

Corrigendum

Corrigendum to “Best approximation in polyhedral
Banach spaces” [J. Approx. Theory 163 (11)
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Abstract

The present note is a corrigendum to the paper “Best approximation in polyhedral Banach spaces”, J. Approx. Theory 163 (2011) 1748–1771.

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Professor G. Godefroy kindly pointed out to us that the implication

“if P_Y is H -u.s.c. on its effective domain then Y is relatively strongly proximinal”

in [1, Theorem 5.1] is false. A counterexample is provided by any proximinal (and hence strongly proximinal) hyperplane $Y \subset X$, given by a norm-one $f \in X^*$ which is not a point of strong subdifferentiability for X^* — this follows easily from Lemma 1.1 and Theorem 2.5 in [2].

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(The gap in our proof consists in the fact that $\|\xi_n\|$ can be in general less than 1, and so Lemma 2.7 cannot be applied at the end of the proof.) Fortunately, for our results which use [1, Theorem 5.1] this implication is not necessary.

Theorem 5.1 in [1] should be substituted with the following proposition which uses the auxiliary mapping $R_Y(\xi) = q^{-1}(\xi) \cap B_X$ ($\xi \in X/Y$), where $q: X \rightarrow X/Y$ is the quotient map (see [1, Section 2]).

Proposition. *Let Y be a closed subspace of a Banach space X . Then the following assertions are equivalent:*

- (i) Y is relatively strongly proximal;
- (ii) R_Y is H -u.s.c. on $q(B_X)$;
- (iii) R_Y is H -u.s.c. at each point of $\Sigma := q(B_X) \cap S_{X/Y}$.

Moreover, if these assertions hold, then the metric projection P_Y is H -u.s.c. on its effective domain.

Proof. The equivalence (ii) $\Leftrightarrow$ (iii) follows from [1, Corollary 2.5]. For the implication (iii) $\Rightarrow$ (i), it suffices to repeat the proof of [1, Theorem 5.1], starting from its third line.

It remains to show that (i) $\Rightarrow$ (iii). Assume (iii) is false. There exists $\xi \in \Sigma$ and a sequence $\{\xi_n\} \subset q(B_X)$ such that $\xi_n \rightarrow \xi$ and, for some $z_n \in R_Y(\xi_n)$ ($n \in \mathbb{N}$) and some $a > 0$, we have $d(z_n, R_Y(\xi)) > a$ for each n . Choose $x \in B_X$ such that $\xi = q(x)$, and notice that $d(x, Y) = \|\xi\| = 1$. Since $\xi_n \rightarrow \xi$, there exist points $x_n \in q^{-1}(\xi_n)$ ($n \in \mathbb{N}$) such that $x_n \rightarrow x$. Now, for each n , $z_n, x_n \in q^{-1}(\xi_n)$ implies that $x_n - z_n = y_n \in Y$. By [1, Observation 2.6], we have $R_Y(\xi) = x - P_Y(x)$. We can write

$$\begin{aligned} \liminf_{n \rightarrow \infty} d(y_n, P_Y(x)) &= \liminf_{n \rightarrow \infty} d(x_n - z_n, x - R_Y(\xi)) \\ &= \liminf_{n \rightarrow \infty} d(z_n + x - x_n, R_Y(\xi)) \\ &= \liminf_{n \rightarrow \infty} d(z_n, R_Y(\xi)) \geq a. \end{aligned}$$

On the other hand, $\|y_n - x\| \rightarrow d(x, Y) = 1$ since

$$\|y_n - x\| \leq \|z_n\| + \|x_n - x\| \leq 1 + \|x_n - x\| \rightarrow 1$$

and

$$\|y_n - x\| \geq \|z_n\| - \|x_n - x\| \geq \|\xi_n\| - \|x_n - x\| \rightarrow 1.$$

It follows that (i) is false.

For the last part, see the first two lines of the proof of [1, Theorem 5.1]. $\square$

In [1, Theorem 4.2], we have proved that if X is polyhedral with (*) then R_Y is H -u.s.c. on $q(B_X)$. Thus, by the above proposition, Theorem 4.3 and Corollary 4.4 remain valid. Notice that Example 6.1 uses only the correct implication of Theorem 5.1.

Remark. Let Y be a proximal subspace of X . Notice that, by [1, Lemma 2.7], P_Y is H -u.s.c. if and only if the restriction of R_Y to $S_{X/Y}$ is H -u.s.c. On the other hand, by the above proposition, Y is strongly proximal if and only if R_Y is H -u.s.c. at each point of $S_{X/Y}$.

References

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