the strategy employed by two different groups, described below.
   Group 1: Toss the die 10 times and compute the empirical probability for each number on the die. If the empirical probability is close to 1/6 then the die is fair.
   Group 2: Toss a die six times. If each number appears once then the die is fair.
   Which group is using an appropriate strategy to make and support a claim about fairness?
   a. Group 1
   b. Group 2
   c. Both Group 1 and Group 2
   d. Neither Group 1 or Group 2

Figure 17. Task 8c – Open-Ended Pedagogy Question Critiquing Student Strategies

These eight tasks were analyzed within each case and also analyzed across all five cases.

With-in Case Analysis

Tasks were analyzed and coded using the guiding frameworks for the three aspects of learning to teach probability. For each task the preservice teacher's responses were analyzed as they pertained to the three aspects – orientations, content knowledge, and pedagogical content knowledge. Depending on the task certain themes regarding each aspect were noted and coded within each case. For example, in Task 5 there were

mainly instances of pedagogical content knowledge however indirectly content knowledge was also addressed and noted.

After the tasks were analyzed, relationships between aspects were studied across the tasks within each case. For each case a diagram was created where all the findings were written down and themes among the aspects were analyzed (see Figure 18).

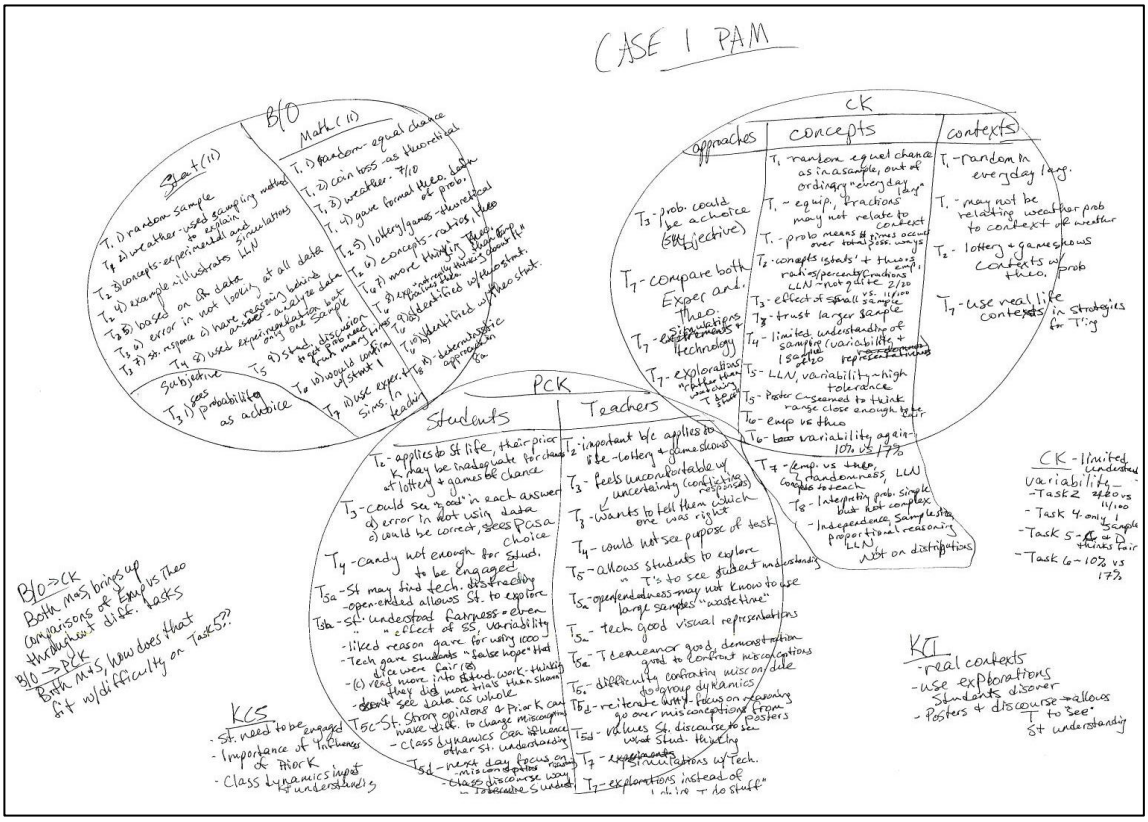


Figure 18. Diagram of Pam's Within Case Analysis

For example, when analyzing students' work on the Schoolopoly Task, Pam noticed that the students claimed the probabilities were spread evenly across all 6

numbers in a trial of 80 rolls. They argued that each number had an equal probability of 11/80 or ~13.75%. This preservice teacher said, "Oh is 80 even divisible by 6? If it was [spread] even it [the total] would have to be 66." She noticed that if the claim was true, that each number appeared 11 times in a simulation of 80 rolls, the probabilities do not add to 1. The students only displayed three of the 6 probabilities and each was 11/80. From this comment the researcher inferred that Pam understands that within an evenly spread distribution the sum of the probabilities should equal one; the denominator should be the total trials. From her analysis of the students work and her interpretation of the students' understanding of probability, it appears that her content knowledge of probabilities allowed her to pick up on the misconception. Thus, her content knowledge influenced her pedagogical content knowledge, specifically her knowledge of content and students.

Cross-case Analysis

The cross-case analysis consisted of a task analysis across the 5 cases as well as a cross-case analysis of the relationships within each case to look for relationships that may exist among all preservice teachers. For the latter, the primary source of analysis was then the diagrams made for each case similar to that shown in Figure 18.

For the analysis of the tasks, the responses from each preservice teacher were compared and similarities and differences were noted. To aid in the analysis, tables were created to compare the responses across the five cases. Within each task the researcher

analyzed common themes that emerged and hypothesized possible reasons for these themes. The focus was on the three aspects of the framework (orientations, content knowledge, and pedagogical content knowledge) as these related to the task. The analysis looked at the effectiveness of the task at illuminating these aspects as well as how the task may have affected the preservice teachers' responses.

Validity

As with any qualitative study, validity threats cannot entirely be removed. What is included in this section is an identification of possible threats that pertain to this study, and how those threats are addressed. Validity threats occur in both the data collection stage and the data analysis stage.

Data Collection

At the data collection phase the researcher kept detailed records with dates and times. The interviews were video-taped and transcribed verbatim. These measures support the credibility of the data. In addition to tapes, each interview was structured by an agenda. Another way to combat validity threats that was used in this study was triangulation. The process of triangulation involves using multiple sources of data in order to verify and clarify findings (Creswell, 2003). Thus multiple forms of data for each case – interviews, test responses, and written documents – were collected. This is important since different contexts may allow for different findings. For example, a

written reflection may give more information that what is collected in an interview setting because the preservice teacher is not constrained by time and place.

Data Analysis

To insure credibility within the analysis stage the researcher used the analytical frameworks that are grounded in theory as has been outlined in the literature review. The data collected was detailed as a result of the verbatim transcriptions, video tapes, and researcher reflections. To insure against researcher bias and reactivity the researcher used peer debriefing (Creswell, 2003) to enhance the accuracy of the account. "This process involves locating a person who reviews and asks questions about the qualitative study so that the account will resonate with people other than the researcher" (p. 196). One peer looked over the coding structure from the first case and the findings with respect to the preservice teachers' orientations, content knowledge and pedagogical content knowledge. Additionally the researcher and advisor met on several occasions at various stages of analysis to discuss the validity of interpretations of the data. Meetings were held during the coding stage for each of the tasks, during the within case analysis, and the cross-case analysis.

Summary

In this chapter the general methodology for this study has been provided. This study is a multi-case study with five preservice teacher cases and eight tasks as units of analysis. Qualitative research methods have been used to collect and analyze data in order

to build a rich, detailed description of the relationships among orientations, content knowledge, and pedagogical content knowledge within each preservice teacher case as well as across preservice teacher cases and tasks. The next chapter presents a description of the findings and analysis for each of the five preservice teacher cases in this study.

CHAPTER 4: WITHIN CASE FINDINGS

Overview

This chapter presents the findings for each of the five cases in this qualitative case study. A case has been defined as a preservice teacher preparing to be a secondary mathematics teacher at a southern university in the United States. The focus in each case is on the preservice teachers' orientations towards probability, their content knowledge of probability, and their pedagogical content knowledge of probability. The findings are presented within each case and organized by eight tasks taken from three interviews and a pre/post test that was given in the course the preservice teachers were taking at the time. For each case a description of the preservice teacher is given followed by the researcher's findings related to the eight different tasks. At the end of each case is a summary of that preservice teacher's orientations, content knowledge and pedagogical content knowledge that addresses the first research question:

> *What is the nature of preservice teachers' orientations, content knowledge, and pedagogical content knowledge?*

This is followed by a description of the analysis of relationships among orientations, content knowledge and pedagogical content knowledge with each case that addresses the second research question:

> *What are the relationships among preservice teachers' orientations, content knowledge, and pedagogical content knowledge for teaching probability?*

Within Case Analysis

Case 1 - Pam

Introduction

Pam was in her junior year as a Mathematics Education major with a focus on the secondary level and was approximately 20 years old. During the time of the study, she had taken one 200 level methods course: Introduction to Teaching Mathematics, and was currently taking a 400 level course on teaching mathematics with technology. At the beginning of the study, Pam expressed that she could not recall being taught probability in high school and also that she felt unsure as to how she could teach it not having been taught it herself.

Pam's responses to the tasks indicated she may have both mathematical and statistical orientations towards understanding probability. As for her content knowledge, Pam seemed to be the case of a preservice teacher that understands how to compute empirical and theoretical probabilities but often displayed an incomplete understanding of expected variability within random situations. As for Pam's pedagogical content knowledge, at the beginning of the study her knowledge of content and teaching was more focused on structure and minimizing uncertainty in the classroom. Whereas at the end of the study she seemed to value more open-ended discussions and letting the students guide instruction. This seemed to be a typical case of novice teachers feeling

uncomfortable with uncertainty, as noted in prior research (Begg & Edwards, 1999; Stohl, 2005).

Task by Task Analysis

Task 1: Interpretation of Randomness and Probability. (See Figure 3) When Pam responded to the question about what random means to her, she mentioned a sample where everyone has an equal chance. "In terms of probability when I think of random I think of a random sample. If you were to have a completely not handpicked group of people but a really open sample and you randomly pick someone from it." Notice, too, in this quote she defined the notion of randomness in the context of a statistical experiment, namely, sampling. Thus, while she was using a mathematical oriented view of random meaning everyone has an equal chance, or all samples are equiprobable, her words also indicated that she was using a statistical view by tying random to sampling methods.

The example that Pam mentioned of something happening in a random way was the lottery. She clarified that the people who bought lottery tickets were not a random sample, "because it depends on whether or not you have the money to buy the tickets" but the chances of winning the lottery are random. Here it seemed her content knowledge of randomness included situations where events are equally likely to occur.

Interestingly, when she was asked if random meant something different in a probability and statistics class she mentioned how the term is over used in general language and communication. "I know in high school everybody would say 'that's really

random', I guess it just means like out of the ordinary." Then she added that random has a more specific meaning in probability and statistics; however she did not elaborate on what that meaning may be. This may indicate that she believed the way the term random is used in everyday language could impact students' understanding as it is applied to probability and statistics. Also Pam's pedagogical content knowledge includes an awareness of how students' experiences affect their mathematical understanding.

When asked about the meaning of the statement 'getting a head on a coin toss is 50%,' she responded: "in a coin toss there's two possible ways that the coin could flip, so when you think of percentages, 100% divided by two is 50%. So the probability of the coin tossing one way or the other is 50%." Here she is applying a mathematical orientation to interpret the meaning of probability in a theoretical way as heads being one of the two possible ways the event can occur. She also may have an understanding of probability that is tied to fractions since she described 50% as 100% divided by two.

Her response to the question about the meaning of the statement '70% chance of rain' also seemed to indicate an association with fractions and a possible equiprobable understanding. "if there were say ten days when the weather was kind of similar and seven of those days it rained, then you could kind of narrow that down to one day… there's probably about a 70% chance of it raining." For this interpretation, Pam seemed to think there would have to be an assumed equiprobable distribution among the days. Also from her response, she seemed to have a statistical orientation because she is using a

hypothetical experiment where a person samples 10 days and finds seven of those days where it rains and thus 70% represents a sample statistic.

One final example indicating Pam used a mathematical orientation can be seen from her response to the question about what probability meant to her. She gave a more formal definition that defines theoretical probability. “I guess probability is probably the number of events, or the number of times that an event occurs out of the number of possible ways that it can occur.” When she described probability she referred to the number of possible ways it can occur. This assumes there is a total finite number of possible ways; however, she did not specifically point this out so it is uncertain as to whether she has made that assumption.

Therefore, in terms of Pam’s content knowledge of random and probability, the meaning of random for Pam seemed to be highly correlated with having an equiprobable distribution. In order for something to be considered random, each event must have an equal chance of occurring. Similarly with her understanding of probability, Pam seemed to have an underlying concept of a known equiprobable distribution. The meaning she gives for probability is the chance of something occurring divided by the total ways something can occur. This assumes a known sample space where all outcomes are equally likely.

Pam’s combined responses to these questions indicate she may have more of a mathematical orientation towards probability yet there were also indications of a

statistical orientation. One underlying concept that Pam has affixed to the meaning of probability as well as randomness is the notion of equiprobability. She also used a formal definition when defining the meaning of probability. One interesting finding is that in a context where equiprobable events are not applicable, as in the case of 70% chance of rain, Pam displayed a statistical orientation to explain probability.

Task 2: Curricular Issues. (see Figure 4) When Pam talked about the importance of teaching probability she referred to the lottery and game shows, both contexts that tend to have already known, or computable, theoretical probabilities. She seemed to believe probability applies to students' daily lives and that, "a lot of kids don't really have a good grasp of what stuff actually means." She said, "They might think their chances of winning are actually a lot greater than they really are. So [teaching probability] maybe helps them get a more realistic sense about probability by knowing more about it." In terms of her knowledge of content and students, Pam may have believed students' prior knowledge of probability is inadequate and recognized the need for teaching probability as it applies to their lives.

Things Pam listed as being important to teach in a high school probability course include: 1) distinguishing between experimental and theoretical probability; 2) an understanding of graphs, plotting points, and trend lines; 3) how to calculate a percentage, a decimal, and a ratio, 4) simulations, and using computers to run simulations, and 5) basics of how to conduct a good experiment. From this response,

Pam's content knowledge of probability seemed to be tied to statistical concepts such as running simulations and experiments, and ways to display and make sense of data. This list also may indicate that she has an appreciation for the usefulness of both a statistical and mathematical orientation because she refers to empirical ideas (empirical probability, running simulations, experiments) as well as theoretical ideas (theoretical probability, percent, decimal, and ratio).

As a follow up question, Pam was asked to explain what she meant by conducting a "good" experiment. She referred to an assignment she was working on for her class related to using graphing calculators to run simulations. "The main point that I'm trying to make in my paper is to get kids to realize that the more experiments you do the closer the experimental probability will be to the theoretical probability." Here she is indicating that using simulations with students is a good way to illustrate the law of large numbers because the calculator can generate many outcomes quickly. Thus, her content knowledge may include an understanding of the law of large numbers, however as she continues on with this response it is evident that this understanding is incomplete:

> Because like if something has a probability of like 11%, then let's say you only do 20 samples and something happens twice out of the 20 times, that will only give you 10%. But if you were to do maybe 100 samples or 100 experiments then it will happen maybe around 11 times so it will be closer to the 11% than your experimental probability was when you only did a few experiments.

While the point she was making was that the more experiments or trials you do the closer you will get to the assumed known probability, she did not seem to expect a large amount of variability with small samples and seemed to believe a sample of 100 would provide the expected 11%.

From this task, it appeared that Pam understood probability in terms of statistical concepts as well as in an empirical context, thus displaying a statistical orientation towards probability. Yet she also listed applications of probability that involve theoretical distributions, thus indicating a mathematical orientation. In terms of her content knowledge, Pam brainstormed general concepts like finding percents and ratios, as well as concepts related to data. In addition she indicated a somewhat incomplete knowledge of the law of large numbers and how that relates to probability. Her knowledge of content and students reflected that she valued the teaching of probability because it can be applied to students' everyday lives. She might also have awareness that students' prior knowledge of probability could be misinformed, thus causing a need to be included in school curriculum.

Task 3: Real world Context and Teaching Situation. (See Figure 5) When asked which car she would buy, Pam chose Honda and based her decision on the Consumer Reports data rather than the friend's statements since the report had "sampled 400 cars for each one and Honda still had fewer problems." She also added that she "would probably trust this data more than just one specific [comment]." From this response it

seemed her decision making was based on the larger data set indicating that she possibly had an understanding that a larger data set will give more reliable information than a few comments. Also, by choosing to rely on the study Pam may have a statistical orientation, basing her decision on the larger data set, rather than a more subjective or personal orientation which would rely more on the friends' responses.

Pam's critique of the hypothetical student group responses gives some insight about her knowledge of content and students. For group (a) Pam makes the assessment that the students made an error by not taking into account the consumer reports study:

> I would probably call it an error, maybe it's not an error in judgment but, by not taking into account all the information they were given I think that's kind of an error… because you can't just base something on just one instance when you have a lot of data to use.

Pam pointed out the importance of the data in the study of 400 cars and although she is unsure if this is an error in judgment, she does state that you can't base probability judgments on just one instance and ignore the data. This may be an indication that she is able to imagine why someone would use a subjective approach to make such a judgment.

In this next comment Pam seemed conflicted by wanting a definitive answer, yet also using some sort of reasoning with the data. Her response to group (c) was interesting because at first she said she liked their answer because "they seem to take everything into account" but then she said, "If I was posed a question like this I wouldn't want to come

out of it indecisive, as like not giving a clear cut answer, this is kind of a really general answer to me." She went on to say that while it is the case that you could flip a coin to decide, "I would rather have some sort of reasoning, some sort of answer in front of me rather than… even after you analyze the whole problem being like, well whatever it doesn't matter." This quote seemed to indicate that Pam realized how data can assist in making decisions about uncertainty rather than remaining indecisive. It also seemed to indicate that she may be uncomfortable with uncertainty, with the fact that the students did not make a decision. This can be a common reaction within probability – teachers, especially mathematics teachers, are uncomfortable with not getting one 'correct' answer.

> Another example of this uneasiness can be seen in the following quote:
>
> Maybe what I'm getting at is that probability is all kind of like personalized. I don't know if I'd actually be that great at teaching it because I kind of like to think about it as more of like a choice, like you're given information then you do what you want with it kind of instead of clear cut like this is, cause I see good things in all of these so I wouldn't be able to really distinguish – I wouldn't be able to tell a class like oh group (b) your right, or group (c) your right. So, I don't know.

This quote is illustrative of Pam's lack of confidence with teaching probability. She seemed unsure about how to deal with the different responses and that probability can be interpreted based on a personal choice about the data. This is an indication that Pam may

believe that many students may have a subjective orientation towards probability. This also indicates that Pam's knowledge of content and teaching, especially how to handle multiple interpretations in a classroom, is still being developed.

Task 4: Experimental Context and Critique/Plan. (See Figure 6) Pam suggested using an experimental approach to make a claim about the probability of selecting a red candy: "I would just take a sample, I think I might do more than 10 but probably no more than 20… if there were like 5 red ones out of 20 then that would be 25%." This use of experimentation and the fact that she would count the frequency of red candies divided by the total in the sample indicates Pam seemed to be able to estimate a probability based on a sample of data. She goes on to say, "So that would be the probability for, or a guess at the probability, of picking a red one out of the whole bowl." Thus, she seemed to understand her method of sampling gives a *guess* at the probability, and that she can extrapolate this sample to the entire data set, the whole bowl. Pam's empirical approach indicates a statistical orientation as opposed to using a theoretical approach that would require her to count all the red candies and how many total candies are in the bowl. She may not have suggested a theoretical approach due to the context of an interview setting where she had a limited amount of time.

However, it could be argued that Pam's understanding of empirical probability is limited because she only used one small sample and no repeated sampling. When asked if she would only do one sample Pam replied, "yeah, probably, I mean actually I didn't

even really think about that, doing it multiple times and like mixing it up. That really didn't even occur to me but yeah, I was originally thinking only to do it once." This may also indicate a limited understanding of variability among samples, as well as a limited understanding of representativeness (i.e., her thinking that 1 sample of 20 would be sufficient to represent the whole bowl).

With respect to her pedagogical content knowledge, Pam's responses to parts b and c show that she would probably not use it in her teaching and could not see the purpose of this task. One reason for not wanting to use this task was, "kids are I think really over exposed to counting situations like this or counting experiments." It is interesting that she equates this type of task with counting situations since there are no combinations or permutations involved and it is unclear what she meant exactly by 'counting experiments'—perhaps that you sample data and count to determine the frequency of an event.

Another reason she gave for why she wouldn't use this task was because she could not see a purpose for the task. "It just seems kind of pointless and I don't think kids would be engaged even though it is candy, everybody likes candy but I don't think they would really be engaged in probability just by doing this, just because it used candy." When pressed to see if she could think of a way to make it less pointless, for example give it a context like quality control for the candy company, she thought for 15 seconds then said she could not and expressed her concern with this for her future teaching. "I'm

kind of worried about being creative… having innovative ways to teach stuff and how to have good ways to teach stuff… I don't feel like I do a very good job with that." She seemed to have an image of a teacher being able to adapt tasks and be creative; this shows she is thinking about the students and how to adapt tasks to make them more engaging for students. Yet with this particular task she could not think of a way to give it a purpose for the students. One more thing to note is that Pam also expressed the idea that, just because a task uses candy, doesn't necessarily make it engaging for students.

Task 5: Schoolopoly Task and Reflection. (See Figures 7 and 8) Pam's initial thoughts of the task were that she liked how open it was and how that makes it open for interpretation. "I think when you limit kids then there's often only one track of thinking… so you could really see where they're all at in their thinking." Here she seemed to be thinking about the task in terms of giving students the opportunity to explore. Also she seemed to see this task as allowing the teacher to gain a better idea of students' understanding and thinking. An interesting thing to note is that she pointed out what she seemed to see as a flaw of limiting students to one track, yet with the Consumer Reports task she expressed unease with a situation where students had conflicting responses.

Also, when Pam was asked if there were any down sides to the task she gave the same reason, the fact that it is open-ended. "… The lack of direction might have some effect on their results because they could maybe only do one at a time and do that 10

times, and then that would take up a pretty large chunk of time." In addition to students seemingly wasting time, she said that some students may not run enough trials and then wouldn't get reliable results. "Maybe you can just hint … you should be trying to do as many trials as you can to get a better idea of how the dice are going to fall…or even more accurate reading." Pam seemed to make the argument that the lack of guidance could hinder students from noticing the need for a large sample size. She also offered the suggestion of the teacher hinting to the students to do more trials; however, by doing this, students would not discover for themselves the need to use large sample sizes. So on one hand she said minimal guidance is beneficial because it gives students the freedom to discover different things as well as allow the teacher to assess students' thinking. And on the other hand she is saying that too little guidance can waste time and hinder students from thoroughly exploring the data.

The learning objectives that Pam listed for this task include: the law of large numbers, sample size, fairness of dice, and analyzing pie graphs and bar charts. As far as anticipating what difficulties students might have with this task, Pam focused on the technology and how the students may be more interested in making colorful charts than focusing on percentages and decimals. She also went on to mention that "having those charts up gives them a good visual representation of it." So she seemed conflicted between the purpose of the different representations – pie graphs, bar charts, and tables –

to give students visual representations of the data, and how these may be distracting to students.

After having the opportunity to analyze the students' work and the video clips, Pam said that she would use this task in her teaching, or one similar to it. "I think it is a phenomenal task, I think it's really, really good. Because you get, as a teacher you get a much better understanding of what your kids are thinking." When pressed further to explain how one gets a better understanding of students' thinking, she brought up the fact that one kid related the task to gambling so he may have some background knowledge on that; also another kid thought a lot of trials were necessary – indicating he has a knowledge of the law of large numbers "without really knowing." Still more reasons this task allows teachers to analyze student thinking were the different aspects students were attending to, namely how some based their conclusions mainly on percentages and numbers while others used the graphs more. "Once the technology is put in front of them you can use it to help mold what they'll get from the task." Here Pam pointed out that the technology can be used by a teacher to assist students with this task, however it is not clear *how* she thinks the technology should be used (other than what she observed in the video clip, see discussion below).

For the second part of Task 5 (see Figure 9) the preservice teachers analyzed four posters displaying students' work with the Schoolopoly Task. The students were in pairs and two groups investigated the dice company called Calibrated Cubes and the other two

groups investigated the Dice-R-Us company. The preservice teachers were asked to critique each poster (both verbally and written on a handout) focusing on the students' understanding of probability and what the students considered to be compelling evidence. After they analyzed the posters the preservice teachers were then asked to compare the posters and say whether they were convinced by any of them.

Poster A. The students that made this poster investigated Calibrated Cubes and they determined the dice were unfair. They based their decision on two trials of sizes 100 and then 1000. The reason they gave for these sample sizes was, "if you're playing for money you're gonna want to win your money back so that's why we ran it 1000 times." The evidence they gave was based on the sample of 1000 trials because "all the numbers started to even out." As seen in Figure 19 the numbers 2, 4, and 6 occurred more than 1, 3, and 5.

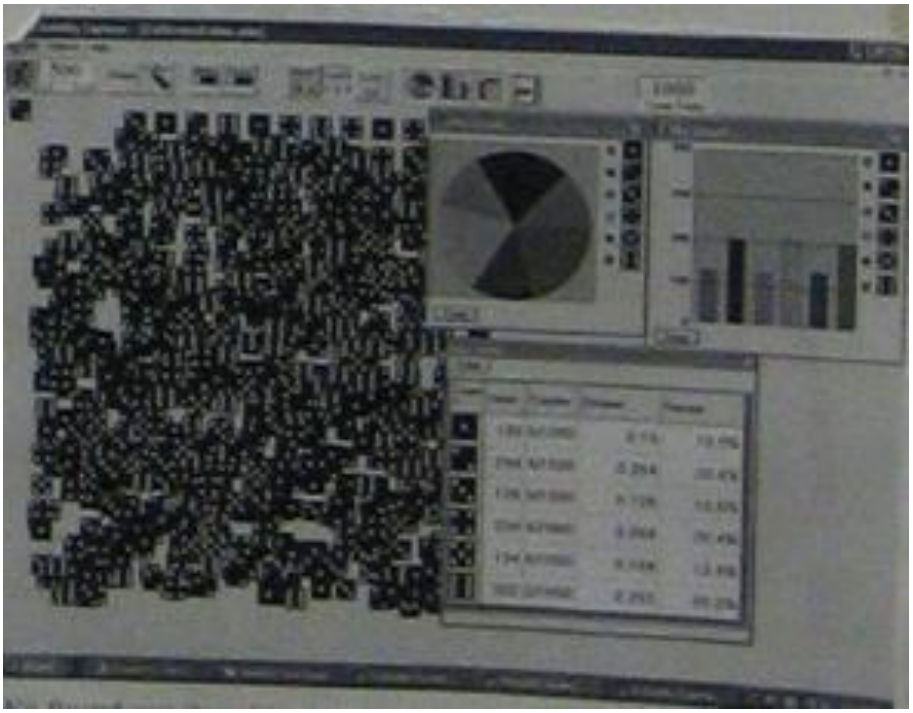


Figure 19. Poster A – Screen Shot Included on Poster

For this first poster Pam said the students had an understanding of the meaning of fairness as being even. “Their understanding may be that a couple numbers have much higher rates, or higher number of times they’ve been rolled, then that would be unfair.” She added that they also had an understanding of the effect of sample size, “they’re on the right track in thinking the higher number of trials rolled, the number of times it hits each number should be like around the same area, not have some way down at 120 and some up at 204.” Here Pam pointed out that the students correctly understood how, with the more trials they ran, the dice should even out and thus this amount of variability was too high for the dice to be fair.

As for the compelling evidence, Pam said "I'd say their compelling evidence is, I mean the only thing they really cited, was that some numbers occurred a lot more than others." She continued by saying that it was hard to tell if they understood percentages because they basically just listed what the chart said. So Pam seemed to think that they listed the numbers occurring a lot more as the compelling evidence, but she could not tell whether or not they understood what percentages meant.

One more thing that Pam mentioned about this poster was that she liked the fact that they explained why they ran it 1000 times. "I guess they're associating it with gambling… I don't really understand what that has to do with anything but I think it's just the fact that they knew running it a lot of times would give you a more accurate reading." With this quote Pam is referring to what the students wrote at the top of their poster (see Figure 20). Pam noticed that the students were placing a context to their reasoning and although she couldn't see where they were coming from, she did appreciate that they justified their reason to run it 1000 times.

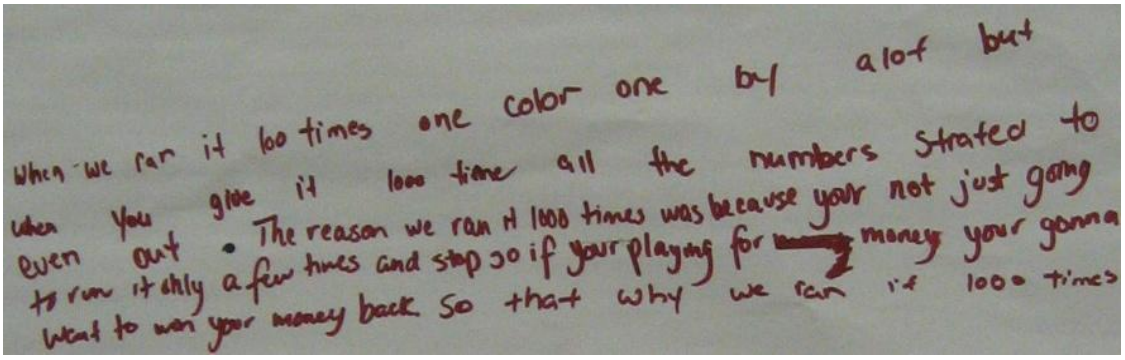


Figure 20. Poster A – Student Writing

With regards to Pam's pedagogical content knowledge she was able to determine the students' understanding of the need for a large sample. Plus her comments go beyond simply stating this to also evaluating their use of a justification. Here too, as in the first task where she pointed out students' understanding of randomness may be impacted by their everyday experiences, she brings attention to the fact that these students also related to their past experiences.

Poster B. The students that made this poster also investigated Calibrated Cubes; however they determined the dice were fair. The evidence they gave consisted of a table displaying only the first three numbers, and a pie chart. They ran 80 trials (see Figure 21).

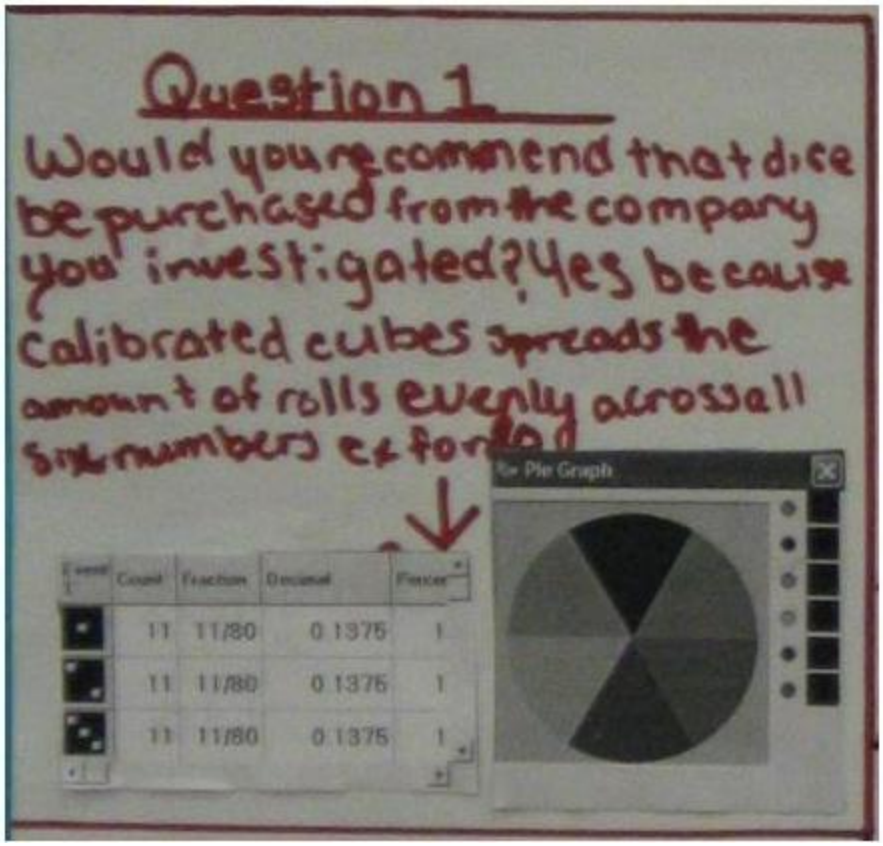


Figure 21. Poster B – Screen Shot Included on Poster

In analyzing this poster Pam seemed to recognize that with empirical probability, due to randomness, there should be some variability within the results and the larger the sample size, the less variability there should be (provided the outcomes are equally weighted). "80 just doesn't really seem like enough trials to do – I was just thinking, usually when we would do stuff in class with die, we would do 300 trials or something like that." She went on to explain this idea with a coin toss, "and you would think it would be around 150 each – but like even still with 300 trials over like the range (pause)

between heads and tails like 143 to 157, so I would doubt that with 80 trials and 6 possible outcomes that, that [getting 11 outcomes for each die] answer's wrong." She recognized that, even with 300 trials, you will have some variability; however, she only seemed to expect minimal variability, since +-3% is small for a sample of 300. Thus although she expects variability to be there, her tolerance for variability was low. This evidence also was present in her analysis of poster D. The likelihood of getting exactly the same answer for each possibility, with a sample size of 80, is minimal – thus her analysis of their answer is that it is wrong.

When explaining what she thought the students understood about probability Pam said they understood that the fairness of the dice is associated with an even spread of even percentages. She added "maybe this set of trials didn't allow them to understand anything really about it because it gave them such a ridiculously even reading… this might give them false hope that it would always be even like that." This quote is another indication of Pam's understanding of variability and the fact that this trial seemed to be "ridiculously even." Also her wording about the trials not allowing the students to understand anything seemed to indicate that she was aware of how students may interpret fairness falsely.

Another interesting finding from Pam's analysis of this poster is that, although she stated that she thought their evidence did not support their claim, she did not notice that, if each number was rolled 11 times, the total favorable outcomes did not add up to

the total sample size. The researcher prompted Pam asking her to "say a little something about, so what they're trying to tell us is that it's spread evenly across the 6 numbers, and there were 80 total…" To which Pam replied, "Oh, ok I see that wow, sorry I didn't get that, it would have to be 66. I didn't even catch that." She then tried to make sense of how this could be and questioned the technology, "it wouldn't count up all the totals?" After the researcher demonstrated within the software how the table could be resized, Pam said that she didn't think middle school kids would "be thinking that business like that they would purposely omit information. But that's kind of what it looks like." Thus Pam seemed to be surprised that they only showed the numbers that were the same. What this exchange demonstrates is Pam first noticed that getting 11 for each number out of a sample size of 80 would be 'wrong' yet she did not think students would purposefully omit data to make it look like it was correct.

Before moving on to the next poster Pam looked back at poster A and was still very confused, she couldn't see how these two groups could get such different results if they were examining the same company. "Does that, I mean, (looking puzzled) work with this program, the way they were weighted, that you could get one trial like that? Or is that because they only did 80 trials instead of 1000?" This seemed to indicate her uneasiness with the lack of variability that occurred in the students' data and her wanting to possibly explain it with the technology program. Or she may be thinking that because the sample size was so small that this seemingly unlikely result is due to randomness. She then

referred back to the previous poster, “because theirs seemed a lot different than this but they were weighted the same because they were the same company…”

As for Pam’s pedagogical content knowledge, this confusion may be impacting her ability to determine students’ understanding. Also with regard to instructional strategies it seemed Pam sees using technology may lead students in the wrong direction by giving them unusually even results.

Poster C. The students’ work that is shown in this poster comes from investigating the Dice-R-Us company. They determined the dice were fair based on comparing results from two trials of sizes 80 and 100. They based their conclusion on the results of the number three which occurred the most times (20/80) in one trial and the least times (10/100) in the other (see Figure 22).

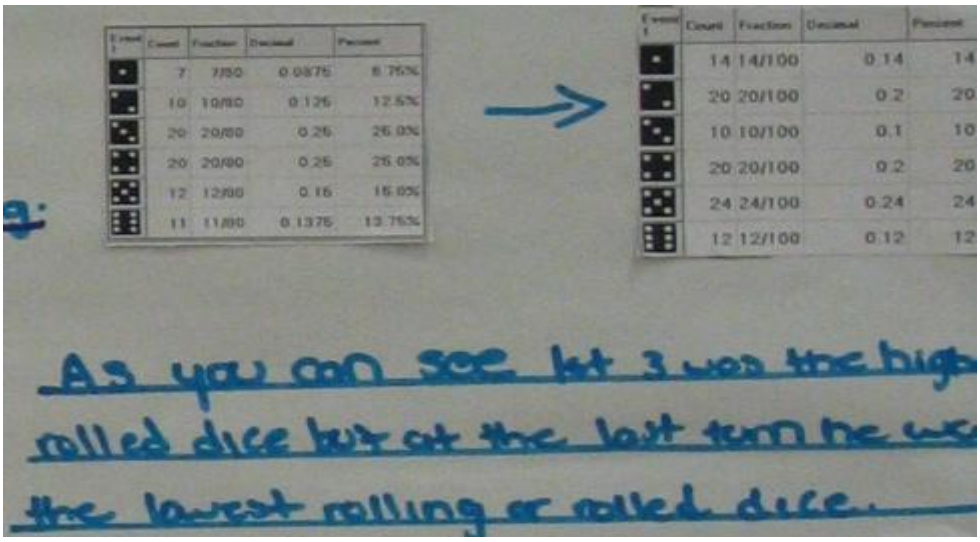


Figure 22. Poster C – Screen Shot Included on Poster

Upon first looking at this poster Pam said that the numbers were fairly close in range "so it kind of looks to me that their dice are fair but their evidence doesn't prove that at all." What is interesting to notice with this poster is the range of values for both trials. Out of 80 the range is between 7 and 20, and out of 100 the range is between 12 and 24; yet those numbers seemed fairly close in range for Pam. This seemed to be another indicator of her misunderstanding of the amount of expected variability for a random event and this relationship with sample size. Where above she stated that in class she learned on a coin toss, out of 300 tosses, she would expect the range to be 143 to 157; here she has a much larger range and considers that to be fair. It may be that because these sample sizes are smaller she is attributing that amount of variation to that fact and not to whether it is random or not.

For student understanding Pam said that she thought they understood that "you need to do a lot of trials." Her rationale for making that assessment was as follows:

> So like it makes me think that this was their first turn, they might have done more sets of 80 or 100 trials in between and then they ended with this one, and maybe they kept track of like one number and say it was 3, and they noticed that this had a low number of 3s rolled and this had a high number or 3s maybe all the others that they did between there fell in between there. So I do think they understand they need to do large number of trials.

Clearly Pam is reading more into the student work than is presented. There is no evidence that the students ran any more trials than the two that were shown on their poster. Pam also said that she thought the students did not understand that they need to look at the set as a whole rather than basing their evidence on just one number. Here Pam seemed to understand that the students were falsely basing their conclusion of fairness on a comparison of a single data point rather than considering the whole data set.

Poster D. In the last poster this group determined that the Dice-R-Us dice were not fair. They ran a sample of 1000 trials and gave the percents, the bar graph, and the pie graph as evidence. For question 1 the reason they gave for not recommending this company was "because 2 of the dice barely are rolled and the rest are rolled a lot." Also for the evidence they wrote, "The percent are not close only one number reached 200. The bar graph is not close almost one number takes up the pie graph." As seen in Figure 23 their screen shot displays all three representations. The percentages, which cannot be read in this figure, were: 12%, 18.6%, 19.4%, 21.4%, 18.3%, and 10.3%

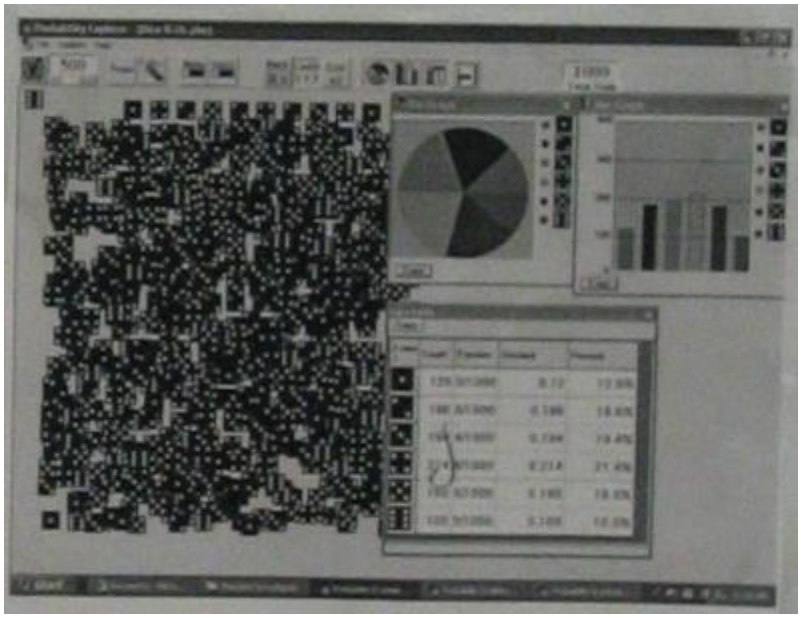


Figure 23. Poster D – Screen Shot Included on Poster

Pam's first assessment of the students' understanding was that they don't know how to analyze graphs because she did not see one number taking up most of the pie graph. "When I look at that pie graph I don't see that one number is taking it up, maybe 1 number is a little bit higher but there always has to be 1 number that's higher in any sort of experiment." Here again Pam seemed to be displaying her understanding of some expected variability within an experiment; and also the amount of variability to her seemed reasonable. This is further evidenced in the following quote:

> Ok it still seems like they're having trouble with like they say this has a very low percent and the others have a high percent. I would consider that difference, you know, to be weird, but extremely low and extremely high? Maybe they thought

because with 1000 trials that was the case. I don't know, I would say those are pretty fair die.

It is interesting that she does not consider that amount of variability to be an indication that the dice are unfair, yet this is consistent with her analysis of the other posters. Also it is unclear what she means by "maybe they thought because with 1000 trials that was the case" since the tolerance of variability should decrease as the trial size increases. It seemed Pam was trying hard to use the students' work to make judgments about their understanding of concepts, but her own understanding may have caused her to make misinterpretations. She incorrectly interpreted the students' conclusion of the dice being unfair as an error because she accepted the variability as being consistent with fair dice.

One thing that Pam thought the students did understand was how to calculate an empirical probability. "So ok, I guess they understand that in this experiment the percentages of how many times a certain number was rolled is the experimental probability for that certain number on the die." Thus Pam seemed to be able to identify that students understand experimental probability and, in her opinion, they do not understand how to measure expected variability for fair dice.

Comparing Posters. The two posters for each company displayed conflicting analysis; one concluded the dice were fair while the other concluded the dice were unfair for their company. This was intentionally designed to push preservice teachers' to

evaluate the evidence and make a claim as to whether they believed the dice were fair or not.

When comparing the two posters that investigated Calibrated Cubes, Pam said she did not understand how the two could be representations of what they did since they came to different conclusions. She said that she would put more confidence in poster A because they did so many trials. She agreed that the dice looked unfair and added "I think these people (poster A) were on the right track when they say some data are rolled in the 120-135 range as opposed to the others being rolled in the 200/204 range." She said that poster B only did 80 trials and also left out "a huge portion of their evidence."

For the two posters that investigated Dice-R-Us, Pam said that she would not be convinced by either of their arguments. "I would conclude the dice are fair but this (poster D) argument seems so opposite of what the data says." And for poster C she again assumes the students did a lot of trials and that the percentages "weren't really that extreme… they were eluding to the fact that 3 would be likely to fall in between the 10 and 25%." She said that she would give a little bit more credit to group C.

What is interesting about Pam's comparison of the posters for each company is the conclusions she makes are based on the amount of variability within the results. Her conclusion that Calibrated Cubes is unfair is based on a range of variability between about 13% and 20%. Yet for poster D, which shows a greater variability range between about 12% and 21%, she feels strongly that the dice are fair. It may be that the fact that

the distribution for poster D is symmetrical is influencing her decision that the dice are fair.

After analyzing the posters the third part of Task 5 (see Figure 10) involved the preservice teachers viewing video clips of students discussing their posters and a teacher intervention. After watching the video clip of the students' discussion Pam made two points – one regarding the student who was talking about winning his money back and the other about the student who said the percentages would be the same regardless. "I still don't really understand what that kid (James) was talking about with the gambling… what he was saying was right. That in order to get a true sense of probability then you would have to run it more times." Thus Pam has identified that he seemed to understand the law of large numbers. As for the other student, Pam said that she did not understand the law of large numbers and pointed out that the other students started clapping.

When asked how she would then direct the conversation, Pam mentioned that James was getting upset "it seemed like he was the little guy that always got picked on maybe." So while she didn't answer the question directly, she did pick up on the group dynamics and offered that as one possible reason the other students did not listen to him. She went on to talk about another student's comment that 80 was a reasonable number of trials to run because in the context of a game he said, "you're not going to be sitting there for four hours rolling dice." What she said she hadn't thought about looking at it that way, that she was thinking about determining "the absolutely correct probability, or to the

best of your knowledge, the best you could get would be to do a lot of trials." It seemed she understood that it was necessary to run a lot of trials in order to get an accurate measure of the probability and thus she did not think about the context of the board game. "So I thought that was a really good point he brought up that I hadn't even considered." Therefore this quote seemed to illustrate Pam may be developing an appreciation for the contextual constraints placed on real life use of probability and how that can have a possible affect on student understanding.

After the researcher prompted her again to explain how she would direct the conversation, she said that she would ask the class what they think the "positives and negatives" are about what James said and also take the focus off of gambling. Additionally, she expressed concern with the girl's seemingly strong belief that the sample size doesn't matter, "that face says don't mess with me… she looks like she feels pretty strongly." Thus, again, Pam seemed to be aware of the classroom dynamics and how students' strong beliefs can be a possible hindrance when trying to confront misconceptions. It was at this point that Pam suggested another approach, "what if we only did four trials? Then… your percentage in no way could correlate to a real percentage." With this question she is causing the students to see a situation where the number of trials would prevent you from getting all of the numbers. She seemed to think this would cause the students to realize the sample size does make a difference when determining the probabilities of each number.

Next, with the critique of the teaching intervention, Pam first pointed out the demeanor of the teacher, "she was pretty authoritative about it which was good because the kids were getting rowdy." She also said that she thought it was good that the teacher let the students guide her and tell her how many trials to run at a time. From this next quote Pam seemed to think this strategy of letting students make decisions was effective:

> They were telling her all different numbers and she complied and let them see exactly what they wanted to see. I think the kids were so strong willed that they were still trying to defend what they were saying before. They kept saying 'can you see' like wanting to try to prove their points. But then like, even towards the end I could've sworn I heard the kid that was saying 80 was reasonable before, I think he's the one that said try 2000. So I think she may have been able to change some people's minds.

Thus Pam noticed that one student who at first did not think a large sample size was necessary, in the end wanted to see the results after a very large number of trials was necessary. While this does not necessarily imply the student's 'mind was changed' it does indicate that he was willing to see what would happen and possibly curious to test his original belief.

Therefore a theme that emerged from Pam's analysis of the video clips is that she tended to focus on the classroom dynamics, such as how one student seemed to be 'picked on' and another student's demeanor suggested an attitude of 'don't mess with

me'. Also she focused on how the teacher's demeanor was necessary since the students were 'getting rowdy.' Pam seemed to be trying to build off the students' dynamics when considering strategies to intervene.

Pam's responses to the reflection (see Figure 11) reiterated many of the points that she made during the interview. Her answer to number one indicated a few things about her pedagogical content knowledge. She mentioned that the open-ended poster creation allowed the teacher to see what, and how, students were thinking. She also thought the class discussion was a benefit because it gave the students a chance to "teach their peers their rationale and communicate areas of disagreement." Her focus in these answers is on student understanding and ways teachers can assess that understanding. As for the drawbacks, she again said the students may not "get to the discussion of the law of large numbers on their own." Here she believes the teacher may need to hint without giving it away, however she does not go into detail about how a teacher would do this.

When asked what she would do the next day, Pam said she would review the discussion from the previous day being sure to "reiterate WHY (her caps) the class came to the conclusions they did." Thus, she would focus on their reasoning and not necessarily their conclusions. This may indicate that she values the process the students went through more so than the final conclusion. She also said she would address the mistakes in judgment and reasoning on their posters; noting that some were underdeveloped and she would want to gain a better understanding of those students'

thinking. Again, Pam was focused on how students' make sense of the task, their reasoning and thinking about the concepts.

Along these same lines Pam pointed out that, from her participation in this task, she learned that students "bring a lot of background knowledge into a lesson and give a lot of input; however it's not always accurate." She went on to mention that this fact makes teaching harder because some students may have strong beliefs that other students may be influenced by the beliefs of others and not question them. Pam is aware that using students' prior knowledge is important to understand their thinking, yet she also realizes how letting students bring that prior knowledge and beliefs into discussions can make teaching more difficult.

Finally when talking about what she has learned about her own understanding Pam felt that her understanding of the content of probability did not change, that she had a "pretty good grasp of the law of large numbers before this activity." Yet from her interpretation of student work in Task 5 indicates she has a misunderstanding of expected variability thus she seemed to not have a solid understanding of this idea. So even her involvement in this task that was purposely constructed to confront this idea, did not seem to help her improve her understanding.

As for the teaching of probability Pam replied, "I learned a LOT about teaching probability. I plan on giving students lots of opportunities to talk to me and their classmates so I can really evaluate what they know and understand. It makes a huge

difference to evaluate where your students are at before you just jump in and start teaching." Pam seemed to value discourse in the classroom as a useful tool for teachers to gain understanding of what the students initially understand. From this she can build her teaching off of their prior knowledge.

Task 6: Interpretation of Meaning of Probability. (See Figure 12) When Pam was asked to comment on each of these statements she said that the first was a "decent way to go about it" and mentioned that this was an experimental probability and defined that as: "how many times a three occurred over how many times you performed the experiment." From this response it seemed Pam has an understanding of empirical, or in her words experimental, probability. Also she expressed that using simulations to find the probability is a "pretty good" way, which may indicate she has a statistical orientation.

For the second statement in the task Pam said "that's not very good reasoning right there." She goes on to say that their guess of 10% is sort of close to 1/6 and that is "not a horrible guess." Similar to the finding in Task 5b where she thought the variance in poster D was indicative of randomness, Pam seemed to be comfortable with the amount of variance between 10% and ~17%. One other point that she made about this statement is the fact, by saying a three doesn't come up very often, "that's not taking into account that all the numbers have the same probability of occurring, assuming it's a fair die." One conclusion from this statement is that Pam seemed to recognize how the

personal orientation does not account for the fact that each number should have the same probability if the dice is fair.

Pam's comments about the third statement are interesting to note: "This is pretty good reasoning, that's finding the theoretical probability. This (points to first statement) is not really thinking about it, this is just doing an experiment but this is more thinking about it I guess." What is interesting about her comment is she seemed to believe finding the theoretical probability requires more thinking than simulating an experiment. This seemed to imply that she places more credibility in theoretical probability than in empirical probability. This is consistent with a mathematical orientation. This is also a common belief held by mathematics teachers as they tend to think deterministically (Lee, 2005) – that actually calculating the probability is 'thinking about it' because you have to know that the die has 6 sides, that is the total sample space, and rolling a 3 is one of those outcomes. And that a common belief is that when one runs simulations the technology does the 'thinking.'

Similarly with the fourth statement she replied "this is like not really thinking about it at all but basing your conjecture on other peoples' opinions." She also focused on "it depends on the die because you don't know if the die has 6 faces, some don't, and you also don't know if it's fair so the probability could be small or high." Here she expressed that this way of determining the probability is just basing your conjecture on other peoples' opinions and you can't be sure if the die is fair. Notice that with the third

statement, where it is assumed that each face has an equal chance of occurring, she did not mention this fact. Similarly with the second approach she did not mention this. This could be prompted by the "depends on the die" part of the statement, either way she is attending to that assumption.

Of all four statements Pam said that she identified with and agreed the most with statement three. "I would go with number 3. I would just make that conjecture, I would just think about that in my head and realize there are 6 sides and one of these is 3 so 1/6." This seemed to imply that Pam has a mathematical orientation towards determining probability, she identifies with the theoretical calculation. Also in this quote she mentioned that she would "think about" which goes back to her previous comments about that statement. She did go on to say that statement 1 would be a good way to confirm it but that she wouldn't do that "if it was just me." This tells us that she may have an understanding of a relationship between empirical and theoretical probability, that one can be used to confirm the other. The fact that she also referred to the empirical probability as a plausible way to confirm the theoretical probability indicates that she could also have a statistical orientation.

For the second part of Task 6 (see Figure 13) Pam discussed each of the four statements. For the first statement Pam said she wouldn't agree with it. She went on to say that for something like statistics and interpreting graphs, those are "more open for judgment, just like to how you interpret it." But when it comes to probability "there is

some interpreting in probability but there's more guidelines, more tools to help you than there are in something like statistics." Thus she doesn't agree with the idea of probability as being a personal judgment and seemed to think there are more guidelines and tools. This might imply that she does not tend to have a subjective orientation towards the meaning of probability since she thinks it is more based on using guidelines than on personal judgment.

The second statement Pam said she did agree with; "favorable would be whichever one you want, or whichever one you're observing." That was all she had to say about that definition. Pam displayed an understanding that the third statement is tied to experimental probability but that, as a definition, it has "a lot of words and it's kind of hard to understand." For the fourth statement Pam agreed that it is a definition of probability but expressed she still had trouble with the idea of randomness.

When asked which statement she would identify with she chose the second statement, "I think that would be the simplest way to explain it." This supports her having a mathematical orientation. The researcher asked a follow up question, "Can you think of any situations where number 2 wouldn't work, as far as trying to find the probability?" The reason for this question was to see if she had an understanding that this definition only works when the complete sample space is known and finite. After thinking for 30 seconds she could not think of a situation.

Finally when asked if Pam had any further comments to make about the statements she applied these ideas to a teaching context. She said that she would bring up the first subjective statement but, from a teacher's standpoint, does not think it is a good approach to take. "I don't think that's a good way to approach it because when you teach something you should kind of give students the rules and the tools they can use to solve problems." She added that from a student's perspective it would seem like the teacher doesn't know what she is doing if she said "well then I'll leave it up to you."

What is significant about this quote is that she feels uncomfortable with exclusively defining probability as 'a personal judgment' and as a teacher should include 'rules and tools' to avoid looking like she doesn't know what she is doing. This again points to wanting to give students a way to determine or calculate probability. This is also an indicator of her belief that a pedagogical approach to probability is best when using objective methods.

Task 7: Curricular Issues Revisited. (See Figure 14) Recall from Task 2 that Pam listed more statistics related concepts than specifically probability related. The concepts she listed were: experimental versus theoretical, graphs, plotting points, trend lines, basic statistics, percents, ratio, decimal, computer simulations, and experiments. In this task she said that she would want students to have a "really solid understanding of what experimental and theoretical probability was and the differences and similarities between them." It seemed here that Pam had a stronger opinion toward the inclusion of these

concepts and talked about the need to understand how the two compare. In addition she listed randomness, real-life applications (more than just coin tosses) and the law of large numbers. She seemed to have developed a somewhat different understanding of what a probability curriculum should include and added the importance of randomness as well as the law of large numbers. Notice too that she did not mention variation or variance in her list of topics. This may correspond to her lack of understanding of this concept as shown in Task 5.

Strategies for teaching probability that she listed were: computer simulations using *Probability Explorer*, *Excel*, and graphing calculators; real life applications; and explorations. Many of these strategies were more than likely related to her participation in the Teaching Math with Technology course. She stressed the importance of making the teaching of probability relate to real-world contexts, and used The Price Is Right as an example.

One other point she made about teaching probability was making sure the students are actively engaged in explorations, as opposed to teaching definitions and having the teacher do it all. "Doing an exploration as opposed to watching the teacher do stuff is I think a lot more effective, and then it allows the students to come up with rules on their own." This gives us insight into her teaching and learning philosophy in general – that students learn better through explorations than through direct instruction. This contradicts

what she implied in her response to the previous task, namely that it is the teachers' role to give students "rules and tools."

An interesting comment she made about her own learning of teaching probability is that she had a hard time thinking about this because she didn't recall having learned it in her own experiences in high school. "I don't think it was ever addressed; which is weird because I think I went through the pretty normal sequence… so it's hard for me to come up with ways to address it when it never was presented to me." Pam made an important point that echoes finding in the literature as well as the importance of including these experiences in teacher education programs. (Garfield & Ahlgren, 1988; Jones, et al., 2007) Especially since the teaching of probability has become more common in MS and HS curriculum. However it depends on the curriculum and textbooks that are used.

Task 8: Multiple Choice and Free Responses from Pre/Post Tests. For each of the four questions in Task 8a (see Figure 15) Pam did not change her answer from the pretest to the posttest. The only question she got incorrect was question 2 with an answer of 95% - 100% of those days; whereas the correct answer was d. This indicates that Pam has an "outcome approach" (Konold, 1989); according to this model a person reasons about uncertainty by predicting the outcome of a single trial. So in this example, because 70% is greater than 50% the outcome is almost certain that it will rain. Using the outcome approach often stems from deterministic thinking.

From these results we can infer that, at the beginning of this study, Pam had an understanding of how to do a simple interpretation of probability (question 1) but had difficulty with a more real world complex interval interpretation dealing with a range of values (question 2). Also this misunderstanding was not changed by her participation in the study and her course on Teaching with Technology.

In addition she had an understanding of the concept of independence (question 3) and recognized that each of the possible sequences was equally likely. This understanding was present at the beginning of the semester and remained correct at the end. With question 4 she displayed correct understanding of randomness as it applied to the situation of coin tosses, that is, outcomes with equal probability.

For the first question in Task 8b (see Figure 16) Pam answered correctly on both her pre- and post-tests. This indicated that she correctly understood that two sample spaces that have distributions that are proportional (6:4 = 60:40) are equally likely.

However with the second question, her pre-test response was c) equal chance (incorrect) and her post-test response was b) Hospital B (10 births a day). On her pre-test, Pam was not taking into account the different sample sizes. This is consistent with findings on other tasks, namely that Pam did not understand the effect of sample size before the study and course but afterwards she seemed to have a better understanding that the smaller the sample size the greater the chance of variation from an expected value occurring. However, she still has a high tolerance for variation within large samples.

Pam's responses to the third question in this task indicates that she did not understand distributions of a sample statistic from a fair coin toss experiment at the beginning but had a better, though maybe not complete, understanding at the end. On her pre-test Pam chose c as the only distribution that best represents data from a coin toss experiment. Whereas on the post-test she actually chose all the other distributions: a), b), and d). Choice a) indicates that she has a representativeness misconception. This means that she thinks a uniform distribution would be representative of a distribution of the sample statistic for number of heads. She does choose the other two correctly, choices b and d, so we can conclude that her understanding has grown somewhat.

For the third part of Task 8 (see Figure 17), on both her pre- and post-tests Pam answered question 1 that a) Group 1 is using an appropriate strategy. This is incorrect because 10 trials is not a large enough sample to conclude whether the dice are fair or unfair. However her critique of the students' reasoning improved from the pretest to the posttest.

Her pre-test response was, "Group 2's strategy doesn't allow for enough trials. I'm not sure what empirical probability is so I'm not sure how to critique Group 1." What is interesting here is that she said she didn't know how to critique group 1 yet she believed they were using an appropriate strategy. Also she seemed to think that 6 trials were not enough, but that 10 were – this shows that she may not have an understanding of the amount of variability that still exists when only 10 trials are performed.

On her post-test she critiqued both groups, "Group 1 should do more trials than only 10 but calculating empirical probability for each number is a good idea. Group 2 needs many more trials; the chance of each number appearing exactly once in only 6 is highly unlikely." Her critique of the students' reasoning improved from the pretest to the posttest because in the posttest she critiqued both groups' strategies and she did point out the fact that group 1 should do more trials. She also displays an understanding of empirical probability since she says that calculating this for each number is a good idea.

Summary of Pam's Case

Orientations. It is shown (Liu, 2005) that one can have multiple orientations towards the meaning of probability, and the findings from the eight tasks reveal that Pam seemed to display both a mathematical orientation as well as a statistical orientation towards probability. Below is a summary table of the number of instances that were coded of Pam's orientations and in which task they occurred.

Table 4 *Summary counts for Pam's orientations by task*

Tasks	Statistical	Mathematical	Subjective	Personal
Task 1	2	4	0	0
Task 2	2	2	0	0
Task 3	3	0	1	0
Task 4	1	0	0	0
Task 5	1	0	0	0
Task 6	1	4	0	0
Task 7	1	0	0	0
Task 8	0	1	0	0
TOTAL:	11	11	1	0

Some indications of Pam having a mathematical orientation include how she described randomness and probability in a theoretical way, relying on an assumption of equal probability. Also when Pam was asked questions regarding the meaning of probability she said that she identified with the responses that were described in a theoretical way. Another indication of Pam having a possible mathematical orientation was her choice of applications for probability, namely the lottery and games, which typically have known, or computable, theoretical probabilities. Also she seemed unable to provide an example of a context with an unknown distribution.

Pam tended to show a more statistical orientation in tasks that required her to assess student understanding or make a claim based on data. She seemed to always take the data or evidence into account and in the third task she stressed this importance. Additionally, Pam said that she would use simulations and experiments in her teaching of probability, which indicated that she would use a statistical approach when teaching probability. Yet she also displayed a mathematical orientation in her belief that, in order for the teacher to not look like they don't know what they are doing, teachers should use a "rules and tools" approach.

There was only one instance where Pam's response was coded as indicating a subjective orientation which occurred in the third task. Pam said that she saw probability as being a matter of choice. It should be noted that while she did make this statement, in the sixth task she said that she thought probability was more than just a personal

judgment. Thus she seemed to be aware of this orientation, but tends to think it is not adequate for a full understanding of probability.

Content Knowledge. Pam seemed to display an understanding of the following probability concepts: how to compute a theoretical probability, the relationship between theoretical and empirical probability, the law of large numbers, independence, and sample size using proportional reasoning.

However one common theme across a few tasks was Pam's possible misconception of variability and the amount of expected variability in relation to sample size in situations involving an equidistribution. She did seem to understand the main point of law of large numbers, namely that as the more trials were run the amount of variability should decrease. Yet as was seen in Task 5 Pam determined one situation, which involved a large number of trials, was fair when the variability indicated it was not. She also concluded another situation, which also involved a large number of trials, was unfair in which the variability was less than the situation she had previously determined was fair. Additionally, in the sixth task Pam mentioned that the variability between 10% and about 17% were "pretty close." It seemed that Pam may need a little more exposure to situations involving experimentation determining fairness.

Pedagogical Content Knowledge. With respect to Pam's pedagogical content knowledge of teaching probability, her knowledge of content and students as well as her knowledge of content and teaching seemed to have changed from the beginning of the

study to the end. Additionally one theme that was related to both teaching and students was her attention to classroom dynamics and how students' beliefs can impact teaching decisions. For example, she noticed that students in the video clip seemed very opinionated and sure in their assertion that percentages would be the same regardless of the sample size. This in effect caused her to notice the teacher needed to be more authoritative and also build on the students' suggestions of sample sizes to demonstrate the behavior with the technology.

Some main themes that emerged from her understanding of students and probability were that she thought probability should apply to students' lives, specifically in contexts of games and lottery. She seemed to think students tend to overestimate their chances of winning and a better understanding of probability may help them make better decisions. When analyzing students' work, Pam had difficulty assessing students' understanding and also seemed to read more information into the students' work and inferred knowledge that was not evident. Also her own misunderstanding of expected variability caused her to incorrectly assess students' as being incorrect when their reasoning was actually correct. Another finding with respect to Pam's knowledge of content and students was that she seemed to believe that students will learn more effectively when they are actively engaged in exploratory tasks.

This notion seemed to lead to some disequilibrium within her beliefs – on one hand she seemed to believe students learned best through exploration, yet she was

uncomfortable with students being given too much freedom. For Pam's knowledge of content and teaching, at the beginning of the study she seemed to want more structure to open-ended tasks. Yet at the end of the study she seemed to value students' discussion and freedom to explore unanticipated paths because that allows the teacher to gain a better understanding of the students' understanding. Additionally, it seemed different tasks elicited different strategies, in Task 7 she said she valued student exploration however in Task 6 she still wants to give the students more structure and rules to use. Another aspect in Pam's knowledge of content and teaching was the inclusion of technology as an effective strategy. She pointed out that technology allows students to focus on various representations which allow them to gain a deeper understanding of the relationships within probability.

Therefore the findings from this study indicate that Pam displayed both mathematical and statistical orientations. Her content knowledge seemed to include concepts from both statistics and probability and this knowledge also seemed to deepen as she participated in the study and course. And perhaps, by her own account, the most dramatic change occurred in her pedagogical content knowledge. She stated that she had learned a lot about effective ways to teach probability and also seemed to gain a greater confidence in her abilities to teach probability in the future.

Relationships Among Three Aspects

Possible relationships among the three aspects were looked for within each case. After describing the preservice teachers' orientations, content knowledge, and pedagogical content knowledge, the researcher wanted to know if any relationships existed between these aspects of knowledge. This analysis addresses the second research question, *what are the relationships between preservice teachers' orientations, content knowledge, and pedagogical content knowledge?*

For Pam it seemed that her evenly strong orientations as both mathematical and statistical may have impacted both her content knowledge and pedagogical content knowledge. For example, her analysis of student work in the Schoolopoly Task (Task 5) indicated her confusion about expected variability within fair dice. It may be that her mathematical orientation influenced her belief that the large amount of variability was acceptable even in large sample sizes. However she expressed confusion, so her statistical orientation may have caused her to be unsure of her analysis.

In terms of her pedagogical content knowledge, Pam seemed conflicted about strategies for teaching probability. Different tasks seemed to evoke different responses from her about how best to teach probability. Her mathematical orientation may have influenced her preference for more structured teaching and giving students rules to work from, whereas her statistical orientation may have influenced her preference for using experimentation and more open-ended explorations. And there is the possibility that she

believes a teacher *should* use experiments and explorations but she is not really comfortable with these strategies.

Her views towards teaching probability seemed to have shifted throughout the study. In the beginning her mathematical orientation may be related to her stating her being uncomfortable with her abilities to address conflicting responses. But later on, after possibly developing a more statistical orientation, she valued classroom discussion as a way for teachers to see students' thinking.

Case 2 – Yasmin

Introduction

Yasmin was in her junior year majoring in Mathematics Education at the Secondary level. She was approximately 20 years old and had completed one 200 level mathematics methods course, Introduction to Teaching Mathematics. She was enrolled in a 400 level mathematics methods course, Teaching Mathematics with Technology during the time of the study.

Yasmin's orientation towards the meaning of probability was both statistical as well as mathematical. At the beginning of the study Yasmin's content knowledge of probability consisted mostly of statistical concepts and in the final interview she was able to list more concepts specifically related to probability. As for Yasmin's pedagogical content knowledge, her knowledge of content and teaching included using experimentation and opportunities for students to explore using technology with some

teacher led instruction. Her knowledge of content and students included her ability to identify student conceptions and misconceptions as well as an awareness of the impact of group dynamics.

Task by Task Analysis

Task 1: Interpretation of Randomness and Probability. Random, to Yasmin, meant "not coming in any particular order, not having a sequence or something you can calculate, like pi is random." When explaining the concept of randomness Yasmin seemed to display a statistical orientation since her response was focused on order and sequences. Her example, the number pi, may be indication of the misconception of thinking that any non-repeating sequence is random. The number pi is an irrational number that does have a particular order just not in a repeating sequence.

The example Yasmin gave of something happening in a random way was weather. She did not elaborate on this but simply gave a one word response. This example is interesting since it is not necessarily something that has no order or sequence; it is an example of something that cannot be known or predicted with certainty. This may indicate that her knowledge of randomness also includes uncertainty. As for the meaning of random in a probability and statistics class, Yasmin responded, "I think it's pretty much the same. It means that each individual thing has an equal chance of occurring." While she said she thought the meaning was the same, she gave another characterization – that of being equiprobable, which is an indicator of having a mathematical orientation.

Another indication of Yasmin possibly having a mathematical orientation can be seen in her response to the question about the probability of a coin toss being heads. She said, “basically there’s two options, you're either gonna get heads or tails. So it's half on one half on the other.” Her interpretation is based on theoretical probability where the favorable outcome is one of two total outcomes. When asked to explain the meaning of 70% chance of rain Yasmin stated that it’s probably going to rain. She went on to explain that “it’s one out of four chances, so if they said, hypothetically speaking if they did it four days, so there's a 75% chance of rain, three out of four days would be rain.” Yasmin rounded 70% up to 75%, or she may have heard 75%, and then explained that percentage in terms of a fraction. Similar to Brad, Yasmin may be associating percent with fractions and simply not connecting it to the application of the context of coins or rain.

Yasmin’s explanation for the meaning of probability was, “it’s the likeliness that something would happen.” She went on to give the example of winning the lottery, saying it is like one in a million or two million. Here she is associating probability with a real world context and again explaining it as the fraction of favorable outcomes over total outcomes. This is a theoretical interpretation, indicating she may have a mathematical orientation.

Task 2: Curricular Issues. Yasmin said that she does think learning probability is important because it is used a lot in the media. “They (students) at least need to have an understanding of it so they can analyze the data and they're not going to be misled by

statistics because they don’t understand what they're saying or how they got the numbers.” In this quote Yasmin gave another application of probability and how it can be used in real life. Also she associated probability with analyzing data and understanding statistics. Similar to her reason for the importance of probability, the concepts Yasmin gave that should be included in teaching probability were all related to statistics. These included: spread, measures of center, scale, and probability. She elaborated on a context she saw in a cartoon concerning the presence of cockroaches in a cafeteria. In the cartoon, the concept of scale was used to illustrate a point saying that the difference between 0 cockroaches and 1 cockroach on a scale of 100 would look different than on a scale of 1000. She said she thought that was important because students may not notice how using different scales can misrepresent the data. These concepts are mainly related to statistics – where probability was listed as one of those concepts – not concepts specifically related to probability. Therefore her content knowledge of probability is limited at this point and seemed to be closely tied to statistics.

Task 3: Real-world Context and Teaching Situation. Yasmin said that she would buy the Honda, “because the consumer reports surveyed 400 people, you only surveyed 3.” Thus she based her decision on the information that had the larger sample size which indicates that she has an understanding of the effect of sample size. She elaborated further to give a more complete description of how a larger sample gives more reliable information:

> The one person you talked to that had the major problems, that's just one person out of the whole total population of thousands of people who probably own Hondas and haven't had that same problem. And it's such a small sample size that you can't use that data… if you look at a larger group you get a more accurate percentage, as you keep running the trials there's going to be less variation in the information.

Here she referred to the entire population of people who may own Hondas; she also explained that the variation between the outcomes reduces as you perform more trials and the percentage gets more accurate. This is an indication of an understanding of the law of large numbers. From her response Yasmin seemed to be displaying a statistical orientation since she is relying on the data rather than the individual responses.

For the student responses she only addressed two of the three, responses (a) and (c). Maybe she did not mention anything about response (b) because it was the same as her response, but that cannot be known for certain. For the first response where the students said they would base their decision on the Honda owner's comment, Yasmin said she would point out the conflicting information with the Consumer Reports and ask them what they thought about that. Here she gives the students an opportunity to think about the report and compare that with their conclusion rather than simply telling them that they should base their decision on the report. This may indicate that her pedagogical content knowledge includes the notion of having students arrive at conclusions through

an inquiry method as opposed to direct instruction, although she did not offer any suggestions for a further activity or task that would allow them to explore the concept more fully.

With the third comment where the students said they might as well toss a coin to decide, her answer was rather puzzling: "I would talk to them about the fairness of a coin and what if it was weighted." On paper she added, "Would they still have a 50% chance?" While she does seem to understand that the students would use a coin because they believe it is equally likely to have troubles with a Honda as it is with a Toyota, it seemed that she is trying to get the students to see how a coin toss may not be a way to determine one choice over the other if it was weighted. However it is not clear how she thinks that would be helpful in teaching the students that they need to use the information from the report that is given. Another point to make is that she did not elaborate as to the purpose of getting the students to think about the possibility of having a weighted coin as it pertains to the context of the cars decision. Unfortunately, the researcher did not ask her to elaborate on her response. Thus, interpretations on her intent are difficult.

Task 4: Experimental Context and Critique/Plan. Yasmin began by moving the candies around in the bowl to see how many different colors; she concluded there were four different colors. She also noticed that there are both watermelon and cherry types of red candy and said she would, "probably count the flavors as 4 flavors and 2 of them are red so I would say you have 2 out of 5 chance of winning a red one." Thus her first

reaction of how to make a claim was more of a theoretical approach, simply observing how many different colors, assuming they are equally distributed, and then taking the favorable divided by the total. It is interesting to note that she interchanged flavors with colors and it is unclear why she said the total was five. On one hand she said there were four different colors, when actually there are only three different colors and four different flavors. This approach is an indication of a mathematical orientation. Also her content knowledge with regards to making a claim about probability given an unknown distribution includes the assumption that it is distributed equally.

For experimentation Yasmin suggested using a repeated sampling method by pulling 10 at a time and "keep repeating that until you get an average of how many were red that you feel was fairly accurate." She also suggested organizing the data into a table. Yasmin's content knowledge could include the concept of repeated sampling yet she did not clarify how many samples she thought would be needed to get a 'fairly accurate' probability. Also she did not give any reason for suggesting samples of size 10.

When asked if she would use this type of task with high school students Yasmin said she probably would because it gives students an idea about randomness, "because they can see the different colors and it makes them think about, are there really more of one color than another." In addition to being able to see the colors she mentioned that they "could actually take samples" and count them. One other advantage to using this task that she mentioned was that the different colors make it very visual. It seemed her

pedagogical content knowledge as pertains to students includes the advantages of using hands-on activities. This type of task, in her opinion, gives the students the opportunity to do experimentation with samples and gives them a context to think about randomness.

In response to the question about how she would use this task, Yasmin said it would be a good opening task and went on to explain why it would be a good task to use. She said there were a "wide range of topics they could actually begin to explore." These topics were: spread, sampling, and randomness. She said the students could draw some samples and they could look at the spread – she did not say how big the samples should be nor how many samples should be used. For randomness she said, "it's good as far as the visual for randomness because there's not, I mean there's little clumps of red ones here and green ones there but they're not in any particular order." Here she is reinforcing the idea that random means not having a particular order. In addition to using this task with repeated sampling she said they could, "actually dump the bowl out and count all the red ones and green ones and calculate the probability if they wanted to take the time to do it." Yasmin suggested a theoretical approach was possible but added that it would be too time consuming. Her pedagogical content knowledge then seemed to include having the students perform explorations by collecting repeated samples. Additionally, she repeated the fact that the candy was colorful and she thinks that color is an advantage for helping students understand randomness.

Task 5: Schoolopoly Task and Reflection. Yasmin stated the learning objectives for this task were to understand the concept of something being weighted. She then used an example, "like if the number of births, are there more males than females, even though they say there's a 50/50 chance, um is one actually more than the other?" This is an example from a task used in the course she was taking on Teaching Mathematics with Technology where they investigated real data about the gender of live births in a collection of counties in NC. The concept of sample size or the law of large numbers was not initially mentioned as a learning objective; however the researcher asked her this same question at the end of the interview and these were mentioned as learning objectives. This may mean that after analyzing student data and seeing how students explored this task she was aware of more objectives the task could meet.

Student difficulties that she mentioned were that the idea itself "is a little abstract." She said that the students might know there is a 1 in 6 chance for each number to come up and getting them to understand if it is weighted that is not the case. "Getting them to actually understand that if it's weighted that it's not really a 1 in 6 chance because the likeliness of one of the events is going to be more than the others." It is not clear as to why she thinks this would be difficult for students to understand. It could be that she thinks that understanding a non-uniform distribution is much more difficult for students.

When asked about the pros and cons of the task, Yasmin only listed advantages. She liked the fact that it is a "real world situation, deciding whether something is fair or

not." She also said that the software "gives them several different ways to look at it, to see the different aspects of it so they can come to a conclusion so they'll be able to understand." When probed to explain what she meant by how the software gives them different ways she mentioned the table with the percents, the pie chart, and the bar chart. "… is pretty much the same information just in a different format, so instead of looking at how big the piece is you see how tall the bar is on the graph." She is pointing out how the technology allows students to see different representations of the data and how this can lead to better understanding.

The final question asked of the preservice teachers – whether they would use this task in their teaching – was actually asked of Yasmin at the end of the interview. Yasmin said she would use this task because she thought it was fun and interesting. She added, "it's neat for them to play with and there's a lot of stuff you know just by playing with it that they'll pick up and learn." Again she is displaying a preference for letting students explore and experiment as a way to learn probability. Another advantage of this task that Yasmin mentioned was that it brings out a lot of discussion and the "topics will just flow from one to the other." She then compared it to teaching Algebra, "if you teach Algebra and you teach one step equations what do you do? One step equations and then the next thing, you move to something different." Whereas this task, "really like flows, it's like piggy backed off of each other, the different concepts." It is not clear which concepts she is referring to or exactly how she believes they build off one another. When she was

asked at the end of the interview what she then thought the objectives for this task were, she listed: randomness, fairness, and the law of large numbers; these may be the concepts she is referring to here.

Poster A. These students investigated Calibrated Cubes and they determined the dice were not fair. They based their decision on two trials of sizes 100 and then 1000. The reason they gave for these sample sizes was, "if you're playing for money you're gonna want to win your money back so that's why we ran it 1000 times." The evidence they gave was based on the sample of 1000 trials because "all the numbers started to even out." As seen in Figure 24, the numbers 2, 4, and 6 occurred more than 1, 3, and 5.

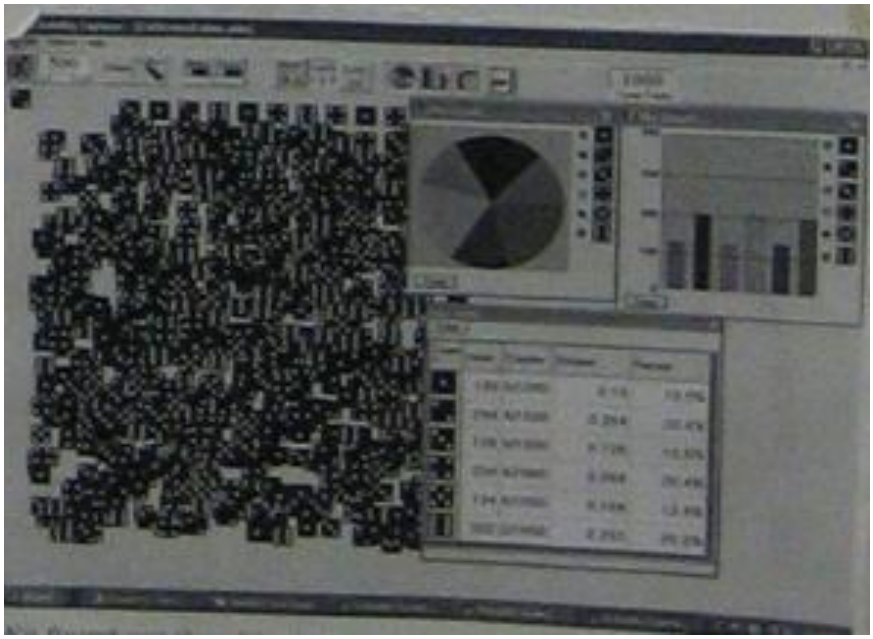


Figure 24. Poster A – Screen Shot Included on Poster

The students also wrote above this picture, “we ran it 1000 times and all the odd numbers had a low percent and even numbers had a 20+%.” In response to the third question they wrote out the percentages for each of the numbers (in order): 13%, 20.4%, 12.6%, 20.4%, 13.4%, and 20.4%. Additionally they referred to the counts, “some numbers were between 120 and 135 and other numbers are between 200 and 204.”

In Yasmin’s analysis of this poster she said, “they ran it a lot of times so they did associate like the law of large numbers and they initially saw it in the beginning that it was all over the place, but they saw the pattern as they ran it more times.” She picked up on the fact that the students were able to conclude the larger sample gave a more accurate representation of the fairness of the dice. The compelling evidence that she thought the students gave were the percentages, “because the percentages that they got, the odd ones seemed to all have lower percentages and the evens were higher, and I think that’s what they based theirs off of.” Thus her analysis of their understanding and compelling evidence was that they understood fairness, the law of large numbers, and the percentages as probabilities. She said that she thought the students had an understanding of what was going on, yet at this point she did not state whether she agreed with the students’ determination of the dice being fair.

This all seemed to indicate that, with regards to Yasmin’s pedagogical content knowledge, she has the ability to interpret student understanding based on their work. Her analysis included the interpretation of their understanding of the law of large numbers

and being able to make conclusions based on the percentages. Yet her statements seemed more declarative as opposed to evaluative.

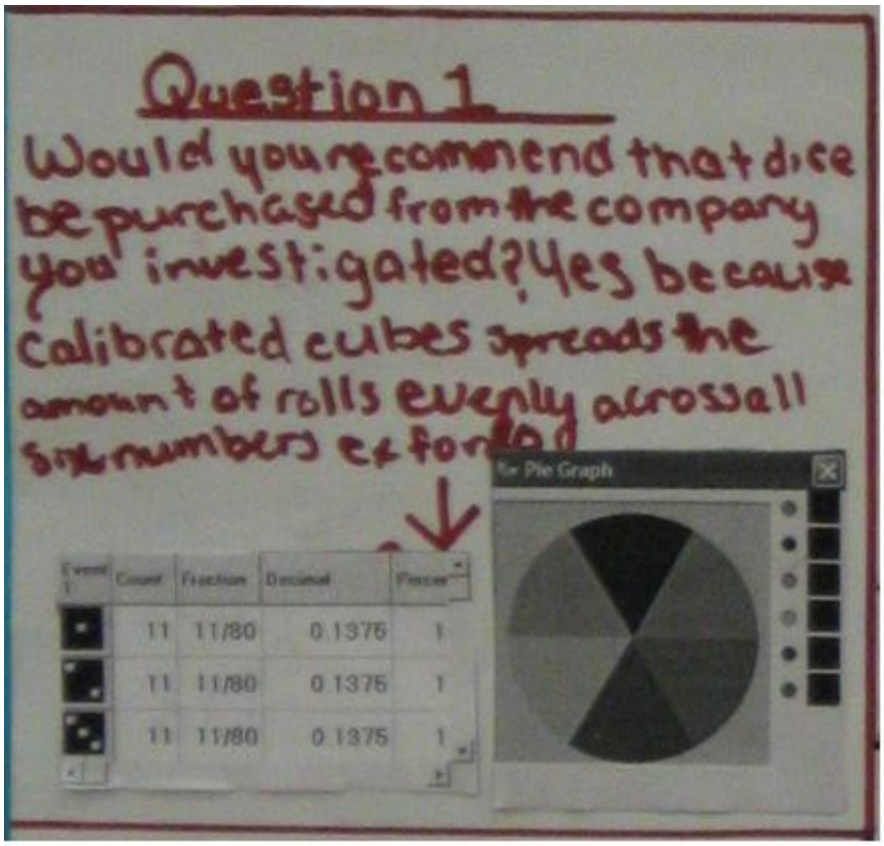


Figure 25. Poster B – Question 1 Student Answer

Poster B. With regards to the students' understanding, Yasmin said that she thought they didn't understand because they pointed out that one shade in the pie graph is a little larger. "They noticed that one piece had a bigger portion than the other but they're still saying the dice is fair so they're not really understanding that they got the numbers for that one being larger would indicate that it's not." In the students' answer to question

2, as can be seen in figure 26 below, they wrote: “basically all the shades are the same except one which is a little bigger… they’re all the same except one of them. That’s why you could see that one shade is a little bigger.” This could be interpreted that the students felt that amount of variability was acceptable for fair dice. It is not clear whether this small variability is acceptable for Yasmin or whether she expects all six numbers to show up equally.

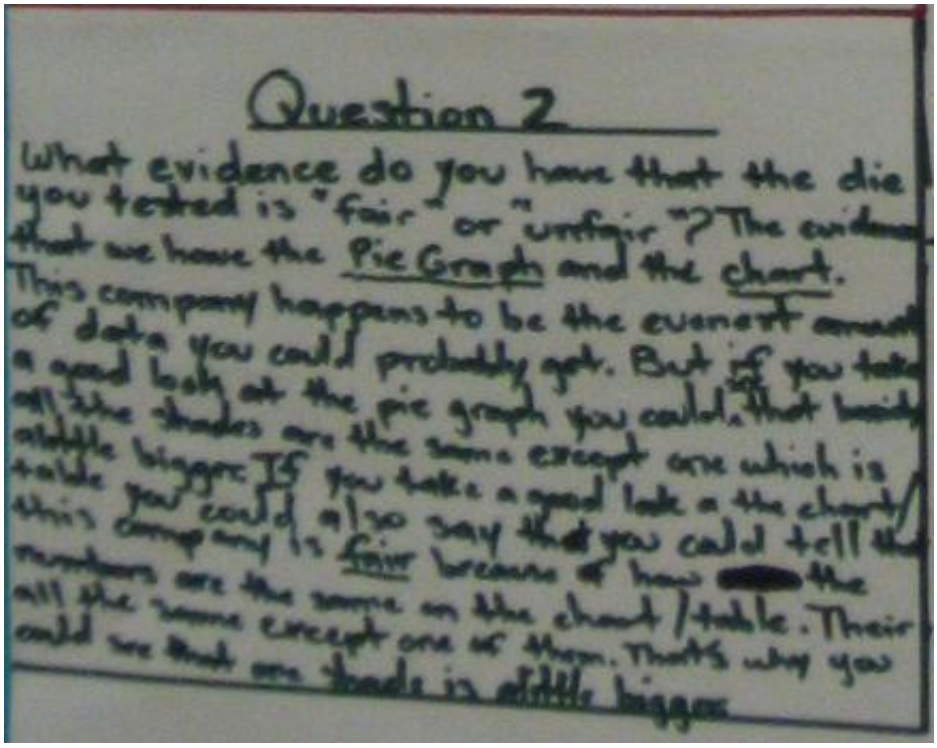


Figure 26. Poster B – Question 2 Student Answer

Additionally she noticed they only listed three of the six numbers in their table asking, “why doesn’t it give the other 6 sides of the dice?” The researcher showed Yasmin in the software how the table could be dragged to show some or all of the

information. She interpreted them displaying only three meant that they "didn't look at all the pieces." It may be that she did not think the students could have done this intentionally. Another point she brought up was that the students only did 80 trials, "they didn't really consider the more trials you ran they might get more consistent answers or more different." An interesting note with her analysis of this poster was that she did not mention the fact that the total number of rolls – assuming each one appeared 11 times – did not add up to the total number of trials. Even though she pointed out that the sample size was small, and in previous statements indicated she understood this would imply more variability, she attributed the one number being a little larger would indicate the dice were unfair. Therefore it is not clear whether she believes with this small of a sample size getting one number more times than the others would indicate the dice being unfair.

When asked to further to explain what she thought the students understood about probability, she said that they understood that for something to be fair all of them should be approximately equal. Then she added, "But I think because there was only one out of place, they didn't really know how to accommodate that … and since the others were pretty much all the same they thought it was fair." Thus, she may be thinking that one number being slightly different was not evidence of unfairness to the students because the other numbers were the same.

As for Yasmin's pedagogical content knowledge, she seemed to be able to interpret student understanding from the work they displayed on the poster. She

mentioned how the students were understanding fairness and it seemed she thought their understanding was incorrect because the one higher number would imply the dice were unfair.

Poster C. The students in this group analyzed Dice-R-Us and claimed that the dice were fair. They based their decision on the fact that the number 3 was rolled "the highest" in one trial but "the lowest" in another trial as can be seen in Figure 27. In the first example the sample size was 80 and 3 occurred 20 of those times, but so did 4. The second example was out of 100 trials and 3 occurred 10 times.

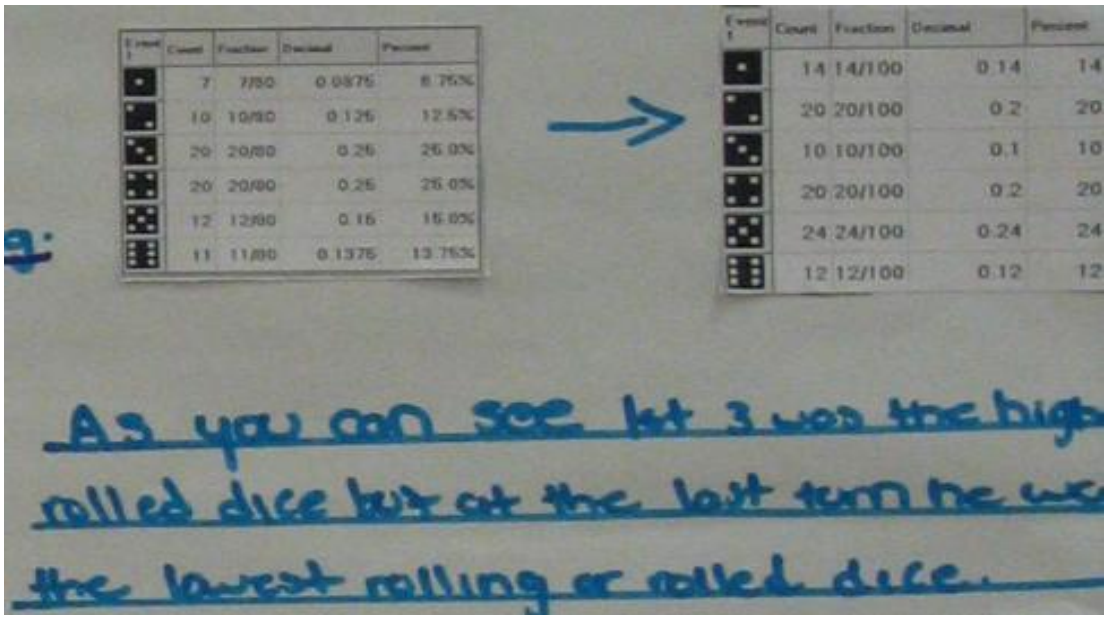


Figure 27. Poster C – Screen Shot Included on Poster

Yasmin expressed difficulty interpreting the students' understanding. "I don't even know what they were thinking; they say 'the evidence that I have to show that this is fair is posted to the right.' This is why they think it was fair but the numbers are completely different." She compared the counts in the two samples and mentioned that she thought the students were basing their idea of fair on randomness:

> I guess they think that it's fair because the numbers appear to be random… to be somewhat randomly dispersed between 10 and 24. They're not understanding the picture as a whole, they're trying to look at individual cases instead of everything and saying, well the numbers came out kind of random so they're fair.

Yasmin pointed out the fact that the students were focusing on the individual cases and not looking at the data as a whole. She seemed to correctly assess that the students were basing their determination of fairness on the individual cases seeming random. Also it seemed she is interpreting the students' response to mean that if there is no order or pattern from one sample to the next then there is fairness. As for her pedagogical content knowledge of students she was able to make claims about their understanding of fairness as being tied to randomness between samples.

Poster D. In the last poster this group determined that the Dice-R-Us dice were not fair. They ran a sample of 1000 trials and gave the percents, the bar graph, and the pie graph as evidence. For question 1 the reason they gave for not recommending this company was "because 2 of the dice barely are rolled and the rest are rolled a lot." Also

for the evidence they wrote, “the percent are not close only one number reached 200. The bar graph is not close almost one number takes up the pie graph.” As seen in Figure 28 their screen shot displays all three representations. The percentages, which cannot be read in this figure, were: 12%, 18.6%, 19.4%, 21.4%, 18.3%, and 10.3%

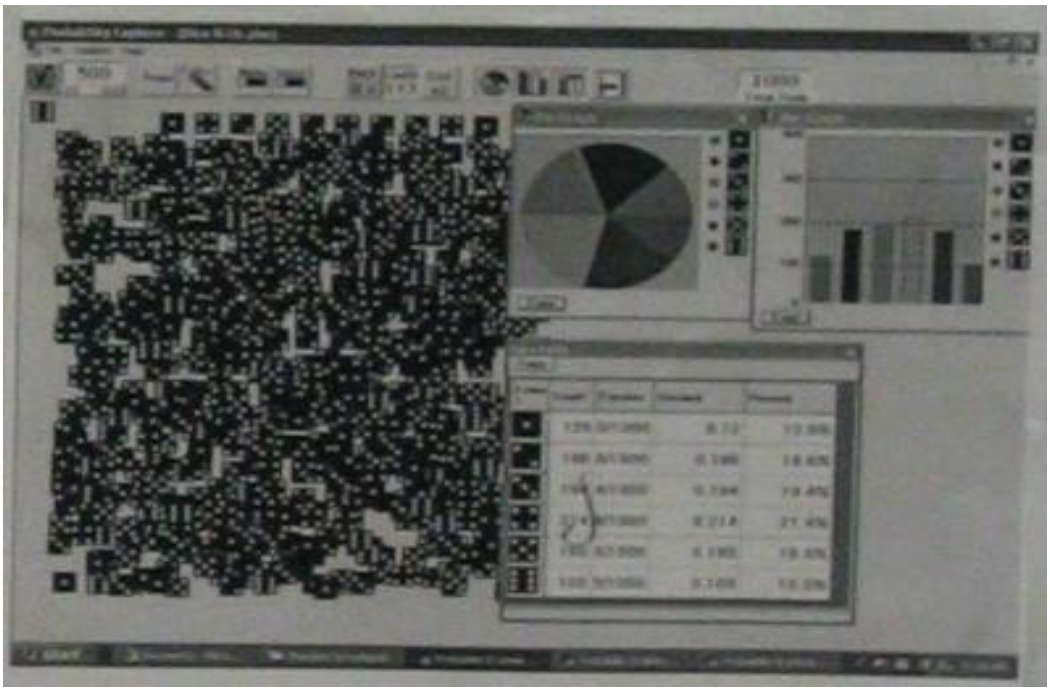


Figure 28. Poster D – Screen Shot Included on Poster

Yasmin’s analysis of the students’ understanding was that they understood how to determine if the dice was fair, and also had an understanding of percents. “I think they understood how to determine if the dice was fair… they were seeing that one number showed up more than the rest.” She also added, “so they pretty much understood the percent. I think they had a pretty good grasp of what was going on.” Interestingly when

the researcher probed her to explain why she said they understood how to determine fair she said, “I don’t think they necessarily know how to determine fair but they know at least how to determine unfair because one number showed up more than the others.” She went on to say that she would hope they would know the opposite is true, “but we don’t know for sure they know that.” Here she seemed to display a keen understanding that when analyzing student work one needs to be careful not to assume too much from what is given. I think this is referring to her prior analysis on the last poster that between the two samples there was no pattern apparent. Need to mention this as an indicator that she able to analyze students’ responses in comparison with other students’ work and her ability to consider a larger picture.

When trying to explain more about their understanding Yasmin brings up the ideas of variation and deviation:

> I’m not sure if they understand exactly why the range of these, that it’s ok for them to vary some. It’s like they want to put a definite line there and say well this is acceptable, this is what it should be, but they’re not really calculating out of 1000 trials they didn’t really calculate what the actual should be to find like the deviation around that.

This comment seemed to display a mathematical orientation because she is referring to the actual, or theoretical, probability that should occur out of 1000 trials. It is not clear whether she thinks students at this level should be able to calculate a theoretical

probability based on empirical data or if they should be able to figure out deviations. This also indicates a statistical orientation because she seemed to want the students compare the theoretical and empirical using deviations; all of which are needed for the law of large numbers.

Comparing Posters. Some interesting findings came out of the analysis of Yasmin's understanding while she was comparing the posters. She was not convinced by either poster A or B but she did say that she thought poster A "had a better sense of what was going on." Then she went on to say that she thought the dice seemed pretty fair,

> See you've got 13%, 20%, they're all pretty close together and there's not a huge difference as far as percentage for a 1000 trials. So I wouldn't, if it was one that showed like a lot more than another I would say not, but I think it's pretty fair based on what they got.

So she said that 13% and 20% was not enough variance to conclude the dice were unfair, yet when she compared poster C to D she said poster D was "on the right track, they found it unfair because of these [points to bar graph displaying a difference between 10% and 20%] and I would say the same." It may be the fact that the distribution is symmetrical in poster D and not in poster A is the reason she viewed D as more unfair, because the range between 13% and 20% is pretty close to the range between 10% and 20%. The actual percentages for these two companies, based on the weight tool in the software, were 13.33%, 20%, 13.33%, 20%, 13.33%, 20% for Calibrated Cubes (poster

A) and 12.5%, 18.75%, 18.75%, 18.75%, 18.75%, 12.5% for Dice R Us (poster D). However these values were never revealed to the participants. So the first poster has more variability, but for Yasmin the last poster is more convincing as being unfair. She seemed to have a high tolerance for variation out of 1000 trials; it is not until the percentages are about 10% apart that she believes the dice are unfair.

Another interesting finding is that at the end of the interview, after watching the discussion among the students about Calibrated Cubes and the teaching intervention, she changed her mind about poster A and decided that company was unfair. This may be a further implication that using student work in conjunction with videos of student discourse as well as teaching can be an effective combination within teacher education to impact preservice teachers' content knowledge of probability. Yasmin went back to poster A and said she thought "the first group was on the right track to find it, but they said it was unfair. Um, how do you think you would overcome something like that?" So she is initially thinking that their conclusion was wrong – that she needed to 'overcome' it. This shows she is thinking about how she would deal with something like that in her classroom.

In the next part of Task 5, after watching the class discussion, Yasmin was asked to explain what she thought the students were talking about. She pointed out that they were talking about the law of large numbers, and that one student "didn't know how to explain it but he understood it made a difference because of a previous experiment I

guess they had done." This particular student had worked on poster A where they described how their results 'evened out' from a sample of 100 trials to a sample of 1000 trials. She also said that other students didn't understand and were saying things like "it (the sample size) shouldn't matter."

With regards to this comment, "it doesn't matter how many times; the percentages will be the same regardless" Yasmin replied, "I think everybody pretty much agreed with her except the one guy. But I think they're associating it with randomness." It seemed Yasmin is thinking that the students' assumption that the percentages will be the same regardless has to do with their understanding of randomness; it is not clear as to how she is making that conclusion. Interestingly without being prompted she went on to explain how this misunderstanding could be addressed,

> I think if they would look at maybe a small case with like 6 times and then 1000 times they can see the difference. Because if you roll the dice 6 times you're not gonna get the numbers 1 through 6. But according to them you should… and so if maybe explained it to them that way and had them look at it maybe they would understand it a little bit more.

She went on to say that if you showed them how, with a small sample size, you're likely to get some numbers multiple times, some numbers not at all, and incorrectly determine the dice are unfair. Then she would have them look at the same dice 1000 times and they

can make a better determination if it's fair or not because they can see it “starting to come to a certain number.”

Thus, when analyzing classroom discourse, Yasmin seemed to be able to determine that some students showed understanding of the law of large numbers while the majority was not convinced. She also was able to offer a way to intervene in order to reach the students that didn’t understand. This shows that her pedagogical content knowledge includes using teaching demonstrations with purposeful examples to confront students’ misconceptions.

In the teaching demonstration video clip, the first thing Yasmin mentioned was that the teacher didn’t go up to 1000 or 2000, but she thought that once this happened the teaching would have been effective. In response to the question about what she thought the teacher did that was good and bad she pointed out how the teacher stopped the running of the trials at 68 and asked the students “what do you think now?” and repeated this tactic at 274. “So I think that could help them associate the randomness in with the law of large numbers and start to see that as you do more trials it should converge to the actual ratio.” By showing the students the results at different stages, Yasmin pointed out students may get a better understanding of how the law of large numbers works. In this quote she is also pointing out that this strategy can aid in their understanding of randomness.

On the effectiveness of this teaching episode Yasmin said that some of the students understood the concept but others didn't want to change their minds. "Others just wanted to believe that it wasn't gonna matter, that one was always gonna stay in the lead even though the others were really close to it, they didn't want to see that. They didn't want to accept it." Yasmin mentioned this fact in the final interview as well, that some students will be resistant to change their mind due to the group dynamics, not wanting to admit they're wrong, even when faced with a demonstration showing how sample size does impact findings and conclusions about fairness.

Yasmin mentioned that the one thing she would have done differently was to run the experiment up to 1000 trials. "… she only went up to like 275 or 300. So I think once she got up higher, with her guiding them to that, they would be able to get to the conclusion." So it seemed that she does not think the sample size of 275 or 300 would be convincing enough for the students but that 1000 trials would. This may be because a few of the posters used this large of a sample size. This also shows that Yasmin herself might not be convinced of the fairness of the dice until the sample size reaches 1000 trials. The issue of how large a sample size needs to be before being able to make a well informed conclusion about fairness is one that many students and teachers find challenging.

In Task 5d, the reflection, a benefit to this task that Yasmin identified was that it "allows students to see how the law of large numbers relates to the probability and learn these things for themselves." This statement is further evidence of her belief that

engaging students in exploratory tasks gives them the opportunity to construct their own knowledge for themselves. As for a drawback, she did not refer to the task but the video instead, “in the video they never made a clear connection to the law of large numbers.” Here she seemed to want a conclusion to the teaching episode that showed a connection to the law of large numbers.

Yasmin wrote that on the next day she would, “talk about the project and then demonstrate how the number of trials relates to the accuracy of the percentage. I would conduct a similar experiment as a class and help make these ideas concrete.” From this response it seemed she would use teacher led instruction which includes experimentation to solidify the idea of the law of large numbers. Thus, she intends to build on the students’ current knowledge by using a similar experiment as a whole class.

What Yasmin said she learned from participating in this interview was a combination between student exploration and guidance. “I learned that students often time can get on the right track, but still not see everything correctly. To teach effectively we must allow the students to figure things out, but still guide them.” This shows her pedagogical knowledge seemed to include both of these strategies and she believes a combination of the two is the best way for students to understand concepts fully.

Yasmin’s answer to the final question was not about her own understanding, but more about what she learned. With respect to content knowledge she wrote that she learned how the number of trials relates to probability. And with respect to pedagogical

content knowledge she wrote, “I learned that teaching this to students can be tricky and we need to make sure that they understand clearly.” Based on her previous responses it may be that she thinks one way to ‘make sure they understand clearly’ is to teach directly and also act as a guide. Also, based on previous responses, she may think ‘teaching this can be tricky’ is because students’ have strong beliefs about probability that may be misconceptions and difficult to confront.

Task 6: Interpretation of Meaning of Probability. When responding to the four statements Yasmin displayed a preference for the empirical statement, saying “well on this one the person actually did experiments so they calculated what the probability of rolling the dice is, so they’re actually like trying.” This idea of actually trying seemed to indicate that she may think using experimentation is a better way of figuring out probability. However when asked which statement she identified with she chose the third statement, “if someone asked me that on the street I’d probably say this, that it's just 1/6th that there's 6 sides.” From these two comments it seemed Yasmin has both statistical and mathematical orientations but favors a mathematical interpretation of probability.

For the second statement Yasmin pointed out that the person was not basing their statement on “concrete data.” Also with the third statement she mentioned that the person was assuming the dice are fair. So she seemed to have the content knowledge that in order to claim the theoretical probability there must be an assumption of equiprobable. Also her knowledge of fair is tied to theoretical probability. The last statement she also

pointed out that the five people assumed the same thing, “that would be assuming the dice are fair,” and that the others were considering that it may not be fair “so it could be a little bit higher or a little bit lower, that they didn’t know.” Her assessment of the statements indicates that she seemed to have an understanding of empirical probability, theoretical probability, and fairness.

When the researcher asked if there were any of the statements that she thought were wrong, she mentioned the second one but that she wouldn’t say the person was wrong:

> …this second one, I wouldn’t say they're wrong because if you ask somebody what they think the chances are you can't really say that their answer is wrong but it's just not based on, they didn’t come to that conclusion in a really reasonable way. So I wouldn’t really rely on what they said that its 10% because they're not really giving any evidence of where they got that from other than the three doesn’t come up that often so there's really no method behind it.

Her reason for not saying their answer is wrong is because it is that person’s opinion, it is what they think. This indicates an understanding and appreciation for a subjective or personal orientation to probability. But she did point out that she wouldn’t rely on their conclusion because there was no evidence or method. Again this may be an indication of a statistical orientation, wanting to base a claim about probability on evidence. One other comment she made about this statement was that, “the others at least put some thought

into it." It seemed that, because they didn't use any method or evidence, she thinks that they didn't put any thought into their claim. However it could be argued that because this person was basing their claim on past experiences that they were in fact "putting thought into it."

For the statements in Task 6b about the meaning of probability she started off her response by saying that she liked the last one, but did not elaborate as to why. She then went to the second statement and said that she thought it was correct. For the first statement she seemed a little unsure:

> The first one I don't know, I think it's correct. I don't really know what to say about that because it's, I mean if you're judging something and your trying to say what the probability is you're gonna relate it to things you know because you can't relate it to somebody else's knowledge. So I mean it could vary depending on you know that person's knowledge of probability or you know life experiences or whatever they're using to base that off of.

From this response it seemed that Yasmin can see how the meaning of probability could be subjective. She pointed out that the meaning could vary depending on the person's knowledge and life experiences. To conclude she said, "I think they're all true." This may indicate that she can appreciate all of the orientations to approaching probability.

While she did not say anything about the third statement, when asked which she would identify with she said, "I would, the second one is probably what I would say. I

think the second and the third." So this statement seemed to indicate her mathematical orientation as well as an appreciation for a statistical approach.

Task 7: Curricular Issues Revisited. Concepts Yasmin said were important to include in the teaching of probability were: theoretical and "experimental" probability, the law of large numbers, "being weighted", fractions, "the chance of something happening over your various alternatives", sample size, and sample space. This list of concepts is more inclusive than the suggestions she gave in the first interview. Also these concepts are related to probability, whereas the concepts she first listed were related to statistics. Thus her content knowledge of probability seemed to have grown. The concepts she listed were related to both theoretical and empirical probability; these are indications that she may have both mathematical and statistical orientations.

The first strategy that Yasmin mentioned she would use to teach probability was, "the experiments are the best way so they can actually see it at work and see how the things relate to each other and that makes a better understanding." Yasmin stated that using experiments leads to better understanding because students can notice relationships and see probability 'at work.' It is not clear what things she is referring to that she thinks are related; but this shows her pedagogical content knowledge for teaching includes using experimentation. Also she seemed to think that student understanding is impacted by their ability to actively engage and run experiments.

She elaborated on how simply "seeing" how probability works does not imply understanding and she used the video of the teaching intervention from the second interview as an example.

> I think sometimes also just seeing it, if you don't really understand it and you don't know what it means, like in the video we were watching the students saw what was happening but they didn't want to see it. They weren't really making the connections that you needed to and I think sometimes that you have to just 'ok, this is what happened' and almost force them to focus on a particular thing … especially in a big group discussion, because they don't want to admit that they're wrong.

It seemed Yasmin has an awareness of how students' strong beliefs can influence their understanding. Also she is pointing out how the classroom dynamics can influence students to be resistant to changing their ideas because they don't want to be wrong in front of their peers. Her suggestion for addressing this difficulty is for the teacher to tell them the correct interpretation. So it seemed she thinks experimentation and exploration are good strategies for students to gain understanding but at some point the teacher needs to intervene and focus the students.

The researcher asked her to say a little more about how she would use experiments. She said she liked "the ones on the computer because they're easy to set up, they don't take a lot of preparation and time." She explained that technology would be

good to use because of the speed, she also went on to say that software could be easily used by students, "especially for younger grades the *Probability Explorer* was really easy because you can just drag and drop the graphs. It didn't require a lot of programming or really computer skills to use it." In addition she pointed out more benefits of the technology, "the fact that you could run either the dice or some basic ones or create your own experiment was a good option in that." For her, having students create their own experiments was beneficial to their understanding,

> For them to create their own experiment they have to understand the concepts, what the alternatives would be and how the things would be. If dealing with something that's weighted how they can accommodate that in their trials so I think that's a good judge is having them create their own experiment give them like, 'well we have this how would you model it?'

This indicates that her pedagogical content knowledge may include the awareness that by requiring students to create their own experiments they have to coordinate many concepts and think about how a situation would be modeled in a simulation.

One strategy she said she would not use was memorization. "I wouldn't want to give them a list of terms to memorize and ask them questions because it's, the terms are similar and just reading it gets confusing without seeing it work it's hard to understand." This is another indication of her knowledge of content of students as well as teaching. She is saying she thinks students learn best by 'seeing it work', which seemed to suggest

that modeling or experimentation as opposed to having students memorize definitions are teaching strategies that she thinks are more effective for students to understand the concepts.

There were three things Yasmin said she learned about probability – empirical probability, the law of large numbers, and sample space. In addition she referred to her own experiences learning probability in middle and high school as being limited to situations that were equiprobable:

> When I was taking it in school I think the teacher may've mentioned that things could not be fair, we all kind of knew that, but we never actually worked with any scenario where it wasn't a 50/50 or an even chance of the possibilities, we were never asked anything like that, we never had to work with it. It was put out there that it's possible, but it was never really worked with or anything. So working with that kind of stuff and seeing how different technology tools can represent that… would be pretty easy to incorporate and to expand upon the basic ideas which was all I was really given in school.

Again she is stressing the importance of having students "work with" probability concepts beyond situations that are equiprobable as well as use technology to expand on the basic ideas. For students to understand fairness they need to be given scenarios involving things are not fair and opportunities to explore these situations.

Task 8: Multiple Choice and Free Response from Pre/Post Tests. For the first two questions in this task Yasmin got them correct on both her pre- and post-tests, which indicates she understands how to interpret simple probability as well as a range of probabilities. For the third and fourth questions she answered incorrectly on the pre-test. Her answer for the third question was b) THHTH and her reason for that answer was a) since the coin is fair you ought to get roughly equal numbers of heads and tails. These answers indicate that at the beginning of the course and study she may not have had an understanding of independence; also she was displaying a representativeness misconception – meaning that she estimated the likelihood of a sample based on how closely it represents the population. On her post-test she answered both of these questions correctly, thus her understanding of independence was impacted.

For Task 8b Yasmin answered correctly for the first question on both tests; understood probabilities as ratios. However on the next two questions she answered incorrectly on the pre-test and correctly on the post-test. For the second question she chose c) both hospitals are equally likely to record 80% more females. Choosing this answer represents she had the misconception of the law of small numbers. On the third question on the pretest she chose answer c) which shows a distribution spread out from 5 to 50. This indicates that on the pre-test she did not show an understanding of sampling variability, but did on the post-test. Similar to case 1 this choice actually may have been due to the placement of the graphs above the letter choice.

The first question in Task 8c Yasmin answered correctly on both her pre- and post-tests. However her responses to the second question were scored as a 1 (out of 4) on the pre-test and 3 on the post-test. For the pre-test her response was, "neither group did enough to determine if the dice was fair. They did get the concept that 1/6 would be the probability for each side." Here she was able to identify the students' misconception of sample size as well as their understanding of theoretical probability representing fairness. On the post-test she responded,

> Rolling the dice 10 times is not enough. Likewise rolling it 6 times is not either. To roll the dice 6 times and get each side once is not very likely, therefore could give bad information. At least group 1 is trying to compute the probability to 1/6.

In this response she again mentioned the students' misconception of small sample size and the theoretical probability. She also added that the likelihood of getting all 6 numbers with a sample of 6 was highly unlikely.

Summary of Yasmin's Case

Orientations. Yasmin seemed to display both mathematical and statistical orientations in her responses across the eight tasks. As the table shows below there were eight indications of a statistical orientation as well as eight indications of a mathematical orientation. There were two instances of knowledge of a subjective orientation; however these examples did not seem to indicate that Yasmin herself had a subjective orientation.

Table 5 *Summary counts for Yasmin's orientations by task*

Tasks	Statistical	Mathematical	Subjective	Personal
Task 1	1	3	0	0
Task 2	0	0	0	0
Task 3	1	0	0	0
Task 4	0	1	0	0
Task 5	1	1	0	0
Task 6	4	2	2	0
Task 7	1	1	0	0
Task 8	0	0	0	0
TOTAL:	8	8	2	0

On several occasions, Yasmin expressed her belief that using experimentation and relying on data were preferable to calculating theoretical probabilities. On Task 6 she said that the person doing the experimenting was "actually trying" and that the person basing their claim on their past experiences had no data to back up their claim. These are some of the examples of her statistical orientation. Yet there was equal evidence of her mathematical orientation – she preferred the theoretical probability statements in Task 6, she used theoretical probability in the candy task, and her responses in the first task indicated a theoretical understanding of probability.

So it seemed she may have a strong preference for objective orientations, mathematical and statistical, yet she also appreciates the subjective approach. In Task 6b she commented on the statement about probability being a degree of belief based on personal judgment and information about experiences as being correct. She said that the

meaning of probability could vary depending on the person's knowledge and life experiences.

Content Knowledge. Yasmin's content knowledge of probability in interview 1 was closely tied to statistics and she did not give probability specific concepts when asked what should be taught in a probability unit. Yet at the end of the study, her content knowledge of probability was more robust and she was able to give multiple concepts that were specifically related to probability. For example on the pre-post test, Task 8b, Yasmin was able to correctly answer the question regarding the law of large numbers on the post-test where she had that wrong on the pre-test. Also, on the question about sampling variability she changed her answer from incorrect on the pre-test to correct on the post-test.

Concepts that Yasmin seemed to display an understanding of include: theoretical probability, effect of sample size, and the law of large numbers. Some concepts that Yasmin may not understand very well include: variability, randomness, and fairness. From her responses during Task 5 she seemed to attribute a large amount of variability to randomness. However, it seemed that this understanding had improved at the end of that task. Also from Task 5 she seemed to display an understanding of fairness as each outcome having to be equal, even in empirical situations. And on the post-test she got the sampling variability question correct, which she had incorrect on the pre-test.

Pedagogical Content Knowledge. With regards to Yasmin's pedagogical content knowledge there were some common themes across the tasks: real world applications, experimentation, and using technology. On several occasions Yasmin mentioned the benefits of using real world applications and experimentation with technology. She expressed the belief that students learn best by being actively engaged and being able to design their own experiments.

These findings indicate that her knowledge of content for teaching is centered on teaching strategies that include exploration, technology, and real world applications. In addition her knowledge of content of students includes an awareness of student misconceptions, the impact of classroom dynamics, and careful analysis of student work. In Task 5 as well as 8 she was able to hypothesize students' understanding as well as misconceptions. Also her responses in Task 5 and 7 showed that she is aware of how students can be resistant to changing their conceptions in group settings. And finally when analyzing student work in Task 5 she was careful to not read too much in to their answers and to not make assumptions about their understanding.

Relationships among Three Aspects

Across all eight tasks some patterns seem to exist about the relationships among the three aspects for probability knowledge for teaching. Yasmin had even indications of both mathematical and statistical orientations, with a few indications of an appreciation for a subjective orientation. This mix may have impacted her abilities to interpret student

work in the Schoolopoly Task. For example, her strong mathematical orientation could have hindered her tolerance for variability among results within large sample sizes. Yet she was able to recognize that large sample sizes were needed, which could be an influence of her statistical orientation. Additionally, on the pre/post tests Yasmin answered several questions incorrectly on the pre-test and then correctly on the post-test. These questions were related to independence on a coin toss, the law of small numbers, and distribution of sampling variability. It could be that with her development of more of a statistical orientation, Yasmin was able to understand these concepts as presented on the post-test.

Another relationship that was found within the case of Yasmin was that her developing pedagogical content knowledge may have influenced her content knowledge. As in the Schoolopoly Task Yasmin changed her mind about the fairness of a poster after she had interpreted student understanding based on video analysis of classroom discussion and teaching. After viewing the students' discussion, Yasmin seemed to realize that the amount of variability in the first poster was too high with that large sample size to conclude the dice were fair. Thus, by participating in the analysis of the discussion as well as the teaching episode, Yasmin's content knowledge was affected.

Therefore, for Yasmin, it seemed her varied orientations impacted her content knowledge as well as pedagogical content knowledge. Her teaching strategies tended to be more aligned with statistical orientations: designing experiments, modeling, using

simulations. Also what she listed as important to teach in probability was mainly tied to statistical concepts. Yet her understanding of expected variability within experiments of large sample sizes was incomplete.

Her appreciation for the subjective approach may have impacted her pedagogical content knowledge as was shown in her comments about the classroom discussion. She pointed out the difficulty in changing students' conceptions due to their strong beliefs which could be based on past experiences. By having an appreciation for the subjective approach it seemed Yasmin was able to notice how students' conceptions of probability were based on their beliefs and past experiences. This ability to notice students possible subjective approach allowed Yasmin to realize the difficulties teachers can face when teaching situations involving uncertainty.

Case 3 – Brad

Introduction

Brad was an approximately 20 year old student in his fourth year as a mathematics education major with a focus at the secondary level. He had taken a 200 level mathematics methods course, Introduction to Teaching Mathematics. At the time of the study he was enrolled in a 400 level methods course, Teaching Mathematics with Technology. Brad mentioned that it had "been a while" since he had probability, and in the final interview he referred to his taking a 300 level statistics course in his sophomore year.

Brad represents the case of a preservice teacher that displayed a slightly more statistical orientation towards probability yet there were also instances of a mathematical and subjective orientation. In response to the tasks and his answers on the pre- and post-tests he seemed to have an understanding of several concepts within the domain of probability with the exception of randomness and sample space. In pedagogical situations Brad displayed difficulty imagining how he would direct classroom discourse that involved conflicting student reasoning. Additionally, on both his pre- and post-tests Brad scored low on analyzing student strategies.

Task by Task Analysis

Task 1: Interpretation of Randomness and Probability. Brad had difficulty explaining the meaning of random, he first remarked that you usually use random in the definition. “It just means it’s jumbled up, there’s no real order to it at all… it just happens, randomly.” The way he explained randomness could be interpreted as having a statistical orientation since he is focusing on the sequence of events not being in any order. The only example Brad could think of for something that happens in a random way was a random number generator; however, he did not elaborate on this. Also when asked if random meant something different in statistics he talked about how, in order to determine causation or correlation between variables, the data needs to be random. “Cause when it’s not random it’s all biased and you can't really get any good data out of that.” The researcher asked him to explain what he was referring to when he said “it’s not

random" and he said the data. This quote reveals that he seemed to have an understanding of random that is tied to fairness, or unbiased data. One thing to note is that he refers to 'it' as being the data and not the process of collection such as random sampling.

Brad's interpretation of the probability of getting a head on a coin toss being 50% was that "there's a half and half chance it's gonna land on heads." When asked to elaborate on that, he said he had nothing to add. This implies that he associates 50% with half. It could be inferred he understands the chance of getting a head is equal to the chance of getting a tail although he doesn't use those words. This comment also shows a hint of an understanding of equiprobability in the context of a coin toss, which may be an indication of a mathematical orientation.

When Brad was asked to explain the probability of 70% chance of rain, interpreting a probability given a real-world context, he gave a simplistic response that exhibits a limited understanding. "It means like 7 out of 10 times out of like any day of the week that it says that it's 70% it will rain." Based on this response it is difficult to say what he understands – it seemed that he means of 10 days when there is 70% chance, 7 of those days it will rain. Both responses to the coin toss and the weather indicate that he may associate 50% and 70% with a part-to-whole fraction and shows no real application of what it can imply about the context of coins or rain.

Brad defines probability as, "the chance a certain occurrence will happen, given a percentage or a fraction that the event will occur." Again Brad is associating probability

with a percent or fraction. Since he doesn't say anything about each occurrence having an equal chance, or anything about doing experiments to determine probability we cannot infer that he has a statistical or mathematical orientation from this definition. What we can determine is that his content knowledge of probability includes percentages and fractions as a way to describe or quantify chance.

Task 2: Curricular Issues. Brad stated that probability is important for students to learn. The reasons he gives are that students will need to use their knowledge of probability in their everyday lives. "Like the 70% chance of rain, you see that pretty much every day on the weather – and it's helpful for students to know probability and just to apply it to real life situations." Thus, Brad views probability as applicable to the real world. Another reason he gives, again how it is applicable to life, has to do with statistical analysis. "I know a lot of newspapers show like a graph that's way out of whack, and if they've had probability they're able to analyze it and look through the bias to look at the actual data." He seemed to value probability and see it as a tool for students to become better informed citizens. This reason, being a better informed citizen, has also been discussed in the research literature (e.g., Ben-Zvi & Garfield, 2004; Gal, 2005).

When Brad was asked which topics and concepts he thinks are important to teach probability he mentioned that it's been a long time since he had probability. The researcher prodded further to say, "when you think of probability, what are some of the big things that come to mind for you?" The only thing he could think of that was directly

related to probability was the chances that something will happen. When he was told it could be for both probability and statistics then he listed the following statistical topics: analyzing graphs, measures of center, applications to real life, sampling methods to eliminate bias. This indicates that his content knowledge related specifically to probability is lacking; yet when he thinks of statistics he can name four concepts or topics.

From Brad's answers to the second task he seemed to display a limited content knowledge of probability since all he could think of was the chances something will happen. Also he could not offer an interpretation of a statement using a percent chance beyond restating the percent as a fraction or ratio (half, 7 out of 10). However, he stated that he believed probability is an important topic to include in students' schooling. Thus, he did seem to recognize ways knowledge of probability concepts can be applied to real life.

Task 3: Real-world Context and Teaching Situation. Brad based his decision about whether to buy the Honda or Toyota on the Consumer Reports data. His reasoning, "I'd probably still buy the Honda even though the friend said that happened because there is a sample of 400 cars up here, this is a real small sample so it doesn't really apply, because this could be an outlier." Brad relied more on the data from the study than on the individual responses. He displayed an understanding that a single data point may not be representative and could be an anomaly since he pointed out that the responses could be

outliers. Thus his content knowledge of probability is tied to the idea that you cannot rely on data from a small sample which may not be representative of the larger population.

When Brad was asked to read the hypothetical student responses he explained that he would have a class discussion about each response, pointing out the strengths and weaknesses "saying there's no right or wrong answer with each of these." He went on to paraphrase and explain how each response could be valid. He seemed to be offering a hypothesis for the students' reasoning and may have been trying to rationalize why the students believed they were right. "With (a) Consumer Reports could be biased towards Toyotas and they put that out there, so you could only go on what your friends say, so (a) would have reasons to go with Toyota because they don't trust consumer reports." Thus, Brad conjectured the students may have been more inclined to trust their friends as opposed to trusting the report because it may be biased towards Toyotas.

Response (b) is similar to the reasoning he gave for choosing Honda. "For the same reasons I put on mine because Consumer Reports had more data than just your friends so since the Hondas were more reliable in consumer reports data they were better." Here he doesn't actually state that he thinks this is a correct response, but he does point out that he agrees with this statement based on the amount of data Consumer Reports reported.

For students' response (c) Brad discusses how this could be correct reasoning based on the fact that Consumer Reports reported 'somewhat fewer' mechanical

problems. Also he mentioned that there are other variables that could affect the car's performance.

> Since the consumer reports said there's not really that much of a difference between Hondas and Toyotas and it's pretty much chance that anything happens with your car, could be the driver was driving it too hard and broke it or there's lots of other variables in there that could affect why the particular car messed up than it just being a Honda problem or a Toyota problem.

Based on his analysis of all three student responses, Brad indicates that he is not clear on how to interpret different conclusions from the data. However, he seemed to be carefully considering how the students were thinking; thus his pedagogical content knowledge may include being able to hypothesize student's thinking based on their responses. Because Brad did not decide on a 'correct' answer and didn't point out any misconceptions in reasoning, the researcher asked him what he thought would be the point of this problem, what would he want the students to take away from the problem. "The end thing I would want them to take away is that just make sure to look at all of the variables involved… so with that you can make an educated decision on what things you want to go with." Thus, Brad would want them to be able to make an educated decision by taking into account things like possible bias in the data and other variables that could affect the outcome. His summary is that all three responses could be interpreted as correct and that may indicate that he has an understanding and appreciation for both a statistical and subjective

orientation. Based on his response, he would base his decision on the larger data set and seemed to have an understanding that a small sample size would not be representative of the population. Also when he is presented with a subjective response he can perceive how that can be a valid interpretation of probability. He did not perceive the students saying 'you may as well toss a coin to decide,' as an erroneous response that basically ignores both the study findings and the individual responses.

Task 4: Experimental Context and Critique/Plan. Brad reasoned that in order to make a claim about the probability of selecting a red candy you would have to know the initial distribution.

> I guess you'd have to know how many different flavors there are in a pack and what are the chances that each of them get in there by the company, if they try and make it even or not. Then take into account how many are in there and how many are red and divide them and that's the chance.

So Brad seemed to display knowledge of how to calculate a theoretical probability. Yet he seemed to be unfamiliar with how to find the probability when you are not given the distribution or the theoretical probability of each color.

When asked if he could think of a way to find the probability if he was not given that information, his first response was that you could count them all "but that would take way too long." Then he suggested using a sampling method of drawing 30 at a time and seeing how many red ones you get every time you draw 30. Even though he is able to

offer a sampling method, he does not state what he would do with this data to estimate a probability, and thus does not display a complete understanding of how to approach the task using sampling techniques. He also offers that "you could just take into account each flavor and then assume each one has an equal chance of being drawn." Here he is assuming each flavor is equally distributed, has the same probability; which in this particular case would be an incorrect assumption. This assumption implies he has a mathematical orientation towards probability because he is assuming equiprobability. Also his first response, that you could count them all, is indicative of a mathematical orientation where you would know the entire sample space. From these responses he seemed more comfortable at applying theoretical techniques for stating a probability and unsure how to use an empirical approach.

With regard to his pedagogical content knowledge, when asked if he would use this type of task and how, he replied that he would use it. The reason he gave was that there is no simple answer; the task requires students to think about different ways they could come up with the probability of getting a red candy. When asked how the students would do that his answer was rather vague: "by just thinking through the problem, coming up with different solutions and testing out each of those solutions." He did not go into detail about *how* the students would come up with different solutions, or *how* they would test out those solutions.

His response to how he would use the task in his teaching was that he would do it "pretty much the same way that it's done here, just give them a real simple exploration." He also suggested that he would give them more information, "this is how many are in a total pack of Jolly Ranchers, I guess that's about it, maybe give them a little more information than just have the bowl and have fun." One thing to note is that he said earlier this task was good because it doesn't have a simple answer; yet he does not seem to make the connection that giving students the distribution of the candies may reduce the task to a simple answer. He talked about having the students do a simple exploration, but he did not go into detail about how the students would set up the exploration, whether they would use actual candy or some sort of simulation, how he would assess their understanding, etc. Thus he seemed to value use of an exploration but seemed more comfortable in structuring it – possibly leading to devaluing experimental/statistical approaches and privileging a theoretical/mathematical approach.

Task 5: Schoolopoly Task and Reflection. For Task 5 Brad participated in a group with Jeff and Sam, cases 3 and 4 respectively. Thus Brad's answers to questions may have been influenced by the other participants' comments. Where applicable the context is noted as well as the comments that Brad is responding to from the other preservice teachers. However the work he wrote down for part b, analyzing student work, and his reflection were done independently and analyzed accordingly.

In responding to the analysis of the Task Brad mentioned a few weaknesses. He said that for this task the students are not given enough direction, "they might look at one thing too much and analyze the wrong part that they're not supposed to." When prompted to explain what he meant by "the wrong part," Brad elaborated that they may focus on the percentages being the same and that "maybe a few more trials of 80 or something, because if they just do it once then the data just could be skewed at one point… if they don't get that then it's wrong." This answer reveals his belief that the task needs to be more guided to keep students focused on the objective. Also it is unclear why he thinks focusing on the percentages being the same would be the wrong thing. It may be that he thought when they don't run enough trials the percentages may be the same, causing students to make the wrong conclusion. Thus, similar to his comments in Task 4, Brad seemed to think that exploratory tasks need to be more structured and he may be uncomfortable with students possibly going off in what he considers to be a wrong direction.

For the next two questions pertaining to the learning objectives and student difficulties Brad did not offer a comment. Jeff and Sam mentioned some objectives might be the law of large numbers, the difference between empirical and theoretical probability, and problem solving skills through analysis of data. Sam pointed out that students may have difficulty if they were not given enough guidance from the teacher they may not

know where to start. These points are only briefly mentioned since they may have impacted Brad's statements later on.

When asked if this is a task they would use, all three said that it was. When prompted to explain why they thought it would be a good task to use, Brad was the only one that responded. He said, "especially because the uh, the weighted is locked up and they can't see it so they're actually testing, like, they're actually doing an actual experiment." He said that this task was good because the students were "actually doing an actual experiment" because the weight tool was locked (students could not see what the theoretical probabilities were for each number on the die). This seemed to indicate that Brad values the fact that students are required to perform an experiment to collect data that can be analyzed.

Therefore for the task analysis Brad at first thought the task was too open-ended and may cause students to focus on the "wrong part." Yet he also liked the task because it did not allow students to look up the weights of the dice, requiring them to do exploratory data analysis. Additionally, he seemed to recognize the fact that students would need to run trials that required large sample sizes, indicating he may have an understanding of the effect of sample size on determining fairness.

Poster A. The students that made this poster analyzed Calibrated Cubes and determined that the dice were unfair. The evidence they gave included percentages, a table, a pie graph, and a bar chart (see Figure 29).

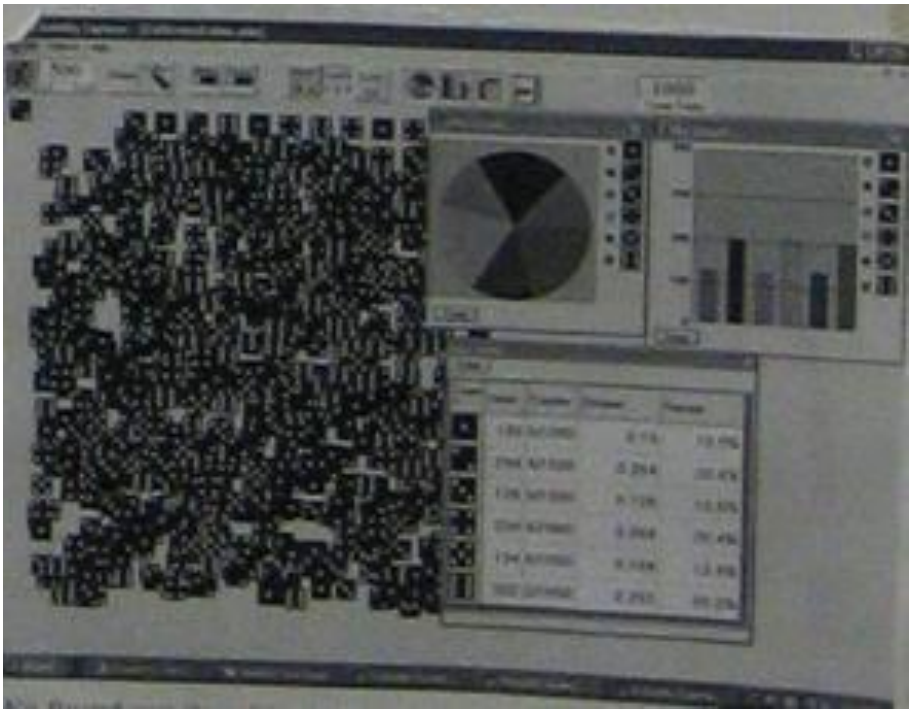


Figure 29. Poster A – Screen Shot Included on Poster

Brad started this discussion by pointing out the fact that the students were "definitely looking at the law of large numbers, because they used 1000 trials and so all their data evened out." He also mentioned that the students seemed to conclude that the even numbers came up more than the odd numbers but the students didn't really know any of the terminology. He referred to their answer to the first question and said that it was confusing as to what they really understood (see Figure 30).

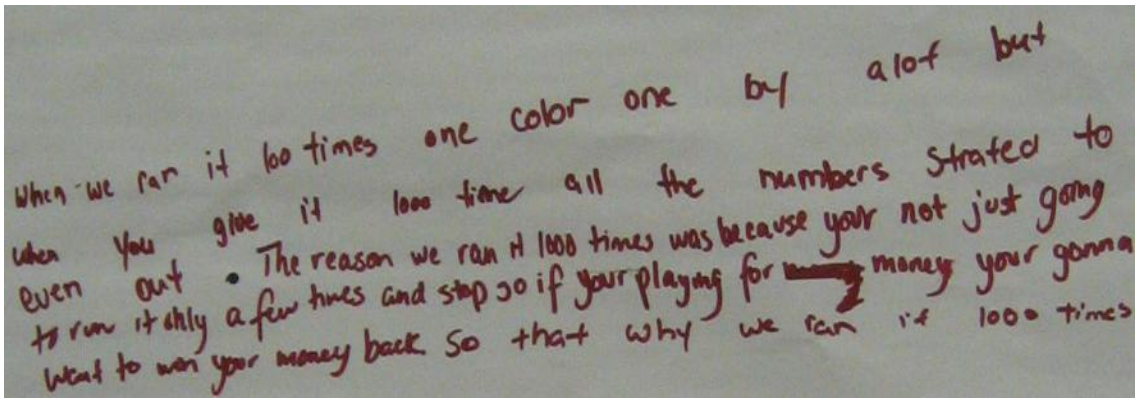


Figure 30. Poster A – Student Writing

What Brad wrote on his handout pertaining to the students' understanding and their compelling evidence was basically the same as what he said. He wrote that the students understood the law of large numbers, did not have terminology to explain their reasoning, and they felt the amount of trials were significant enough to draw conclusions. Thus, Brad seemed to have an understanding of how the law of large numbers played a role in the students' reasoning and determining that the dice were unfair. It is interesting that he mentioned the fact that they did not have the correct terminology; this may imply he had the assumption that they should have.

Poster B. This poster showed another pair of students' analysis of the same company. However, this pair of students concluded the dice were fair and showed a table displaying only the first three numbers. They also included the pie graph as evidence of the company being fair (see Figure 31).

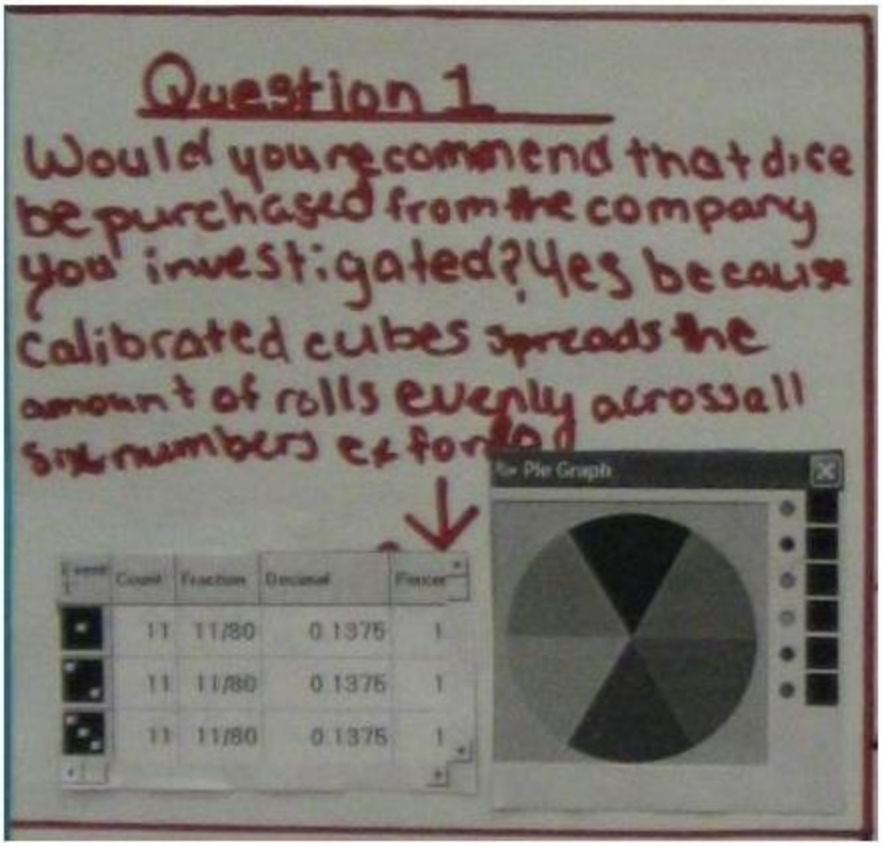


Figure 31. Poster B – Question 1 Student Answer

What Brad wrote down in response to this poster was that the students only looked for the same chance of each die, instead of the "big picture." Also he pointed out that they felt if the numbers were basically the same then that was enough evidence. In his writing he did not mention the fact that the sample size was only 80 trials, however during the discussion he referred to the fact that if all numbers occurred 11 times the sum would not add up to 80.

The students also wrote that one of the sections in the pie graph was bigger than the others and Brad said “it seems they got five of them are 11 out of 80 and then one of them different so you'd think it’d be 25 if all the others are 11 out of 80… that’s a pretty big difference out of 80 trials.” Thus, he seemed to understand that if five numbers were 11 then one number would have to be 25, much more than the others, and within 80 trials there may not be sufficient evidence to conclude fairness.

The preservice teachers also pointed out the fact that the students only used a small portion of the poster (see Figure 32).

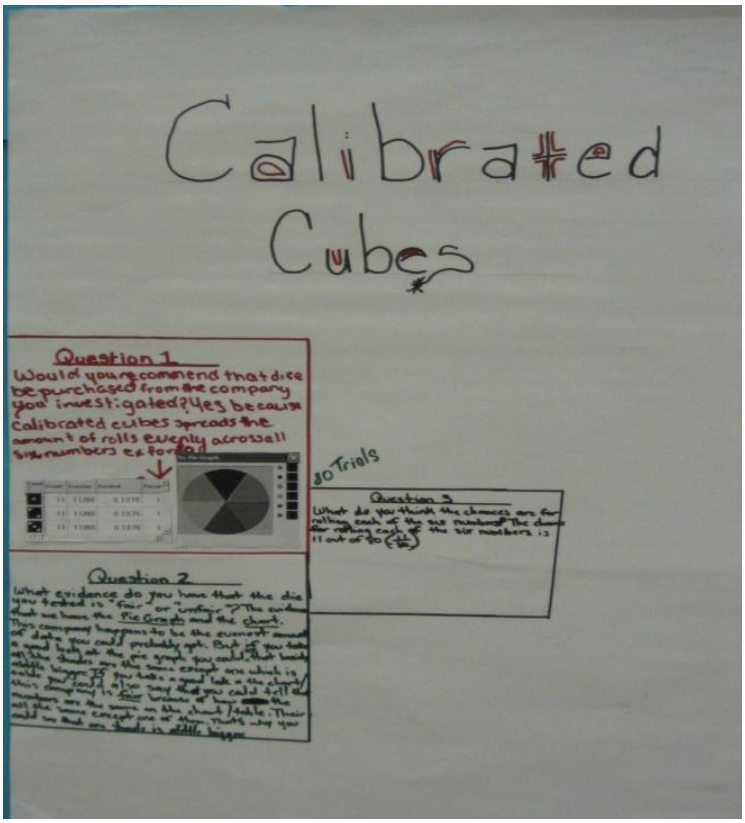


Figure 32. Poster B

What is interesting is that Brad then said, “if it was a little bit bigger… it probably would’ve showed the rest of the table.” This may imply that he thinks the reason the students only showed part of the table was due to space and not their intentional choice to only show the numbers that were exactly the same.

Poster C. This poster displayed analysis of the company Dice-R-Us. The pair of students determined the dice were fair based on comparing results from two trials of sizes 80 and 100. They based their conclusion on the results of the number three which occurred the most times (20/80) in one trial and the least times (10/100) in the other (see Figure 33).

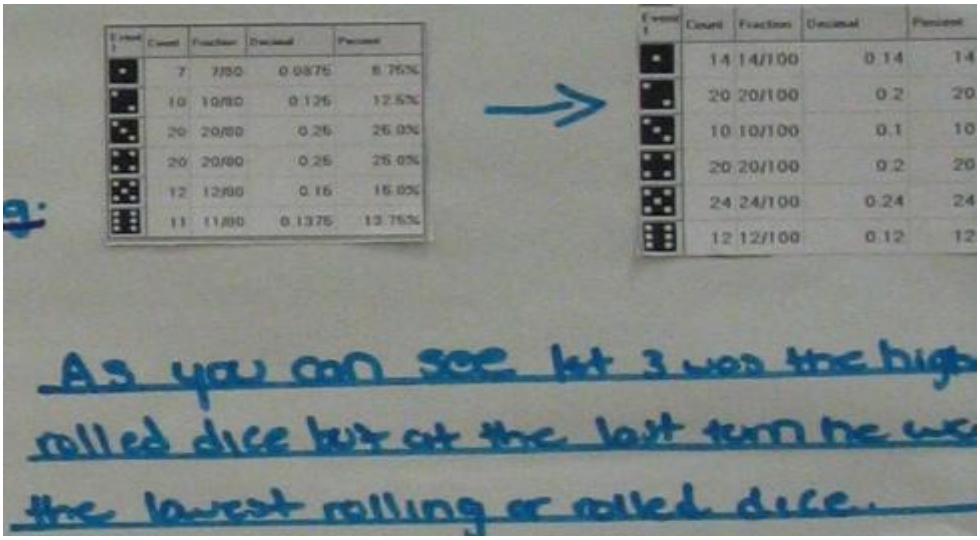


Figure 33. Poster C – Screen Shot Included on Poster

In the discussion of this poster Sam pointed out how the percentages differed from one trial to the other, “what is this 7 out of 80? that’s less than 10% went up to 14%.

Both of these are 20 out of 80, 25% and this went down to 10%?" Brad replied to this comment that he thought that was what the students were looking at to see if it was fair. Brad also pointed out that just using the two examples was "definitely not enough" to show whether the dice were fair.

Another aspect of the students' work that Brad pointed out was the fact that the students only focused on the number of times the number three occurred in each trial and they did not look at the percentages. "They didn't talk about the difference in the percentages they just looked at it as the number of trials compared to the rest of them." Brad noted what they did not attend to, the percentages, but rather that the students were comparing frequencies of 3's across the data sets without regard to the fact that they had changed the number of trials they ran. Brad seemed to pick up on this fact when he pointed out that they changed the number of trials; but he went on to say "I don't know if that really matters… because it's a lot easier to compare the same amount of trials." This indicates that he may believe that in order to compare two data sets the sample size must be the same.

On the handout Brad wrote that the students understood different trials can give different results but they only focused on one aspect. He also wrote that they felt the randomness between the trials was compelling enough. Thus, Brad's analysis of the students' work showed that he seemed to understand the need to look at the whole data set and not just compare one number to another. He also seemed unsure as to whether

two data sets have to be the same size in order to compare them, or at least he thought it is easier to compare them when they are the same size. Also he seemed to display an understanding of the students' over reliance on randomness (i.e., no pattern from one set of trials to the next) to determine fairness.

Poster D. The pair of students that made this poster also looked at Dice-R-Us and they concluded the dice were unfair. The evidence they gave was mainly focused on the percentages from an experiment of 1000 trials. Their pie graph and bar chart were also mentioned as evidence that the dice were unfair.

During the discussion Brad pointed out that the students used 1000 trials and said they understood the law of large numbers. He added that they *seemed* to be getting at the idea, "they don't really explain the law of large numbers they just said only one of them reached 200." Brad wrote down that the students used the law of large numbers and he also wrote that they did not "come out and say it." This statement is similar to what he said in his analysis of poster A where he pointed out that the students did not have the correct terminology. Further on in the discussion Brad seemed to hint at the idea of using a sampling method. He said that the students were concluding "each of the chances being exactly what they got, instead of having more trials and figuring out the average percentage of each one." This could imply that Brad thinks the best way to determine the probabilities is to run multiple trials and then take the average of those percentages.

He also wrote that the students did not understand what % means. This is because they mixed percents and fractions (see Figure 34).

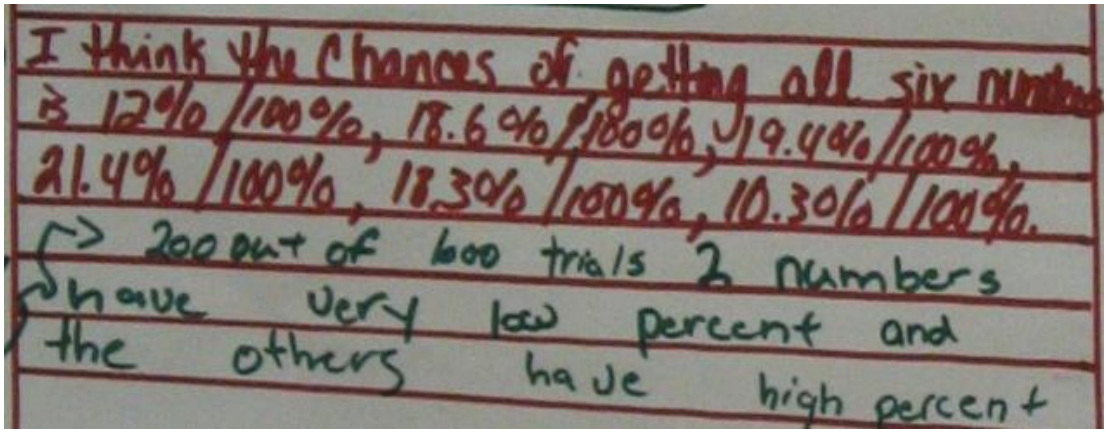


Figure 34. Poster D – Question 3 Student Answer

Thus Brad's interpretation of the students' understanding was that they understood the law of large numbers, but did not know how to explain their reasoning. Brad was also able to identify what they might not understand which was how to express percentages without using fractions. Another finding was that Brad seemed to have an understanding of using a sampling method to determine the percentages of each number rolled.

Comparing Posters C & D. For the comparison of the posters the preservice teachers started with looking at the Dice-R-Us company. They were asked if they were convinced by either of the posters and Brad said he would go with poster D because of "the amount of variation from 10% to 21% and the 1000 trials." This comment seemed to show that Brad has an understanding of the law of large numbers, namely the fact that

this amount of variability is too high to claim the die have an equidistribution with that large of a sample size.

Comparing Posters A & B. For the comparison of the Calibrated Cubes posters the discussion was mainly between Sam and Jeff. However, Brad did refer to the number of trials being too low. He also said that their [poster B] findings were not believable, "they drew the conclusion that each chance of six is 11 out of 80, I don't think I would believe that." From this statement it seemed that he is displaying an understanding of the effect of sample size, namely that for this small of a sample the likelihood of these results is not believable.

As for Brad's pedagogical content knowledge with regards to analyzing students' work Brad seemed to be able to draw some conclusions of student understanding. His comment about the sample size implies that he may have thought the students' incorrectly concluded that each dice would be rolled the same number of times. And while he did not find that believable, he seemed to imply that the students did.

For the analysis of the class discussion video clip in Task 5c to direct the conversation Brad suggested having the students calculate the theoretical probability. "…go in to what would a fair percentage be… and how close their data was actually to that number and how skewed." It seemed that by having them calculate the theoretical probability he might think that would illustrate how varied the empirical results would be from the expected. It seemed also that he may be thinking the students were comparing

percentages within the data set (i.e. the percent of one will be the same as the percent of two, etc.) as opposed to comparing percentages of different sample sizes. This comment may also indicate that he is trying to combine mathematical and statistical approaches. By computing a theoretical probability this would bring explicit attention to what students should expect from an equidistribution. Thus they could use the theoretical probability as a base comparison in making their judgments form the data.

For the video clip of the teaching demonstration, Brad commented that in regards to the effectiveness, the students were allowed to guide the teacher on the number of trials to run. He pointed out the fact that one student asked her to run it six times and when they ran it again with different results the student said, "oh so I guess my theory wasn't right." Thus Brad seemed to be saying that the strategy of letting students suggest strategies could be effective for allowing them to confront misconceptions. This was the only contribution Brad made during the discussion about this video clip.

For his reflection on the Schoolopoly interview (see Figure 11), some benefits of the task that he listed were: it allows the students to explore and come up with their own conjectures, it gives the students a different look at the standard dice problem – how it can apply to a real life situation. The drawback that he saw in this task was that it lacks direction, "they were a little too free to explore and they were coming up with some off the wall stuff that was not really aimed at what the task was trying to get at." During the final interview he was asked what he meant by 'off the wall stuff' and he mentioned that

some of the things they wrote on their posters he never would have thought of. For example, focusing on one number and when that came up the most and the least the students concluded it was fair (Poster C); and the poster where they only showed 3 of the 6 results (Poster B). This could mean that "off the wall" to him means strategies that he views as not aligned with the objective, or something he can not anticipate. This could be yet another indication of his uneasiness with giving students too much freedom in an exploration. It also seemed that he may not see how giving students the freedom to explore could be a way for the teacher to see what students understand about a topic. He did not seem to view this task as a way to assess students' understanding, but rather simply as a way to teach them a particular concept.

Brad's answer for what he would plan for the next day was to compare the theoretical probability to the empirical findings, which is similar to how he responded to the video clip of the students' discussion when he said he would ask them to calculate the theoretical probability. This is an indicator of him using both a mathematical and statistical orientation by using the theoretical distribution to make judgments about the empirical findings. He also listed several questions he would ask, "I would ask questions like: How many times should this number come up? If it doesn't happen that way, why is that so? Is this difference significant enough?" These are good questions to get students to think about how the theoretical compares to the empirical. However, the question remains as to how Brad would explain why you'd get different results and what a

'significant difference' would be. His comment may be indication that he did not think all the students were understanding the law of large numbers, and differences between empirical and theoretical, which he perceived as the objective of this lesson (mentioned by Sam and Jeff in the first part of this task). Thus, he was planning on an intervention to focus the students on the objective based on his understanding of these concepts, and not necessarily what the students had displayed on their work on the posters and in the class discussion.

Brad focused on the students and also teaching as two areas that he learned from participating in this task. As for students he replied, "I learned that most students do not have the vocabulary to explain their thinking thoroughly. Also students are very creative and really stand by their solutions." By giving Brad the opportunity to analyze student work and view a class discussion, he learned that students can be creative when they don't have the vocabulary. From the class discussion he saw that students really stand by their solutions and this caught his attention. "I also took away a different approach to teaching a new subject. I think this task could be very useful in the classroom and has a lot of potential and could see myself using this task in my classroom." Also by seeing this task 'in action' Brad saw the benefits and now has a task idea that he can apply to his future teaching.

As for Brad's assessment of his own understanding of probability and the teaching of probability was that he, "learned that I am not as sharp on probability as I

used to be, I forgotten some of the terms and this task reinforced that there is not just one solution to any problem." Thus, this task reinforced his comments from the first interview, namely the cars task, that there is not one right answer.

Task 6: Interpretation of Meaning of Probability. Brad's comments on the statements in Task 6a indicate that Brad seemed to have an understanding of empirical probability and the law of large numbers. "… (#1) is finding more of the empirical probability of coming up with a three. So out of the however many times he's getting close to the theoretical probability and there's so many rolls so it's gotta be pretty accurate." Brad demonstrates understanding of empirical probability and its relationship to theoretical probability. This seemed to be another indication of the value he places on theoretical probability as being accurate, since this statement does not mention anything about theoretical probability. He also seemed to understand that the larger the sample size the less variability there will be in one's estimate of a probability.

In response to the second statement Brad said, "This guy just doesn't play enough I don't think, because 3 should come up evenly unless he's not playing with a fair die." Brad may believe that it is not bad to make an estimate based on experience but, depending on one's exposure to the context, they may not have had an opportunity to gather enough data. It seemed that Brad is aware that one can have a subjective approach to probability and offers two reasons why the student may be getting the estimate they make – not having enough experience and not playing with a fair dice. Also he seemed to

have an image of what the answer should be if it is a fair die and that 10% is too far off, thus his rationale about sample size.

Brad's comment on the third statement indicates an understanding that a characteristic of theoretical probability is an assumed equiprobable distribution. "The third person came up with the theoretical probability pretty accurately. But he didn't say anything about it being fair." Here he focuses on the fact that the person didn't explicitly say anything about the die being fair, thus he seemed to have an understanding of the importance of such an assumption.

He also mentions using experimentation to determine fairness as a viable strategy. This finding comes from his response to the last statement, "And then the last person … probably should've done a little more experimenting trying to find an unfair dice, or fair vs. unfair to try and figure out how many times three came up." It is a possibility that through his participation in the previous Schoolopoly Task Brad now recognizes that using experimentation to determine fairness may be a better strategy than just surveying a few people's opinions. Additionally, by suggesting using experimentation, this may indicate Brad values a statistical approach.

Brad chose the third statement as the one he would identify with, "um the third one, I'd just say there are 6 different sides as long as they're all weighted evenly and it's a fair die it should be 1/6." When asked if he could think of reasons why that approach might not work in general he seemed confused about the question. The researcher

reworded the question, “can you think of other situations besides dice that we may run into problems with this approach?” To this he responded,

> It probably should (work) for most cases but depending on the fairness of whatever you’re using like for spinners they could eventually like the middle thing starts wearing out so they don’t spin around as well. Um and for coins they could be weighted differently to where heads or tails could come up more often.

One thing to notice from his response is what is missing – he does not state why the different weights make it hard to do a theoretical approach nor did he mention the need to collect data to make an estimate. Additionally, when he was prompted to think of something besides dice he mentioned spinners and coins which are other random number generators. He also says that it depends on the fairness which indicates that he may understand that theoretical probability has an assumed equiprobability. Thus, overall he seemed to value a mathematical approach to estimating probability; however, his responses to other tasks seem to indicate that he can see how and why the other approaches would be needed.

Brad’s comments of the four statements in Task 6b illustrated his understanding of probability as being applicable to more than experiments:

> the top one, that could be applied to just running your experiments but I don’t think they would be able to generalize an answer with their definition of

> probability and they wouldn't be able to apply it to other situations which I think is something that probability should be able to do.

Here he expresses how he thinks that the first definition of probability is too limited and his belief that probability should be applicable to other situations.

One interesting comment that Brad made about the second definition is that it depends on what the person means as favorable. "… that one just depends on what you mean by favorable. Or what the person means by favorable." When prodded further to explain what he meant Brad discussed the complex probability of getting one through four on a die.

> So I guess well it just depends on their definition of favorable outcomes like if they wanted a one through four to come up whenever rolling a dice that would be the favorable outcomes over the total outcomes. That would be out of the six that one through four would come up.

This quote provides insight into Brad's content knowledge of theoretical probability. It seemed that he considers favorable outcomes as being either one outcome or multiple outcomes. Also it is unclear whether he understands how to calculate the probability of getting one through four on a six sided die.

With the third statement Brad recognized that it referred to the law of large numbers, "… they're referring here to stabilizing as the law of large numbers so as the number of trials gets larger it gets closer to the theoretical probability." From this quote it

seemed Brad is assuming the hypothetical number is the same as the number computed using a theoretical approach, which is another indication that he may have a mathematical orientation.

For the last statement – probability is the measured randomness of an occurrence – Brad said that he did not know how one could measure randomness. This implies that he may not think of probability as a tool that we can use to measure randomness. “I guess it’s just the measure of, what they think is going to happen in a random case could be what they’re thinking, I'm not sure.” Here he seemed to be thinking that probability can predict or measure what is going to happen in a random instance. This seemed to be very similar to the idea of measuring the randomness of an occurrence.

For the second part of this task Brad chose the second definition, reiterating that it depends on how one defines favorable outcomes, and that it will give you the theoretical probability. He also commented that the third definition regarding empirical probability could be used to find the hypothetical number for the probability of something happening.

The researcher asked him if he could think of situations when one [definition] would be better than the other. The purpose for this follow up question was to see if he recognized that in order to use the first statement one would need to know the sample space; also each outcome would need to be equally likely. “I guess the third one would be better for testing out stuff and… the first one would be the best to find the actual outcome

of a specific die or coin or spinner or whatever you're using." He then went on to say that the second one would be better for applying to a "general case… I think 2 would be more favorable for generalizing like for a coin there's 2 sides so if you want heads to come up that's your favorable outcome so it'd be 1 out of 2." This seemed to indicate that he is showing an appreciation for how experiments can be a valuable tool for seeing how an object behaves but that a theoretical approach is more generalizable.

Another important finding that his responses are indicating is that, although he stated in Task 2 (and reiterated in Task 5) that probability is important because it applies to real life, all of his examples seem to be about common devices used in schools. This could be due to lack of exposure to situations where probability is applied to real life, that most of his experiences have been with common devices such as coins, dice, and spinners.

Task 7: Curricular Issues Revisited. Topics that Brad listed he would want students to learn were: randomness, sample size, law of large numbers, the difference between empirical and theoretical probability, and applications to studies and society. He also mentioned that he would want his students to be able to design their own experiments. With each concept he listed he made reference to students being able to apply their knowledge in real life situations, "I would definitely want them to learn randomness so they can eliminate bias, so they can look at studies that have come up and look and see if they are random enough and to be able to apply to situations", "…how all

this can apply to studies and other things that have been found within society", and "so they can actually, if they want to they can design their own experiments and stuff like that." Here his content knowledge of probability is closely tied to its usefulness in statistics and his pedagogical content with regards to teaching includes applications and experiments. This all implies that Brad sees a need for the students to have this knowledge to be able to apply it to their life. These comments also indicate a statistical orientation since he would want students to develop an understanding of probability that is related to empirical data.

Strategies for teaching probability that Brad listed were using technology and having students design and create their own experiments. One reason he gave for wanting to use technology was the speed that technology affords and then he explained advantages of using different technologies, namely *Probability Explorer* and *Excel.* "… with the random number generator in *Excel* you can show them all the different numbers that can be created and how to apply that to an experiment that they have or whatever they need to do." Specifically with *Probability Explorer* he explained how that could be used to demonstrate the law of large numbers, "Using *Probability Explorer* you can show the law of large numbers and how 10 trials is, or 100 trials is better than- the more trials you have the more accurate your predictions are gonna be." He goes on to explain how using *Probability Explorer* can aid in teaching students the difference between empirical and theoretical probability. "…explaining the differences between what they found and

then what the theoretical, what they should've found, and if it's not, should we change the theoretical probability? If it's not an accurate prediction of what the data should be." This last quote indicates that Brad may have the belief that the theoretical probability is what the 'correct' probability is and that the empirical should match that probability. Yet he also gives the option of 'changing the theoretical probability'. This may indicate that he has a belief that the theoretical probability should be 'an accurate prediction of the data' and that it is just a model for the probability that is really unknown to us.

As for students designing and creating their own experiments, Brad pointed out that he would want them to build on their own interests. "…Let them come up with something they want to find out about and, then let them go in and use all the stuff that they learned about earlier and apply it to, and find out whatever their hypothesis was." Again Brad is thinking about how probability can be used in data analysis, experiments, and hypothesis testing, which indicates that he may have an understanding of the link between probability and statistics. This statement also indicates Brad's knowledge of content and students includes building on students' interests.

When Brad was asked what he had learned about the teaching and learning of probability he mentioned how to teach it, specifically with reference to technology. "I definitely know how to teach it better… and *Probability Explorer* demonstrates the law of large numbers and randomness and you can use that to explain a lot of things in probability." From this quote Brad displayed understanding of ways he could use the

technology within his teaching as well as what concepts these technologies would be useful to assist in teaching. When Brad reflected on what he learned about his own knowledge he said it was surprising how much he had forgotten since he took his statistics class his sophomore year. He went on to say, "I learned how to use the technology to be able to demonstrate the topics… with the technology it'll make it a lot easier and demonstrate it a lot better than just telling the class." This seemed to reiterate his belief that using technology to teach probability will be more effective than merely stating rules of probability.

Task 8: Multiple Choice and Free Responses from Pre/Post Tests. For the multiple choice questions Brad gave correct answers on all four questions in both his pre and post tests. What this shows is that Brad understands how to interpret probabilities as both an estimate and a range. He also understands independence as it applies to a sequence of coin tosses.

For the second part of Task 8, Brad got the first and last questions correct on both his pre and post tests. Thus he understands that the probability of selecting a blue marble is the same when the proportions are equal, as in the first question. Also he is able to recognize a reasonable range for the distribution of a sample statistic. For the second question he was incorrect on his pre test with the response that the two hospitals are equally likely. On his post test he gave the correct answer; this indicates his understanding of how expected variability depends on sample size.

For the third part of Task 8 on both his pre and post tests Brad chose answer a) that Group 1 is using an appropriate strategy. Brad's pretest critique was, "Group 1 may get skewed data but has a better grasp on fairness then group 2. There is no effect from 1 roll to the next so each side has an equal chance of coming up." With this response he does not include any of the strengths and weaknesses or any reference to sample size. His reasoning that each roll is independent and thus has an equal chance indicates that, at the time of the pretest, he did not see the effect of sample size on determining fairness. Whereas his posttest critique was, "Group 1 is trying to eliminate chance and find the fairness of the die while group 2's die could come out to be unfair when it is actually fair because it could be all 1s or any other combination." Here he gives a strength of group 1 as trying to find the fairness, however he does not mention the sample sizes as being too small nor does he mention comparing empirical to theoretical probabilities.

In terms of Brad's pedagogical content knowledge, it is interesting that in many of the previous tasks as well as earlier parts of Task 8, namely the multiple choice questions; he displays understanding of the effects of sample size and the law of large numbers, except in the hospital task question in Task 8b on the pretest. Yet when he is asked to critique student strategies he does not pick up on a weakness of a lack of data in both groups' strategies. So it seemed that Brad may not be able to fluently apply his knowledge of probability to pedagogical situations.

Summary of Brad's Case

Orientations. With respect to his orientations, Brad tended to exhibit more of a mathematical orientation but also, to a lesser degree, a statistical orientation. There was one indication of a subjective orientation. The table below displays the number of instances that were coded as mathematical, statistical, subjective, and personal in these tasks.

Table 6 *Summary counts for Brad's orientations by task*

Tasks	Statistical	Mathematical	Subjective	Personal
Task 1	1	1	0	0
Task 2	0	0	0	0
Task 3	1	0	1	0
Task 4	1	3	0	0
Task 5	1	2	0	0
Task 6	1	5	1	0
Task 7	2	0	0	0
Task 8	0	0	0	0
TOTAL:	7	11	2	0

Throughout the tasks Brad seemed to value computing theoretical probabilities over using an empirical approach. Comments such as "theoretical is more accurate" and "probability is more than just experiments" show that he may believe probability should be seen from a more theoretical perspective. In both the candy task and the Schoolopoly Task he stressed the importance of comparing theoretical probability with empirical probability. The most evidence of a mathematical orientation occurred in the sixth task when he was asked to interpret several statements about calculating probability and the

meaning of probability. In both the first and second parts of the task he identified with the theoretical statement. Additionally in the statement that was related to empirical probability in Task 6, he interpreted it as being related to theoretical probability.

There seemed to be evidence of a statistical orientation in all of the tasks except 2 and 8 (which didn't offer evidence of any of the orientations). Probably the most evidence that Brad may have a statistical orientation is in his comments about how he thinks probability should be taught and why he thinks probability is important for students to learn. He said that he would want students to be able to design their own experiments and when he teaches probability in the future he wants to include experimentation. So although his knowledge seemed to be closely tied to a mathematical orientation, it may be that through his experiences in this study as well as the course on technology he is embracing more of a statistical orientation when applied to teaching.

As for a subjective orientation it is unclear as to whether this is an orientation that Brad displays or whether he is simply aware that this can be an orientation one takes. In task three when he was asked to comment on the three student responses he was able to hypothesize why this approach would be useful.

Content Knowledge. Some of the concepts that Brad seemed to understand include: the law of large numbers, how to find empirical and theoretical probability, and the effects of sample size (Brad did not understand this concept on the pre-test, but did on the post-test in Task 8b). Brad's understanding of the meaning of probability seemed to

be limited to percentages and fractions with little ability to apply probability to a context. Also Brad's knowledge of probability seemed to be closely tied to what would he considered traditional school experiences, namely, using dice, coins, and spinners. His understanding of randomness was tied to experiments not having any bias. He also stated that he had a hard time explaining randomness without using random in the definition. In addition, with the seventh task when he was asked to list topics he felt should be included in a probability course, he mentioned that he thought randomness should 'definitely' be included and the reason he gave was to eliminate bias.

Another interesting note with respect to Brad's content knowledge of probability is his understanding of probability in the context of weather. In the first task when Brad was asked to explain what 70% chance of rain means his response seemed to indicate that he did not understand the meaning other than as a fraction of 7 out of 10. However in Task 8a when he was given a choice of ranges of probabilities he correctly chose the range that represented the most accurate prediction. Implications for this finding are that the different tasks may elicit different responses. Students may make better sense of a probability prediction when given a range of choices as opposed to having to explain the meaning. While the multiple choice question did not allow for the option of interpreting the statement merely as a fraction, that was in the previous question on the test.

Pedagogical Content Knowledge. Brad's understanding of pedagogical concepts within the teaching and learning of probability, by his own account as well as evidenced

in his responses to the tasks, seemed to deepen from the beginning of the study to the end. For example, when he was asked to list concepts to teach in probability in the first interview (Task 2) he listed statistical concepts and mentioned chance as the only thing specific to probability. Yet in the final interview (Task 7) the topics he listed included randomness, sample size, law of large numbers, the difference between empirical and theoretical probability, and applications to studies and society. Clearly his curricular knowledge of what to teach has expanded.

As for his knowledge of content and students, Brad seemed to place importance on making probability applicable to students' lives. He also mentioned using students' interests and allowing them to design their own experiments. In Task 5, when he analyzed the students' work, he seemed to be able to determine students' possible understanding and also indications of students' misunderstandings. And in Task 7 he mentioned that he would use technology and experiments when teaching probability because the students can "see" the ideas and understand the concepts more deeply than if they were to use more traditional paper and pencil tasks. This shows that Brad may believe students, specifically within the learning of probability, gain more knowledge by being actively engaged with technology and experimentation.

Brad's knowledge of content for teaching was highly tied to using technology and performing experiments where students would be able to collect and analyze data. He indicated a preference for exploratory tasks and experiences for students that required

them to make their own decisions; however he seemed uncomfortable giving students too much freedom and wanted tasks to be more structured. Brad also seemed to want to give his students a different experience learning probability than he had. For example, in Task 7 when he talked about what he had learned, he reflected on how much he perceived he had forgotten in just two years and then added that with technology "it'll make it a lot easier and demonstrate it a lot better."

Relationships Among Three Aspects

Within the case of Brad there seemed to be evidence of ways his orientations may have impacted both his content knowledge and pedagogical content knowledge. For example, with the cars task, he seemed to have difficulty responding to conflicting interpretations of data which may be due to his more mathematical orientation. Also with the candies task, Brad was quite vague in his explanation of how he would use this task and he tended to devalue empirical probability and privilege the theoretical. While he said he liked that the task was open ended, when explaining how he would use it, he said he would give the students information that would reduce the task to calculating theoretical probabilities.

Therefore, it seemed that Brad may have originally had a somewhat limited knowledge of probability that was mainly mathematically oriented. But, after participating in tasks such as making claims given data and analyzing student work with an exploratory task, Brad seemed to display more of a statistical approach. Additionally,

there were several examples of Brad combining the mathematical and statistical orientations where he would compare the theoretical with the empirical. It seemed that these two approaches would be influential in his teaching as evidenced by his preference for using experiments and technology to teach students probability that would be applicable to their lives.

Case 4 – Jeff

Introduction

Jeff was a traditional aged junior majoring in Mathematics Education at the Secondary level. At the time of the study he has completed one 200 level math methods course, Introduction to Teaching Mathematics. He was also enrolled in a 400 level math methods course of how to teach mathematics with technology.

Jeff displayed a strong statistical orientation with a few instances of a mathematical orientation. His content knowledge seemed to expand from the beginning of the study to the end. In the first interview he could only list a few topics to include in teaching and in the last interview he was able to list quite a few more. His pedagogical content knowledge also seemed to deepen quite a bit after participating in the study and the methods course. At the beginning, his strategies for teaching probability included mainly the traditional games of chance, whereas at the end he listed experimentation and using technology as strategies he would use.

Task by Task Analysis

Task 1: Interpretation of Randomness and Probability. In response to the first question, similar to the other cases, Jeff mentioned that defining randomness was difficult to put into words. He explained that he thought randomness was like "blindly picking something out of a pile", and "not repetitive." He also stated that the sample you are choosing from has to be fair in order to be a random selection; he referenced the bowl of candies, to be used in a later task, which was on the table. These responses indicated that Jeff has both mathematical and statistical orientations due to the fact that he included fairness which applies to theoretical probability as well as 'not repetitive' and sampling which both apply to simulations and experiments.

In addition to being fair and not repetitive, Jeff also mentioned that random could mean being unexpected. An example he gave of this was how it is used in everyday conversation, "well that's random." This may be an example of how everyday language use of a word can cause conflicting interpretations in a statistical sense. Thus, Jeff's content knowledge of randomness includes ideas such as fair, non repetitive, and unexpected. As for an example of the meaning of random in a probability and statistics class Jeff said, "What's the probability of having one out of you know taking a random guess." This response is rather vague, yet it could indicate that his content knowledge regarding randomness as it applies to probability and statistics includes thinking of

random as guessing. Also the phrase, “one out of,” may be an indication that he was trying to apply an equiprobable distribution.

When asked to explain the meaning of ‘the probability of getting a head on a coin toss is 50%’ Jeff first stated that the sequence of tosses would not be alternating, that they are independent of each other: “it doesn't mean HTHT back and forth.” He also used the law of large numbers to explain “over a long period of time, if you kept on flipping heads and tails, if you did it hundreds and hundreds of times, you would get, you would keep on getting closer and closer to 50%.” He also mentioned that if the sample is small there is a good chance that it’s not going to be 50%. These responses indicate he seemed to understand the concept of the law of large numbers – both in terms of small and large sample sizes. This also is an example of a statistical orientation since he is using experimentation and empirical probability to describe the meaning of a probability of 50% in a coin toss. This is a common frequentist interpretation of probability. Jeff went on to describe the theoretical probability that you have two choices and the chance of getting 1 is 50%, which aligns with a mathematical orientation. Thus, he was able to offer two ways of describing the meaning of 50% probability that indicate a comprehensive understanding of how probabilities are often used and interpreted.

For the meaning of 70% chance of rain Jeff said that it means “you have a better chance of it raining than it won't.” He added that it is not certain to rain, “there’s a good chance but it's not inevitable.” Here he is not describing the meaning in terms of an

outcome approach (Konold, 1989) because he does not think 70% means that it will inevitably rain. He does not define it with a sampling method, such as 7 out of 10 days, but more in general as an indication of the possibility of rain. It seemed Jeff understands this concept as being a greater chance of occurrence but still realizes it does not mean rain will definitely occur. He seemed to have the ability to see probability on a continuum of more or less likely, as opposed to discrete (i.e., that anything greater than 50% means the event is certain to happen).

Jeff defined probability as "the chances of things happening… if you do one thing over and over again a certain number of times, it's how many times you would get that result," and "what are the odds that you'll get one of them out of others." The way he described the meaning of probability indicates a statistical approach by referring to doing things over and over and then counting the results of a particular outcome. He also referred to probability as being the odds of something occurring; here he seemed to equate odds with probability. He also referred to odds many times throughout the other tasks, thus it seemed his content knowledge of probability is tied to the notion of odds. It is not clear that he understands the technical difference in definitions between odds and probability. This also could be because odds and probability tend to be used interchangeably in everyday language.

Task 2: Curricular Issues. For Jeff one reason he thought probability was important for students to learn is because they will need to use probability in everyday

life and that it is something that is commonly used. Examples of how students would use probability in everyday life included the context of games of chance and the weather. He mentioned the same "70% chance of rain" as was used in the prior task and playing cards in a casino. This indicates that Jeff did think it is important for students to learn probability as well as recognized ways in which probability applies in real life. Also his content knowledge includes situations involving chance like games and weather.

Concepts that Jeff listed as important to include in teaching were using coins and dice as well as being able to conduct surveys and using M&Ms for sampling (none of which are concepts but more about strategies and useful devices). The reason he gave for using coins and dice was because, "when I think of probability that's the first thing I come back to because it's used so often," and,

> I think it's important to realize that even though you could get heads three times flipping a coin it doesn't necessarily mean you're definitely gonna get tails the next time but it just means that overall in a long period of time it will end up hitting towards 50%.

From this quote it seemed Jeff has an understanding of independence, and although he did not use that term, seemed to believe it is an important concept for students to learn. Also this indicates he may have a statistical orientation because he refers to 'over a long period of time' and the empirical probability as well as the sequence of events being independent.

As for doing surveys he gives the example of wanting to know the probability that a person you meet is going to be left handed (he is left handed). Another example he gives is sampling from a bag of M&M's. Again he stated that this was something that is used often when teaching probability: "just because, I mean they'd always do that, they'd pull out a bag of M&M's and they'd give them to each person and see what the odds are of having like a brown M&M." He went on to say that he felt this was important for students to learn, "I think that's kind of good 'cause students kind of, it gives them a better grasp of probability." Another reason he listed for using this type of activity was that it lets them do a "hands on activity" and can "better interest them in the subject." This response indicates more of his pedagogical content knowledge as it relates to students; he places importance on using activities that are hands on, as well as recognizes the importance of considering students' interests. For all three things – coins, dice, and M&Ms – he mentioned he would include these because they are used so often. He seemed to be drawing on his prior schooling experiences.

One interesting thing to note about Jeff and this task is that at various times throughout he expressed frustration that he couldn't think of more topics:

> I know there's like so many important ones that I just can't think of right now… woo, it is hard to think of something off the top of my head… I'm sure there's something else I can think of… let me think, aw come on, I'm sure there's

something else… I know there's so many concepts that are important for students to know.

The reason he gave for this difficulty is because, "it's been a while since I've done probability." Jeff's response to this task indicates that teaching strategies and commonly used devices are the most prominent way he could describe important topics and concepts. However, embedded within his comments are evidences that support a belief that collecting data from real experiments (e.g., flipping coins, conducting surveys, choosing an M&M) are what he considered to be most valuable.

Task 3: Real-world Context and Teaching Situation. Jeff answered the first question by stating that he would chose a Honda and he dismissed the one statement from the Honda owner saying, "I don't think you should base your opinion on that one person… because that's just one person, you never know, it could just be bad luck." He went on to say that of 400 cars there have been fewer problems with Hondas so, "in the long run Hondas have fewer problems than Toyotas." He gave a rationale for his decision that is based on the larger sample size and pointed out the long run behavior. Furthermore he explained why basing your decision on the statements of your friends is not a good strategy. This rationale seemed to indicate that Jeff has a statistical orientation because he was basing his decision on the data, rather than a subjective orientation which would be based on personal statements.

When addressing the student responses, Jeff stated that he would try to get the students to consider the survey which has "more than a large enough sample size." For the first response he said that he would ask the students if they could base their decision on "anything else that's concrete"; and he would explain to them that just because one person had that problem doesn't mean they will. From this he explained how he would "go into well, what can we use to find out the possibility of me having those same problems… what information do we know that can help us?" Thus he would steer the conversation towards basing your decision on the data of the 400 cars.

When asked how he would address response (c) he mentioned that the students were "stuck between two things – the report and based on the people you know." He mentioned that he would do similar things, but did not elaborate on what these things might be or how he would be able to help students understand the difference between the data in the report versus the personal comments. He talked about how there's always a chance, "but what would give you the greater chance of not having those problems? So that way I'll be trying to lean towards the main idea." He also stated that the main idea of the task was to get the students to see that, "one bad experience doesn't mean you're bound to have the same problem." It seemed he is aware there is not a causal relationship, pointing out that one person's problem is not directly related to another's possibility of a problem. His other comments also suggest he considers the main point of the task is for students to learn to value a larger set of data for making decisions.

Some things to note about how he answered the second part of this task are that he talked about what his role as the teacher was in directing the conversation, yet he did not mention anything about having the students do some activity to aid in their understanding of the law of large numbers. Nor did he offer ways to help students understand the concept of why it is not a good strategy to base your decision off of one person's experiences. He did say he would try to steer the conversation towards getting the students to realize that the larger sample size is more of an indicator for which type of car to buy. At the end of the first interview he referred back to this task as a good example of how important probability and statistics is for students to learn; because they will need to use that knowledge in everyday life to make informed decisions.

Task 4: Experimental Context and Critique/Plan. How Jeff would make a claim about the probability of a selecting a red candy was at first theoretical. He said that he would count how many different colors to get a vague idea of the probability. He did point out however that the distribution of the colors would have to be even, he did not simply assume they were. "You would just evenly distribute it out, like if there was five of them there's a 20% chance of getting either one of them." He went on to explain that in order to get more specific results he would separate them all out and count up each color and divide by the total. This strategy involved calculating the theoretical probability of selecting a red candy out of the entire sample space. From this response it seemed Jeff is applying a mathematical orientation because he first stated he would use a way to

compute a theoretical probability as opposed to doing experimentation. It is interesting that this task evoked such a response when he seemed to have been emphasizing the importance of experimentation in prior responses. One possible reason for his switch in approach is that he was presented with a countable context – although there were many candies in the bowl, it was possible to count the candies and make an estimate of a probability based on a computational technique.

Following up with how he could use experimentation, Jeff used a teaching scenario for doing sampling, "you could go around and ask just kinda bring it around the classroom and ask each student to pull one out and then see what the odds are that they pull out a different kind." He described how he would have the students sample the data and also mentioned that he would need a large sample size, "if it was a large enough survey of people I would write down on the board and then say out of each divided by the total number that gives us the odds of pulling that one out." So he seemed to have the content knowledge of how to use experimentation to make a claim using a large sample size. Note again he used the word 'odds' interchangeably with probability.

Jeff said that this was a task that he would use with high school students and his original suggestion was to use a counting technique by going around the class having each student select a candy and then adding that up to compute the empirical probability. But as he elaborated he said that a problem with the task is, "that there's so many different colors you'd have to do it so many times in order to find somewhat accurate

results." What he was referring to was that this strategy would be too time consuming in a classroom setting. However he did offer a solution where "you could just try to maybe separate them into groups, to give each one of them a pile and see how many they added up to." Here his pedagogical content knowledge seemed to include the awareness of the difficulty of using experimentation and sampling within the time frame of teaching given the importance of being able to collect a large amount of data and that using a counting technique ("added up to") to compute a theoretical probability might be easier.

Task 5: Schoolopoly Task and Reflection. For the analysis of the task, Jeff pointed out one strength of the task is that *Probability Explorer* software saves time and allows students to roll the dice "a bunch of times." As for the objectives of the task he said one thing would be the law of large numbers. The researcher asked him to say a little more about that and he said, "The larger number of attempts you try the more you get to the approximation of the probability it will reach each possible outcome." This response is another indication of Jeff's knowledge of the law of large numbers as well as his awareness of how the task can be used to teach students this concept. Jeff also said that he would use this task, however he did not elaborate on why.

For the analysis of the posters each preservice teacher had a handout where they wrote down 1) what they could say about the students' understanding of probability, and 2) what they thought the students considered being compelling evidence. The preservice

teachers filled these out after they had a brief discussion of each poster before moving on to the next poster.

Poster A. During their discussion Jeff agreed with a comment Brad made about how the students were looking at the law of large numbers because they used 1000 trials. Jeff said, "like you said, it seemed they grasp the concept of using the highest number of possible outcomes they could get using 1000." He also added that maybe the students didn't fully understand it and referred to what the students wrote at the top of their poster (see Figure 35). "They didn't really know of a way to word it exactly. Going off of what's up here [on the poster]." What the students wrote was that the reason they ran it 1000 was because, "if you're playing for money you're gonna want to win your money back." So they were referring to a gambling context which Jeff did not specifically point out but said they didn't know how to word it exactly. Perhaps Jeff thought that because they were using this context that they did not quite understand the concept but were instead relating it to a situation where they would want to use a lot of trials.

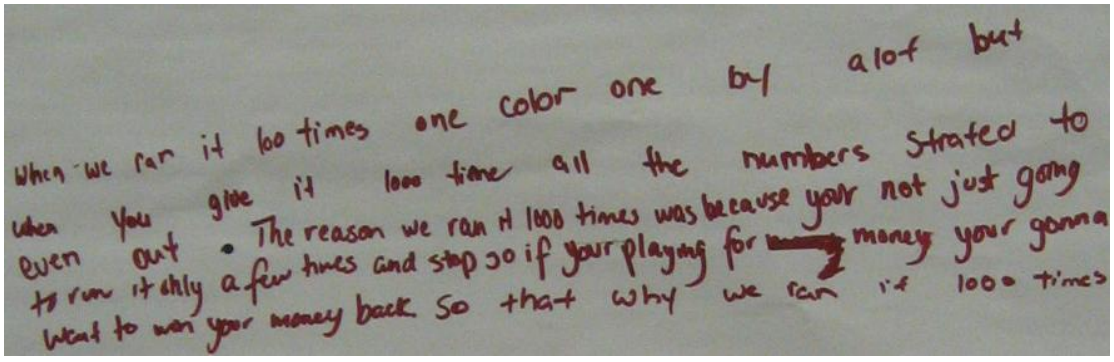


Figure 35. Poster A – Student Writing

While he did not discuss aloud with Sam and Brad what the students considered to be compelling evidence, he wrote on his personal sheet: "they consider the percentage for each die to be uneven to be compelling evidence that the die are not fair, using 1000 attempts." What Jeff was referring to was what the students wrote in response to the third question in the task (see Figure 36). The percentages they found were 13%, 20.4%, 12.6%, 20.4%, 13.4%, and 20.2% for 1-6 respectively. This seemed to indicate that Jeff was interpreting the students' understanding of fairness as being tied to evenness. That is, he seemed to think they understand the percentages have to be even in order for the dice to be fair. He also mentioned the sample size of 1000 which may be an indication of his interpretation of the students' understanding of the importance of a large sample size.

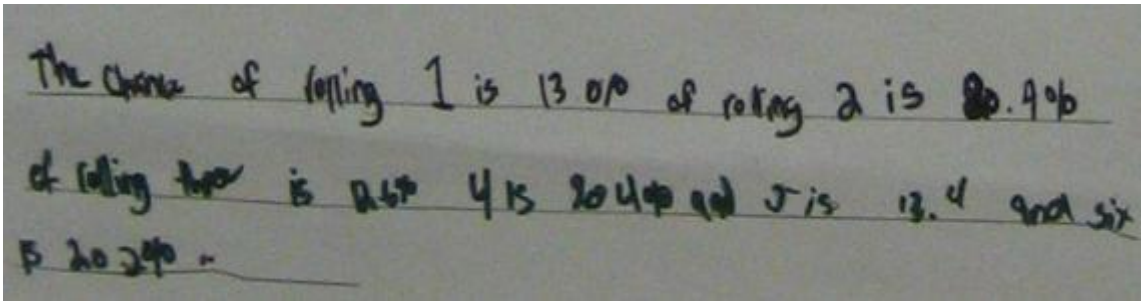


Figure 36. Poster A – Question 3 Student Answer

Poster B. The students that made this poster also investigated Calibrated Cubes; however they determined the dice were fair. The evidence they gave consisted of a table displaying the results of dice 1-3, and a pie chart. They ran 80 trials (see Figure 37).

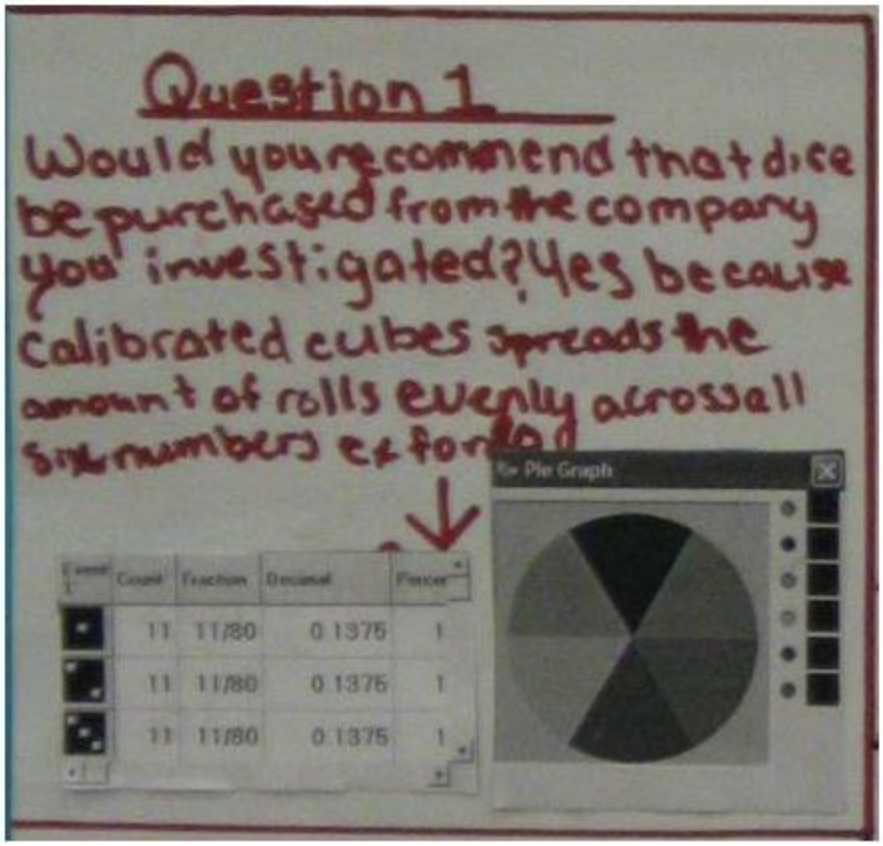


Figure 37. Poster B – Screen Shot Included on Poster

Sam began the conversation about Poster B by pointing out the sample size was only 80 trials. Jeff added to this the fact that "all together that's only 66 outcomes." He was extrapolating the students' statement based on the 11 times 1-3 occurred, that if the rolls were evenly spread across all six numbers then the total sample space should be only 66 outcomes. He continued on to point out that they didn't show the results for the other three numbers on the die. With respect to the pie graph (see Figure 37), the students said one of the shades was larger than the others and Jeff said "they didn't really go into

how much of a difference it is." This may indicate that he has an understanding of the significance of the amount of variation needed to determine fairness. Additionally, after Sam points at the graph, Jeff says it all looks evenly distributed. What is interesting is that he also adds "unless every other one is slightly bigger in each one." It is possible that he is making this inference based on the data shown in Poster A. Jeff made one more statement about the fact that they didn't focus on the difference and he suggested that it looked like they were trying to avoid it, "they might've just been like 'well, these ones look the same so let's just focus on these ones'." Here he is pointing out the fact that they got the same number of rolls for numbers 1-3 may have caused them to want to ignore the other numbers. Rather than focusing on the data as a whole, they only looked at half of the numbers because those three numbers were exactly the same. Thus Jeff's interpretation of the students' understanding is that they were not focused on the whole data set, also that the students may have been convinced by the evenness of the first three numbers.

Poster C. The students' work that is shown in this poster comes from investigating Dice-R-Us Company. They determined the dice were fair and justified this reasoning by displaying two trials, one size 80 and one size 100, and pointing to the fact that the number 3 was rolled the most in one and the least in the other (see Figure 38). This, for them, was evidence that the dice were fair.

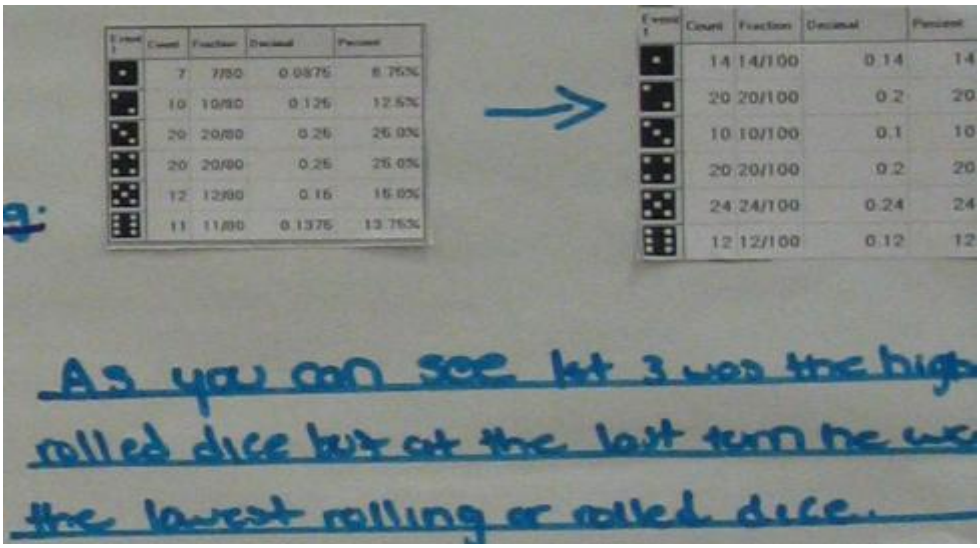


Figure 38. Poster C – Screen Shot Included on Poster

When analyzing this poster Jeff did most of the talking out of the group; first he pointed out the fact that the students were comparing the two sets focusing on "kind of the variation between the two," and that their approach was different than what the others did. Sam actually said to Jeff "you explain what that means" in reference to the students comment that 3 was the highest and lowest rolled in the two trials. Jeff said they were just focusing on the variation for number 3 and that they didn't really go into the other ones. This indicates that Jeff's ability to interpret the students' understanding since he was able to explain how their approach was different from the others. Also it can be seen that Jeff is comparing these students' work with the previous groups.

For their compelling evidence, Sam pointed out the percentages and Brad said the students were looking at that to say it is fair. Jeff added "just the variation of the

percentages." He seemed to be able to analyze the students' understanding of fairness as being based on the variation of the percentages for only one of the numbers on the die. What this says about Jeff's pedagogical content knowledge is that he is able to make a claim about the students' understanding of fairness through his interpretation of their work. He added that the students were focused on the variation of the percentages, thus he was able to clarify what it was about the percentages that convinced the students that the dice were fair.

Poster D. These students also looked at the Dice-R-Us company but they determined the dice were unfair. They ran 1000 trials and based their decision on the percentages as seen in Figure 39.

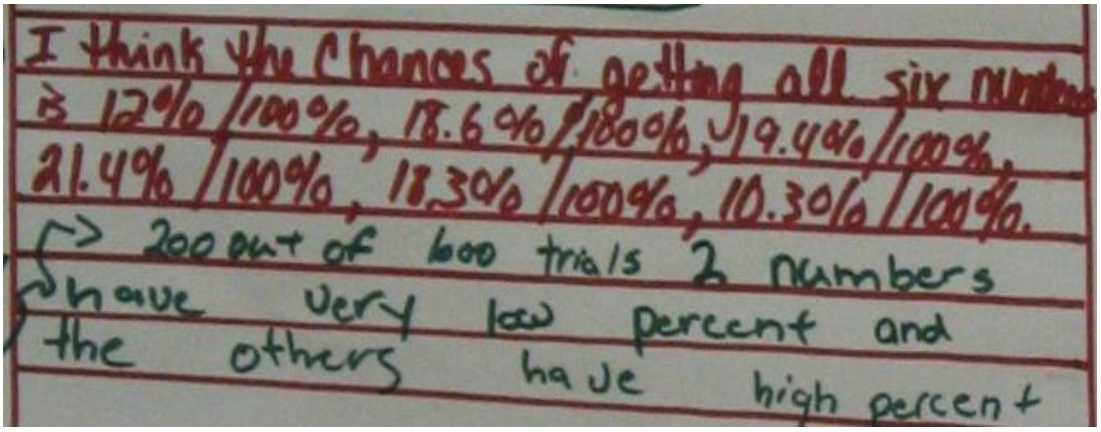


Figure 39. Poster D – Question 3 Student Answer

The first thing Jeff mentioned about this poster was how the students were mixing fractions with percents, how the expression 12%/100% while technically correct is not conventionally correct. The preservice teachers then pointed out the sample size, Brad

says it is out of 1000, but Jeff thinks it is 500 based on the screen shot. In the software the students ran two sets of trials of 500 and displayed the accumulated results. The researcher pointed out the 1000 in the upper right hand corner which indicated the total sample size (see Figure 40).

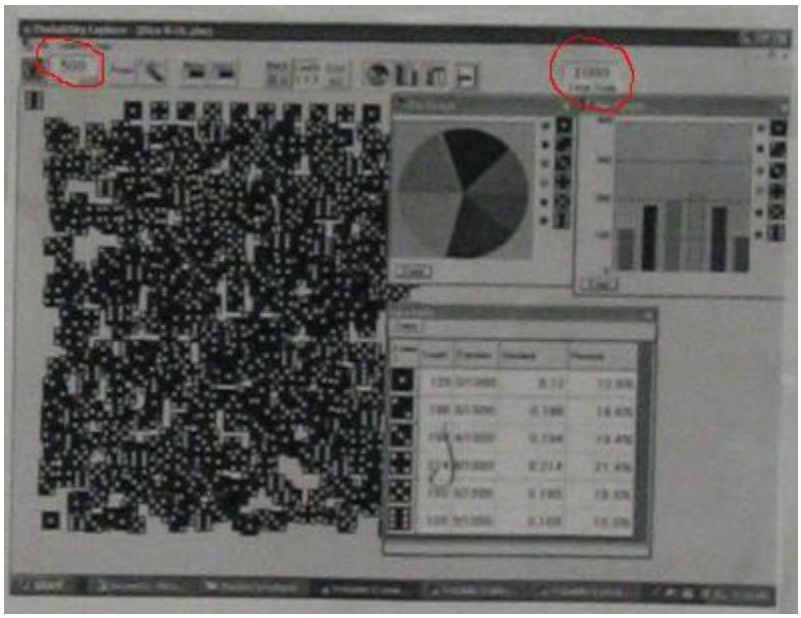


Figure 40. Poster D – Sample Size

This distinction seemed to be important to the group, Brad mentioned that the fact they used 1000 shows they used the law of large numbers. To which Jeff added "they kinda realize after such a large number of different trials that's the best approximation you're gonna get." This is another indication that Jeff seemed to understand the effect of sample size. Additionally, he referred back to the previous two posters (C and B) saying these students seemed to have a better understanding of this concept. Again, as to Jeff's

pedagogical content knowledge, he seemed to be looking at the students' work in relation to what the other students have done. He is taking a more global approach to interpreting the students' work.

What Jeff thought the students' put as their compelling evidence was the fact that the first and last numbers had lower percentages than the middle ones. He then agreed with their conclusion, "which means that they aren't fair because they used such a large number of trials." He also pointed out that the evidence on this poster was "much more compelling" than on the previous example. His ability to interpret the students' understanding of fairness shows that he has a fairly strong pedagogical content knowledge as it relates to students. He was able to identify what the students thought was compelling evidence and also offered a judgment on the correctness, saying that he agreed with their conclusion and that the evidence was compelling to him. Again too he is comparing this groups' work to the previous groups'.

Comparing Posters C & D. When the preservice teachers compared poster C to poster D they all agreed that they were more convinced by poster D and not convinced by poster C. Perhaps because they did some comparisons during their analysis of poster D there was not much discussion at this point. In fact, all Jeff said was "yeah" in agreement with Sam and Brad; however from his previous comments he expressed this opinion that he would agree with poster D.

Comparing Posters A & B. In comparing posters A and B Jeff started the discussion stating he preferred poster A over poster B. The reason he gave was because of the number of trials they did, that poster A would be more accurate. In addition he said that in poster B they "just wanted to make them look even." Sam asked Jeff if he was saying that he'd be convinced the dice are unfair and he said yes. These statements all point to the fact that Jeff seemed to have an understanding of the effect of sample size and is convinced by the students who used larger sample sizes.

In the third part of Task 5, the discussion of the first video clip regarding the students' discussion, Sam and Brad spoke the most. Jeff did offer a comment about one student that seemed to understand the law of large numbers but that he didn't know how to put it into words, "he seemed to be getting across the idea… the larger number of times it's the real percentage of what it's going to be." Again Jeff is referring to the law of large numbers and how the empirical probability will be more representative of the "real percentage" with the more data that is collected.

As with the discussion of the first clip, Jeff did not offer much during the preservice teachers' discussion of the teaching intervention. He started the conversation by saying the teacher in the video seemed to have a good idea with her approach of using a "really low number of trials to start out with" and then comparing it to using a large number of trials. He also added towards the end of the discussion that it was hard to determine if the teaching had been effective or if they were getting more confused about

the concept. Here he was able to critique the teaching and voiced his agreement with the strategy that was used. He also stated that he could not tell from the video whether the students were grasping the concept or becoming more confused. What he attended to in this clip was both the teachers' actions as well as the students understanding. These seem to be indications of his ability to attend to both considerations for teaching strategies in a probability lesson, as well as a cautious approach to making statements about students' understanding without more explicit evidence.

In Task 5d, the reflection, Jeff listed one benefit of the task – that it allows students the opportunity to see the law of large numbers themselves. One drawback that he mentioned was that, "the exploration may confuse students on the concept while they are working on the task if they strongly believe that the sample size is irrelevant." This statement was most likely influenced by the video of the class discussion as many of the students emphatically expressed this belief. Jeff did say that, despite this drawback, "these students would learn the concept at the end of the lesson and retain the information better in the long run." Here it seemed Jeff appreciates having students do an exploratory task and that this type of activity will lead to better retention of the information. He also seemed to understand that by giving students the opportunity to explore they will learn the concepts for themselves, as opposed to being told.

The next day Jeff said he would go over different examples involving the law of large numbers to see how well they have retained the information. He did not elaborate

on what types of examples he would use or exactly how he would 'go over' them, whether he would use another activity or direct instruction.

What he learned, in his opinion, was that, "exploration activities can be eye-opening experiences for students and even allow them to understand a concept without being told." Again this is reiterating his opinion that using exploration activities is a better way to teach students than direct instruction. He also wrote that he learned that "technology can greatly strengthen an exploration by giving students more compelling data with far less time-consuming grunt-work." His main reason for students using technology seemed to be for the time saving aspect that it acts as an amplifier. But he also went on to explain other benefits of the use of technology that would constitute using it as a cognitive reorganizer (Ben-Zvi, 2000; Pea, 1987). "I have learned that technology is an essential tool to teaching probability, as it provides the possibility for students to explore data and discover significant concepts themselves, such as the law of large numbers." Thus, for Jeff technology is an important tool to use in teaching probability concepts, indicating that he would use technology as part of his teaching repertoire.

Task 6: Interpretation of Meaning of Probability. Jeff agreed with the first statement and pointed out that it seemed to be an accurate percentage, that a million rolls would be more than enough for a large sample size. For the second statement he said that it was an inaccurate assumption, just a "rough guesstimate." Also he pointed out that, by the nature of the problem, the outcome of a 3 on the dice was not going to show up that

often, “Because, I mean of course it doesn’t come up often because there are six different possibilities.”

How Jeff critiqued the third statement is another indicator of his statistical orientation. He recognized that “this is just pretty much the theoretical probability” and he went on to say, “it’s not based on any evidence any samples so, it’s just kind of you know it’s not backed up by anything.” This statement gives evidence of him valuing empirical probability and that he may think that the theoretical probability by itself is inadequate.

Jeff further displayed this preference for empirical probability in his critique of the last quote by his focus on sample size and the use of the qualifier, just, when addressing the theoretical probability (italics added to indicate emphasis in tone):

> This person decided just to go around and ask some people about it and like a little over half of them *just* gave the theoretical probability… the other one was based on a small sample size, it could be small or high if it was a small sample size.

He interpreted the part in the quote where it mentioned ‘it could be small or high’ as being dependent on using a small sample size. This implies that he could be thinking in terms of experimentation and finding the empirical probability, although this was not directly mentioned in the given statement.

The statement that Jeff said he agreed with most was the first one; again indicating a statistical orientation. “Even though this looks very time consuming, I'd say this one is the better one, backed up by the most evidence.” He places importance on having evidence to be able to back up the probability claim. In addition to his comments on the second statement, and stating that that was the one he most agreed with, Jeff also downplayed the theoretical probability statement (italics added): “This one is *just* theoretical probability, no sign of actually using a sample or you know testing it out in any way.” Other than the fact that the statement did not mention using any data or evidence, Jeff did not refer to the limitations of theoretical probability – namely needing a finite sample space and each outcome being equally likely. However, as you will see in part b, he did display knowledge of these limitations.

Additionally, when Jeff was asked if he thought any of the statements were incorrect he said that all were but the one with which he agreed. For the second statement he said, “definitely this one, just because it's just based off an assumption, just thinking back and being like, ‘oh I only rolled, I don’t remember rolling a three very often so I think it’s 10%’.” Thus, Jeff seemed to believe that basing a claim about probability solely on past limited experiences is not sufficient. Also with the third statement he said that the theoretical probability was good but that it doesn’t go into the empirical probability. Therefore, Jeff’s content knowledge of probability seemed to require attention to

empirical probability and having data or evidence to back the claim. Similar findings were evident in his responses to the second part of this task.

For Task 6b, interpretations of the meaning of probability, Jeff did not agree with the first statement saying that the way it was worded, "makes it sound like it's just kind of a chance of being anything." For the second statement Jeff brings up the limitations for this definition, "this one would work if there wasn't a weight of either one option, if it was an equal, you know evenly distributed thing." He also added that this would work for a coin toss but not any other thing and that it is "very limited in that way." It is interesting to note that he did not mention these things in response to the specific statement about theoretical probability of dice, but he did in this more general context. Perhaps the context being more general allowed him to recognize the limitations, or at least bring attention to them. In addition, he could be over-generalizing the need for an equiprobable distribution when using theoretical probability. For example, he did use theoretical probability in the context of the candy task where each individual candy has an equal chance of being selected, however each event (i.e., a specific color) could have an unequal chance.

The third statement Jeff said was a little confusing, but he reworded it as, "over a large number of times the frequency reduces… over a period of time the more samples you do the closer, I mean it reduces the variation over the increase in sample size." Jeff

recognized that this statement reflects the law of large numbers, and he also said that it is "definitely a part of probability."

As for the fourth statement, Jeff initially expressed frustration not knowing how to evaluate it, "I don't know I mean it kind of works in the fact that, ahh I don't know." As he talked some more he agreed that it would work and again brought up the notion of the law of large numbers. "Each sample achieves a random result whether like with one has a better chance than another one… in a way it is a measure of randomness of an occurrence over a large period of time."

To explain which statement(s) he agreed with or identified with, Jeff began by eliminating the ones with which he did not agree. He ruled out the first one simply saying, "I'm not big on the first one."He also ruled out the second one for the same reasons mentioned above – that it is too limited. Then for the third and fourth statements he expressed agreement: "I would use parts of the third one, in a way it is trying to explain the law of large numbers so I do agree with it in that way," and "the fourth one I do agree with too, because it is kind of the measured randomness of an occurrence like how many out of you know like a thousand random results, how many are going to achieve this result." Both reasons for agreeing with the third and fourth statements are tied to the law of large numbers and collecting a large number of results – indicating a statistical orientation.

The researcher went on to ask Jeff if he could think of any situations where the theoretical probability statement would not work, trying to see if he would also mention the fact that the sample space had to be finite and known. What was interesting was he gave the example of rolling 2 dice, which actually can be calculated using theoretical probability. He knows that this definition requires the probabilities of each outcome to be equally likely, and the fact that a 6, 7, and 8, came up more often than a 2 or 12 was the reason he gave that this definition wouldn't work in this example. He is confusing the frequency of the outcomes, with the actual probabilities. Each outcome can be calculated and the sample space is known, but the probabilities of the sums are not equal. This is another example where Jeff may be over generalizing the need for an equiprobable distribution to use theoretical probability and not seeing that he could use counting techniques to find the ways of getting a specific event (i.e., a 12 with two dice).

One additional thing to note, when Jeff was asked if there was a way to figure out the probability when we could not use the theoretical he did say that we could use empirical probability or technology to create experiments to measure the probability.

Task 7: Curricular Issues Revisited. Similar to Task 2 when asked what concepts to include in teaching Jeff was uncomfortable with the fact that he could only come up with a few ideas. There were several (6) long (greater than 15seconds) pauses while he tried to think of more to add and several times he said that he couldn't think of any "off the top of his head." However his self-perception of not being able to come up with ideas

was counter to reality as he did list (in his order) quite a few: the law of large numbers, theoretical and empirical probability, sample size, weighted difference, sample space, and randomness. The researcher believes that if this question were given in a different setting than an interview, Jeff may have at least felt more confident in his knowledge of probability concepts. But the list he did give was quite complete, although he did not go into detail defining each concept he listed. Thus the researcher cannot say for sure whether he had a correct knowledge of these concepts. However, he was asked to explain what he thought sample space meant and his reply was, "seriously I can't think what sample space would be, maybe the limit of how many samples you do to achieve the true probability from the example?" It is not clear what he means by "true probability" but he may be referring to theoretical probability. Here he still seemed to be thinking in an experimental setting by saying, "how many samples you do" rather than thinking of sample space as the collection of all possible outcomes. This may also be related to how he could not think about how it was needed to develop the entire sample space for the two dice context in order to compute a theoretical probability.

The strategies that Jeff mentioned were: using real life examples to "grasp their attention more," using technology to "have them discover for themselves the law of large numbers," and having students design their own experiments. In addition, he mentioned using M&M's "just because M&Ms are always fun for kids." The strategy of flipping a coin Jeff said he was "kind of iffy on," but he did say it would be a good way to show

how the sample size relates to empirical probability and that it is a, "possibility of what to use." The strategy he said he would not want to use was, "a sample using die without using technology. Just because I think that would be way too time consuming; trying to get a large enough sample size would at least take up a whole class." Note also that at the beginning of the study the first things he listed were "coins and dice" but now he doesn't think those are as effective, or as important to include.

These strategies and reasons for his strategies indicate that Jeff values student interest by using real life examples and fun activities. His reasons also indicate that he seemed to believe students learn best or gain more from using technology and designing their own experiments as opposed to simply flipping a coin several times. Similar to his comments in Task 5d, these responses indicate he may believe giving students experiences where they are experimenting and building on their own interests will lead to better learning and are best teaching practices. Also the numerous indications of a statistical orientation may be influencing his decision to use experimentation and empirical probability.

What Jeff listed as things he had learned about the teaching and learning of probability were the law of large numbers and how technology can be used as a pedagogical tool to help students discover things for themselves. "That would be one big thing, the fact that it can really save the students time, instead of spending so much time just on the procedure it has them immediately go through the procedure and analyze the

results." Here he is pointing out the fact that technology allows the focus to be on the results and how to analyze and interpret, as opposed to simply going through the procedure of collecting the data. Thus he implied that he learned how technology can both amplify by speeding up the process of data collection, and also be a cognitive reorganizer by bringing the focus on the results.

Task 8: Multiple Choice and Free Responses from Pre/Post Tests. For all four questions in Task 8a Jeff answered them correctly on both his pre- and post-tests. This shows at the beginning of the course and study he had an understanding of how to interpret simple probabilities as well as a range of probabilities. He also had an understanding of randomness and the independence of events in a sequence of coin tosses. Thus, he demonstrated understanding of the meaning of probability and how to interpret probability.

For the first question in Task 8b Jeff gave the correct answer on both the pre- and post-tests, demonstrating knowledge of proportional reasoning as it pertains to probability. However on the second question with regards to sample size and variability, Jeff answered (c) on his pre-test and correctly answered (b) on the post-test. This indicates that his knowledge of sample size had grown to include the understanding that the larger the sample size, the less likelihood there is of obtaining results that vary from the theoretical probability.

The third question from Task 8b was designed to assess the preservice teachers understanding of expected variability within a distribution of a sample statistic from a fair coin toss experiment. Jeff chose both (b) and (d) on both his pre- and post-tests. Thus, he displayed an understanding of these concepts both at the beginning and end of the study and course.

With Task 8c Jeff also answered correctly on both tests. As for his critique of the strengths and weaknesses, on both tests he scored two out of four. Two of the possible strengths and weaknesses of the students' strategies were not included. His pre-test answer was, "Group 2's strategy is way off because there is a good chance that a number will appear more than once in six tosses. Group 1's strategy is better than group 2's but ten tosses is not enough for a six sided die." Jeff did recognize that the small sample size in both groups is a weakness however he does not recognize that the students are comparing empirical probabilities to theoretical probabilities, which is a strength of both groups.

On his post-test his answer was, "For group 1 tossing a six sided die ten times is not enough to always achieve an empirical probability 1/6 if the die is fair. It could work if the die was tossed 100 times or more. For group 2, there is a very good chance that each number would not appear by only tossing a die six times, even if the die is fair." While Jeff's score did not increase with this post-test response, he does add more to his critique. Namely the fact that using a small sample size will give a greater chance of

indicating something is not fair when it may actually be fair. He also added that not only is ten times not enough, but 100 times or more would be enough trials. Thus, Jeff's answers collectively on both his pre- and post-tests indicate he had a strong content knowledge base before and after the course.

Summary of Jeff's Case

Orientations. The summary table below shows that Jeff seemed to display more of a statistical orientation in his responses across the eight tasks. There were ten instances of a statistical orientation while only three of a mathematical orientation and no indications of subjective or personal orientations.

Table 7 *Summary counts for Jeff's orientations by task*

Tasks	Statistical	Mathematical	Subjective	Personal
Task 1	3	2	0	0
Task 2	1	0	0	0
Task 3	1	0	0	0
Task 4	0	1	0	0
Task 5	0	0	0	0
Task 6	4	0	0	0
Task 7	1	0	0	0
Task 8	0	0	0	0
TOTAL:	10	3	0	0

Characteristics in Jeff's responses throughout the tasks that indicated he had a statistical orientation include his preference for empirical over theoretical probability, using empirical probability to explain meaning of a theoretical probability, numerous references to the law of large numbers, and relating probability to a statistic determined

from collected data. An example of how he used empirical probability to explain the meaning of a theoretical probability occurred in the first task when he used a frequentist approach to explain a coin toss having 50% probability. An example of his preference for empirical over theoretical probability occurred in the sixth task where he identified with the empirical statements. In addition he also seemed to understand how theoretical probability is limited to certain situations and having more of a statistical orientation may be contributing to this awareness.

The three indicators of a mathematical orientation occurred in Tasks 1 and 4. When he explained what random meant to him Jeff used the word fair, which is tied to outcomes being equi-probable. He also gave the theoretical probability of the coin toss as being 1 out of 2 and on the candies task initially he used a theoretical approach. These indications of a mathematical orientation are evidence that he understands theoretical probability and that he may prefer a mathematical approach when given the choice of a problem solving strategy (as in Task 4).

Content Knowledge. Jeff's content knowledge of probability seemed to be rather strong as shown in the first few tasks as well as the pretest. Additionally, it seemed to deepen from the beginning of the study to the end.

He expressed difficulty in brainstorming concepts to include in the teaching of probability both during the first task as well as the seventh, the first and final interviews, respectively. He also was frustrated and really struggled to think of more concepts; this

seemed to really bother him – knowing there was more to be said but not being able to think of what that was. While Jeff still was frustrated during Task 7, he did show growth in his understanding from the beginning as he was able to list more concepts and not merely rely on instructional strategies (i.e., using dice and coins) as a way to describe important concepts.

One area of probability that Jeff seemed to have a strong understanding of is the law of large numbers. He references this concept in all but one of the tasks. In the first interview he explained this concept, but when asked specifically about the law he answered that he did not know what it was. We may be able to infer that his past experiences learning probability taught him that the more data you collect the less variability there will be, yet was never explicitly taught using the terminology of 'law of large numbers.' In his analysis of the students' work in the Schoolopoly Task Jeff was able to notice the variation within the results as well as the different sample sizes and how that impacted the findings. Thus, not only did he display knowledge of large sample sizes and how they affect probabilities, but he also was able to express the effects of small sample sizes.

Another concept that appeared throughout was that of the relationship between empirical probability and theoretical probability. He was able to explain how the variability between empirical and theoretical decreases as the sample size becomes larger. He also was able to point out how theoretical probability is limited to situations with

equi-probable outcomes. He never explicitly mentioned the other limiting property of theoretical probability – that the sample space must be known and finite. However we cannot infer that he does not have that knowledge simply by the absence of him mentioning it. In addition, the concept of sample space was not well explained by Jeff as indicated in the example of the candy task and the sum of two dice. He seemed to be over generalizing the equiprobable distribution idea for how to compute a theoretical probability based on the known sample space. In fact he seemed to not be able to think about how to construct a sample space for a compound experiment like two dice.

Pedagogical Content Knowledge. With regards to Jeff's pedagogical content knowledge pertaining to probability, it seemed to develop from the beginning of the study to the end.

At the beginning of the study, in the first interview, when he was asked to list important concepts to include in the teaching of probability he said "coins and dice" which are not concepts but random generating devices. At the end of the study he down played this importance and in fact said that he would not use dice unless it was with some sort of simulation software; he also said that he was "iffy" on the use of coins but he still felt they were important as a way to teach students the law of large numbers. Another point to mention is that in the first interview he was responding to the question of *what* to include in a probability course, whereas in the last interview he was responding to the question of *how* to teach probability. This subtle difference in inquiry combined with his

experiences in the course and this study may have impacted this change of opinion. Additionally, it seemed in the first interview he could not adequately describe what to teach about probability without using examples of how to teach probability.

As far as teaching strategies, at the beginning as well as the end, Jeff stated that he felt it was important for students to have "hands-on" activities. In addition, he reiterated the importance of including experiences that would interest students, which pertain to their lives, thinking this would increase engagement. Also he felt the Schoolopoly Task was advantageous to include in teaching because of how it "required students to think," and was a real world problem. He did seem to have a more robust understanding of various ways to teach probability. He even explicitly stated that fact in the seventh task when he described what he had learned about teaching and learning probability.

With respect to his knowledge of students, Jeff seemed to have the belief that students learn and retain information best when they are given the opportunity to explore data on their own. As for analyzing student understanding, he did not do this when presented with the student responses in the cars task; however in that task he was simply asked how he would direct the conversation and not explicitly instructed to comment on the students' understanding. However when he was asked to analyze student understanding by examining student work in the Schoolopoly task he seemed to be able to determine what students did, as well as did not, understand. Also when analyzing the students' work he compared the work of one group to the work of another, something the

other cases did not do. Another indication of his knowledge of students was his understanding of the importance of recognizing student interests and using real world applications.

Relationships Among Three Aspects

Some common themes that were apparent throughout the eight tasks for Jeff were the ideas of large sample sizes, experimentation, and real world applications. It seemed that his strong statistical orientation does relate to his content knowledge of probability and his pedagogical content knowledge. His statistical orientation can be viewed as having both a positive and negative impact on how he thinks about probability and the teaching of probability.

Some examples of how his statistical orientation may be influencing his content knowledge in a positive way include his responses regarding the importance of teaching probability and his responses within the Schoolopoly task. While he mentioned using games of chance and using things like coins and dice, he broadly talked about how probability is something that should be connected to the real world in teaching. Also when asked about strategies for teaching probability he mentioned using real life examples and experimentation as ways to engage students. These ideas align with a statistical orientation because statistics is tied to a context.

Another way his statistical orientation can be seen as being related to his thinking about probability is his understanding of the law of large numbers. He frequently referred

to this concept across different contexts: when answering the cars task, when analyzing student work, when analyzing the Schoolopoly task, and when explaining strategies for teaching probability. An explicit example of how his statistical orientation could be helping Jeff is in his responses within Task 5, the Schoolopoly task. His strong conceptual understanding of the law of large numbers helped him in analyzing student work and noticing more of the details within the representations.

In contrast, there were indications where Jeff's strong statistical orientation may have had a negative impact on his content knowledge. When he was asked to give an example of something that could not be computed theoretically, his example was the probability of the sum of two dice – an event that can be computed theoretically. It may be that while he has an appreciation of the need for theoretical probability, his conceptual understanding could be limited to only how theoretical is related to empirical. He may not have a strong understanding of sample space and how to compute theoretical probabilities. Even with the first task when asked to explain the probability of a coin toss he talked about it in terms of frequencies and within an experimentation context.

Also it is important to note what was missing from Jeff's knowledge across the three aspects. There was no evidence of Jeff ever illustrating a personal or subjective orientation and in fact he doesn't seem to value those as possible ways students' may approach probability. This could be a hindrance when trying to interpret student work or being able to anticipate student responses in class discussions.

Therefore, with regards to all three aspects of knowledge (Orientations, Content Knowledge; and Pedagogical Content Knowledge) Jeff's more statistical orientation seemed to influence his content knowledge as seen in his weak interpretation of theoretical probability, his importance of using real world examples, and his strong understanding of the law of large numbers. This statistical orientation may also have influenced his pedagogical content knowledge as seen in his preference for using experimentation in his teaching repertoire, in his understanding of possible increased student engagement through using experiments that are related to their interests, and in his interpretation of student work.

Case 5 – Sam

Introduction

Sam was a graduate student enrolled in the licensure only program in 9-12 mathematics education. He reported that he was getting his masters degree in operations research and statistics at the end of the current semester. He plans to teach high school or junior college, but did not have any prior teaching experience. Sam had taken a 200 level mathematics methods course, Introduction to Teaching Mathematics and was currently enrolled in a 400 level mathematics methods course on Teaching Mathematics with Technology.

Throughout the eight tasks, Sam seemed to indicate more of a mathematical orientation; however with his strong background in statistics there were also some

indications of a statistical orientation. Sam's content knowledge of probability was closely tied with statistics and seemed to be fairly complete; on the pre and posttests he got all the answers correct. However his pedagogical content knowledge was limited but seemed to expand as the study progressed.

Task by Task Analysis

Task 1: Interpretation of Randomness and Probability. For Sam, random means "events that happen by chance, there's not any predetermined outcome." And the examples he gave for random were tossing a coin or throwing a die, also he clarified that the coin and die would have to be fair. He exhibited the same understanding of randomness when he described it's meaning within a probability and statistics class: "… if you take a sample of a certain population you don't pick for instance a certain age group or just all males or all females, the chance of anyone being selected is the same." Thus, Sam has an understanding of randomness as something that is fair and not predetermined. This definition, combined with the examples he gave are indications that he may have a mathematical orientation towards probability.

When Sam described the meaning of the probability of a coin toss, he said that "when you flip a coin you can expect half the time it will come up heads." Sam seemed to understand 50% probability as "half the time"; the way he described the probability in terms of time and "when you flip a coin" could indicate that he is thinking of probability as performing experiments with coin tosses. This may be a hint at a statistical orientation.

In response to the question regarding 70% chance of rain, Sam said he questioned how that applied to the weather forecast. “I don’t know how exactly they're meaning of it… if it’s going to rain over 70% of the area or whatever. But as far as probability it means that most, I guess, it’s more than likely it will rain” He also added the question, “does that mean there's a 30% chance it's not going to rain at all?” There are two findings within these quotes: one, he offered an interpretation that 70% refers to the area of coverage; and two, in terms of probability he said that it means it is more likely to rain. This may indicate that he is unsure how to interpret probability statements in the context of weather predictions. And also that he describes 70% as having a “more than likely” chance of happening

After he was asked to explain the meaning of probability, Sam paused for 12 seconds and then said, “Probability is a way to estimate events happening that you're unsure of the outcome.” First of all, his long pause may mean that he was thinking carefully before answering the question. Secondly, he seemed to understand probability as a way to measure the likelihood of unknown events happening. This correlates with his definition of random being events that are not predetermined. It is hard to say from this definition if his orientation is more mathematical or statistical since he did not indicate how he thinks probability is measured. However by referring to probability as applying to events where you are unsure of the outcome, this could indicate a statistical orientation because the theoretical probability would be unknown.

Task 2: Curricular Issues. Sam explained that he thought it was important for students to learn probability for "a lot of practical reasons." Those reasons were "starting a savings account or any sort of investment you may take, they're inclined to gamble." The researcher asked him to explain how probability was related to investments, "people like to speculate on the short term whether something is going to rise in value and probability shows you that often there's a trend." He also added, "Not everything in the world can be explained for certainty and so probability gives you a good estimate of what may happen." Thus, Sam thinks probability is important for students to learn because it gives them a way to deal with situations involving uncertainty however it is unclear if he has an understanding of *how* probability could help.

The topics Sam listed that he thought were important to include in teaching probability tended to be more related to a mathematical orientation. These topics included: randomness, counting techniques such as combinations, independent versus dependent events, conditional probability, and expected values. While this is a fairly extensive list, Sam said "I'm missing a lot here obviously," indicating he thinks there should be more included but he couldn't think of more at the time. And then went on to add topics that are more related to statistics, "Sampling, data representation, histograms, box plots, graphs, quantitative measures, looking at averages, spread." It's possible that Sam's background in statistics has helped him to recall many topics related to probability

and statistics. However, by simply listing these topics we cannot know how well he may understand the topics.

Task 3: Real-world Context and Teaching Situation. In response to the question regarding which car he would chose, Sam said he would buy the Honda based on the information from the Consumer Reports data. He said that 400 cars is a good sample size, but that the phrase "somewhat fewer mechanical problems" doesn't give you a good idea of "what the differences were or how spread apart those differences were." As for the three friends' comments, Sam said that was a small sample size and the problems could be attributed to "bad luck" and added, "You don't want to base your judgment on one friend who had a Honda and who had problems." From his response Sam indicated he had an understanding of the effect of sample size on the reliability of the information. Also he seemed to understand that basing one's judgment on a single instance is not advisable.

When Sam read the hypothetical student responses he first replied that he liked how they responded to the reason why they felt one brand was better than the other. He also said he would be interested in learning the students' reasoning. This implies that his knowledge of content and students includes a desire to understand how students reason and make sense of data they are given.

Sam went into detail about how he would confront group (a)'s misconception, "I would ask them if that would change if for instance there were 99 other people who you

didn't know about that didn't have any problems with the Honda." Here he is having the students consider the possibility that there may be others they are not aware of and thus it is faulty to base their decision on this small sample. He then explained how, "This leads into the law of large numbers and sample size so I would reiterate that, the importance of sample size." Again Sam brings up the importance of sample size so this seemed to be a concept that he understands and that he wants students to understand.

For the other two responses Sam said that group (b) "is right on" and that group (c) "is interesting too." Then Sam said, "I would like them to tell me what would make a difference if they would prefer one over another." Thus, Sam would have the students think about their decision making process and what would be required to convince them to chose one over another.

Therefore, with respect to Sam's pedagogical content knowledge, he offered a few questions that he would ask to probe students' thinking and push them to think further about how to make claims based on data. He seemed to be interested in understanding students' thinking and was able to identify concepts he saw as related to this task, namely, the law of large numbers and sample size.

Task 4: Experimental Context and Critique/Plan. To make a claim about selecting a red candy Sam said the first thing to do is make sure the bowl is mixed up. Next he suggested taking a sample because "it would be time prohibitive to pick them all out and make counts of all of them." Sam then reached into the bowl, while making a point to

look the other way, and took out a handful of candies. He suggested that 30 candies would be a good sample size "30 just for the reasons, that good old probability reasons." He made a face indicating he didn't really know how to explain it but in statistics 30 is an agreed upon convention of being a large enough sample size. Sam separated his sample into two piles – "count the total number of red ones you have versus everybody else." In his sample there were 11 red and 19 non-red; he calculated the percentage on his scratch paper, almost 37%. Sam then suggested that, while this was a "decent sample size but you probably want to repeat it again and maybe repeat it 5 times depending on time, and then take the average of the proportions that you got." Sam seemed to display an understanding of repeated sampling and decided to do experimentation to determine the probability of a red candy. These methods are indicative of a statistical orientation.

Sam said that the task "may be a good introduction to understanding proportion and likelihood of an event happening." He would use this task in his teaching or something similar, "something easily sampled and you have several groups do it and they all get different answers and get them to understand why that occurs." Sam went on to discuss how he would separate students into groups and have each take a sample. He explained that this task could lead into a discussion of sample size and then repeated samples. He added they could also calculate the mean and look at the spread, "you may have a spread of, well based on mine, maybe 6 to 15 or something like that." From these comments it seemed Sam understands the concept of repeated sampling, sample statistics,

and sampling distribution, and how that can be taught using this task. He appears to have good understanding of how to use such a task to expose students to informal ways to use the data from repeated samples to infer something about the distribution in the population (bowl of candies).

Task 5: Schoolopoly Task and Reflection For this task, Sam participated in a group interview with Jeff and Brad, due to scheduling restrictions. Brad and Jeff's comments are noted if they seemed to influence Sam's responses.

For the objectives of this task Sam said, "Maybe the difference between theoretical and empirical probability, plus it's an analysis exercise so just from that standpoint they're given a problem, it's a problem solving exercise." Sam pointed out the fact that this task has students solve a problem and analyze data. This goes along with what he listed as a strength of this task, "I like the fact that it's a real problem, and it's not just um rolling the die… they put a problem behind it." It seemed Sam believes having a real world context for the task is beneficial.

As for student difficulties, Sam said that students may not know where to start; it depends on how much has been covered before hand and how much guidance the teacher gives. "Some may see, if they get minimal guidance, some may just look and say, 'ok so what? I'd rather have the number 2 on my die.'" Sam seemed to be saying that students may have difficulties with this task if they are given too little guidance. However one of the strengths Sam mentioned was that it's open ended. "Students can decide out of the

trials that are run, what's important, what data is important and make their own determination." Sam reiterated this opinion in response to Brad's comment about not being given enough direction, "so you want it more structured? See I like the fact that it wasn't structured." So Sam seemed to be aware of students needing guidance with this task, but values the fact that it is open ended so students can make their own decisions about what data is important and how to make sense of the data to answer the problem.

Sam said that he would use this task and also he agreed with Brad who said he liked the fact that the weight tool [true distribution on the die] was hidden, "like they're actually testing, they're actually doing an actual experiment." This reiterates Sam's placing importance on a task being a real problem for students as opposed to merely an exercise.

Poster A. The students who created this poster analyzed Calibrated Cubes and determined the dice were unfair. The evidence they gave included a pie chart, bar graph, table, and percentages (13%, 20.4%, 12.6%, 20.4%, 13.4%, and 20.4%).

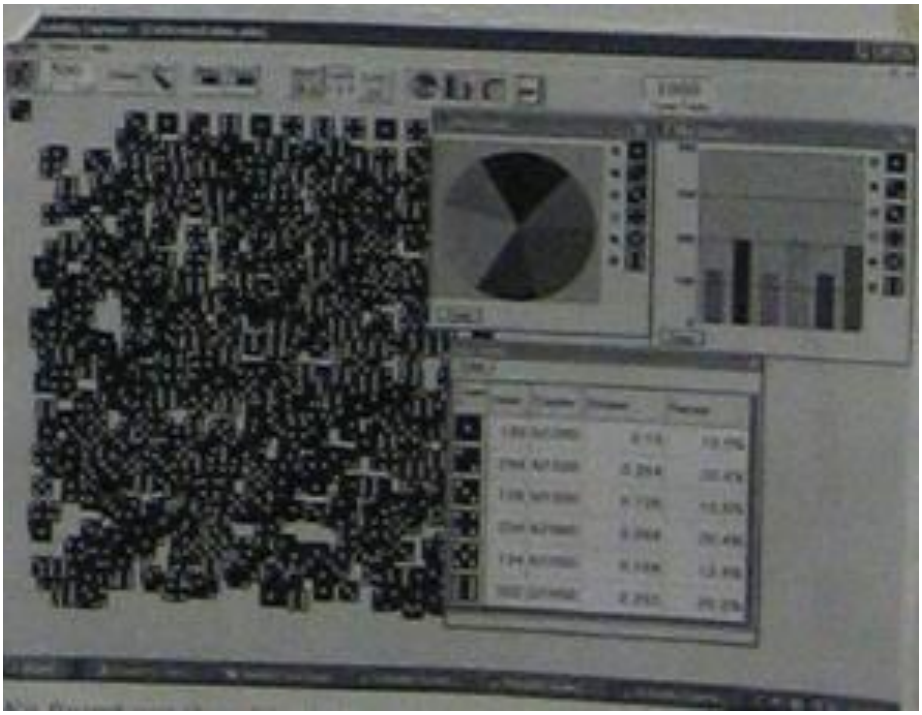


Figure 41. Poster A – Screen Shot Included on Poster

The students also wrote “we ran it 1000 times and all the odd numbers had a low percent and even numbers had a 20+%.” Sam said he liked that they pointed out this fact but he wasn’t sure if they understood how significant the difference is in comparison to how many trials they ran. “So they explained the gap. I don’t know if they understand how significant that is in comparison to how many trials they ran.” From this response it seemed Sam may have an understanding of the amount of expected variance in relation to sample size. He also seemed to be careful not to assume too much about students’ understanding.

Sam went on to reiterate this thought, “See this is a good exercise, but you don’t really know what the students understand until you go over this.” It may be that Sam doesn’t believe he can analyze much about students’ understanding based only on the posters. He also seemed to believe that the task, in and of itself, is not enough to promote student understanding, that you need to also include dialogue. “Then you go over the terminology, and what exactly this means and ask them pointed questions. But, you know, this is a good opening exercise but you can’t really say, after they do this you know, ok, they understand this.” Again, Sam seemed reluctant to make conclusions solely based on the students’ written work.

Poster B. For this poster students analyzed Calibrated Cubes and concluded that the dice were fair. (See Figure 42).

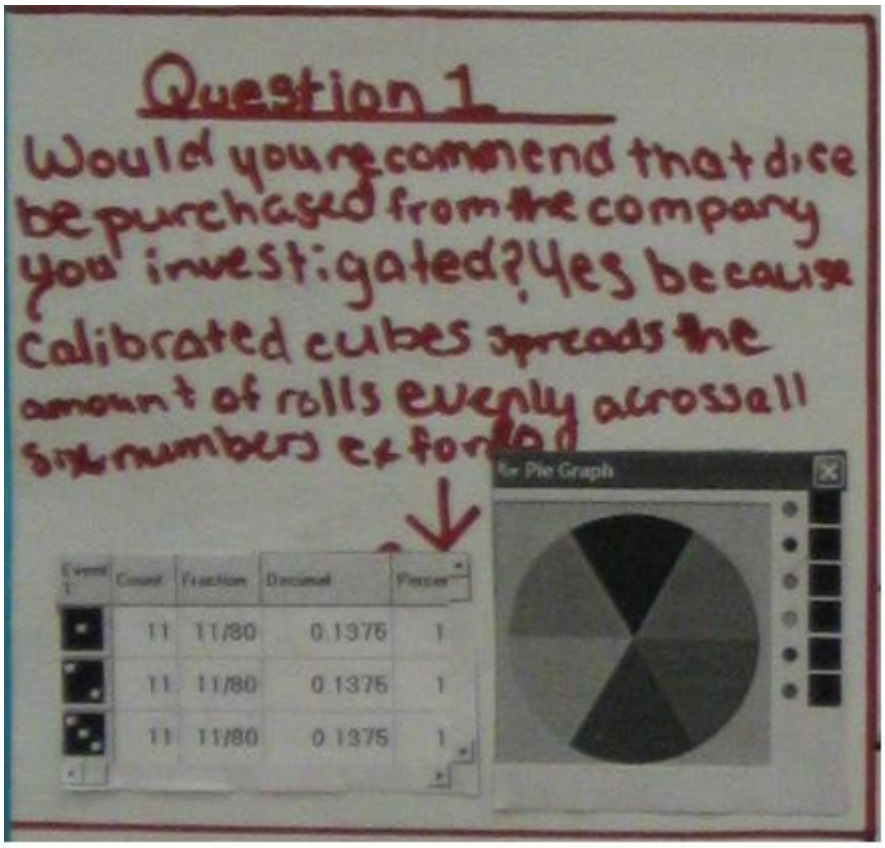


Figure 42. Poster B – Question 1 Student Answer

The first thing Sam mentioned about this groups' work was that they only ran 80 trials. He also said that "it looks like they were looking for any possible difference; you know they said one shade bigger, darker?" But Sam thought the pie chart looked all even and that the table data did not match the "bar graph" referring actually to the pie chart (since there was no bar graph displayed). "Look, cut it in half, cut it in half, cut it in half." He motioned on the poster how the pie looked like it was separated into equal sixths.

Another point that was made was that the students' data did not add up. Brad said, "11 out of 80 with 6 possibilities doesn't you know…" and Sam said "yeah that's something you would hope they would check that all the probabilities add up to one." The preservice teachers at that point began speculating as to why the students did not use all of the poster space. Sam offered the suggestion, "or they ran out of time and put up what they had." He then added, "I'd give them another shot, tell them why it's incomplete, maybe explain a couple things to them and have them slightly redo it. Then maybe we could understand exactly what they understand."

From this analysis, Sam seemed to understand first that 80 is too small a sample size, second that the probabilities must add up to one. As for Sam's pedagogical knowledge, he indicated that he could not say what the students' understood and that he would give them another chance. But he also said he would tell them why it was incomplete. He did not elaborate on what he would tell them as to why it was incomplete, so it is unclear as to how this strategy would help illuminate the students' thinking.

Poster C. The students that created this poster analyzed the Dice-R-Us company and concluded the dice were fair.

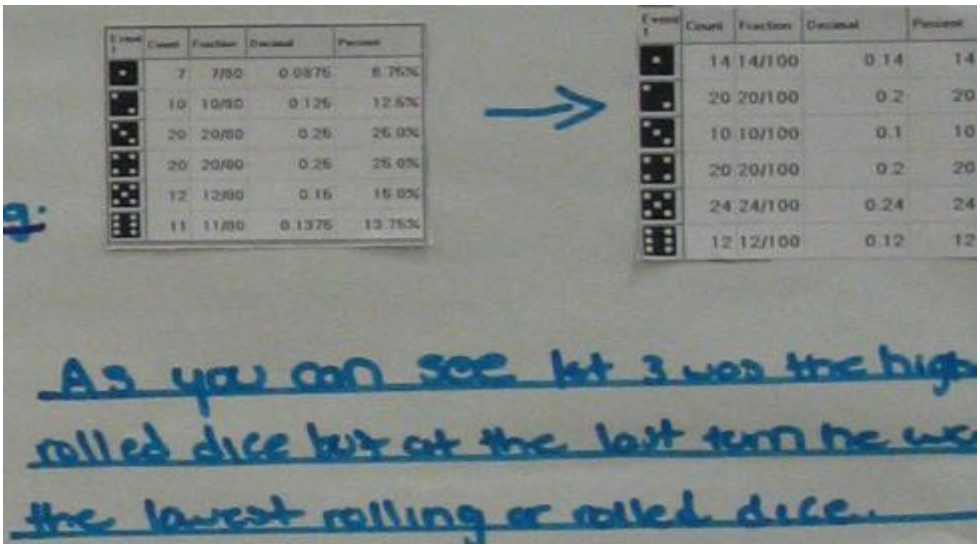


Figure 43. Poster C – Screen Shot Included on Poster

Sam started the discussion saying, "this is what they ran 80 times and then 100 times." Thus he is pointing out the sample size, but did not indicate a judgment about this. When trying to make sense of their argument comparing the two samples, Sam turned to Jeff and said, "Ok, you explain what that means." Jeff simply reworded what the students wrote, saying they thought it was fair because 3 was rolled the most and least in the two samples. Sam then pointed out the percentages, "that's (number 1) less than 10% went up to 14%. Both of these (numbers 3 and 4) were 25% and this (number 3) went down to 10%... and they're saying because of that they thought the dice is fair."

When prompted to explain what this meant in terms of the students' understanding of probability Sam said that they didn't understand much, just the fact that you can get different results. He also said that they concluded the dice were fair because

they thought “anything is possible when it’s random.” Here it seemed Sam was able to see that the students related fairness with randomness; but they did not have a clear understanding of how the two are related. He then brought up the fact that they went from 80 trials to 100, which he said was good, but that there was no evidence they understood anything about the law of large numbers.

Poster D. This group also analyzed Dice-R-Us, but they concluded the dice were not fair.

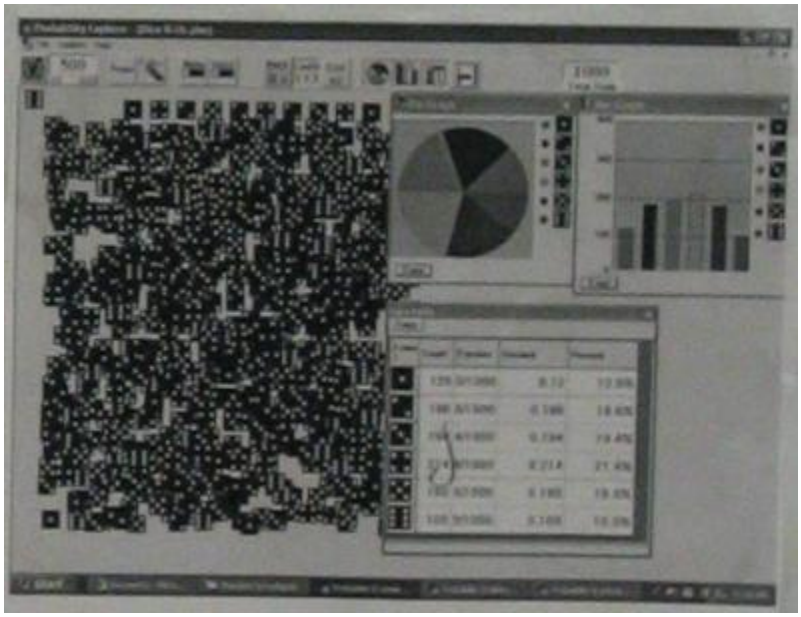


Figure 44. Poster D – Screen Shot Included on Poster

For this poster Sam pointed out that the students “took their empirical data for theoretical probabilities, so for instance, whatever they got they said, ‘oh yeah that’s what the probability is’… so there's no evidence that they understand that your results are

going to vary." He was referring to what they wrote in answer to question 3 (use your data to estimate the probability of each outcome) in the task, see Figure 45.

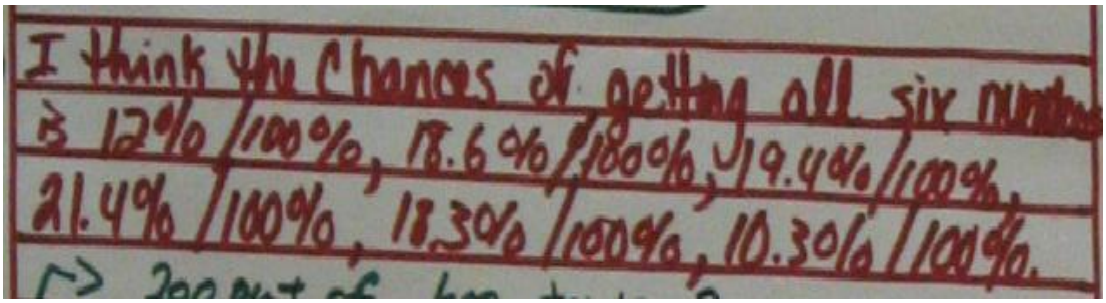


Figure 45. Poster D –Question 3 Student Answer

It is interesting that he concluded there is no evidence they understand the results will vary and it is unclear as to what this may imply about Sam's understanding of the relationship between empirical and theoretical probability.

Finally, Sam said that it seemed this group understood a little more than the group that did poster C, but "not that much more." Also he thought they came to a good conclusion, used the data well and looked at the evidence. Thus, Sam seemed to be able to analyze the students' understanding of probability as being based on the empirical evidence, but he seemed to think it was incomplete due to the fact that they listed the empirical probabilities as, what he thought, were theoretical probabilities.

Comparing Posters. For the comparison of posters Sam did not offer much comment. He seemed to agree with Jeff who pointed out that, from posters A and B, he would be convinced the dice were not fair. Sam added that the evidence in poster B was incomplete since the table only showed results for numbers 1, 2, and 3.

For the comparison of posters C and D, Brad pointed out that poster D used 1000 trials to which Sam agreed saying "and then the number of trials they did." This may indicate that Sam was more convinced by poster D, although he did not explicitly say that. He did however say that he was not convinced by the results in poster C. This it seemed for Sam that sample size plays a large role in his being able to make a conclusion about the fairness of the dice.

In summary, Sam's pedagogical content knowledge seemed to include thinking carefully about students' work and being sure not to jump to conclusions. It also seemed that he does not always know how to interpret students' work. This may be an indication of a lack of confidence in interpreting students' work. Also he seemed to value student dialogue and sees that as a way to gain insight into students thinking.

For the class discussion video clip in Task 5c Sam said he would first ask them why they think the percentages would be the same and then suggested giving the students a different example to help them understand that the percentages would not always be the same. However the example he chose did not seem to relate to the concept:

> Take weather for example. Look at how many times it rains in a week and maybe it was a wet week and it rained 5 out of the 7 days, so ok does that mean that it rains almost every day in the summer? And some kids are going to say, 'well no we have a lot of sun,' and then we'll say well you can't just look at a week and then expand upon that.

It seemed Sam is trying to make the point that you can't base your conclusion on a small sample. As for the effectiveness of this example, weather may actually be more confusing to students than understanding fairness of dice. While this is a good analogy in the sense that the theoretical probabilities are unknown, it does not quite explain the concept of the percentages changing based on the sample size.

On the other hand, Sam seemed to be displaying knowledge of students by wanting to use an analogy that they may relate to. Weather is something every student has experienced and he may see that as something to build on their previous knowledge.

Teaching Intervention. For the video clip of the teaching intervention, Sam said he wanted to see what the teacher said after the demonstration stopped. "I think that it's nice working with this technology… they did enough of that, ok stop that and then get to the teaching part about it, to the teaching lesson and what's significant about it." He seemed to be thinking that using the technology is not "the teaching part" and maybe he wants the teacher to give the students more direct instruction about what they should be learning. Even though there was teacher input and dialogue between the students and the teacher, Sam seemed to think this is not "the teaching part." One other point he made was that he thought having the students decide how many times to run the experiment was good.

In Task 5d, the reflection, the benefits Sam saw for this task were that it was a real world problem that required students to analyze data. He added, "This type of

problem is much more beneficial than a problem with only a mathematical solution. The problem required students to think critically." It seemed Sam values tasks that have a context and give students the opportunity to think critically and analyze data. The drawback he listed was that the posters were too time consuming to make. "Perhaps students could list their findings and their recommendations while they orally support their answer. Either the students or the teacher could then list small bullet comments regarding the students' findings to be discussed later." From this statement it seemed that Sam may not see the value of having the students display their work in the poster format, which requires students to include evidence to support their claims. However Sam did offer an alternative to using posters that he thought was better. This seemed to indicate that Sam's pedagogical content knowledge includes various teaching strategies, one of those being discussion where the teacher summarizes the information on the board. He also mentioned that the bulleted points could be discussed later, showing he may be thinking about how to use the information in future instruction.

For the next day, Sam said he would lead a guided discussion and have several key concepts "for student understanding that I would guide the students to discover." He did not elaborate on exactly what those key concepts would be or how he would guide the discussion. This response seemed to indicate that Sam feels there needs to be a summarizing discussion on the main points of the exploration that the students were to learn.

What Sam said he learned was that students' thinking varies significantly and their thought processes are unique. Also that "Real-world applications without simple solutions, done properly, can be tremendous learning exercises for students. It is vital that follow-up discussion used to solidify key concepts is done." Thus Sam recognized that students' thinking processes present challenges for teachers to understand how students are thinking about the problem. He also said that a teacher's role is to anticipate areas where students might have misconceptions. "You need to get into the minds of students in order to see things from their perspective. This is a challenging endeavor, but if successful, is most beneficial for the students' understanding." Clearly Sam sees this as an important part of a teacher's role. He also pointed out that follow up discussion is needed, indicating that simply using a real world exploratory activity is not sufficient for students' understanding of a concept or topic.

Task 6: Interpretation of Meaning of Probability. In response to this task Sam revealed a somewhat strong mathematical orientation. He preferred the theoretical statement and for the first statement he said, "I think you should probably say definitively what the theoretical probability is and then relate your simulation and how close that actually did come to your theoretical probability." He went on to add, "So it's not a bad statement but I, well I don't really like it." He also didn't seem comfortable with the fact that, using empirical probabilities, you would get different results:

> I wouldn’t state it like that because someone else rolls the die a million times well they're gonna get a different result. So they're gonna have a different result and their probability of getting a 3 they’ll say it's about something else which is different so you need to clarify that.

It is interesting that he points this out since, with a sample size that large, the variance in those results should be negligible. This seemed to imply Sam’s preference for a “definitive theoretical probability” as opposed to different results that may result from sampling approaches.

For the second statement, Sam said that it doesn’t show anything that whoever wrote it “knows how to figure out the theoretical probability so it's kind of a silly statement.” Again Sam is talking about theoretical probability and dismisses this statement as ‘silly’. Therefore, in Sam’s opinion, just basing a claim about probability on past experiences is not as good as being able to compute a theoretical probability. Further evidence of this preference for being able to find the theoretical probability can be seen in his response to the third statement. Sam said, “Yeah, there you go, there’s the theoretical probability I was talking about. I like that one, that’s nice.” And he put a checkmark next to it; clearly he seemed to favor the theoretical over the others. Also he does not have an appreciation for how probability could be thought of subjectively and even dismisses this approach.

Lastly with the fourth statement Sam pointed out the fact that basing your decision on the most popular answer is not always right. He then gave an example from a game show that he had watched recently. The contestant polled the audience for help in answering a trivia question on the correct spelling of genealogy. "… like 72% of the audience chose the wrong spelling. And you know they went with it so there you go, if you go with the majority opinion that's not always, doesn't always work out to be." Here Sam related this statement to his own prior knowledge of the most popular answer not always being the correct answer.

Sam said he most identified with the third statement before being asked, "3 is the best, it's the best sentence out of all of those." From these responses to this task Sam clearly seemed to favor the theoretical probability over the others, which may be an indication of having a mathematical orientation.

When Sam read the statements in the second part of Task 6 on how to define probability he put a checkmark next to the second statement, "… mostly number 2 it's the fraction of favorable outcomes over the total outcomes. That's uh, you know, most in line with the definition." And added that "in the mathematical sense this is exactly how you want to define probability." This again clearly shows he may have more of a mathematical orientation.

For the first statement Sam said it was not a bad layman definition but "in a mathematical sense it's not correct… if you asked me for the probability of something

that is not definitive." It is not clear what he means by definitive, but he may be referring to the theoretical probability. He then went on to describe a situation from his past experiences where he based probability on his personal judgment:

> I can remember when I was a lifeguard outdoors I used to get pretty good just watching the clouds at the other end of the lake when they were coming and could tell whether or not we were gonna have a storm coming… I'll look up at the clouds and based on my past knowledge and personal experiences I'd give you a degree of what my, of the probability.

Again Sam is relating to past experiences, and while he did not explicitly point it out, he was using weather as an example of something that cannot be determined using theoretical probability.

In fact, when he was asked if there was any reason the second definition (the one he chose as most mathematically correct) wouldn't work, Sam had difficulty coming up with an answer. "(pause 10 seconds) Where it wouldn't work? (15 seconds) Ok, I'm still thinking. (20 seconds) I um, (5 seconds) I don't think so I think that's going to work in all circumstances." So even though he just gave an example of predicting the probability of the weather, when specifically asked he could not think of a situation where the theoretical definition wouldn't work. Here is an example of where his orientation may be influencing his content knowledge. He seemed to have a pretty strong belief that using theoretical probability is the correct way to figure out probabilities and thus he can't give

an example of when this would not be valid, even though he used such examples several times throughout the study.

The researcher continued to probe further and asked if he could explain what would be needed for that definition to work (such as knowing the sample space and having equiprobability). When the question was worded in this way he pointed out that "you need to determine the number of total outcomes" and then went on to describe a situation where that might not be possible.

> In some circumstances like um let's see, you have a worker and I don't know he's doing something, stacking boxes, and he has so much left to do and you ask 'hey what do you think, what's the probability he's gonna be done by 5 o'clock? You know, or is he gonna have to work overtime? Um, there counting your total outcomes is pretty difficult. So, I guess in a circumstance like that it's gonna be difficult using that definition to come up with the probability.

In this example it is unclear what Sam thinks the favorable outcomes would be, other than the worker finishing on time. He went into further detail that there are other variables such as variability in the sizes of boxes and that determining the total outcomes "is next to impossible." The researcher asked if he could think of another approach to figuring out the probability in this example, such as running simulations, and he could not think of anything, "I'm kind of stumped by that."

For the third statement Sam said that this was referring to the law of large numbers and illustrated the concept by drawing Figure 46.

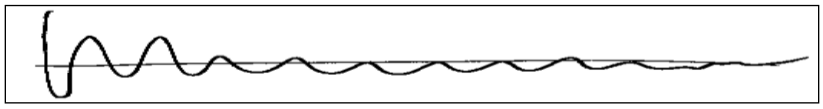


Figure 46. Sam's Illustration of the Law of Large Numbers

He explained, "what I get out of that is like it's the law of large numbers where here's your average [draws a horizontal line] you're basically you're approaching it and your wave so to speak is stabilizing." His illustration and explanation seem to indicate he understands that the variability decreases as the number of samples increases. Thus he seemed to have an understanding of how the law of large numbers works. Additionally he explained that the "hypothetical number" meant the expected value. "It says it's the hypothetical number, that's not probability, the hypothetical number, what it's talking about there is your expected value, so that's wrong too."

As for the last statement Sam said "no, no that's not probability at all. That's the variance so that one's totally false, so, just get rid of that guy," and he crossed it out on the paper. Sam seemed pretty sure that this statement could not be used to define probability. Therefore Sam's combined responses for both parts of this task seem to indicate he has a mathematical orientation.

Task 7: Curricular Issues Revisited. In explaining what concepts he would include Sam said he would start with randomness, underlined the word twice as well as

put a checkmark next to it. After a 40 second pause he added, “ways to count events and theoretical probability.” He added “everything that’s tied into empirical or experimental probability, that goes into variation.” Finally he reiterated the importance of counting events, “counting leads to favorable outcomes and knowing how to count the total number of outcomes.” These concepts are similar to what he suggested in the second task from the beginning of the study, however, here he included empirical probability. It may be that his participation in Task 5 influenced this addition. Yet most of these concepts seemed to be related more to a mathematical orientation.

As for what strategies he would use, Sam said that using different representations is important. He listed different ways data could be represented – histograms, box plots, and charts – and added, “There may be a better way to do that, um but it's just kind of the standard, ‘ok we have data, ok well how do we show it and take some measures of it?’” He seemed to be referring not specifically to probability but more to data analysis. The researcher asked what he meant by “a better way” and he suggested doing an activity where students “get your results and have them list their results and then take it from there and take those results and how you can represent them differently.” It seemed he thinks probability would be taught better if students collected their own data and then looked at different representations; rather than being given data. He did not elaborate on how students would get their results, whether they would run simulations of experiments or research to find a data set.

In response to what Sam thought he learned from his experiences in this study he said, “I think it's given me the time, or I guess a chance, to look into probability in more detail and to actually consider what's important for teaching at an early age.” Further probing into what Sam learned about what's important for teaching revealed, “you have to be flexible.” The reason for this flexibility he said was because students learn differently. “There's a certain way that people understand things best; that doesn’t work out for everybody else, or for everybody. So, yeah just be flexible and open to the understanding that the students currently have.” He elaborated that as a teacher you may be thinking in a certain way and that students may have a good understanding but it is just not the way you think about it. “It's not how you would picture it to be and you gotta be flexible and be able to understand that, I guess learning to see how students and that group that you're dealing with view things.” And also that students’ peers may be able to explain better than the teacher.

From these comments it seemed Sam’s pedagogical content knowledge includes the importance of making sure to address different ways students learn. Also he seemed to believe that it is important to realize that students may interpret things differently than the teacher, but this doesn’t necessarily imply they don’t understand.

Task 8: Multiple Choice and Free Responses from Pre/Post Tests. Each of the four questions in this first part of Task 8 Sam got correct on both his pretest and posttest. This indicates that at the beginning and end of the course Sam had an understanding of

how to interpret simple probabilities as well as a range of probabilities and independent events.

On the second part of Task 8 Sam got all three questions correct on both his pre and posttests. Thus Sam had an understanding of the effect of sample size, proportional reasoning, as well as the expected variability in a distribution of a sample statistic from a fair coin toss experiment.

However for the third part of this task, the questions related to pedagogy, Sam got the first question wrong on both the pre and posttests. He chose (a) that group 1 used an appropriate strategy. What is interesting though is that in his pretest critique of each strategy he said that group 1 "although a good strategy, lacks a sufficient sample size to draw accurate conclusions." And in his posttest critique he said, "Group 1 is using an appropriate strategy, however, the number of trials is too low to realistically expect the empirical probabilities to match the theoretical probabilities." So although he said that he thought they were using an appropriate strategy, he pointed out that the sample size they chose was too small. Perhaps when he said they were using an appropriate strategy he was referring to the fact that they said they would compute the empirical probabilities and compare those to the theoretical probability.

On the pretest the weaknesses Sam pointed out for group 2 was that their sample size was too small and "the criteria used to determine if the die is fair is faulty statistically." As for the posttest Sam said mostly the same thing, adding, "While you

would expect each number to appear $1/6^{th}$ of the time, this doesn't mean in 6 trials each number will appear. The trials are independent and shows prior results have no bearing on subsequent rolls." Thus it seemed Sam has an understanding of independence as it applies to rolling dice and also that in 6 trials it is not likely each number will appear.

Thus in terms of Sam's pedagogical content knowledge, he seemed to have difficulty analyzing student work. He indicated that group 1 had an appropriate strategy yet pointed out that their strategy was faulty.

Summary of Sam's Case

Orientations. Sam seemed to display more of a mathematical orientation in his responses across the eight cases. See Table 8 below for counts of indications of different orientations within each task.

Table 8 *Summary counts for Sam's orientations by task*

Tasks	Statistical	Mathematical	Subjective	Personal
Task 1	2	2	0	0
Task 2	0	1	0	0
Task 3	1	0	0	0
Task 4	1	1	0	0
Task 5	0	0	0	0
Task 6	0	3	0	0
Task 7	0	1	0	0
Task 8	0	0	0	0
TOTAL:	4	8	0	0

The concepts that Sam listed as being important to teach tended to be more mathematically oriented – using theoretical probability and counting methods. Also, in

Task 6, when he analyzed the different statements he seemed to show a strong bias towards theoretical probability and he seemed to be unable to think of a situation where theoretical probability could not be applied. It is interesting that with his strong background in statistics that he would display more of a mathematical orientation than a statistical one. However, there were a few instances where he seemed to be more statistically oriented. For the cars task he based his decision on the data and for the candies task he chose to use a sampling method. It may be that the way Sam was taught statistics did not afford many connections with probability that were not theoretical in nature.

Content Knowledge. With respect to his content knowledge Sam seemed to have a strong understanding of statistics and a fairly strong knowledge of probability. This is not surprising given his background in statistics. On his responses to tasks where he had to make a claim based on data, Sam was able to correctly describe how to employ a repeated sampling technique. He was also able to explain why a larger sample size gives more reliable information about probabilities.

In many of the tasks he referred to the sample size and explicitly the law of large numbers, in both tasks 3 (cars) and 4 (candies) as well as the Schoolopoly task and Task 8 when he drew a diagram to illustrate his point. Thus, this concept seemed to be well understood by Sam and also an important concept within probability.

Pedagogical Content Knowledge. While Sam had a fairly strong content knowledge of probability due to his educational background, he stated that he learned more about how to teach probability.

With regards to his knowledge of content and teaching, Sam expressed that he liked activities that are open ended and allow students to explore and build off their own ideas. However, he also expressed that he thought the actual teaching accompanying these types of activities should be more guided. For example, in the video clip, he thought that after the exploration the teacher should have guided the discussion more and focused the students on the correct ideas. Another important part of teaching that Sam values is the need to be flexible because students learn in different ways. Thus, it seemed Sam has an understanding of teaching probability that includes using exploratory activities, guided instruction and discussion, and an openness to being flexible in order to accommodate different learning styles.

As for Sam's knowledge of content and students, Sam seemed to have some difficulty analyzing students' work in Task 5. He was unsure, from the information given, how to determine students' understanding. He also seemed to be aware of how difficult it is to determine what it is students understand. As can be seen in his comments in Task 7, a teacher may have one way of presenting information, but students may interpret that differently or learn better from their peers. It seemed clear that Sam has a belief that it is

important for a teacher to have a good understanding of what it is their students do and don't understand and that one way to discover this is through dialogue.

Relationships Among Three Aspects

Some common themes that occurred across the tasks were the importance of the law of large numbers, using real world examples, and student thinking and reasoning. Sam had a stronger mathematical orientation towards probability which seemed to impact his content knowledge and pedagogical content knowledge. Additionally there were inconsistencies between his content knowledge and pedagogical content knowledge.

One way his mathematical orientation may have had an influence on his content knowledge was that he could not think of a context in which a theoretical definition of probability would not work. Also in response to the statements about probability he strongly favored the theoretical interpretation and seemed to be uncomfortable with the empirical statement.

Another relationship that could be seen was in the Schoolopoly task where he tended to have difficulty interpreting students' work. This may be due to his mathematical orientation and wanting to know what the 'correct' answers were in order to compare students' answers to some known theoretical distribution. He seemed to be unable to interpret their work and made statements like "we don't know what they understand from this." It could be that because he didn't know what the correct answers

were then he couldn't take a statistical orientation to analyze their understanding from a perspective of making claims based on the data they had collected.

As for the inconsistencies between his content knowledge and pedagogical content knowledge, Sam said that he valued students' different approaches but his content knowledge indicates that he doesn't show appreciation for different approaches to probability. What was noticeably missing was that Sam saw no appreciation for the need for a subjective orientation. He expressed strongly that this approach was "silly" and wrong. This may impact his pedagogical content knowledge in that he may not be able to recognize how students might use this approach when doing probability tasks. Thus this is contradictory to his belief that a teacher needs to be flexible to students' different understandings.

Therefore, with respect to all three aspects of knowledge (orientations, content knowledge, and pedagogical content knowledge), Sam's mathematical orientation seemed to have an impact on both his content and pedagogical content knowledge. Also there seemed to be conflicting beliefs between valuing different student approaches and not valuing a subjective approach.

Chapter Summary

This chapter has presented the findings within each case of a preservice teacher. Analysis of each of the eight tasks was given as well as summaries of the nature of each aspect of knowledge and relationships between these aspects within each case.

Pam represented a case of a preservice teacher that has both a statistical and mathematical orientation towards probability as well as an appreciation for a subjective orientation. These orientations seemed to have an impact on both her content knowledge and pedagogical content knowledge; which at times seemed to be incomplete as in the example of her tolerance for a high amount of variability in a large sample size. From the beginning of the study to the end, by Pam's own account, her pedagogical content knowledge of probability had grown.

Yasmin, in many respects represented a similar case as Pam. She too had equally strong statistical and mathematical orientations yet also showed an appreciation for a subjective orientation. Additionally Yasmin had difficulty assessing the posters and displayed a content knowledge of variability that seemed to be lacking. At the end of the study Yasmin displayed understanding of probability concepts such as the law of small numbers and variability among samples as evidenced on her post-test responses.

Brad represents the case of a preservice teacher that has slightly more of a mathematical orientation than a statistical orientation. He too showed an appreciation for a subjective orientation. In several tasks Brad said that he valued real-world contexts in teaching probability, and he gave the specific examples of newspapers and reading graphs as a real-world application for probability. Strategies that Brad would use when teaching probability included explorations and using technology, yet he expressed he would want more structure.

Jeff represents the case of a preservice teacher that has more of a statistical orientation with some indications of a mathematical orientation as well. Jeff did not show an appreciation for a subjective view and even viewed this orientation as wrong, or incorrect. One concept that seemed to be prevalent across tasks within the case of Jeff was the law of large numbers. As for strategies to teach probability, Jeff suggested using real life examples, technology and experiments.

Sam represents the case of a preservice teacher that has more of a mathematical orientation with some indications of a statistical orientation. His strong mathematical belief may have had an impact on his not being able to see a need for the subjective approach. Sam displayed content knowledge of the effects of sample size, the law of large numbers, and sampling methods. Sam also revealed his knowledge for content and students includes the importance of student thinking and reasoning, as well as the importance of including classroom discourse. One final comment about the case of Sam is that he believes teachers need to be flexible in order to address different reasoning and thinking from students, thus be able to handle situations where students may have conflicting viewpoints.

In the next chapter cross case analysis is given. The focus of the cross case analysis is on the similarities and differences across cases within each task. Additionally relationships across the cases among orientations, content knowledge, and pedagogical content knowledge across cases are presented. .

CHAPTER 5: CROSS CASE ANALYSIS

Introduction

This chapter is organized into two sections: the first section contains cross-case analysis of the preservice teachers' responses to each of the 8 tasks; the second section contains analysis of the relationships among the three aspects of knowledge (orientations, content knowledge, pedagogical content knowledge) across the five cases. For each task, the discussion includes similarities that were evident among all preservice teachers, as well as findings that were unique to one or two cases. Within the second section some common relationships across the cases were related to their orientations and how that related to both their content knowledge and pedagogical content knowledge.

Within Task Analysis

Task 1: Interpretation of Randomness and Probability

Task 1 consisted of several questions about randomness and probability. These included defining randomness and probability in the preservice teachers' terms, giving examples of things that happen in a random way, and interpreting meaning of probability. The following Table 9 shows a brief description of responses to each question within Task 1. The bulleted points represent a brief description of each individual preservice teacher's responses to that question, as described more comprehensively in the analysis in Chapter 4. For example, Pam said that random meant randomly picking a sample.

Table 9 *Brief description of preservice teachers' responses to Task 1*

Question	**Pam**	**Yasmin**	**Brad**	**Jeff**	**Sam**
Defn. of random	• Randomly pick sample • Equal chance	• No order or sequence, • Can't calculate	• Jumbled up- no order, • Expressed difficulty	• Blindly picking a sample, fair • Not-repetitive • unexpected • Difficulty	• Events that happen by chance • Not predetermined
Examples of random	• Lottery (the chances, not the people)	• Weather	• Random # generator	• Everyday lang.	• Tossing a coin or throwing a die - fair
Ex in Stat class	• Everyday lang. (could impact S under. in P&S)	• Equal chance of occurring	• Need random data to avoid bias, to determ. Causation	• One out of a random guess	• Sample – chance of everyone being picked is same
Coin toss	• 50% is 100% divided by 2, tied to fractions	• Half on one half on other, fractions	• Half and half chance (hint of equiprob.)	• Sequence of tosses wouldn't alternate, independent • Over long period of time, • If small SS good chance not 50%	• When toss coin expect half time come up heads (could be thinking of hypoth. exper.)
Weather	• Fractions, equiprob., 7/10	• "means it's gonna rain" • 3 out of 4 days it rained, fractions	• 7 out of 10 times, part to whole	• Better chance of raining than won't • Good chance but not inevitable	• Didn't know – 70% of area? • It's more than likely it will rain
Defn. probability	• # of events out of # possible	• Likeliness something will happen (example lottery, fractions)	• Chance something will happen • Percentage or a fraction that it will occur	• The chances of things happening • If do thing over and over how many times get that result	• A way to estimate events happening that you're unsure of the outcome

Across all cases the preservice teachers as a whole presented a fairly complete conception of randomness, one that included equal chance, not repetitive, no order, unexpected, and not predetermined. They were able to give both 'real world' as well as typical or more theoretical (coins and dice) examples of randomness. The preservice teachers' interpretation of the probability of a coin toss and a weather statement of 70% rain tended to be related to a part-whole interpretation, indicating a tendency to use a more mathematical orientation to interpret a percent chance of an event occurring. In addition, their interpretation of the weather example was more varied, it seemed some did not have a complete understanding and may not have related the interpretation back to the context. And with the meaning of probability, most of them referred to the chance of something happening with the exception of Pam who gave a more classical definition.

One pattern that was found is that for the examples of things that happen in a random way, Brad and Sam mentioned random number generators and coins and dice, whereas the others mentioned real world applications. One possible reason for this is that both Brad and Sam tended to have more of a mathematical orientation towards their understanding of probability and it may be that their content knowledge of randomness is mainly tied to more concrete ways of generating random numbers such as coins, dice, and random number generators.

In all cases the examples given reflected the definitions they gave – Pam said lottery and gave the meaning of random as having an equal chance. Yasmin said weather

and gave the meaning of random as something that couldn't be calculated. Brad's example was a random number generator and his definition included the notion of not having any order. Jeff said everyday language and in his definition of random included the notion of unexpected. And Sam gave the example of coins and dice and his definition of random was events that happen by chance. It could be that this task, sequencing the questions where they have to first define the meaning of random and then give an example, influences their responses. It would be interesting to see responses if the order of questioning were reversed.

In response to the meaning of 50% on a coin toss, as well as interpreting 70% chance of rain, Pam, Yasmin, and Brad connected the meaning with fractions. This seemed to indicate their understanding is tied to part to whole conceptions. Jeff and Sam referred to a hypothetical experiment of a coin toss, which would be expected of Jeff because of his strong statistical orientation but was possibly unexpected with Sam who was mostly mathematical. It could be that this example of a coin toss is so commonly used that the interpretation of the meaning is not impacted by one's orientation.

Yasmin, Brad, Jeff, and Sam all mentioned that probability was the likeliness or chance of something happening. Pam gave a more classical definition of the number of events out of the number possible. Jeff added that if you do something over and over the probability is how many times you get that result. Sam gave what could be characterized as the most sophisticated answer, "a way to estimate events happening that you're unsure

of the outcome." So across all 5 preservice teachers they seemed to have an understanding that probability is a measure of the chance of something happening.

As for the effectiveness of this task, it seemed that having the preservice teachers talk about their understanding of the meaning of randomness may have influenced or had some kind of an impact on their ability to give examples of random events. Also asking preservice teachers to interpret a question about the meaning of 70% chance of rain in their own words seemed to reveal they think of this as a fraction that is devoid of the context. This may mean that more emphasis on interpreting probabilities in real word contexts and as an indicator of a range of possibilities (expected variation) around the probability (e.g., 60-80%) is needed in instruction.

Task 2: Curricular Issues

The second task consisted of two questions pertaining to curricular issues. The first question asked the preservice teachers to explain why they think it is important for students to learn probability, and the second question asked them to brainstorm topics they thought should be included in teaching probability. In the following table key points from the preservice teachers' responses are given.

Table 10 *Brief description of preservice teachers' responses to Task 2*

Question	Pam	Yasmin	Brad	Jeff	Sam
Why important to learn prob.	• Lottery, • Game shows	• Media – analyze data statistics, not be misled	• Weather, • 'to apply to real life situations' • Newspapers, look through the bias	• Used in everyday life • Weather, • Games of chance (casino)	• 'a lot of practical reasons' • Savings account • Investments • Gambling • Life is uncertain
Topics to teach	• Exp. vs. Theo. • Graphs, trend lines • %, dec., ratio • Simulations • How to conduct exper.	• Spread, • Measures of center, • Scale • probability	• the chances something will happen • analyzing graphs • measures center • sampling methods	• coins, dice, • conducting surveys • M&Ms • Coins – H doesn't imply T next, (independence)	• Randomness • Counting techniques • Ind. Vs Dep. • Conditional prob. • Expected values

For reasons as to why it is important for students to learn probability, common responses were that 1) it applies to everyday life such as the weather and real life situations like investments, 2) helps students made sense of data that is in the media and newspapers, and 3) it applies to games of chance: game shows, lottery, and gambling. By asking the preservice teachers why they thought it was important to teach probability, we can analyze their pedagogical content knowledge. It seemed that with these preservice teachers, they related reasons for teaching probability to students' needs and how probability is applicable to their lives. This shows that their knowledge of content and students includes reasons as to how probability applies to students' lives.

The topics they thought were important tended to be more related to statistics rather than solely to probability; in fact Yasmin and Brad listed probability, or the chance of something happening, as one of the topics. The answers were varied with no one thing being common among all five. What this implies about the preservice teachers' knowledge of content and teaching is that their concepts for what to teach within probability are limited mainly to statistical concepts. Sam seemed to be the only one to give a list of probability specific topics and this may be due to his background knowledge of probability and statistics.

Thus as a task, Task 2 revealed that this group of preservice teachers believed teaching probability was important, yet had a limited understanding of what topics to teach within probability. These questions may be helpful as an introduction to teaching probability for preservice teachers to be aware of their own conceptions towards probability. Watson (2001) included similar questions on a survey designed to profile teachers' competence and confidence to teach probability and statistics.

Task 3: Real World Context and Teaching Situation

The third task was an activity that was set in a real world context where the preservice teachers needed to make a decision based on information they were given about the mechanical problems of Toyotas versus Hondas. This task also asked them to comment on hypothetical student answers and discuss how they would direct the conversation in a classroom setting. The following table shows brief descriptions of each

preservice teacher's responses to the questions in Task 3. At times summaries of the researchers' findings are also included.

Table 11 *Brief description of preservice teachers' responses to Task 3*

Question	Pam	Yasmin	Brad	Jeff	Sam
Which car would you buy, why?	• Honda – 400 cars, trust the data	• Honda – 400 cars	• Honda – 400 cars, comments could be outliers	• Honda – can't base it on one person, 400 cars	• Honda – 400 cars, plus other is small SS & 'bad luck'
Response to student answers	• a) may be an error not taking CR into account (unsure if error or not) • c) liked they took everything into account but rather have an answer • uneasy – wouldn't be able to tell class b you're right or c you're right	• a) point out the CR conflicting data and ask them what they thought about that • c) talk about fairness of coin, could be weighted (although she did not say how that would help with this context of cars)	• gave possible reasons for each answer 'no right or wrong': • a) CR could be biased • b) same reason he put • c) could be correct based on 'somewhat fewer' being vague and other possible variables • uneasy – look at all the variables and make educated decision	• a) 'what information do we have that can help us?' refer them back to CR • c) there's always a chance, but what would give you the greater chance of not having problems? • Not a causal relationship (1 bad comment doesn't mean you will also have a bad car)	• Liked that they said why, would be interested in their reasoning • a) ask if there were 99 other people you didn't know about…? LLN and sample size • b) 'right on' • c) ask them what *would* convince them?

In response to which car they would chose, all five of the preservice teachers said they would buy the Honda and based their decision on the Consumer Reports study. They specifically pointed out the fact that the study used a large sample of 400 cars. Three of the five, Brad, Jeff, and Sam – also mentioned the other comments as being "outliers" or "too small a sample size" and "bad luck." Therefore, it seemed that all of these preservice teachers have the content knowledge that one should base decisions on larger sample sizes when interpreting data.

Also this question was in the context of a real-world situation where they were not given the actual results of the study to explore but had to make conjectures based on the reported findings. This type of task is indicative of what students may encounter when needing to make decisions based on information reported in the media or newspapers. Thus, it seemed that these preservice teachers may be able to apply an understanding of sample size to interpretation of results from a study in making a personal decision.

The second part of this task asked the preservice teachers to comment on hypothetical student responses and then explain how they would direct a classroom conversation. One preservice teacher, Pam, explicitly stated her uneasiness with this part of the task and said, "I wouldn't be able to tell the class (b) you're right or (c) you're right." And Brad seemed uneasy with the differing responses and said there was no right or wrong answer; but he was able to offer hypotheses as to why the students might think their answers were correct. This suggests that preservice teachers' pedagogical content knowledge for responding to such varied responses may be lacking. Additionally, their confidence in dealing with varied responses seemed to be low.

Some of the strategies the preservice teachers said they would use to direct the conversation were: to direct students' attention back to the study, ask questions to lead them to a correct conclusion, and to talk about fairness. Two of the preservice teachers (Jeff and Sam) had an inquiry approach to directing the conversation and, rather than telling the students what to look at in the problem, they asked questions that would

require the students to think about their own answers. For example, Sam asked "what if there were 99 other people you didn't know about?" This question would confront the students' misconception that they should base their decision on their friends' comments. Therefore, this task revealed that the preservice teachers' knowledge of content and teaching included several different teaching strategies for dealing with varied teacher responses.

In light of the preservice teachers' uneasiness with having conflicting responses, this task might be useful in a teacher education course to promote discourse among preservice teachers. This would give the preservice teachers' several strategies for handling these types of situations in their own classrooms, and therefore possibly increase their confidence level in engaging students in a discussion based on their intuitive ideas.

Task 4: Experimental Context and Critique/Plan

For the fourth task the preservice teachers were given a bowl of candy and asked to make a claim about the probability of choosing a red candy. They were then asked questions regarding the task: would you use this task in teaching, and how would you use this task? The following table contains abbreviated version of each preservice teachers' responses to the task, as discussed in detail in Chapter 4.

Table 12 *Brief description of preservice teachers' responses to Task 4*

Question	Pam	Yasmin	Brad	Jeff	Sam
How make claim about red candy?	• Sample 10 but not more than 20 • Do that once	• Make sure mixed up • 4 flavors – 2 red, so 2 out of 5 • Interchanged flavors and colors	• You'd have to know the initial distribution (seemed unfamiliar w/how to make claim) • Could count all but time	• Count how many diff colors – the distr. Would have to be equal (didn't assume was) • Separate out and count all	• Make sure mixed up • Take sample b/c time prohibitive to count all
Experimentation	• yes	• repeated samples of 10 • 'keep repeating until get average you feel accurate' • Organize data in table	• Sampling method drawing 30 at a time • (suggests this but does not say what do w/data) • Could count each flavor and assume equal chance	• Responded w/teaching scenario – go around and have each student draw a candy • Teacher writes results on board	• Takes a sample of 30 and counts 11 red and 19 others, ~37% • Probably want to repeat again many 5 times a take avg. of proportions
Would you use this type of task? Why?	• No – kids are overexposed to counting situations • Couldn't see the purpose • Concerned she couldn't adapt the task	• Yes b/c idea of randomness • They could actually take samples • Colors make it very visual	• Yes – no simple answer, requires students to think about diff. ways • "by just thinking through the problem, coming up w/diff solutions and testing out each soln."	• Yes – so many diff. colors you'd have to do it many times to find 'somewhat accurate' results •	• May be a good introduction to understanding proportion and likelihood of an event
How would you use this task?	• Wouldn't	• Good opening task • Topics to explore: spread, sampling & randomness • Good visual for randomness • Could dump out and count but time-consuming	• "pretty much the same way it's done here, give them a simple exploration" • Give them more information	• Separate them into groups, give each one a pile	• Get several groups to get diff answers and talk about why that occurs • Could lead into discussion of SS and repeated samples • Look at spread

When asked to make a claim about selecting a red candy from a bowl where they did not know the distribution, most of the preservice teachers said they would use an experimental sampling approach. Yasmin and Sam pointed out that first they would need to make sure the bowl was mixed up in order to get a random sample. The level of sophistication with a sampling method varied over the five preservice teachers. For example, Brad initially seemed unsure how to make a claim and suggested counting all of the candies and only after being asked if one could use experimentation did he offer a sampling method. Pam said she would take a sample, but when asked if she would repeat the process she said she said no. So Pam had a somewhat limited understanding whereas Yasmin and Sam seemed to display a more complete understanding of repeated sampling by saying they would repeat the process and look at the average of the proportions.

Four of the preservice teachers said they would use this task, or a similar task, when teaching probability, all except for Pam. She stated that she thought students were overexposed to tasks like this and she could not see the purpose. This relates to Pratt's (2005) assertion that a task should have utility to be most effective. While Brad said he would use the task, he was vague in his explanation as to how he would use it other than to say he would have them do "a simple exploration". Additionally, Yasmin and Sam said they thought this may be a good opening task to introduce students to concepts such as spread, randomness, proportions, and the likelihood of an event. They both indicated that

they would have students take repeated samples and collect the data. These strategies give an indication of their pedagogical content knowledge as it relates to teaching, namely, that they would have the students work cooperatively and experiment with the data.

Therefore, these preservice teachers were able to suggest a way to make a claim about a probability from an unknown distribution. Most of them saw the usefulness of a task like this as well as ways to use it within their teaching. This task offers a context where there is a large but countable set and the students can physically sample from the data. A quality that Yasmin said was an advantage for this task. Thus, this task offers teachers a hands-on activity that could be used to introduce students to various concepts within probability and could be used to have teachers consider different sampling methods and benefits or limitations of the methods.

Task 5: Schoolopoly Task

This task was given in four parts, analysis of: the task, student work, a teaching episode, and a written reflection. The findings therefore will be organized by these four parts.

Task 5a: Analysis of Schoolopoly Task

The fifth task was broken into 4 parts. The first part the preservice teachers were asked to analyze the task and discuss the objectives, anticipated student difficulties,

strengths and weaknesses, and if it is a task they would want to use. The following table shows brief descriptions of the answers given by the preservice teachers for Task 5a.

Table 13 *Brief description of preservice teachers' responses to Task 5a*

Question	Pam	Yasmin	Brad	Jeff	Sam
Learning objectives? Purpose?	• LLN, sample size, fairness of dice, analyzing pie & bar (note asked at end after looked at posters)	• Understand concept of something being weighted • LLN and sample size mentioned at end of inter.	• [no comment]	• LLN	• Difference between exper. And theo. • Analysis exercise • Problem solving exercise
Student difficulties?	• Focused on technology – may be more interested in creating colorful charts than focusing on probabilities	• Idea itself is a little abstract – they may understand 1/6 but hard time if weighted	• [no comment]	• [no comment]	• Might not know where to start • May need more guidance
Strengths & weaknesses ?	• Like how open it is "could really see where they're all at in their thinking" • Open-ended as downside as well-lack of direction could 'waste time' • Suggests T hint at using large samples	• Liked real world situation, deciding fair or not • Liked software – gives different ways to look at it	• Not given enough direction, might analyze the wrong part • May focus on %s being the same • May not do large enough samples	• Software allows them to roll dice "a bunch of times"	• Real problem – not just rolling the dice, there's a problem behind it • Students can decide how many trials to run, what data's important, make their own deter-mination
Would you use this task? Why?	• Yes "I think it's a phenomenal task, really really good" • b/c get better understanding what St. think • background knowledge gambling	• would b/c fun and interesting • brings out a lot of discussion • "topics flow from one to the other" (randomness, fairness, LLN)	• Yes – good b/c weighted is locked – so they're actually testing, actually doing an actual experiment	• Yes – didn't elaborate as to why	• Yes – liked Brad's comment about it being a real experiment

Some of the common objectives that the preservice teachers thought the Schoolopoly task was meant to address were: the law of large numbers, and sample size. It should be noted that Pam and Yasmin were asked this question at the end of the interview and thus added objectives that were most likely influenced by their analysis of the posters and video clips. For example, Pam gave the objective of analyzing pie graphs and bar charts and Yasmin mentioned the concept of a probability being unequiprobable. One conclusion that could be drawn is that, for this task preservice teachers may have a greater understanding of the possible objectives after seeing student work and video of class discussion.

For the student difficulties there were no common answers among the preservice teachers, however neither Jeff nor Brad specifically answered this question. The strengths and weaknesses tended to be contradictory. That is, they liked that this was an open-ended exploratory task yet at the same time thought that the students might waste time by not being focused on using large samples, and thus the teacher may need to give more guidance. On the other hand, Sam mentioned how he liked the minimal guidance and the fact that the students could decide how many trials to run and therefore make their own determination.

Besides the task being open ended, another strength that was given by three (Yasmin, Brad, and Sam) of the five preservice teachers were that it was a real world problem and not just an exercise. They also liked that the students were "doing an actual

experiment" which again relates to what Pam said in Task 4 and what Pratt (2005) discusses in his research about the need for a task to have a purpose.

One final note about the task analysis is that the use of the software was given as both strength and a weakness. Pam thought the students might get distracted by the technology and be more concerned with creating charts rather than focus on the probabilities, whereas Yasmin liked the fact that the software allowed the students to see different representations. Also Jeff brought up the fact that with the use of technology, students were able to run many trials in a short amount of time. From these responses it seemed the preservice teachers' pedagogical content knowledge of teaching includes the ability to recognize and consider positive and negative features of using technology for teaching probability. They also were able to offer some ideas about students' difficulties with the task as well as the concepts.

Task 5b: Analysis of Student Work on the Schoolopoly Task

The preservice teachers were shown four posters of students' work on the Schoolopoly task. They were asked to give their analysis of students' understanding and what they thought the students considered compelling evidence. They then were to compare the first two posters to each other, and the second two posters to each other, and then tell whether or not they were convinced by any of the posters. The next table shows how each preservice teacher analyzed the students' work. Each poster is included as well

as their comparisons of posters A & B, and C & D. Also their responses to whether they were convinced are given.

Table 14 *Brief description of preservice teachers' responses to Task 5b*

Question	Pam	Yasmin	Brad	Jeff	Sam
Poster A	• Understood fairness as being even • Understood effect of SS • Variability b/w 120 and 240 too high • Associating it w/ gambling	• Ran a lot of trials so assoc. with LLN • They saw pattern as ran more trials • Their evidence was percentages	• Students were looking at LLN b/c they used 1000 trials • Students didn't have correct terminology •	• Students understood to use a lot of trials but "didn't know how to word it" • Consider percentages to be uneven to be compelling evidence	• Students explained the gap "don't know if they understood the significance in comparison to # of trials" • Sam careful not to assume understanding
Poster B	• 80 not enough samples, • Fairness associated with even spread • Didn't Notice didn't add up to 80 • Confused how two posters same company	• They didn't understand that one shade in pie graph being larger indicated not fair • They didn't look at all numbers • Only did 80 trials • Yasmin did not pick up on sum $66 \neq 80$ • They understood to be fair all have to be approx. equal	• Students only looked at same chance of each die and not the 'big picture' • Students felt the numbers were basically the same was evidence for fair • If all numbers were 11 this wouldn't add up to 80 • One number would have to be around 25 – big difference	• "all together that's only 66 outcomes" • Students said one shade in pie graph was larger but didn't say by how much • Seems to be evenly distributed "unless every other one is bigger" – could be referring back to A's results • Students didn't look at whole set	• They only ran 80 trials • Looking for any possible difference • Data did not add up "you would hope they check that probabilities add to 1" • Suggested students ran out of time and only put up 1-3 • Would give them another chance to redo

Table 14 Continued

Poster C	• "numbers are fairly close in range" so fair, but not for reasons they give • Understood need to do a lot of trials • Recognized St. falsely basing conclusion on comp. single pt.	• Expressed difficulty in interpreting students' understanding • Students don't understand picture as whole • Made claims about stud. Understanding fairness tied to randomness	• Students were looking at difference of one number and concluding fair • Using two examples "definitely not enough" to conclude fair • They didn't look at the percentages, only focused on number of times 3 occurred	• Students focused on variation between two sets of trials • Their approach was diff. than others (again comparing posters) • Students were focused on #3 and didn't look at others • Variation of %s was compelling evidence	• Students didn't understand much, just fact you can get diff. results • Students saying b/c variation in %s between sets of trials dice are fair • Concluded fair b/c "anything is possible when its random" • Good that went from 80 to 100 but no evidence they understood LLN
Poster D	• They didn't know how to analyze graphs – look even to Pam • Did understand how to calc. emp. prob. • Pam's CK may be hindering her PCK	• Students understood dice were unfair b/c one number showed up more than rest • Can't tell if they understood fair (Yasmin displayed careful analysis) • Understood percents • Variation and deviation from theoretical	• Students seemed to be getting at LLN b/c used 1000 trials • Brad hinted at idea of sampling method • Students didn't understand percentages	• Students were mixing %s with fractions • Large sample size – two sets of 500 and accumulated results – shows LLN • Jeff understands effect of sample size • Referred back to posters C&B and said these students had better understanding	• Students "took empirical data for theoretical probabilities" • No evidence they understand your results are going to vary • This group understood more than group C • Students came to a good conclusion, used the data well and looked at the evidence

Table 14 Continued

Comparing A&B	• Couldn't understand how two posters did same company • Agreed with A b/c more trials	• A "had a better sense of what was going on" • Disagreed with A and thought dice were fair • Seems to have high tolerance for variation	• Poster B's findings were not believable • Number of trials in B was too low	• Preferred A over B b/c of number of trials A did • Poster A was more "accurate" • In poster B they "wanted to make it look all even"	• Agreed with others that more convinced by A than B that dice were unfair • Evidence in B was incomplete
Comparing C&D	• She concluded dice fair but said D was opp. Of what data said • Said give a little more credit to C	• Poster D was "on the right track"	• Convinced by poster D b/c amount of variation from 10% to 21% in 1000 trials	• All agreed D was more convincing and not convinced by C	• D understood more than C • Number of trials in D 1000 • Not convinced by C
Are you convinced?	• Agreed CC was unfair (from A) • Not convinced by either C or D – concluded DRU were fair • Interesting – range of variability contradictory	• Not convinced by either A or B • A had more variation than D, but she thought D was unfair and A fair • Note: after watching videos said A was unfair	• Convinced by D – shows an understanding of LLN	• Agreed with D Evidence on D was "much more compelling"	• Convinced dice in posters A&B were unfair • Not convinced by poster C • Agreed with others that D was convincing

The analysis of the posters was the part of Task 5 where quite a few conclusions could be made about the preservice teachers' pedagogical content knowledge as it relates to students. It seemed that all five cases had varying degrees of ability to assess students' understanding of the concepts based on the work that they displayed. Additionally it seemed that the preservice teachers' content knowledge and orientations may have impacted their overall ability to analyze the students' work.

One example of how content knowledge influenced knowledge of content and students was with Pam and her analysis of the students' work in poster D. It seemed that she had a higher tolerance for variability even with high samples and thus erroneously concluded that the students' understanding was incorrect. Another example of how orientations affected knowledge of content and students was with Sam. He seemed to have difficulty drawing conclusions about the students' understanding solely based on what they had displayed on their posters. Having a stronger mathematical orientation may have made it more difficult to analyze student work where he did not know what the correct answer was.

Some other findings from the analysis of the preservice teachers' critiques of the student work were that, for Pam and Yasmin, having the two contradictory posters caused them some confusion as to whether the dice were fair or unfair. They both seemed to be confused as to the amount of variability that was displayed as well as the strategies the students used. This design factor for the task of analyzing student work – namely having two contradictory posters – can thus be effective in confronting preservice teachers' notions about sample size, variability, randomness, and evidence of fairness. Also as noted in the table above, Yasmin changed her mind about poster A after watching the video clip. More discussion of this finding is in the next section.

Task 5c: Analysis of Schoolopoly Teaching Episode

For the third part of Task 5 the preservice teachers were shown two video clips – the first one was of a class discussion and they were asked how they would direct the conversation at the end (when a student said that the percentages would be the same regardless and the other students agreed). The second clip showed the teacher demonstrating with *Probability Explorer* and different sample sizes. They were asked to critique the demonstration and say whether they thought it was effective and what the teacher could have done differently. In the next table responses to Task 5c are given. For some questions a few of them did not have a response because of the fact that they were in a group setting.

Table 15 *Brief description of preservice teachers' responses to Task 5c*

Question	Pam	Yasmin	Brad	Jeff	Sam
Critique discussion	• Student talking about gambling understood need to run many trials • Other student didn't under. LLN • Thought kid got 'picked on' – awareness of group dynamics • Appreciate contextual constraints	• One kid understood LLN • Others didn't understand , said things like "sample size shouldn't matter" • Yasmin thought they were assoc. with randomness	• May have thought students were comparing % w/in data set rather than to a theoretical	• One student seemed to understand LLN but didn't know how to put it into words	• [No comment]

Table 15 Continued

Direct conversa-tion	• Ask class pos. and neg. of James comments • Take focus off gambling • Aware of strong belief that number of times doesn't matter – what if only 4 trials?	• (w/o being prompted) Have them look at size 6 and then 1000 and compare • Purposeful examples to confront students' misconceptions	• Suggested having students calculate theoretical and compare that to the results (combining M and S approach)	• [no comment]	• Ask them why they think the %s would be the same regardless • Suggested giving a diff. example like weather – relating to students prior knowledge
Critique teaching clip	• Pam noticed the demeanor of teacher – authoritative – good considering class dynamics • Thought strategy of letting students make decisions was effective • Build off of students' under-standing	• if the teacher had gone to 1000 or 2000 it would've been effective • Good that the T stopped during the experiment and asked "what do you think now?" • That could help them assoc, random with LLN • Some students were resistant to change minds	• Good that students were allowed to guide the number of trials • Picked up on one students saying "I guess my theory wasn't right" after seeing different results with different sample sizes	• Teacher had a good approach of using low numbers at first and then a large number of trials • Could not tell from video whether students understood or were getting more confused • Attended to both teacher actions and student understanding	• Wanted to see what teacher said after demonstration stopped • "they did enough of that- stop that and get to the teaching lesson and what is significant" • Thought having the students decide how many times to run experiment was good

The critique of the video clips of class discussion and the teaching episode offered insight into the preservice teachers' content knowledge of both students and teaching. When asked how they would direct the conversation, before having the chance to view the second clip of the teaching episode, some suggestions were to ask questions and to offer other examples that were analogous to help the students understand that the percents will not be the same regardless of the sample size. Therefore their knowledge of content and teaching includes using inquiry and similar examples that relate to students' prior

knowledge. For example, Sam suggested using the weather as a similar example to which students could relate. Posing this question to preservice teachers seemed to be useful for getting them to think about ways they could address similar situations in their classrooms.

Some of the comments about teaching after viewing the clip were related to the demeanor of the teacher, the strategy of stopping the simulation to ask the students what they thought at that time, and the teacher had a good approach of using small numbers at first and large trial sizes. These are factors of teaching that may not have been brought to the preservice teachers' minds had they not viewed the video clips. This lends support for using real examples of teaching in teacher education.

For the preservice teachers' content knowledge of students most of them said that the one student seemed to understand the law of large numbers. Additionally, Pam and Jeff brought up the fact that the student didn't have the correct vocabulary and used the context of gambling to explain his reasoning. Thus these two preservice teachers were able to make a hypothesis about this particular student understanding the law of large numbers even though the students didn't use the 'correct' vocabulary. One other aspect of knowledge of content and students that was displayed in this part of Task 5 was the belief that having students choose the sample size was beneficial in confronting students' misconceptions. By giving the students control over the size of the experiments, the students can better understand the concepts.

This part of the task, watching videos of practice, seemed to have an impact on the preservice teachers' content knowledge as well. In particular with Yasmin she initially did not agree with poster A, saying that groups' conclusion that the dice were fair was incorrect. However after watching the students' discussion and the teaching videos Yasmin said that she changed her mind and agreed that poster A was correct.

Thus it seemed that after watching these video clips the preservice teachers were able to add to their pedagogical content knowledge and in one case their content knowledge. By seeing the teaching in context and in action, several preservice teachers' gave more of a critique of the task as it related to both the teaching and students' understanding. It also seemed that these preservice teachers now may have a better sense of how to include this type of task in their future teaching repertoire.

Task 5d: Reflection

The fourth part of the Schoolopoly task was a written reflection where the preservice teachers answered the following four questions: 1) what are the benefits and drawbacks if you were to teach this task?, 2) based on the students work and class discussion, what would you do the next day?, 3) what did you learn by participating in these activities?, and 4) what did you learn about your own understanding of probability and teaching probability? Summary of the preservice teachers' responses to the reflection are given in the following table.

Table 16 *Brief description of preservice teachers' responses to Task 5d*

Question	Pam	Yasmin	Brad	Jeff	Sam
Benefits and drawbacks	• Open-ended poster → see student thinking • Class discussion → "chance to teach peers and voice disagreements" • Drawback – may not get to LLN on their own	• Allows students to learn LLN for themselves – (engaging students in explorations lets them construct own knowledge) • Drawback – video never made clear connection to LLN	• Allowed students to explore and make own conjectures • Apply to real life situation • Drawback – too free to explore and came up with "off the wall stuff"	• Allowed students to see LLN for themselves • Drawback Exploration may confuse students if they strongly believe that the SS is irrelevant • Despite drawback students would learn concept by end and retain info. in long run	• Task more beneficial than problem with one solution – required students to think critically • Drawback – posters too time consuming – alternative: have students list findings and orally support answer, then teacher write on board
What do next day	• Reiterate WHY the class came to the conclusion they did (focus on reasoning, values process) • Address mistakes in judgment and reasoning on posters – want to get better understanding of student thinking	• Demonstrate how # of trails relates to accuracy of % • Do similar experiment as a class (build on students' current knowledge as a class)	• Compare theoretical to empirical • Ask questions: "how many times should # come up, if it doesn't happen that way then why, is this difference significant enough?"	• Go over different examples involving LLN to see how well they retained the information (did not elaborate on 'go over')	• Lead guided discussion on several key concepts (did not specify) • Feels there needs to be a summarizing discussion on the main points
What you learned	• Students bring background knowledge and give input that's not always accurate • Students strong beliefs may influence others and makes teaching harder	• Learned student exploration and guidance – "students can get on right track but not see everything correctly" • Teacher must allow students to figure things out but still guide	• Learned most students do not have vocabulary to explain their thinking • Students are creative and stand by their conclusions • "took away a diff. approach… would use this task"	• "exploration activities can be eye-opening for students and allow them to understand concepts w/out being told" • Technology can strengthen an exploration – more compelling data, save time	• Student thinking varies significantly, their thought process are unique • Real world applications can be "tremendous learning experiences" • Vital to follow up w/discussion

Table 16 Continued

What learned about own under-standing	• Said her own understanding of concepts did not change (thought she had a good grasp of LLN) – however this task, which was designed to improve, did not • Learned a LOT about teaching – plan on giving students opportunity to talk in order to evaluate understanding	• Learned the number of trials relates to the probability • "teaching students can be tricky, need to make sure they understand clearly" • (teaching may be tricky b/c students have strong beliefs that may be difficult to confront)	• "not as sharp on probability as I used to be… forgotten terms" • This task reinforced that there is not just one solution to any problem	• "Learned that tech. is essential to teaching probability b/c gives opportunity to explore data and discover concepts on own like LLN"	• "my understanding of probability is very refined" • A teacher's role is to anticipate students' misconceptions • A challenging endeavor to "get into the minds of students to see things from their perspective"

The responses to the reflection questions offer more evidence of the preservice teachers' pedagogical content knowledge. After having some time to reflect, and for three of them (Brad, Jeff and Sam) the opportunity to think individually, some additional points were made. For example, Pam said that one benefit of using this task is that it gives students the chance to teach their peers to voice disagreements. Also Sam suggested that the posters were too time-consuming, an opinion that he did not share in the group when analyzing the work. One possible hypothesis for Sam not wanting to do posters is that he seemed uncomfortable making assessments of students' understanding based on the evidence the students chose to put on their posters.

And finally asking the preservice teachers what they learned in this process as well as the meta-cognitive question about what they learned about their own learning, gave them the opportunity to think about the task as a whole and gain insight into their own thinking. However, some (Yasmin and Jeff) of the preservice teachers answered the final question in the same way they answered the third question. Both Sam and Pam believed that their understanding of probability was strong before working on the task and that their learning was more related to the teaching aspect of probability. While this may have been the case with Sam, it was not true with Pam. In fact her content knowledge of variability within large sample sizes was lacking as evident in the way she analyzed student work. And her participation in this task, which was designed to possibly confront misconceptions, did not change her understanding or awareness of this misunderstanding. This lends support to the argument that strongly held notions and understandings can be difficult to change (Fischbein & Gazit, 1984; Philipp, 2007; Thompson, 1992; Watson & Moritz, 2003).

Task 6: Interpretations of Calculating and the Meaning of Probability

The sixth task was related to interpreting the meaning of probability and was in two parts. In the first part the preservice teachers were given four statements about calculating probability that they were to comment on and then identify the statement they agreed with the most. In the second part the preservice teachers were give four statements

about the meaning of probability to comment on and then identify the statement they agreed with the most.

Task 6a: Interpretations of Calculating Probability

The following table shows the preservice teachers' responses to the first part of Task 6. In the first row their comments are numbered to correspond with the statements in the task.

Table 17 *Brief description of preservice teachers' responses to Task 6a*

Question	Pam	Yasmin	Brad	Jeff	Sam
Comments on calc. prob. stmts.	• 1. exper. Probability, "pretty good way" • 2. not very good reasoning, 10% not a "horrible guess" • 3. pretty good reasoning, theo. Probability, "this is more thinking about it" • 4. not thinking about it at all, basing on others' opinions	• 1. "person did the experiment so they're actually trying" (opp. of Pam) • 2. Not basing it on concrete data but wouldn't say they're wrong • 3.person assumes dice fair • 4.5 people assumed dice fair, others considered that it may not be fair	• 1. Empirical probability, getting close to theoretical, "so many rolls it's going to be accurate" • 2. "this guy doesn't play enough, or not using fair die" • 3. Theoretical, "but didn't say fair" • 4. "should've done more experimentation"	• 1. Agreed, accurate percentage, more than enough SS • 2. Inaccurate assumption rough guesstimate • 3. Theoretical, not based on evidence – not backed up by anything • 4. "just surveyed people" It could be small or high if small SS	• 1. Not bad, but didn't like it "should state the theo. 1st then compare" • 2. "kind of a silly statement" • 3. Theoretical, "yeah, there you go… I like that one, that's nice" • 4. Can't base decision on most popular answer, it's not always right
Identify with	• 3rd statement – "I would think there are 6 sides and one of those is 3, so 1/6"	• 3rd statement – "if someone asked me I'd say 1/6"	• 3rd statement – "as long as it's a fair die it should be 1/6"	• 1st statement – "even though time consuming it's backed up by evidence"	• 3rd statement – "that's the best sentence out of all of those"

One interesting finding from this first part of Task 6 is that all of the preservice teachers said they identified with the 3rd statement, with the exception of Jeff. Thus the theoretical explanation for determining the probability of rolling a 3 on a die is the most preferred way. One explanation for Jeff's preference for the 1st statement could be that he has such a strong statistical orientation, whereas the other four were either more mathematical or more evenly split between a mathematical and statistical orientation.

Another finding with this task was that Pam and Yasmin had different views about the theoretical and empirical statements. Pam said that the theoretical way required "more thinking about it" whereas Yasmin made the statement that the person doing the experiment was "actually trying." Considering these two preservice teachers' exhibited the same number of indications of mathematical orientation as a statistical orientation this finding may imply that Pam slightly favors a mathematical orientation and Yasmin slightly favors a statistical orientation.

Task 6b: Interpretations of the Meaning of Probability

This part of task 6 included four comments about the meaning of probability. These were different from the first part because they were pertaining to the meaning and not specifically about how to calculate probabilities. The responses from this part of task 6 revealed that the preservice teachers mostly preferred the second statement which was mainly theoretical. The following table displays the preservice teachers' responses to Task 6b.

Table 18 *Brief description of preservice teachers' responses to Task 6b*

Question	Pam	Yasmin	Brad	Jeff	Sam
Comments on meaning prob. stmts.	• 1. wouldn't agree, "w/prob. There is some interpretation but more guide-lines and tools" • 2. Did agree with • 3. experimental • 4.agreed but said she still had trouble w/randomness	• 1. Seemed unsure – "I think it's correct, it could vary depending on that person's knowledge or life experiences" • 2. Thought it was correct • 3. [No comment] • 4. liked	• 1. "I don't think they would be able to generalize an answer or apply to other situations" • 2. Depends on what person means as favorable" • 3. Referred to LLN • 4. Not sure how one could measure random	• 1. Didn't agree "makes it sound like chance of being anything" • 2. "this would work if it was equal, evenly distributed", very limited in that way • 3. LLN, "definitely a part of probability" • 4. Unsure, then agreed – related it to LLN and samples	• 1. Not a bad layman definition, but not mathematically correct • 2. Put a checkmark next to it, "most in line with the definition", "that is exactly how you want to define prob." • 3. LLN, illustrated • 4. "that's not probability at all, totally false"
Identify with	• 2nd statement – "simplest way to explain it"	• 2nd statement, and 3rd statement	• 2nd statement – reiterated that it depends on how define favorable	• 3rd and 4th – b/c relate to LLN, ruled out 1st and 2nd	• 2nd statement – [based on comments above]

As was the case with the first part of Task 6, the preservice teachers tended to prefer the second statement with the exception of Jeff. In fact ,Jeff ruled out the second statement saying that it was too limited because it assumed the events were equally probable. Both Jeff and Yasmin expressed agreement with the third statement which is related to a statistical orientation.

Therefore this task, both parts a and b, can give insight into how preservice teachers (and students in general) interpret how to calculate probabilities as well as the

meaning of probability. By offering different ways to think about probability students may realize there is more than one way, or depending on their orientations, they may see one way as correct and the others as incorrect.

Task 6 does not appear to be that useful at getting preservice teachers to consider different approaches to probability. When compared to the Schoolopoly task this task seemed to be less effective and this could be due to the fact that it is simply asking them to make comments about different statements that are presented to them. With the Schoolopoly task they were able to see how a statistical approach to probability could be used to understand the concept of fair dice.

Task 7: Curricular Issues Revisited

In Task 7 the preservice teachers were again asked what major concepts they thought should be taught in probability. They were also asked what strategies they would use to teach those concepts. And finally what they had learned about the teaching and learning of probability. The following table displays the preservice teachers' responses to Task 7.

Table 19 *Brief description of preservice teachers' responses to Task 7*

Question	Pam	Yasmin	Brad	Jeff	Sam
Major concepts to teach	• Experimental, theoretical , and difference b/w • Randomness • Real-life applications • LLN	• Theoretical and experimental probability • LLN • Being weighted • Fractions • Sample size • Sample space	• Randomness • Sample size • LLN • Difference b/w empirical and theoretical • Applications to real-life	• Uncomfortable • LLN • Theoretical and empirical • Sample size • Weighted difference • Sample space • Randomness	• Randomness (underlined twice and checkmark) • Ways to count events • Theoretical and empirical prob. • Variation
Strategies	• Computer simulations w/ Probability Explorer, Excel, calculators • Real life contexts • Explorations "rather than watching the teacher do stuff"	• Experiments – "see it at work and how things relate to each other" • Use technology b/c speed and easier for students "drag and drop graphs" • Students create own experiments • Would not use memorization	• Have students design own experiments • Using technology: Probability Explorer, Excel and explained how these could be used to teach concepts he listed	• Using real life examples "grasp their attention" • Technology to discover LLN • M&Ms and flipping coin (iffy-too time consuming)	• Different representations – histograms, box plots, charts (data analysis) • Students collect their own data and then look at different representations
Things learned	• Had a hard time b/c she didn't recall learning prob. in HS "I don't think it was ever addressed"	• she learned: LLN, empirical probability, and sample space • Own experiences in HS and MS limited to equiprobable	• he learned how to teach prob. especially w/technology • he was surprised how much he forgot since last took statistics	• LLN • How technology can be used as a pedagogical tool for students to discover • Speed up process so can focus on results	• Gave him a chance to consider what's important for teaching prob. • Learned you have to be flexible and open to students' understanding

What is interesting in the findings from this task across the cases is how the preservice teachers' knowledge of the topics to teach in probability seemed to expand and

deepen. At the beginning of the study, with Task 2, many of the preservice teachers had a limited notion of what to concepts to teach that specifically pertained to probability. In fact many of them gave more concepts related to data analysis and statistics. However at the end of the study, as can be seen in this Task 7, they were able to offer more probability related concepts.

Common concepts across the five cases included: theoretical and empirical probability and the relationship between the two, the law of large numbers, randomness, and sample size. Two of the five mentioned the concept of weighted probabilities (Yasmin and Jeff), and using real life applications (Pam and Brad). It would seem that having participated in the previous tasks had an influence on these preservice teachers' pedagogical content knowledge of teaching probability. Concepts that were not mentioned initially, such as the law of large numbers and randomness, are now ideas that they see as important to teach within probability.

As for the strategies they would use to teach probability, all the preservice teachers mentioned something about using experimentation and data. Several said they would use technology and real life contexts. Another common theme was that they would have students design their own experiments and collect their own data. These strategies align with the guidelines presented in the GAISE report (2005). Again, the participation in the tasks from this study as well as the teachings from their course on teaching math with technology most likely influenced their suggestions of strategies for teaching.

Task 8: Multiple Choice and Free Responses from Pre/Post Tests

This task was separated into three parts: a) multiple choice test questions about (state concepts); b) multiple choice questions about sample size, proportional reasoning, and expected variability; and c) open ended pedagogical questions. The following table summarizes the preservice teachers' responses.

Table 20 *Brief description of preservice teachers' responses to Task 8*

Question	Pam	Yasmin	Brad	Jeff	Sam
8a – 4Qs on calc. prob.	• 1, 3, & 4 correct on pre /post • 2. Incorrect Outcome approach pre & post	• 1&2 correct on pre/post • 3&4 incorrect pre, correct post	• All correct on pre/post	• All correct on pre/post	• All correct on pre/post
8b – SS, prop reasoning, distributi on	• 1. Correct pre/post • 2. Incorrect pre, correct post • 3. Incorrect pre, ½ correct post	• 1. Correct pre/post • 2. Incorrect pre, correct post • 3. Incorrect pre, correct post	• 1. Correct pre/post • 2. Incorrect pre, correct post • 3. Correct pre/post	• 1. Correct pre/post • 2.incorrect pre, correct post • 3. Correct pre/post	• All correct on pre/post
8c – pedagogy question	• Group 1 strategy appropriate (wrong pre &post) • Said 6 was too small, but 10 was ok • Post – says should do more than 10	• Correct pre/post • Said neither group ran enough trials • each side once on 6 rolls is not very likely	• group 1 strategy appropriate (wrong pre& post) • group 1 better grasp than 2 • post – group 1 is trying to eliminate chance	• correct pre/post • pre- group 2 "way off", group 1 better but not enough trials • post- added that 100 trials would be better	• group 1 strategy appropriate (wrong pre& post) • pre & post said # trials was too low for 1 & 2

The findings from the pre and post tests indicate that among these preservice teachers their content knowledge of probability was strong at the beginning of the study, or improved. In the first part of this task three of the five preservice teachers were able to

answer all of the questions correctly on both the pre and post tests. For Pam she did not improve on the question about interpreting the statement about rain, indicating she has more of an outcome approach. Yasmin answered incorrectly on questions 3 and 4 on the pre-test but then got them correct on the post-test thus demonstrated that she learned about independence as it relates to a coin toss.

For the second part of this task, all but Sam answered the second question incorrectly at first and then got it correct on the post-test, therefore they learned the concept of the effect of sample size – namely that there is more variability within small samples than there is within large samples. Therefore either through the study, the class, or the combination of the two, they learned this concept. It could be that the analysis of the posters in the Schoolopoly task assisted in their understanding of the effect of sample size.

The last part of Task 8 was assessing the preservice teachers' pedagogical content knowledge of student answers. Three of the five answered incorrectly on both pre- and post-tests, while the other two answered correctly on both tests. So there was no change in their answer as to whether a group had used an appropriate strategy. All five, however, improved on their analysis of the students' strategies. So while they did not all realize that both groups' strategies were inappropriate, they were better able to critique the students' answers on the post-test.

It could be that the preservice teachers' were actually comparing the two groups to each other and since the first group had a better strategy and made explicit connections between empirical and theoretical probability they felt that was appropriate. However, the option in the task of saying neither groups' strategies were appropriate likely contradicts this hypothesis.

One possible reason for why the preservice teachers' improved on their analysis of the students' strategies could be due to their work in previous tasks within this study – namely Task 3 (the Cars task) and Task 5 (the Schoolopoly task). Both of those tasks required the preservice teachers to look at student work or responses. While Task 3 used hypothetical responses and the question simply asked them how they would proceed with the conversation, the preservice teachers had to analyze the responses to answer the question. And clearly Task 5 required them to analyze students' work more explicitly.

Analysis of Relationships across Cases

One of the research questions for this study was to look at what relationships may exist among preservice teachers' orientations, content knowledge, and pedagogical content knowledge. Within the cases, there were certain relationships that were evident between these three aspects. This section addresses some common patterns in the relationships that were found between the cases as well as relationships that seemed to be unique to a particular case.

The following table shows the instances of the different orientations across these five preservice teachers.

Table 21 *Summary counts of all five cases on all eight tasks*

Task	**Stat**					**Math**					**Sub**		
	Pam	Yas.	Brad	Jeff	Sam	Pam	Yas.	Brad	Jeff	Sam	Pam	Yas.	Brad
1	2	1	1	3	2	4	3	1	2	2	0	0	0
2	2	0	0	1	0	2	0	0	0	1	0	0	0
3	3	1	1	1	1	0	0	0	0	0	1	0	1
4	1	0	1	0	1	0	1	3	1	1	0	0	0
5	1	1	1	0	0	0	1	2	0	0	0	0	0
6	1	4	1	4	0	4	2	5	0	3	0	2	1
7	1	1	2	1	0	0	1	0	0	1	0	0	0
8	0	0	0	0	0	1	0	0	0	0	0	0	0
Total	11	8	7	10	4	11	8	11	3	8	1	2	2
Total	40					41					5		

This table gives a global view of how the different orientations were evident in the different preservice teachers. There were almost the same number of instances of statistical orientations as there were mathematical, and only a few indications of subjective orientations. Therefore in answer to the question of the nature of preservice teachers' orientations, from these findings and with these five preservice teachers, it is clear the orientations tend to be objective and equally represented by both statistical and mathematical. Also, although at a fraction of the frequency, the subjective orientation was also apparent.

If we were to quantify the five preservice teachers' orientations across the four possible orientations (Statistical, Mathematical, Subjective, and Personal) we could map

them onto a continuum as illustrated in Figure 47 below. Since the subjective and personal orientations rarely occurred across the five cases, this diagram only considers the percentage of occurrences of statistical versus mathematical.

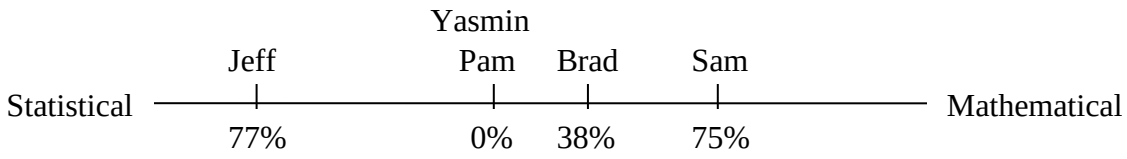


Figure 47. Continuum of Ratio of Statistical and Mathematical Orientations

In this figure only the statistical and mathematical orientations are taken into consideration since the subjective orientation showed in only a few instances. Here we have Statistical on the right hand side and Mathematical on the left. So someone with more of a statistical orientation would fall closer to the statistical side and vice versa. For example, both Pam and Yasmin would be located in the middle of these two orientations since the numbers of instances within the data for each orientation were equal. The percentages were found by dividing the number of mathematical or statistical instances by the total of both the mathematical and statistical. So Jeff was assigned a value of 77% towards the statistical side because he had 10 indications of a statistical orientation out of 13.

What this figure illustrates is that both Sam and Jeff have relatively strong mathematical and statistical orientations respectively; Pam and Yasmin are equally mathematical and statistical, and Brad, while close to the middle, leans slightly more

towards mathematical. This way of thinking about these preservice teachers' orientations can be helpful in looking at the relationships between orientations and content knowledge as well as how orientations can relate to knowledge of content and teaching, and knowledge of content and students.

For example, with the case of Jeff, he seemed to be successful in the Schoolopoly task, designed to align more with a statistical approach, and was able to explain things to the other preservice teachers. Conversely, Sam seemed confused at times, and in one instance in particular, he asked Jeff to explain what the students meant. In the Cars task, both Pam and Brad indicated difficulty addressing conflicting responses from Task 3. This may be due to their mathematical orientations and being more comfortable with situations where there is one right answer.

Another interesting finding when looking across all five preservice teachers is that ones that displayed an appreciation for the subjective approach are all in the middle – Jeff and Sam did not indicate a subjective orientation and they were either more mathematical or statistical. This may suggest that having both a statistical and mathematical orientation, as opposed to one being more dominant than the other, may make it more possible for one to also view probability from a subjective approach.

Different tasks also seemed to illuminate these relationships better than others. For example Task 5, by design, showed relationships that existed between content

knowledge and pedagogical content knowledge as well as orientations and pedagogical content knowledge.

There seemed to also be influences across tasks. In Task 7 the preservice teachers were asked what topics are important to teach in probability. This same question was asked in Task 2 and between that time the preservice teachers completed Task 5 which may have influenced the change in responses. For example, Brad in Task 2 had difficulty listing concepts to teach within probability and actually listed probability as one of the topics. However, in Task 7 he mentioned topics such as the law of large numbers, randomness, and empirical vs. theoretical probability. One conjecture is that by participating in the Schoolopoly task may have Brad increased his knowledge of topics within probability. This finding was also seen in the cases of Jeff, Yasmin, as well as Pam.

The following chapter discusses the conclusions and implications from the findings in this study. The specific research questions are answered and a discussion is presented of how these findings can inform the mathematics education community about the teaching and learning of probability within teacher education.

CHAPTER 6: CONCLUSIONS & IMPLICATIONS

Introduction

The purposes of this study were threefold: 1) to explore preservice teachers' orientations, content knowledge, and pedagogical content knowledge of probability, 2) to describe any relationships that may exist among these three aspects, and 3) to explore how the use of different types of tasks may influence these aspects and how these tasks give teacher educators insight into preservice teachers' understanding. Using the frameworks of Hill, et al. (2008), Kvatinsky and Even (2002), and Garuti, et al. (2008) to analyze responses to the eight tasks, the researcher focused on describing preservice teachers' understanding of teaching and learning probability.

The previous two chapters presented an analysis of each case and cross-case analysis between cases of preservice teachers' work on the tasks and between the cases of the preservice teachers' relationships among their orientations, content knowledge, and pedagogical content knowledge. The purpose of this chapter is to provide conclusions to the findings as they answer the research questions and offer implications for teaching, especially within mathematics teacher education.

This chapter is organized in three main sections: conclusions, limitations of the study, and implications. Within the conclusions is the discussion of the findings as they pertain to individual cases as well as relationships across the cases. The implications from this study are discussed as they relate to methods of researching teachers' knowledge and

also as they relate to teacher instruction. Suggestions for future research are given at the end of this chapter.

Conclusions

Preservice Teachers' Knowledge of Teaching Probability

The first research question that was addressed in this study asked: *What is the nature of preservice teachers' orientations, content knowledge, and pedagogical content knowledge for teaching probability?* The findings for these preservice teachers indicated that they had multiple orientations towards probability. All five preservice teachers displayed at least two different orientations. Previous research has determined that a person can take more than one approach towards understanding probability, and thus this finding supports that claim (Garuti, et al., 2008; Liu, 2005).These particular five preservice teachers represented a broad range of orientations when considering them on a continuum between mathematical and statistical. It should also be noted that both of these perspectives are objective and there was only some evidence of appreciation or considerations of use of a subjective perspective.

In addressing the question of the nature of preservice teachers' content knowledge of probability, it should be restated here that the preservice teachers in this study had self-selected to do a probability related course project and also showed an interest in learning more about probability. So when we make conclusions about their content knowledge we need to be cognizant that their interest in the subject is most likely higher than a typical

preservice teacher. With that said, there were findings that indicated some of these preservice teachers had more of a complete understanding than others about certain topics. Also the consistency of their content knowledge throughout the eight tasks tended to vary.

One conclusion that arose from this study relates to the concepts of sample size and variation. Kahneman and Tversky (1972) pointed out that the idea of sample size and it's relation to variance is not intuitive. Two of the preservice teachers had difficulties with these concepts, Pam and Yasmin, as was shown in their analysis of the students' work in the Schoolopoly task. However it should be noted that Yasmin, after watching the discussion among the students and the teaching episode, changed her opinion about the fairness of the dice and thus the amount of variation that was acceptable for large and small samples. Pam, on the other hand, stayed firm in her opinions and even stated that she perceived herself as understanding the law of large numbers. Haller (1997) found that teachers' understanding of the effect of sample size and the law of large numbers was lacking. Additionally, Lee and Mojica (2008) found that middle school teachers' knowledge of sample size was similar to the findings in this study.

Although we've focused on Pam's and Yasmin's misconceptions, the findings from this study when looking at Jeff, Brad, and Sam, can be contrasted with Haller's study with middle school teachers. These three secondary preservice teachers displayed a

rather complete understanding of the law of large numbers, especially Jeff who referred to this concept, and even offered examples, throughout the study.

Another finding regarding the nature of preservice teachers' content knowledge of probability was in regard to the lists of concepts they gave that were related to teaching probability. These lists tended to be mainly related to statistics and not specific to probability at the beginning of the study. However, at the end they were able to list more topics specific to probability, such as, empirical vs. theoretical probabilities, randomness, sample size, and sample space.

And one final conclusion about the nature of their content knowledge is that all preservice teachers expressed the importance of learning probability because it is applicable to daily lives. They listed contexts where probability is used in the real world such as, weather, lottery, games of chance, and the media. These contexts however are common school experiences or were used in prior tasks, so it is not clear how well they understand how probability is used in the real world. But they do seem to have an awareness of this connection. In agreement with what was outlined in the GAISE report (2005) and suggested by Shaughnessy (2002), using real-world contexts within teaching probability is something these preservice teachers' feel is important.

The nature of preservice teachers' pedagogical content knowledge was studied in two separate categories – knowledge of content and students, and knowledge of content and teaching. Some conclusions that can be drawn from the findings of preservice

teachers' knowledge of content and students include the following common themes: build instruction off of student interests, use exploration activities, and have students design their own experimentation with a real problem (probability distribution unknown to students). These themes echo the guidelines that were presented in the GAISE report (2005) in the fact that real data should be used, as well as explorations.

As for critiquing student work, that seemed to vary across the cases. It seemed, not surprisingly, that this was where the connections between content knowledge and pedagogical content knowledge emerged. If the preservice teacher seemed to display a commanding knowledge of the content, they were better able to assess students' understanding. And vice versa, those that had a less complete knowledge were less equipped to make claims about students' understanding based on their work. With regard to anticipating student difficulties that may arise with different tasks, the preservice teachers seemed to focus on the delivery of the content as well as to the content itself. For example, with the Schoolopoly task, Pam suggested that the technology may be distracting for students; and Yasmin said they may have a hard time with the idea on the dice not being equally weighted and what that means. Similarly, in his study with preservice teachers, Groth (2008) found that they tended to offer discussion about content as well as content-specific pedagogy.

With respect to the knowledge of content and teaching, the preservice teachers in this study exhibited the following understanding: use inquiry method – questioning, use

similar examples to illustrate same concept, use experimentation and explorations – but with guidance, and the importance of class discussions. The strategies the preservice teachers' said they would use were experiments and explorations and these would have to be with guidance. One aspect of the Schoolopoly task some preservice teachers perceived as a drawback was how open-ended it was and expressed concern that the students would 'waste time' or not understand the point of the exploration.

Classroom dynamics and class discussions are also areas that fall under knowledge of content and teaching; a teacher needs to be able to control the dynamics in the class as well as facilitate discussions. With Task 5 the preservice teachers were able to view a video clip of students discussing the problem and this seemed to be effective in getting them to think more deeply about what the students understood. Pam and Yasmin talked about the group dynamics and how one or two students with strong beliefs can affect the whole class. Thus, their knowledge of content and teaching seemed to include the awareness of the effects of group dynamics. That is not to say that they have (or don't have) the sufficient knowledge to teach groups where this dynamic may arise.

The second research question that was studied was, *what are the relationships among preservice teachers' orientations, content knowledge, and pedagogical content knowledge for teaching probability?*

As was seen in the findings within cases relationships were evident among these three aspects of knowledge. Furthermore, relationships could be seen when looking at the

group of preservice teachers as a whole. For some their orientations seemed to relate to their content knowledge and how they answered certain task questions.

For a preservice teacher with a stronger mathematical orientation, they may have difficulty when dealing with probabilistic situations that are more statistical in nature. Specifically with the Schoolopoly task, those that had a more mathematical orientation seemed to have trouble determining students' understanding as well as concluding whether the dice were fair or unfair. For example, Sam seemed to be uncomfortable assessing understanding when he himself didn't know the "right" answer. So this is an example of how orientations can impact pedagogical content knowledge.

The relationship between content knowledge and pedagogical content knowledge has been discussed in the literature and findings from this study support the literature (Ball, 2000; Ball & McDiarmid, 1990; Ball, et al., 2008). For example, Pam seemed to have a misconception about the amount of expected variability in a large sample. This misconception then led her to analyze students' work incorrectly, saying they were wrong in concluding the dice were unfair because, in her mind, that amount of variability was acceptable for fair dice. This finding supports Groth's (2008) claim that using case-based (sample student work) curriculum within teacher education may potentially challenge preservice teachers' persistent misconceptions.

So the findings from this study answer the research question about relationships by describing how the relationships exist. Not only do they exist, but they seem to be

inter-related. That is, orientations can impact content knowledge, yet at the same time ones content knowledge can influence their orientations. Similarly the findings from this study show how orientations relate to pedagogical content knowledge, and how content knowledge relates to pedagogical content knowledge.

The following directional diagram illustrates these relationships:

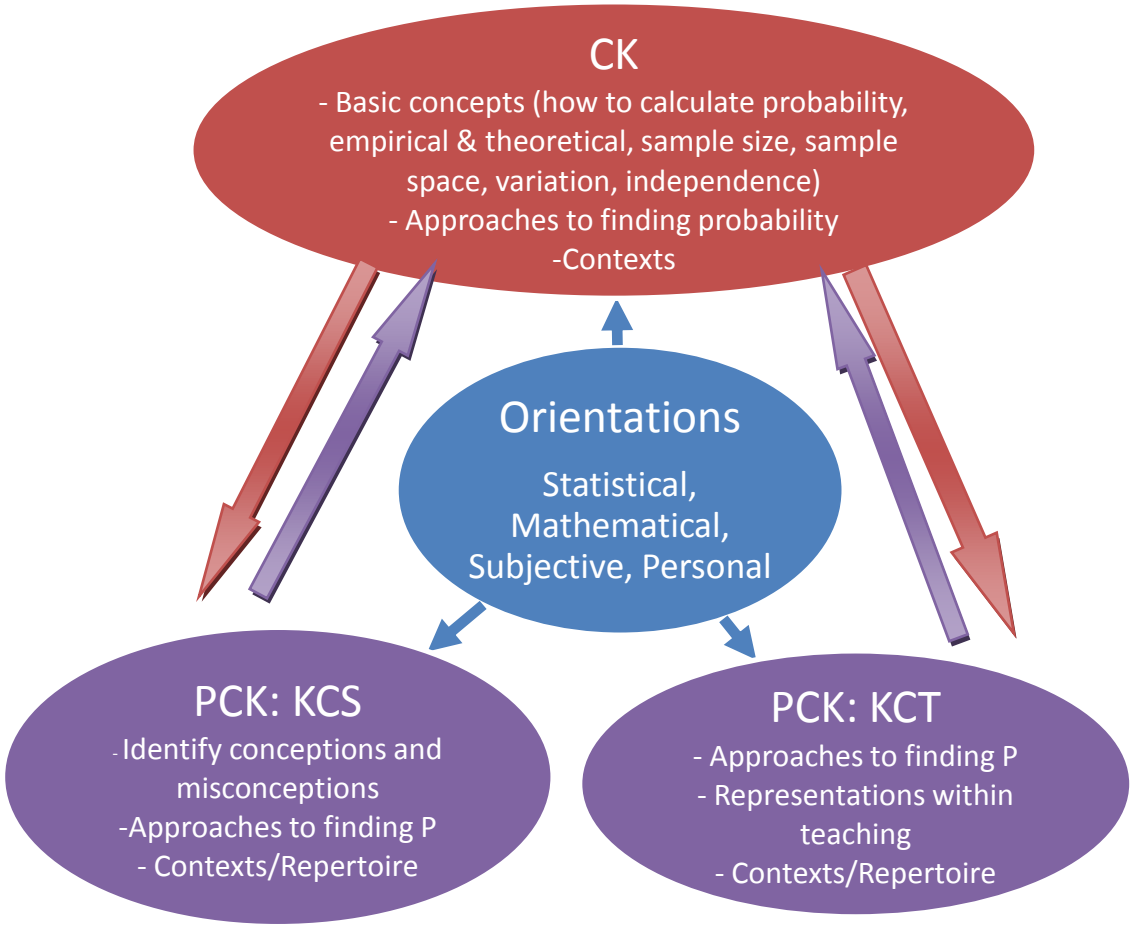


Figure 48. Relationships among Orientations, Content knowledge, and Pedagogical Content Knowledge of Probability.

The third research question asked, *how do different types of pedagogical tasks (open-ended questions, problem solving with and without a context, analyzing student work, analyzing teaching videos, multiple choice questions) elicit different aspects of preservice teachers' orientations and knowledge for teaching probability?*

In answering this third research question, the findings from the eight different tasks, across the cases are discussed. There were certain tasks that seemed to elicit more information and other tasks that weren't as effective at either eliciting aspects or having an effect on knowledge and relationships. First examine Table 22 with respect to the instances of preservice teachers' orientations towards probability within each of the eight tasks.

Table 22 *Summary counts of all five cases on all eight tasks with row totals*

Task	**Statistical**					**Mathematical**					**Subjective**			**Total**
	Pam	Yas	Brad	Jeff	Sam	Pam	Yas	Brad	Jeff	Sam	Pam	Yas.	Brad	
1	2	1	1	3	2	4	3	1	2	2	0	0	0	20
2	2	0	0	1	0	2	0	0	0	1	0	0	0	6
3	3	1	1	1	1	0	0	0	0	0	1	0	1	9
4	1	0	1	0	1	0	1	3	1	1	0	0	0	9
5	1	1	1	0	0	0	1	2	0	0	0	0	0	6
6	1	4	1	4	0	4	2	5	0	3	0	2	1	27
7	1	1	2	1	0	0	1	0	0	1	0	0	0	7
8	0	0	0	0	0	1	0	0	0	0	0	0	0	1
Total	11	8	7	10	4	11	8	11	3	8	1	2	2	
Total	40					41					5			

Tasks 1 and 6 revealed the most about these preservice teachers' orientations, whereas Tasks 2, 5, and 8 revealed little. Task 8 was the multiple choice and free response pre-

post-test. This task was not effective at eliciting information about preservice teachers' orientations towards probability. The types of tasks that gave the most information about preservice teachers' orientations were the interview questions, then the Cars and Candies tasks, followed by the curricular questions and the Schoolopoly task (Tasks 2, 5, and 7). This table reveals that the design of the tasks was successful in purposefully eliciting information about orientations.

Another conclusion related to the question about tasks is that when put in different contexts, and asked to do different types of tasks, some preservice teachers orientations seemed to be more related to the task at hand. For example, a preservice teacher, when responding to the Schoolopoly task which is intended to promote the use of a statistical approach to a probability task, could display a belief in and use of a statistical orientation. Conversely on a task such as Task 6 where they had to comment on different statements about approaches to probability, this task did not seem to impact their orientation.

For Task 8, the pre-post test, three of the five preservice teachers answered correctly in the pre- and post-tests. This seemed to be indicative of their work on other tasks. For example, Yasmin was incorrect on the pre-test question about the expected variability and she also had difficulty analyzing students' work in the Schoolopoly task. This addresses what Peard (2005) referred to as one trouble with the frequentist approach – that students' misconceptions may be reinforced due to the fact that oftentimes short

term frequencies give vastly differing results from long term frequencies. In the students' posters the work they displayed as evidence was small trials that reinforced their misconception.

As far as pedagogical content knowledge, the tasks that involved using artifacts of practice were the most useful, namely, the Schoolopoly task as well as the cars task and the pedagogical question on the pre-post tests. The Schoolopoly task included multiple artifacts that were genuine, that is, were taken from actual student work and real student discourse. This seemed to have the most profound effect on the preservice teachers' pedagogical content knowledge. The cars task included hypothetical student responses as did the pre-post tests.

Limitations

There are some limitations to this study that need to be addressed. First, the study was purposely conducted with secondary preservice teachers who have limited field experiences and do not have any formal teaching experience. So when making conclusions about their pedagogical content knowledge we need to take this fact into account. One other limitation in this study is the time factor. The entire study took place during a limited span of time, a period of eight weeks. It is assumed that this amount of time is sufficient to gain an understanding of the preservice teachers' knowledge related to probability. However, it is possible that a more in-depth analysis could be done over a longer period of time. And finally the impact of the technology course the students were

enrolled in at the time is a limiting factor to be considered. Some of the responses may have certainly been influenced by their experiences in the course.

These factors limit the results of the findings in this study and should be considered when discussing implications for teacher education as well as research methodology. When considering implications for teacher education, the fact that the preservice teachers in this study did not have any teaching experience could limit ways these findings could be used. For example, some of the implications discussed below may be more applicable for courses within teacher education that occur in the beginning of preservice teachers' education. As for discussion on research methodology, the fact that these preservice teachers' were influenced by their participation in a technology methods course may limit the ways in which these tasks could be applied to other studies. For example, with another group of preservice teachers, not in a similar course, these same tasks may reveal quite different findings.

Many teacher education programs may not have in depth opportunities for preservice teachers to explore data and probability. Materials such as those by Lee, et al. (2007) offer these experiences and would be useful to include in the preparation of mathematics teachers.

Implications

Implications for Teacher Education

The research presented here has implications for teaching future teachers how to teach probability. One implication is the need to include experiences similar to those in this study in teacher education programs. The findings about the nature of preservice teachers' content knowledge lend support to Shaughnessy's (2003) argument that teaching connections between sample space and variation is important for teacher education. The idea of sample size and its relation to variance is not intuitive (Kahneman & Tversky, 1972), requiring explicit instruction of this concept within teacher education.

As was evident in the preservice teachers' responses and work on Task 5, being engaged in various pedagogical tasks was – by their own accounts – effective in teaching them about how to teach probability. The combination of the four parts of this task is one of the reasons it so powerful. Preservice teachers were first engaged in an analysis of the task where they had to weigh the pros. and cons. and determine what the objective the task was. Secondly they were given examples of student work to critique. And what made this set of student work even more impacting on the preservice teachers' knowledge was the fact that it included conflicting work. Two groups that investigated the same company came to two different conclusions. Note also, in Task 3, the cars task; they were given hypothetical student work. Using real students' work can be more effective for preservice teachers since it is more authentic and they may believe it is possible for them to

encounter a similar response or work when they are teaching. The third part of Task 5 was watching classroom discussion and then a teacher demonstration. Again, using real student work and real student experiences seems to be an effective means of educating preservice teachers (Crespo, 2000; Hiebert, et al., 2007; Kazemi & Franke, 2004). And finally the last part was a reflection where they were given some time to think further on their work on the task.

Within probability, this use of real teaching practices is even more important since many preservice teachers have not had similar experiences in their own schooling to draw upon. Therefore it is imperative that these types of tasks be included in the teaching of probability to future teachers. Hollebrands, Wilson, and Lee (2007) described how a video case of middle school students' working on a data analysis task, along with their written work, was effective at getting preservice teachers to analyze student work.

In addition to Task 5, Tasks 1, 2, and 7 would be good to include in a methods class at the beginning and then again at the end of the semester. Tasks 1 and 2 are good questions to ask in order for the preservice teacher to become more aware of their own perceptions of randomness and probability. This also may raise their awareness of how little they know about teaching probability and may increase their desire to learn how to teach probability. At the beginning of a methods course where probability is being taught, teacher educators could either give these questions as a survey or as a class discussion. It may be more effective to have preservice teachers answer the questions themselves (to

get an idea of their own knowledge of these concepts) and then have a class discussion. Then at the end of the semester, after they have had experiences similar to Task 5 (Schoolopoly) and Tasks 3 (cars) and 4 (candies), asking them to revisit Task 2 and also ask about strategies can be an effective way to teach preservice teachers how to teach probability.

Another implication for teacher education that arose from the findings within this study is how the relationships between orientations, content knowledge, and pedagogical content knowledge can be used in teacher education. The awareness of how these areas are related and influence each other is essential for teacher educators. In order to address possible misconceptions within preservice teachers, it is important to be aware of how underlying orientations may be affecting those misconceptions and use that to intervene. For example, suppose a preservice teacher was having difficulty understanding the relationship between variation and sample size. The teacher educator could use a statistical orientation and run simulations to demonstrate how the variation should stabilize as the sample size increases.

Implications for Methods of Researching Teachers' Knowledge

Some implications for methods of researching teachers' knowledge include using these various tasks in different settings. This study researched preservice teachers' knowledge both directly and indirectly through the use of various tasks in individual interview settings. Some of the responses may have been different given a different

setting. For example, when Jeff was asked to give concepts related to teaching probability, he had difficulty thinking on the spot and seemed very uncomfortable. Perhaps in a different setting – say given as a survey or in a group class setting – the responses would reveal more about their content knowledge of probability.

Another implication from this study for methods of researching teachers' knowledge is that, through the use of tasks where preservice teachers analyze students' work, the researcher can gain insight into both the preservice teachers' content knowledge as well as their pedagogical content knowledge.

Implications for Future Research

Some areas for future research that are suggested from these findings include looking at inservice teachers' orientations, content knowledge and pedagogical content knowledge. Professional development workshops could be conducted where tasks similar to Task 5 were used, teachers may even bring in their own students work to be analyzed.

Other areas of research that could build off of this study are how effective these tasks are at changing or impacting orientations. Also, as stated in the implications for methods, areas of future research include using these tasks in different settings and in different groups. One specific group that could be studied is preservice elementary teachers. In this study, with these five secondary preservice teachers, none of them displayed a personal orientation. It would be of interest to see whether this would still be true if one studied preservice elementary teachers, or inservice elementary teachers.

The framework developed in this study could be used to further explore relationships and impacts that orientations towards probability have on content knowledge and pedagogical content knowledge. Combined with the tasks that have been presented, this framework offers a valuable tool for researchers to characterize and describe teachers' orientations towards probability, content knowledge and pedagogical content knowledge of probability.

It is the hope of the researcher that the teaching and learning of probability will become more of a focus within the mathematics education community. This area of mathematics is becoming more prevalent in schools' curriculum, in some high schools probability and statistics is a requirement for graduation. And thus research pertaining to the education of preservice teachers within the area of probability is, in the researcher's opinion, of utmost importance. The findings from this study and the use of the framework developed within this study can aid in this area of future research.

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APPENDICES

APPENDIX A: INFORMED CONSENT FORM

Sarah Ives, Principal Investigator
Dr. Hollylynne Stohl Lee, Faculty Sponsor

You are invited to participate in a research project. The purpose of this project is to understand how preservice teachers learn to teach probability. You will contribute to this research by participating in:

1. a series of task-based interviews looking at sample student work and sample lesson plans focused on topics within probability, and
2. a focus group in which you will view and discuss a sample video of teaching probability.

All interviews and focus groups during this project will be videotaped for recall and analysis purposes by the researcher. In addition, these interviews and focus group meeting(s) will take place outside of class time. For your time I would like to offer you your choice of a campus bookstore gift certificate, or a software site license for a mathematics education software application.

There will be no risk associated with your participation in the research study. Your grade in your course will not be affected by your decision to participate in the study. The knowledge the researcher will gain from your experiences will add to the knowledge base in mathematics education, especially with regard to how preservice teachers learn how to teach probability. The information derived from the interviews and focus groups will be kept strictly confidential, with your name removed from all work. It will be stored securely in a locked file and will be made available only to the researcher, faculty sponsor, and other researchers in the mathematics education department at NCSU unless you specifically give permission in writing to do so. No reference will be made to your name in either oral or written reports and transcripts that could link you individually to the study.

If you have any questions at any time, you may contact Sarah Ives at (919) 649-9549, sarahives@gmail.com, or Dr. Hollylynne Lee at 513-3544. If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Dr. Matthew Zingraff, Chairperson of the NCSU Human Subjects Committee, Box 8101, NCSU Campus.

CONSENT

I have read and understood the above information. I have received a copy of this form. I agree to participate in this study.

Participant's signature _____ Date _____
Investigator's signature _____ Date _____
Faculty Sponsor's signature _____ Date _____

APPENDIX B: PRE-POST TEST QUESTIONS

Probability Related Questions (numbering is based on original test, correct answers are bolded)

2) The following message is printed on a bottle of prescription medication.

   Warning: For applications to skin areas there is a 15% chance of developing a rash. If a rash develops, consult your physician.

   Which of the following is the best interpretation of this warning?
   a. Don't use the medication on your skin, there's a good chance of developing a rash.
   b. For applications to the skin, apply only 15% of the recommended dose.
   c. If a rash develops, it will probably involve 15% of the skin.
   d. About 15 out of 100 people who use this medication develop a rash.
   e. There is hardly a chance of getting as rash using this medication.

3) The Springfield Meteorological Center wanted to determine the accuracy of their weather forecasts. They searched the records for those days when the forecaster had reported a 70% chance of rain. They compared these forecasts to records of whether or not it actually rained on those particular days.

   The forecast of 70% chance of rain can be considered *very* accurate if it rained on:
   a. 95% - 100% of those days.
   b. 85% - 94% of those days.
   c. 75% - 84% of those days.
   d. 65% - 74% of those days.
   e. 55% – 64% of those days.

7) Two containers, labeled A and B, are filled with red and blue marbles in the following quantities:

Container	Red	Blue
A	6	4
B	60	40

Each container is mixed thoroughly. After choosing one of the containers, you will reach in and, without looking, draw out a marble. If the marble is blue, you win $50. Which container gives you the best chance of drawing a blue marble?

a. Container A (with 6 red and 4 blue)
b. Container B (with 60 red and 40 blue)
c. Equal chances from each container

8) The following sequences represent the results from tossing a fair coin 5 times. Each sequence displays the results in the order in which they occurred. Which sequence is *most* likely?

a. H H H T T
b. T H H T H
c. T H T T T
d. H T H T H
e. All four sequences are equally likely

9) Select the one explanation that best describes your reason for the answer you gave in item 8.

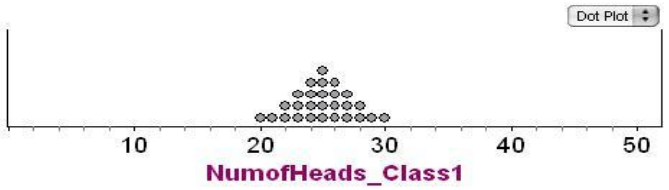
a. Since the coin is fair, you ought to get roughly equal numbers of heads and tails.
b. Since coin flipping is random, the coin ought to alternate frequently between landing heads and tails.
c. If you repeatedly flipped a coin five times, each of these sequences would occur about as often as any other sequence.
d. If you get a couple of heads in a row, the probability of a tails on the next flip increases.

10) Half of all newborns are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?

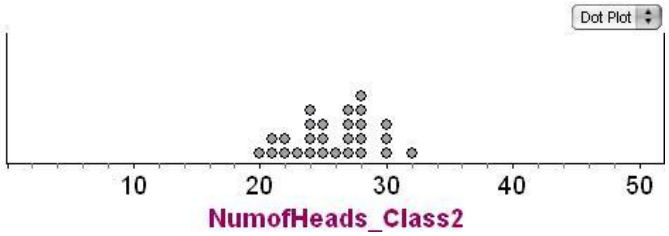
a. Hospital A (with 50 births a day)
b. Hospital B (with 10 births a day)
c. The two hospitals are equally likely to record such an event.

15) Each student in a class tossed a penny 50 times and counted the number of heads. Suppose four different classes produce graphs for the results of their experiment. There is a rumor that in some classes, the students just made up the results of tossing a coin 50 times without actually doing the experiment. **Please select each** of the following graphs you believe represents data from actual experiments of flipping a coin 50 times.

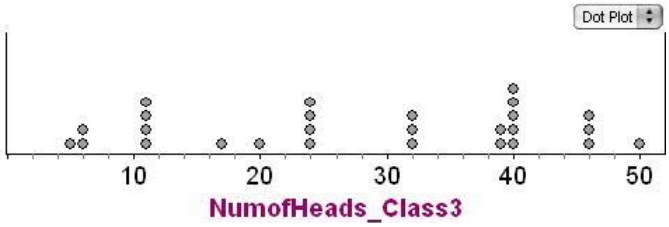
a.



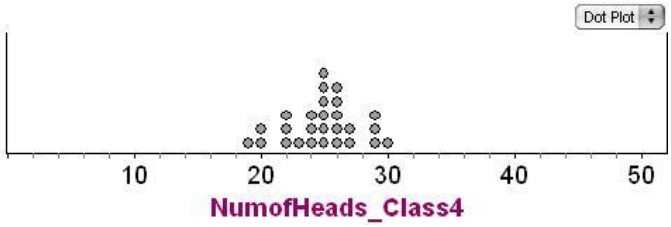
b.



c.



d.



16) Students are presented with a six-sided die and asked to experiment with it to make a claim as to whether the die is fair or unfair. You observe the strategy employed by two different groups, described below.

Group 1: Toss the die 10 times and compute the empirical probability for each number on the die. If the empirical probabilities are each close to 1/6 then the die is fair.

Group 2: Toss a die six times. If each number appears once then the die is fair.

Which group is using an appropriate strategy to make and support a claim about fairness?

a. Group 1
b. Group 2
c. Both Group 1 and Group 2
d. **Neither Group 1 or Group 2**

17) Write a brief critique about the strengths and weaknesses of each strategy described in question #16.

APPENDIX C: INITIAL INTERVIEW PROTOCOL

1. What does random mean to you?
    a. Give an example of something that happens in a random way.
    b. What does random mean when used in a probability and statistics class?
2. What does it mean to say "the probability of getting a head is 50%"?
    a. What does "70% chance of rain" mean?
    b. If you had to explain what probability means, what would you say? [general as measure of something; uncertainty long-term relative likelihoods]
3. Why do you think it is important for students to learn probability? [if don't respond about usefulness prompt: In your opinion, what is the usefulness of probability: in stats, or is it something separate? Counting]
4. Take a few minutes to brainstorm what topics or concepts related to probability you think are important to teach at the MS/HS level. [After listed topics, ask why they believe these are important.]
    a. [blank for their response]
    b. Suppose you are teaching a unit on probability in MS/HS. Here are some possible topics that you could include.

On a scale of 1 to 5: 1 being not important at all and 5 being essential, how important do you think each topic is for students to learn in a probability unit? Why?

i.	Randomness	1	2	3	4	5
ii.	Chance	1	2	3	4	5
iii.	Theoretical probability	1	2	3	4	5
iv.	Empirical probability	1	2	3	4	5
v.	Law of large numbers	1	2	3	4	5
vi.	Sample space	1	2	3	4	5
vii.	Sample size	1	2	3	4	5
viii.	Variation	1	2	3	4	5
ix.	Independence	1	2	3	4	5
x.	Proportional reasoning	1	2	3	4	5
xi.	Counting (permutations /Combinations)	1	2	3	4	5

5. Here are 5 strands that are currently included in mathematics curriculum. Please indicate, by percent, the amount of coverage you think each strand should have at the MS/HS level.

	Grades 6-8	Grades 9-12
Number		
Algebra		
Geometry		
Measurement		
Data Analysis & Probability		
Total:	**100%**	**100%**

6. a) [adapted from Garfield: ARTIST] You are trying to decide between two types of cars. You first consult an issue of Consumer Reports, which compared rates of repairs for various cars. Records of repairs done on 400 cars of each type showed somewhat fewer mechanical problems with Hondas than with Toyotas.

   You have two friends that own Toyotas and one friend that owns a Honda. Both Toyota owners reported having a few mechanical problems, but nothing major. The Honda owner, however, exploded when asked how he liked his car: "First, the fuel injection went out, that cost $250 bucks. Next, I started having trouble with the rear end and had to replace it. I finally decided to sell it after the transmission went. I'd never buy another Honda."

   Given what you currently know, which car would you buy? Justify your response.

6. b) You are teaching a class and have posed this problem. The following are three different groups' responses. **How would you proceed with the class discussion?**
    a. We would recommend you buy the Toyota, primarily because of all the trouble your friend had with his Honda. Since you haven't heard similar horror stories about the Toyota, you should go with it.
    b. We would recommend you buy the Honda in spite of your friend's bad experience. That is just one case, while the information reported in Consumer Reports is based on many cases. According to that data, the Honda is somewhat less likely to require repairs.
    c. We would tell you that it does not matter which car you buy. Even though one of the models might be more likely than the other to require repairs, they could still, just by chance, get stuck with a particular car that would need a lot of repairs. You may as well toss a coin to decide.
7. [On table a bowl has 500 wrapped hard candies: 20% are purple, 50% are red – watermelon and cherry, and 30% are green; interviewee does not know these percentages.]

    Given this bowl of candies, explain how you could make a claim about the probability of selecting a red candy.
    a. Is there any way to do experimentation to support the claim about the probability of selecting a red candy? What would you do with that data?

b. Would you use this task with MS or HS students? Why, why not?

c. How would you use this task?

APPENDIX D: SECOND INTERVIEW PROTOCOL

I. Schoolopoly Task Analysis

Plainfield Middle Schoolopoly

Background
Plainfield Middle Schools are planning to create a board game modeled on the classic game of Monopoly. The game is to be called "Schoolopoly" and, like Monopoly, will be played with dice. Because many copies of the game will be sold as a fundraiser, several companies are competing for the contract to supply dice for Schoolopoly. Several companies, however, have been accused of making poor quality dice. These companies are to be avoided since players of Schoolopoly need to know that the dice they are using are actually "fair."
Each dice company has provided a sample die for analysis. You will be assigned one company to investigate:

Perfect Polyhedra	Calibrated Cubes
Dice R' Us	Dazzling Dice
Delta's Dice	Dice Depot

Your Assignment
Investigate whether or not the die sent to you by the company is fair. That is, are all six outcomes equally likely to occur?

You will need to create a poster to present to the School Board. The following three questions should be answered on your poster:

1. Would you recommend that dice be purchased from the company you investigated?
2. What evidence do you have that the die you tested is "fair" or "unfair"?
3. What do you think the chances are for rolling each of the six numbers?

Use Probability Explorer to collect data from simulated rolls of the die. Copy any graphs and screen shots you want to use as evidence and paste them in a Word document. Later, you will be able to print these to use on a poster. You will give a brief presentation pointing out the highlights of your group's poster.

Schoolopoly Task (Maher, et al., 2006)

1. Open discussion on activity: 'Tell me your thoughts on this activity."
    a. What do you see as the learning objectives associated with this task? What is the purpose of the task, what is it students should learn by doing this task?
    b. What difficulties might students have with this task?

c. [If not already discussed:] In general what are the strengths and weaknesses of the task? Pros and cons?

d. Is this a task you would use if you were teaching probability? Why or why not?

II. Analyze Students' Posters

1. Analyze each poster individually answering the following questions provided on a handout:
    a. Based on the posters, what can you say about the students' understanding of probability?
    b. What do you think they consider to be compelling evidence?
2. Compare posters A and B, then compare posters C & D:
    a. Are you convinced by any of these posters? Why or why not?

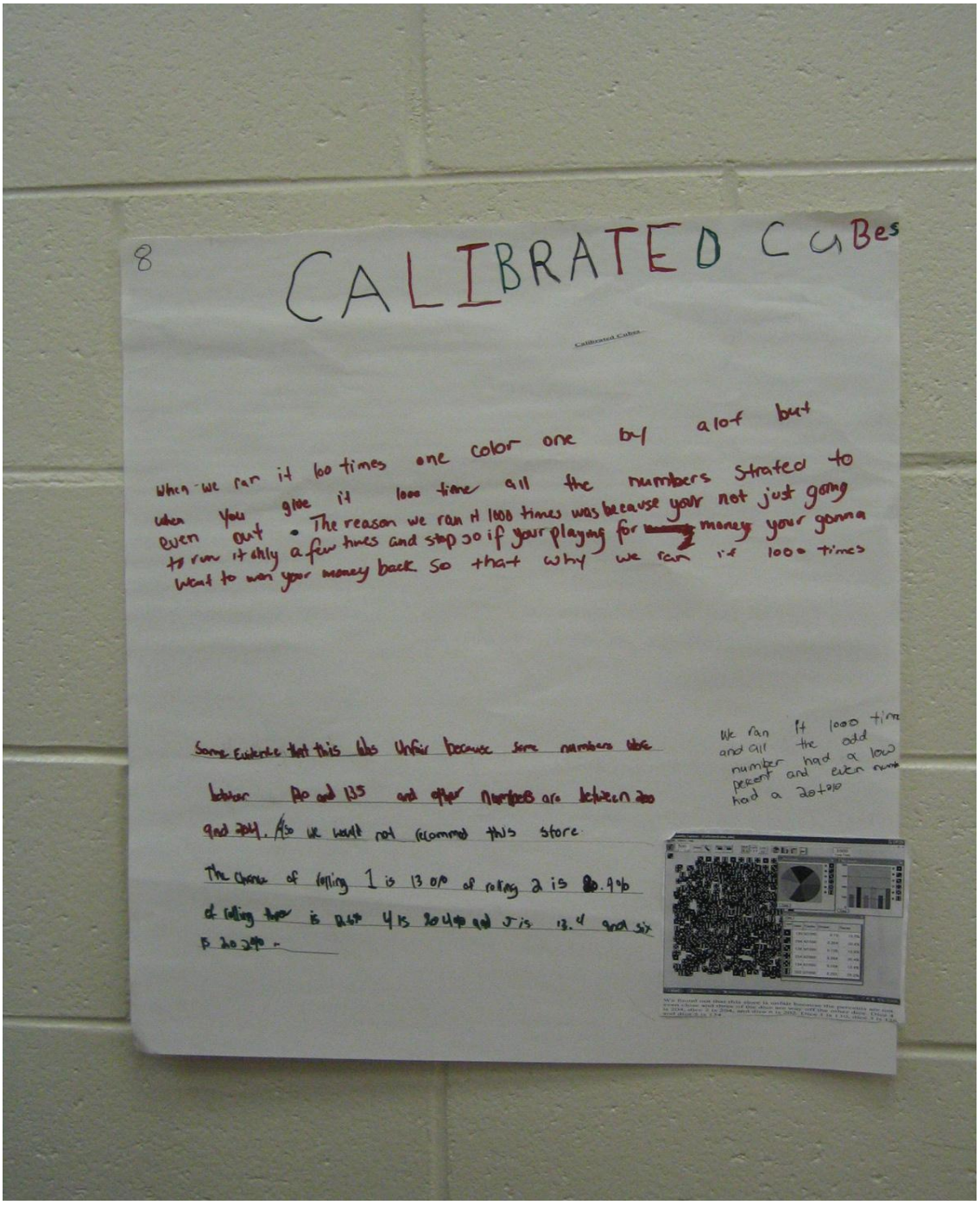


Figure 49. Poster A – Calibrated Cubes

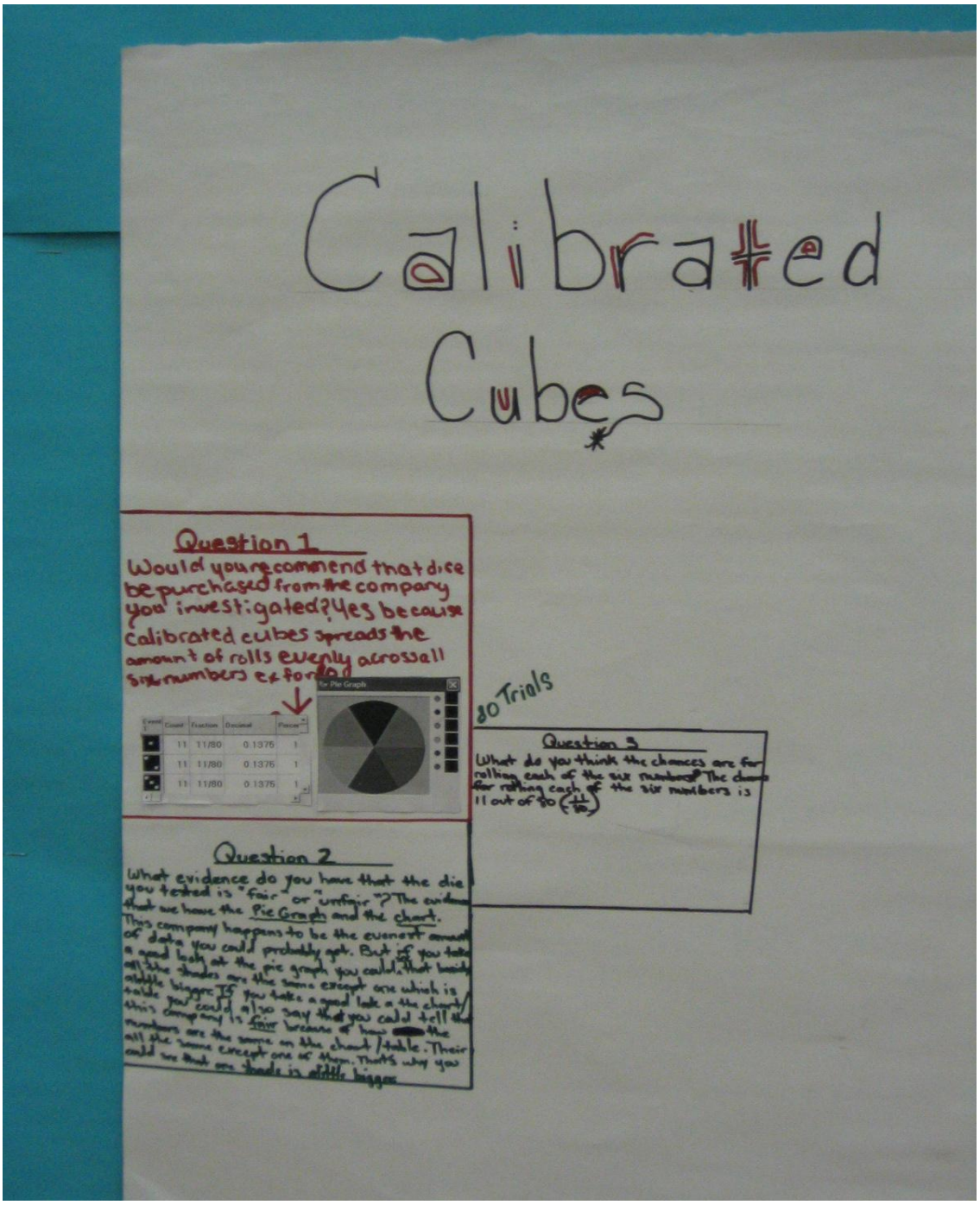


Figure 50. Poster B – Calibrated Cubes

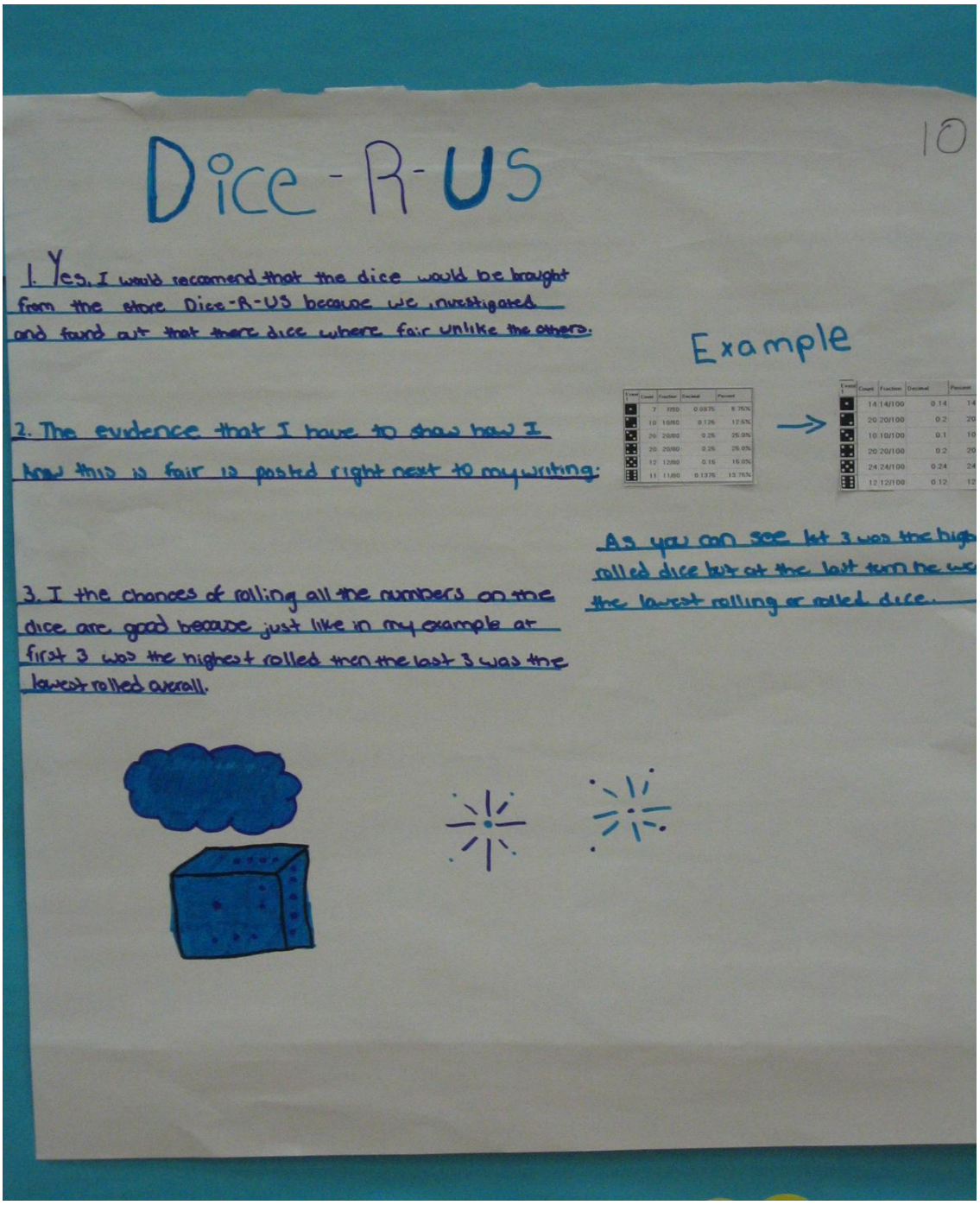


Figure 51. Poster C – Dice-R-Us

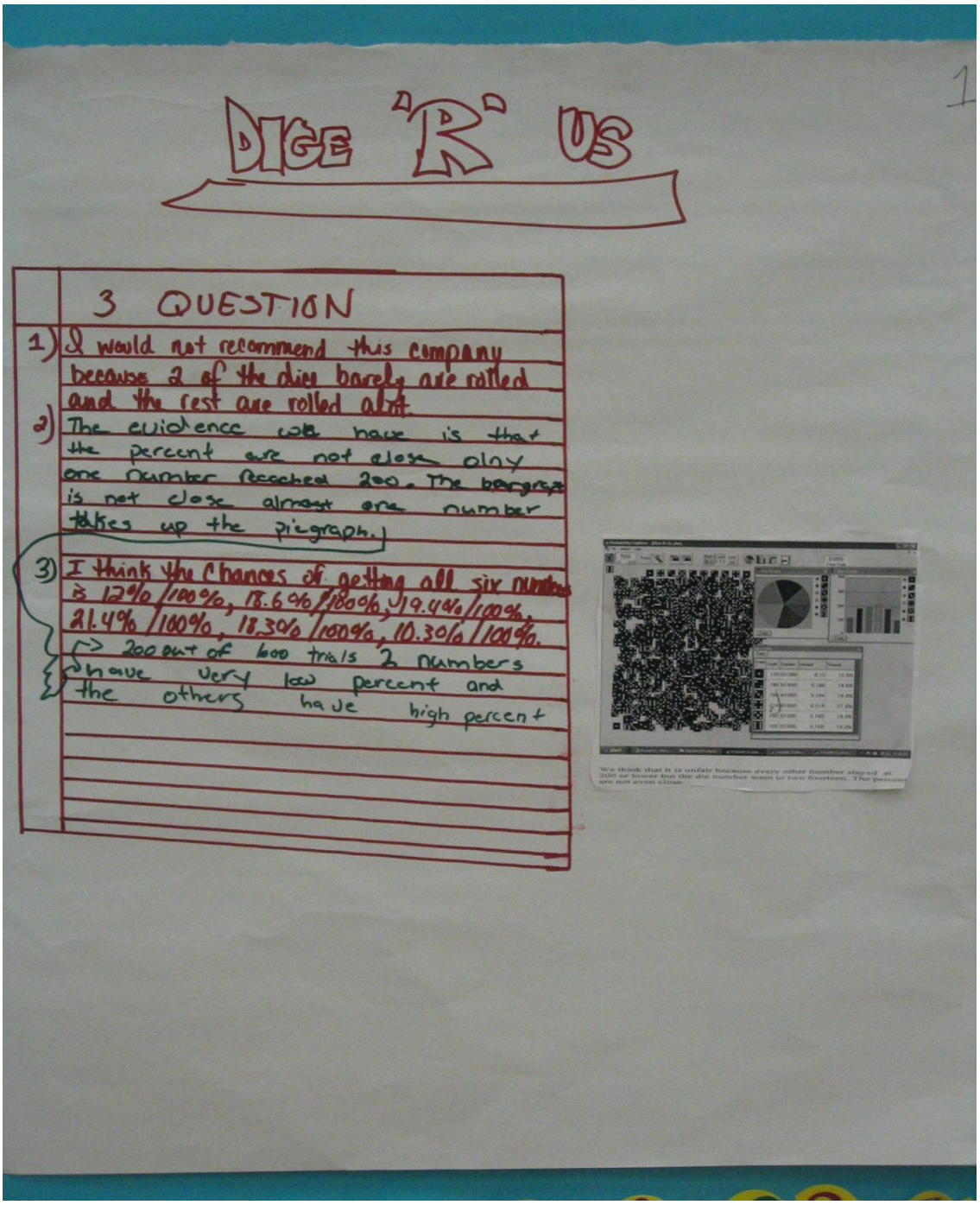


Figure 52. Poster D – Dice-R-Us

III. Analyze Video Clips

1. Whole class discussion about sample size – "80 seems reasonable," "Are you going to quit or are you going to keep playing to try to win your money back?," "It doesn't matter how many times the percentages will be the same regardless."
    a. How would you direct the conversation at this point?"
    b. "What is her misconception?"
    c. "What ways could you intervene to address this misconception?"
2. Teaching intervention – the teacher does a demonstration with the *Probability Explorer* software where she uses suggestions from students about sample size.
    a. Pros and cons – do you think this demonstration was effective? How can you tell?
    b. What could the teacher have done differently?

IV. Wrap up

1. Opportunity for preservice teachers to add any final comments regarding the task, posters, student discussion, and teaching intervention.

APPENDIX E: SECOND INTERVIEW REFLECTION

During the second interview you participated in analyzing a task that was used to teach probability to middle school students. You looked at the task, students' work, as well as two video clips of class discussion related to the task.

1. What do you see as the benefits and drawbacks if you were to use this task to teach probability?
2. Based on the students work and the class discussion, if you were the teacher of this class, what would you do the next day?
3. What did you take away from the experience? Write a few sentences about what you learned by participating in these activities.
4. What did you learn about your own understanding of probability and of teaching probability?

APPENDIX F: FINAL INTERVIEW PROTOCOL

1. Further clarification from first two interviews. [Varied by case]

Common Questions

2. (a) Read the following statements and say a little bit about each one.
    i. I want to figure out the probability of rolling a 3 on a die. I simulated it and out of 1,000,000 rolls and 3 showed up 166,549 times. So the probability of getting a 3 is about 166,549/1,000,000 = .166549.
    ii. I think the probability of rolling a 3 is not very good. Based on my experience playing games with dice 3 doesn't come up very often. I'd say the probability of rolling a 3 is around 10%.
    iii. I want to know what the probability of rolling a 3 on a six sided die. I know there are six faces possible and 3 is one of those faces, therefore the probability of rolling a three is 1/6.
    iv. I want to find the probability of rolling a 3 on a die. I asked five of my friends as well as my brother and parents. Of the 8 people I talked to 5 of them told me it was 1/6. The other three told me it depends on the die, the probability could be small or high, you never know. So I am not sure, but based on what most of the people I asked said, I would say its 1/6.
2. (b) Of these four statements, which one do you identify with, or agree with the most?

3. (a) Read the following statements and say a little about each one.
    i. Probability is a degree of belief based on personal judgment and information about experiences related to a given outcome.
    ii. Probability is the fraction of favorable outcomes over the total outcomes.
    iii. Probability is the hypothetical number towards which relative frequency tends when stabilizing.
    iv. Probability is the measured randomness of an occurrence.
3. (b) Which of the four do you identify, or agree, with the most?
4. As a teacher, if you want students to develop a deep understanding of probability, one that is applicable to life, what are the major concepts you would want them to learn?
5. What strategies would you use to teach these concepts?
6. What has been the most meaningful for you throughout this research process?
7. What are some of the things you have learned about the teaching and learning of probability?
8. What have you learned about your own learning?

APPENDIX G: TASKS

Task 1 – Interpretation of Randomness and Probability

1. What does random mean to you? Can you think of an example of something that happens in a random way? Does random mean something different in a probability and statistics class?
2. What does it mean to say the probability of getting a head on a coin toss is 50%? What does 70% chance of rain mean? If you had to explain what probability means what would you say?

Task 2 – Curricular Issues

1. Why do you think it is important for students to learn probability?
2. Take a few minutes to brainstorm what topics or concepts related to probability you think are important to teach at the MS/HS level.

Task 3 – Real world Context and Teaching Situation

[adapted from Garfield: ARTIST] You are trying to decide between two types of cars. You first consult an issue of Consumer Reports, which compared rates of repairs for various cars. Records of repairs done on 400 cars of each type showed somewhat fewer mechanical problems with Hondas than with Toyotas. You have two friends that own Toyotas and one friend that owns a Honda.

Both Toyota owners reported having a few mechanical problems, but nothing major.

The Honda owner, however, exploded when asked how he liked his car: "First, the fuel injection went out, that cost $250 bucks. Next, I started having trouble with the rear end and had to replace it. I finally decided to sell it after the transmission went. I'd never buy another Honda."

Given what you currently know, which car would you buy? Justify your response.

You are teaching a class and have posed this problem. The following are three different groups' responses. **How would you proceed with the class discussion?**

a. We would recommend you buy the Toyota, primarily because of all the trouble your friend had with his Honda. Since you haven't heard similar horror stories about the Toyota, you should go with it.
b. We would recommend you buy the Honda in spite of your friend's bad experience. That is just one case, while the information reported in Consumer Reports is based on many cases. According to that data, the Honda is somewhat less likely to require repairs.
c. We would tell you that it does not matter which car you buy. Even though one of the models might be more likely than the other to require repairs, they could still, just by chance, get stuck with a particular car that would need a lot of repairs. You may as well toss a coin to decide.

Task 4 – Experimental Context and Critique/Plan

[A bowl on the table has 500 wrapped hard candies: 20% are purple, 50% are red – watermelon and cherry, and 30% are green; interviewee does not know these percentages.]
Given this bowl of candies, explain how you could make a claim about the probability of selecting a red candy.

a. Is there any way to do experimentation to support the claim about the probability of selecting a red candy? What would you do with that data?
b. Would you use this task with MS or HS students? Why, why not?
c. How would you use this task?

Schoolopoly Task (Maher, et al., 2006)

Plainfield Middle Schoolopoly

Background
Plainfield Middle Schools are planning to create a board game modeled on the classic game of Monopoly. The game is to be called "Schoolopoly" and, like Monopoly, will be played with dice. Because many copies of the game will be sold as a fundraiser, several companies are competing for the contract to supply dice for Schoolopoly. Several companies, however, have been accused of making poor quality dice. These companies are to be avoided since players of Schoolopoly need to know that the dice they are using are actually "fair."
Each dice company has provided a sample die for analysis. You will be assigned one company to investigate:

Perfect Polyhedra	Calibrated Cubes
Dice R' Us	Dazzling Dice
Delta's Dice	Dice Depot

Your Assignment
Investigate whether or not the die sent to you by the company is fair. That is, are all six outcomes equally likely to occur?

You will need to create a poster to present to the School Board. The following three questions should be answered on your poster:

1. Would you recommend that dice be purchased from the company you investigated?
2. What evidence do you have that the die you tested is "fair" or "unfair"?
3. What do you think the chances are for rolling each of the six numbers?

Use Probability Explorer to collect data from simulated rolls of the die. Copy any graphs and screen shots you want to use as evidence and paste them in a Word document. Later, you will be able to print these to use on a poster. You will give a brief presentation pointing out the highlights of your group's poster.

Task 5a – Schoolopoly Task Analysis

5. a) **Discuss activity**: Begin with an open discussion (2-3mins) "tell me your thoughts on this activity." Then ask more direct questions:

a. What do you see as the learning objectives associated with this task? What is the purpose of the task, what is it students should learn by doing this task?
b. What difficulties might students have with this task?
c. [If not already discussed:] In general what are the strengths and weaknesses of the task? Pros and cons?
d. Is this a task you would use if you were teaching probability? Why or why not?

Task 5b – Analyze Schoolopoly Student Work

5. b) **Analyze Posters**: Posters A & B (Calibrated Cubes), posters C & D (Dice R Us)

a. Based on the posters, what can you say about the students' understanding of probability?
b. What do you think they consider to be compelling evidence?
c. Compare posters A & B, and C & D. Are you convinced by any of these posters? Why or why not?

Task 5c – Analyze Schoolopoly Teaching Episode

5. c) **View whole class discussion**: The class discussion is about sample size and Calibrated Cubes, some comments from the students were: "80 seems reasonable" "are you going to quit or are you going to keep playing to try to win your money back?" "It doesn't matter how many times the percentages will be the same regardless"

a. How would you direct the conversation at this point?
b. What is her misconception?
c. What ways could you intervene to address this misconception?

View intervention and discuss: The teacher uses the software to demonstrate the effect of large and small sample sizes starting with a sample size of 10.

d. Pros and cons – do you think this demonstration was effective? How can you tell?
e. What could the teacher have done differently?

Task 5d – Reflection on Second Interview

5. d) **Reflection**: During the focus group you participated in analyzing a task that was used to teach probability to middle school students. You looked at the task, students' work, as well as two video clips of class discussion related to the task.

a. What do you see as the benefits and drawbacks if you were to use this task to teach probability?
b. Based on the students work and the class discussion, if you were the teacher of this class, what would you do the next day?
c. What did you take away from the experience? Write a few sentences about what you learned by participating in these activities.
d. What did you learn about your own understanding of probability and of teaching probability?

Task 6a – Interpretations of Calculating Probability

a) Read the following statements and say a little bit about each one.

1. I want to figure out the probability of rolling a 3 on a die. I simulated it and out of 1,000,000 rolls and 3 showed up 166,549 times. So the probability of getting a 3 is about 166,549/1,000,000 = .166549.
2. I think the probability of rolling a 3 is not very good. Based on my experience playing games with dice 3 doesn't come up very often. I'd say the probability of rolling a 3 is around 10%.
3. I want to know what the probability of rolling a 3 on a six sided die. I know there are six faces possible and 3 is one of those faces, therefore the probability of rolling a three is 1/6.
4. I want to find the probability of rolling a 3 on a die. I asked five of my friends as well as my brother and parents. Of the 8 people I talked to 5 of them told me it was 1/6. The other three told me it depends on the die, the probability could be small or high, you never know. So I am not sure, but based on what most of the people I asked said, I would say its 1/6.

b) Of these four statements, which one do you identify with, or agree with the most?

Task 6b – Interpretations of Meaning of Probability

a) Read the following statements and say a little about each one.

1. Probability is a degree of belief based on personal judgment and information about experiences related to a given outcome.
2. Probability is the fraction of favorable outcomes over the total outcomes.
3. Probability is the hypothetical number towards which relative frequency tends when stabilizing.
4. Probability is the measured randomness of an occurrence.

b) Which of the four do you identify, or agree, with the most?

Task 7 – Curricular Issues Revisited

1. As a teacher, if you want students to develop a deep understanding of probability, one that is applicable to life, what are the major concepts you would want them to learn?
2. What strategies would you use to teach these concepts?
3. What are some of the things you have learned about the teaching and learning of probability?

Task 8a – Multiple Choice Test Questions Assessing Interpretations of Probability and Understanding of Independence

5) The following message is printed on a bottle of prescription medication.
Warning: For applications to skin areas there is a 15% chance of developing a rash. If a rash develops, consult your physician.
Which of the following is the best interpretation of this warning?
   a. Don't use the medication on your skin, there's a good chance of developing a rash.
   b. For applications to the skin, apply only 15% of the recommended dose.
   c. If a rash develops, it will probably involve 15% of the skin.
   d. About 15 out of 100 people who use this medication develop a rash.
   e. There is hardly a chance of getting as rash using this medication.

6) The Springfield Meteorological Center wanted to determine the accuracy of their weather forecasts. They searched the records for those days when the forecaster had reported a 70% chance of rain. They compared these forecasts to records of whether or not it actually rained on those particular days.
The forecast of 70% chance of rain can be considered *very* accurate if it rained on:
   a. 95% - 100% of those days.
   b. 85% - 94% of those days.
   c. 75% - 84% of those days.
   d. 65% - 74% of those days.
   e. 55% – 64% of those days.

7) The following sequences represent the results from tossing a fair coin 5 times. Each sequence displays the results in the order in which they occurred. Which sequence is *most* likely?
   a. H H H T T
   b. T H H T H
   c. T H T T T
   d. H T H T H
   e. All four sequences are equally likely

8) Select the one explanation that best describes your reason for the answer you gave in item 3.
   a. Since the coin is fair, you ought to get roughly equal numbers of heads and tails.
   b. Since coin flipping is random, the coin ought to alternate frequently between landing heads and tails.
   c. If you repeatedly flipped a coin five times, each of these sequences would occur about as often as any other sequence.
   d. If you get a couple of heads in a row, the probability of tails on the next flip increases.

Task 8b – Multiple Choice Test Questions Assessing Sample Size, Proportional Reasoning, Recognizing Random Distribution, Expected Variability

4) Two containers, labeled A and B, are filled with red and blue marbles in the following quantities:

Container	Red	Blue
A	6	4
B	60	40

Each container is mixed thoroughly. After choosing one of the containers, you will reach in and, without looking, draw out a marble. If the marble is blue, you win $50. Which container gives you the best chance of drawing a blue marble?

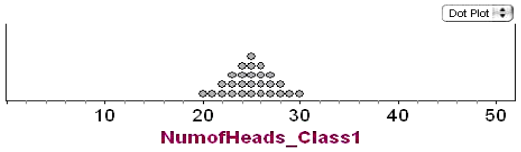
a. Container A (with 6 red and 4 blue)
b. Container B (with 60 red and 40 blue)
c. Equal chances from each container

5) Half of all newborns are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?

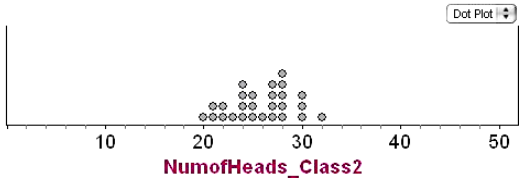
a. Hospital A (with 50 births a day)
b. Hospital B (with 10 births a day)
c. The two hospitals are equally likely to record such an event.

6) Each student in a class tossed a penny 50 times and counted the number of heads. Suppose four different classes produce graphs for the results of their experiment. There is a rumor that in some classes, the students just made up the results of tossing a coin 50 times without actually doing the experiment. **Please select each** of the following graphs you believe represents data from actual experiments of flipping a coin 50 times.

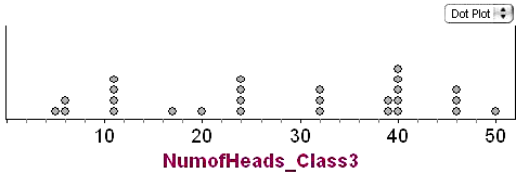
a.



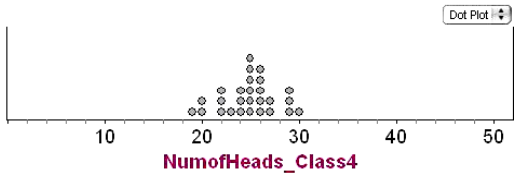
b. **correct**



c.



d. **correct**



Task 8c – Open-Ended Pedagogy Question Critiquing Student Strategies

3. Students are presented with a six-sided die and asked to experiment with it to make a claim as to whether the die is fair or unfair. You observe the strategy employed by two different groups, described below.

   Group 1: Toss the die 10 times and compute the empirical probability for each number on the die. If the empirical probability is close to 1/6 then the die is fair.

   Group 2: Toss a die six times. If each number appears once then the die is fair.

   Which group is using an appropriate strategy to make and support a claim about fairness?

   e. Group 1
   f. Group 2
   g. Both Group 1 and Group 2
   h. Neither Group 1 or Group 2

4. Write a brief critique about the strengths and weaknesses of each strategy described in question #1.