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Corrigendum

Corrigendum to "On 3-dimensional complex Hom-Lie algebras" [J. Algebra 555 (2020) 361–385]

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We wish to fix an error in [1]. The list of representatives of the isomorphism classes of skew-symmetric products $\mu : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ for a complex 3-dimensional vector space $\mathfrak{g}$ given there in Proposition 3.5 corresponding to the rank-2 degenerate products characterized by (2) in Proposition 3.3, were collapsed into a single representative, whereas a one-parameter family of representatives should have been obtained, as it is well known from the classification of 3-dimensional Lie algebras. The mistake affects the statements of Propositions 3.5 and 5.4. This note gives the correct statements of these two results. The rest is unaffected.

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The correct statements 3.5 and 5.4 in [1]

The description of the $\mathrm{GL}(\mathbb{C}^3)$-orbits given in the statement of Proposition 3.3 is correct. In the proof, however, we should have obtained a one-parameter family of orbits associated to the degenerate product μ of rank 2 corresponding to the case $\pi_2(\mathbf{a}_\mu) = 0$ with $\mathbf{a}_\mu = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \neq 0$. In fact, the product for such an $\mathbf{a}_\mu$ takes the form $\mu = \begin{pmatrix} 1 & -z & 0 \\ z & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Then, we may choose $g = \begin{pmatrix} g_{1*} \\ g_{2*} \\ g_{3*} \end{pmatrix} \in \mathrm{GL}(\mathbb{C}^3)$ with $g_{3*} = z^{-1}\mathbf{a}_\mu$ and complete g with the rows g_{1*} and g_{2*} so as to have g in the isotropy subgroup G_2. The point is that, according to (10), the matrix associated to $\mathbf{a}_{g\cdot\mu}$ is exactly the same as the matrix associated to $\mathbf{a}_\mu$ itself. Therefore, the product depends on the non-zero parameter $z \in \mathbb{C}$. We thus write,

$$\mu = \mu''_{4;z} := \begin{pmatrix} 1 & -z & 0 \\ z & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

One would then have to consider separately the cases $1 + z^2 \neq 0$ and $1 + z^2 = 0$ in order to obtain product representatives. This fact changes statement (1) of Proposition 3.5 in the following way:

(1) *If* $\operatorname{rk} S_\mu = 2$, *there are three non-equivalent products:*

$$\mu''_1 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mu''_2 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mu''_3 := \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

together with the one-parameter family,

$$\mu''_{4;z} = \begin{pmatrix} 1 & -z & 0 \\ z & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad z \in \mathbb{C} - \{0\}.$$

If $z \neq \pm i$, *this is equivalent to the one-parameter family,*

$$\mu''_{4;\lambda} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \lambda & 0 \end{pmatrix}, \qquad \lambda \neq 0, \pm 1.$$

If $z = \pm i$ *there is one additional degenerate case:*

$$\mu''_{4;\pm i} := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Statements (2) and (3) of Proposition 3.5 are correct and remain the same. However, a proof of the statement regarding $\mu''_{4;z}$ must be added to Proposition 3.5. The proof goes as follows:

Proof. Let $\{e_1, e_2, e_3\}$ be the basis of $\mathfrak{g}$ under which the product corresponding to case (2) of Proposition 3.3 takes the form $\mu''_{4;z}$ with $z \neq 0$. It is clear that $\mu''_{4;z}$ is equivalent to $\mu''_{4;-z}$. There is a basis $\{e'_1, e'_2, e'_3\}$ given by, $e'_1 = \gamma\, e_3$, $e'_2 = i\, e_1 + e_2$, and $e'_3 = -i\, e_1 + e_2$, with $\gamma \neq 0$ with respect to which,

$$\mu''_{4;z} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i(1+iz)\gamma \\ 0 & i(1-iz)\gamma & 0 \end{pmatrix} \tag{1}$$

Now, if $z \neq i$, take $\gamma = -i(1+iz)^{-1}$, to obtain:

$$\mu''_{4;z} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \lambda & 0 \end{pmatrix}, \quad \lambda = \frac{1-iz}{1+iz} \neq \pm 1.$$

It is also true that, $\lambda \neq 0$. Otherwise, $z = -i$, but in this case one redefines γ to end up with a similar result with $\lambda' = \lambda^{-1}$ in the 23-entry of $\mu''_{4;z}$ and 1 in its 32-entry. This argument also shows that if $z = -i$ one may choose $\gamma = -i/2$ and obtain the degenerate form of the product given in the statement. $\square$

Having changed the products $\mu''_{4;z}$, section 5.1.4 must be changed by changing the statement of Proposition 5.4 whose proof changes only slightly because the isotropy subgroups $G_{\mu''_{4;\lambda}}$ and $G_{\mu''_{4;\pm i}}$ are both given by the same matrices as before; namely,

$$G_{\mu''_4} := \left\{ \begin{pmatrix} 1 & 0 & 0 \\ c & a & 0 \\ d & 0 & b \end{pmatrix} \,\middle|\, ab \neq 0;\; c, d \in \mathbb{C} \right\}.$$

The correct statement of Proposition 5.4 is therefore the following:

5.4. Proposition. *Let $\mathfrak{g}$ be a complex 3-dimensional vector space and fix bases so that the product representatives $\mu''_{4;z}$ take either the form $\mu''_{4;\lambda}$ ($\lambda \neq 0, \pm 1$) if $z \neq \pm i$, or the form $\mu''_{4;\pm i}$ given in Proposition 3.5 (1).*

A. *If $z \neq \pm i$, the pair $(\mathfrak{g}, \mu''_{4;z})$ is a solvable non-nilpotent Lie algebra whose first derived ideal is two-dimensional. Moreover, any $T \in \mathrm{HL}(\mu''_{1;z})$ is equivalent to a one and only one of the following canonical forms:*

(1) *If* $\det(\Theta - T_{11}\mathbb{1}_2) \neq 0$, *then*

$$T \simeq \begin{pmatrix} T'_{11} & 0 \\ 0 & \Theta' \end{pmatrix}, \quad \text{with } T'_{11} \in \mathbb{C}, \text{ and } \det(\Theta - T_{11}\mathbb{1}_2) \neq 0.$$

(2) *If* $\det(\Theta - T_{11}\mathbb{1}_2) = 0$, *then*

$$T \simeq \begin{pmatrix} T'_{11} & 0 \\ v' & \Theta' \end{pmatrix}, \quad \text{with } T'_{11} \in \mathbb{C}, \text{ and } v' \in \mathbb{C}^2, \quad \det(\Theta' - T'_{11}\mathbb{1}_2) = 0.$$

B. *If $z = \pm i$, then $(\mathfrak{g}; \mu_{1;-i})$ is a solvable non-nilpotent Lie algebra whose first derived ideal is one-dimensional. Moreover, any $T \in \mathrm{HL}(\mu_{1;-i})$ is of the form,*

$$\begin{pmatrix} T_{11} & w^t \\ v & \Theta \end{pmatrix}, \quad T_{11} \in \mathbb{C},\ v \in \mathbb{C}^2,\ w = \left(\begin{smallmatrix} T_{12} \\ 0 \end{smallmatrix}\right),\ \Theta \in \mathrm{Mat}_{2\times 2}(\mathbb{C}).$$

Furthermore, if T is equivalent to $T' \in \mathrm{HL}(\mu_{1;-i})$, then either $T_{12} = T'_{12} = 0$ or $T_{12}T'_{12} \neq 0$. If $T_{12} = 0$, then T is equivalent to one and only one of the canonical forms given in (1) and (2) in **A**. *If $T_{12} \neq 0$, then T is equivalent to,*

$$\begin{pmatrix} T'_{11} & 1 & 0 \\ T'_{21} & 0 & T'_{22} \\ T'_{31} & 0 & T'_{33} \end{pmatrix}.$$

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References

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