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Corrigendum

# Corrigendum to "The recognition problem for table algebras and reality-based algebras" [J. Algebra 479 (2017) 173–191]

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ABSTRACT

This note reports and corrects an error in the above article.


An error has been discovered in the proof of Lemma 1 of [1]. Taking the union of the set of nondiagonal matrix units in $M_n(\mathbb{C})$ with an arbitrary RBA-basis of the commutative subalgebra of diagonal matrices will not always produce an RBA-basis of $M_n(\mathbb{C})$, because the requirement that $\lambda_{ii^*0} = \lambda_{i^*i0}$ condition (v) of the RBA definition in [1] may fail to hold. The authors would like to express their sincere gratitude to Harvey Blau for informing them of this error.

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Lemma 1 is used later in [1] for the conclusion of Theorem 2, which is later utilized for the conclusions of Theorem 3 and Corollary 4. The key point in Theorem 2 is that $M_n(\mathbb{C})$ has a rational RBA-basis consisting of the union of the set of nondiagonal basis elements with a rational RBA-basis $\mathbf{D} = \{d_0 = 1, d_1, \ldots, d_{n-1}\}$ of the commutative subalgebra of diagonal matrices, considered as an RBA whose involution acts trivially on $\mathbf{D}$. The previously overlooked condition (v) of the definition requires that the coefficient of $d_0$ in the expression of the diagonal elementary matrix $E_{ii}$ in the basis $\mathbf{D}$ be positive and independent of $i$. The following Lemma, which is due to the reviewer of an earlier version of this note, ensures suitable rational RBA-bases of the subalgebra of diagonal $n \times n$ matrices will exist for all positive integers $n$.

**Lemma 1.** *Let $\mathbb{Q}^n$ be the commutative algebra of $n$-dimensional column vectors under coordinatewise multiplication.*

*(i) There exist bases $\mathbf{D} = \{d_0, d_1, \ldots, d_{n-1}\}$ of $\mathbb{Q}^n$ such that $d_0 = (1, \ldots, 1)^\top$ and the elements of $\mathbf{D}$ are pairwise orthogonal with respect to the usual inner product $\langle \cdot, \cdot \rangle$.*
*(ii) Any such basis is a rational RBA-basis of $\mathbb{C}^n$ with the property that for all $i = 0, 1, \ldots, n-1$, the coefficient of $d_0$ in the expression of $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)^\top$ in the basis $\mathbf{D}$ is $1/n$.*

**Proof.** (i) Since $d_0 = (1, \ldots, 1)^\top$ is a nonzero vector, it is contained in a basis of $\mathbb{Q}^n$. Applying the usual Gram–Schmidt process to a basis $\{d_0, d_1, \ldots, d_{n-1}\}$ of $\mathbb{Q}^n$ will result in an orthogonal basis of $\mathbb{Q}^n$ that contains $d_0$. This proves (i).

(ii) Let $\mathbf{D} = \{d_0, d_1, \ldots, d_{n-1}\}$ be an orthogonal basis of $\mathbb{Q}^n$ with $d_0 = (1, \ldots, 1)^\top$. Then $\delta_k = \langle d_k, d_k \rangle > 0$ for all $k$, and $\langle d_i, d_j \rangle = \delta_{i,j}\delta_i$ for all $i, j$, where $\delta_{i,j}$ denotes the Kronecker delta. Under componentwise multiplication, the structure constants $\lambda_{ijk}$ relative to $\mathbf{D}$ satisfy

$$\lambda_{ijk} = \delta_k^{-1} \langle d_i \cdot d_j, d_k \rangle, \text{ for all } i, j, k.$$

Since the usual inner product satisfies $\langle d_i \cdot d_j, d_k \rangle = \langle d_i, d_j \cdot d_k \rangle$, we have that the coefficient of $d_0$ in $d_i \cdot d_j$ is $\lambda_{ij0} = \delta_{i,j}\delta_i\delta_0^{-1} = \delta_{i,j}\delta_i/n$. It follows from this observation that $\mathbf{D}$ is a rational RBA-basis of $\mathbb{C}^n$, where the involution acts trivially on $\mathbf{D}$.
Let $e_i = \sum_j x_{ij} d_j$ be the expression of the primitive idempotent $e_i$ in terms of the basis $\mathbf{D}$. Then $1 = \langle e_i, d_0 \rangle = x_{i0}\delta_0$ implies $x_{i0} = \dfrac{1}{n}$. As this is independent of $i$, (ii) holds. $\square$

## References

[1] A. Herman, M. Muzychuk, B. Xu, The recognition problem for table algebras and reality-based algebras, J. Algebra 479 (2017) 173–191.