International Journal of Solids and Structures 51 (2014) 3689–3697

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Love wave dispersion in pre-stressed homogeneous medium over a porous half-space with irregular boundary surfaces

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ARTICLE INFO

Article history:
Received 17 February 2014
Received in revised form 4 July 2014
Available online 23 July 2014

Keywords:
Love wave
Porosity
Homogeneous medium
Irregularity
Dispersion equation
Phase velocity

ABSTRACT

The paper investigates the existence of Love wave propagation in an initially stressed homogeneous layer over a porous half-space with irregular boundary surfaces. The method of separation of variables has been adopted to get an analytical solution for the dispersion equation and thus dispersion equations have been obtained in several particular cases. Propagation of Love wave is influenced by initial stress parameters, corrugation parameter and porosity of half-space. Velocity of Love waves have been plotted in several figures to study the effect of various parameters and found that the velocity of wave decreases with increases of non-dimensional wave number. It has been observed that the phase velocity decreases with increase of initial stress parameters and porosity of half-space.

1. Introduction

The nature of different seismic waves is studied in theoretical seismology and also has some practical importance in the field of Civil Engineering, Rock Mechanics and Geophysical Prospecting. The propagation of Love waves in a homogeneous medium over semi-infinite porous medium has importance in earthquakes engineering and seismology on account of the occurrence of porosity, inhomogeneity in the crust of the Earth as the Earth is supposed to be made up of different layers. In the beneath of Earths surface the porous layer is naturally found. In general the pores contain hydrocarbon deposition such as gas and oil. Most oil and gas deposits are found in sandstone or limestone is very much like a hard sponge, full of holes but not compressible. These holes or pores can contain water or oil or gas and rock will be saturated with one of these three. The holes are much tinier than sponge holes but they are still holes and they are called porosity and the layer is called porous layer. The studies of Love wave propagation in a liquid saturated porous medium play an important role in the field of Geophysical problems leading to the exploration of oil and underground water. The propagation of Love wave in elastic media with irregular boundary surfaces is also important leading to better understanding and prediction of seismic wave behaviour at continental margins, mountain roots, etc. Propagation of surface waves in a homogeneous medium over an inhomogeneous elastic half-space are well known and prominent feature of wave theory. Quite a large amount of information about propagation of seismic waves is documented in well-known books written by Biot (1965), Ewing et al. (1957), Gubbins (1990), etc.

The stress generated in a medium, referred as initial stress and may be developed in media due to natural phenomena or by any artificial stress. The Earth may be assumed as an elastic solid layered medium under high initial stresses. These initial stresses contribute a significant influence on elastic waves produced by earthquakes. Many researchers have devoted their work to solve various problems on the propagation of surface waves. Dey et al. (1996) discussed about propagation of Love waves in heterogeneous crust over a heterogeneous mantle. They (Dey et al., 2004) also studied about propagation of Love waves in an elastic layer with void pores. The influence of anisotropy on the Love waves in a self-reinforced medium was formulated by Pradhan et al. (2003). Sharma (2004) established a mathematical expression about wave propagation in a general anisotropic poroelastic medium with anisotropic permeability phase velocity and attenuation. Kalyani et al. (2008) have made finite difference modeling of seismic wave propagation in monoclinic media.

Propagation of seismic waves in layered media bounded by different forms of irregular boundaries has been investigated by many authors. Wolf (1970) observed the propagation of Love waves in layers with irregular boundaries. The dispersion equation for Love wave due to irregularity in the thickness of non-homogeneous crustal layer was obtained by Chattopadhyay

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http://dx.doi.org/10.1016/j.ijsolstr.2014.07.002
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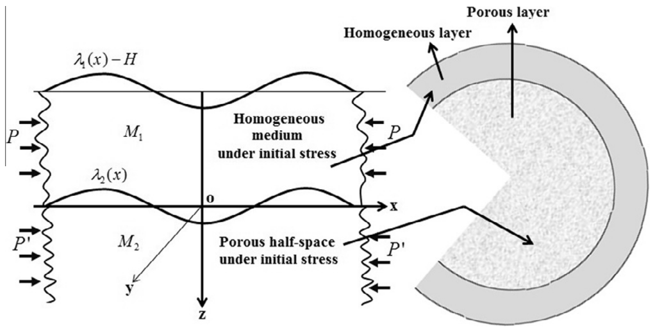

Fig. 1. Geometry of the problem.

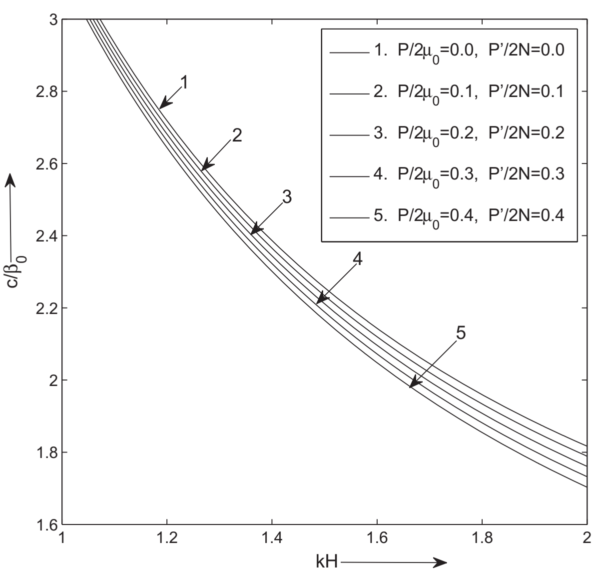

Fig. 2. Case I: dimensionless phase velocity $\frac{c}{\beta_0}$ as a function of dimensionless wave number kH of Love waves for different values of initial stresses $\frac{P}{2\mu_0}$ and $\frac{P'}{2N}$.

(1975). The influence of irregularity and rigidity on the propagation of torsional wave was developed by Gupta et al. (2010a). Chattaraj et al. (2013) introduced the dispersion equation of Love wave, propagating in an irregular anisotropic porous stratum under initial stress. Chattopadhyay et al. (2010) studied about propagation of SH waves in an irregular non-homogeneous monoclinic crustal layer over a semi-infinite monoclinic medium. Propagation of Love wave at a layered medium bounded by irregular boundary surfaces was developed by Singh (2011). Dispersion of horizontally polarized shear waves in an irregular non-homogeneous self-reinforced crustal layer over a semi-infinite self-reinforced medium was derived by Chattopadhyay et al. (2013). Liu and He (2010) studied the properties of Love waves in layered piezoelectric structures.

A good amount of research has been done by many authors in the field of Love wave propagation. Ke et al. (2006) studied the Love waves in an inhomogeneous fluid saturated porous layered half-space with linearly varying properties. Ghorai et al. (2010) shown the Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity. Propagation of Love waves in an orthotropic Granular layer under initial stress overlying a semi-infinite Granular medium was established by Ahmed and Abd-Dahab (2010). Gupta et al. (2010b) discussed the effect of initial stress on propagation of Love waves in an anisotropic porous layer. Disturbance of SH-type waves due to discontinuity of shearing stress in a visco-elastic layered half-space was formulated by Pal and Sen (2011). Kielczynski et al. (2012) observed the effect of a viscous liquid loading on Love wave propagation.

Recently numerous papers have been done by many researchers in the field of wave propagation. Such as Gupta et al. (2013a) discussed about the propagation of Love waves in a non-homogeneous substratum over an initially stressed heterogeneous half-space. Love waves in the fiber-reinforced layer over a gravitating porous half-space was investigated by Chattaraj and Samal (2013). Possibility of Love wave propagation in a porous layer under the effect of linearly varying directional rigidities was introduced by Gupta et al. (2013b). SH-type waves dispersion in an isotropic medium sandwiched between an initially stressed orthotropic and heterogeneous semi-infinite media were studied by Kundu et al. (2013). Manna et al. (2013) formulated Love wave propagation in a piezoelectric layer overlying in an inhomogeneous elastic half-space. Bacigalupo and Gambarotta (2014) discussed about second-gradient homogenized model for wave propagation in heterogeneous periodic media. Propagation of Love wave in fiber-reinforced medium lying over an initially stressed orthotropic half-space was obtained by Kundu et al. (2014).

In this paper, the propagation of Love wave in a homogeneous irregular layer over an elastic porous half-space has been briefly studied. Both the layer and half-space are considered under the effect of initial stress. The dispersion relations have been derived in some particular cases by taking irregular boundary surfaces, $a_1 cos(bx)$ and $a_2 cos(bx)$, where a_1 and a_2 are amplitudes of the surfaces. The influences of porosity, initial stress parameters and corrugation parameter have discussed graphically.

2. Mathematical formulation of the problem

We consider an initially stressed (P) elastic homogeneous layer $M_1 : \lambda_1(x),\ \lambda_1(x) - H \leqslant z$ over an initially stressed (P') porous half-space, $M_2 : \lambda_2(x) \leqslant z \leqslant \infty$ as shown in Fig. 1, where H can be assumed as the average thickness of the upper layer. The x axis and y axis are considered as two perpendicular Cartesian coordinates lying horizontally and vertically coordinate with positive direction pointing downward can be taken as z axis. Where $\lambda_1(x)$ and $\lambda_2(x)$ are continuous functions of x independent of y and consider as the irregular boundaries of the layer. The x axis is parallel to the direction of propagation of waves. So, the non-zero field of quantities representing the motion are only function of $x,\ z$ and time t.

The functions, $\lambda_j(x)$ can be taken as periodic in nature and their Fourier series expansions are provided as Singh (2011)

$$\lambda_j(x) = \sum_{n=1}^{\infty}\left(\lambda_n^j e^{inpx} + \lambda_{-n}^j e^{-inpx}\right), \quad j = 1,2. \tag{1}$$

Here the Fourier series expansion coefficients are λ_n^j and λ_{-n}^j, the wave number is p, $\frac{2\pi}{p}$ is the wavelength, n is the order of series expansion and $i = \sqrt{-1}$. We assumed a very small amplitude of irregular boundaries compared with the wavelength.

3. Dynamics of upper homogeneous layer

The upper layer of the formulated problem is considered as initially stressed homogeneous medium. Let $u_1,\ v_1$ and w_1 be the displacements along $x,\ y$ and z directions respectively. First, we look for the equations governing the propagation of Love wave in homogeneous elastic medium. In this medium waves are propagating along x axis. The equations of motion for a homogeneous elastic solid in the absence of body forces in component form are (Biot, 1965)

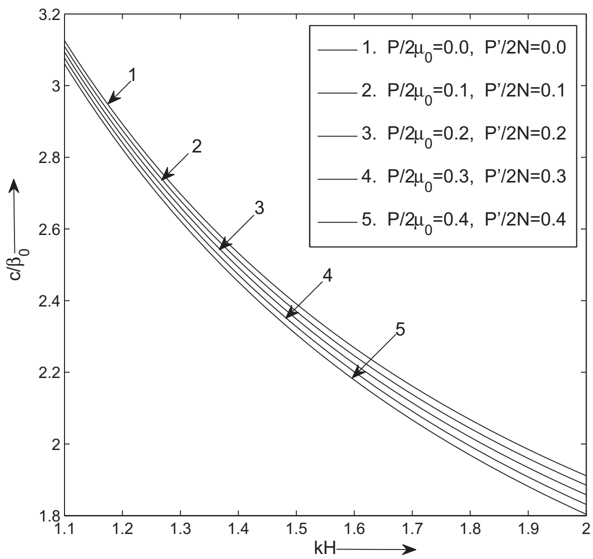

Fig. 3. Case II: dimensionless phase velocity $\frac{c}{\beta_0}$ as a function of dimensionless wave number kH of Love waves for different values of initial stresses $\frac{P}{2\mu_0}$ and $\frac{P'}{2N}$.

$$\left.\begin{aligned}&\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}+\frac{\partial\sigma_{13}}{\partial z}-P\left(\frac{\partial w_z}{\partial y}-\frac{\partial w_y}{\partial z}\right)=\rho_1\frac{\partial^2 u_1}{\partial t^2},\\&\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}+\frac{\partial\sigma_{23}}{\partial z}-P\left(\frac{\partial w_z}{\partial x}\right)=\rho_1\frac{\partial^2 v_1}{\partial t^2},\\&\frac{\partial\sigma_{31}}{\partial x}+\frac{\partial\sigma_{32}}{\partial y}+\frac{\partial\sigma_{33}}{\partial z}-P\left(\frac{\partial w_y}{\partial x}\right)=\rho_1\frac{\partial^2 w_1}{\partial t^2},\end{aligned}\right\}\tag{2}$$

where $w_x,\ w_y$ and w_z are the rotational components along $x,\ y$ and z directions respectively. Here σ_{ij} are the incremental stress components and ρ_1 is the density of the material in this medium.

In case of homogeneous medium Hooke's law gives the equation

$$\sigma_{ij}=\lambda\Delta\delta_{ij}+2\mu_1\varepsilon_{ij},\tag{3}$$

where $\lambda,\ \mu_1$ are Lame's constants, and $\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$.

Applying the Love wave conditions $u_1=0=w_1$ and $v_1=v_1(x,z,t)$ for the medium, then the strain components will be

$$\varepsilon_{11}=0,$$
$$\varepsilon_{12}=\frac{1}{2}\frac{\partial v_1}{\partial x},$$
$$\varepsilon_{22}=0,$$
$$\varepsilon_{13}=0,$$
$$\varepsilon_{23}=\frac{1}{2}\frac{\partial v_1}{\partial z},$$
$$\varepsilon_{33}=0.$$

The volume strain Δ or cubical dilatation, is the change in volume per unit volume. For small strain $\Delta=\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}$.

The stress components will be calculated as

$$\left.\begin{aligned}&\sigma_{11}=\lambda\Delta\delta_{11}+2\mu_1\varepsilon_{11}=0,\\&\sigma_{12}=\lambda\Delta\delta_{12}+2\mu_1\varepsilon_{12}=\mu_1\frac{\partial v_1}{\partial x},\\&\sigma_{22}=\lambda\Delta\delta_{22}+2\mu_1\varepsilon_{22}=0,\\&\sigma_{13}=0,\\&\sigma_{23}=\mu_1\frac{\partial v_1}{\partial z},\\&\sigma_{33}=0.\end{aligned}\right\}\tag{4}$$

Using the stress, strain relations and components condition for Love waves, the equation of motion (2) is written as

$$\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial z^2}-\left(\frac{P}{2\mu_1}\right)\frac{\partial^2 v_1}{\partial x^2}=\frac{1}{\beta_1^2}\frac{\partial^2 v_1}{\partial t^2},\tag{5}$$

where shear velocity, $\beta_1=\sqrt{\frac{\mu_1}{\rho_1}}$.

Let us assume the solution of the above equation be $v_1(x,z,t)=V_1(z)e^{ik(x-ct)}$, where k is the wave number and c is the phase velocity of Love waves. Therefore Eq. (5) takes the form

$$\frac{d^2V_1}{dz^2}+k^2q^2V_1=0,\tag{6}$$

where $q=\sqrt{\left[\frac{c^2}{\beta_1^2}+\zeta-1\right]}$ and $\zeta=\frac{P}{2\mu_1}$.

Therefore the displacement of the upper homogeneous medium is

$$v_1(x,z,t)=\left(B_1e^{ikqz}+B_2e^{-ikqz}\right)e^{ik(x-ct)},\tag{7}$$

where B_1 and B_2 are arbitrary constants.

4. Dynamics of lower porous half-space

We consider an initial stress anisotropic porous half-space. Let (u_2,v_2,w_2) are components of displacement vector of solid and (U_x,V_y,W_z) are components of displacement vector of the liquid part of a porous material in the direction of $x,\ y,\ z$ respectively. Neglecting the viscosity of water, the dynamic equations of motion in a porous layer under the compressive initial stress P', in the absence of body forces, can be written as Biot (1965)

$$\left.\begin{aligned}&\frac{\partial\tau_{11}}{\partial x}+\frac{\partial\tau_{12}}{\partial y}+\frac{\partial\tau_{13}}{\partial z}-P'\left(\frac{\partial\omega'_z}{\partial y}-\frac{\partial\omega'_y}{\partial z}\right)=\frac{\partial^2}{\partial t^2}(\rho_{11}u_2+\rho_{12}U_x),\\&\frac{\partial\tau_{21}}{\partial x}+\frac{\partial\tau_{22}}{\partial y}+\frac{\partial\tau_{23}}{\partial z}-P'\left(\frac{\partial\omega'_z}{\partial x}\right)=\frac{\partial^2}{\partial t^2}(\rho_{11}v_2+\rho_{12}V_y),\\&\frac{\partial\tau_{31}}{\partial x}+\frac{\partial\tau_{32}}{\partial y}+\frac{\partial\tau_{33}}{\partial z}-P'\left(\frac{\partial\omega'_y}{\partial x}\right)=\frac{\partial^2}{\partial t^2}(\rho_{11}w_2+\rho_{12}W_z),\end{aligned}\right\}\tag{8}$$

$$\frac{\partial\tau'}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{12}u_2+\rho_{22}U_x),\quad\frac{\partial\tau'}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}v_2+\rho_{22}V_y),$$
$$\frac{\partial\tau'}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w_2+\rho_{22}W_z),$$

where $\tau_{ij}\ (i,j=1,2,3)$ are the incremental stress components of solid and τ' is the stress vector due to liquid part of porous. This stress vector τ' is related to the fluid pressure p by the relation $-\tau'=fp$, where f is the porosity of the layer. The angular components $\omega'_x,\ \omega'_y$ and ω'_z are defined as

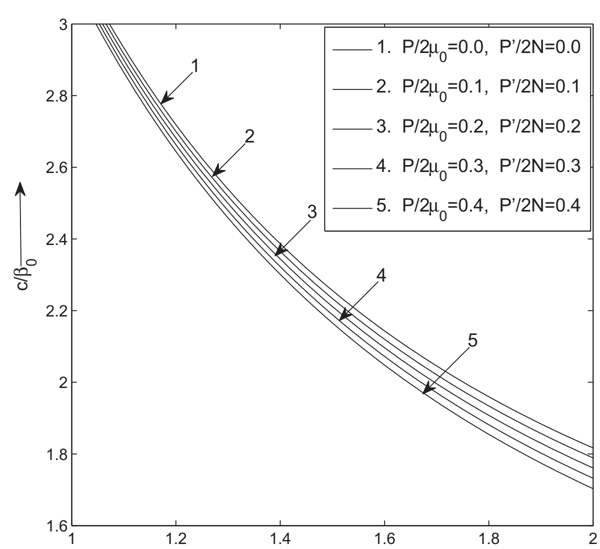

Fig. 4. Case III: dimensionless phase velocity $\frac{c}{\beta_0}$ as a function of dimensionless wave number kH of Love waves for different values of initial stresses $\frac{P}{2\mu_0}$ and $\frac{P'}{2N}$.

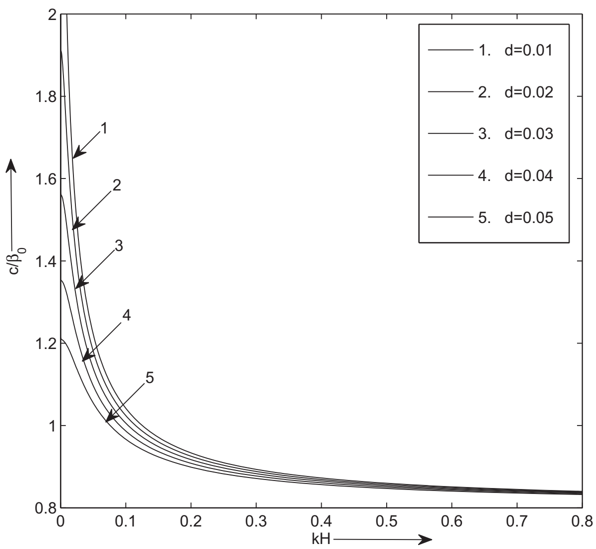

Fig. 5. Case I: variation of phase velocity $\frac{c}{\beta_0}$ with wave number kH of Love waves for different values of d.

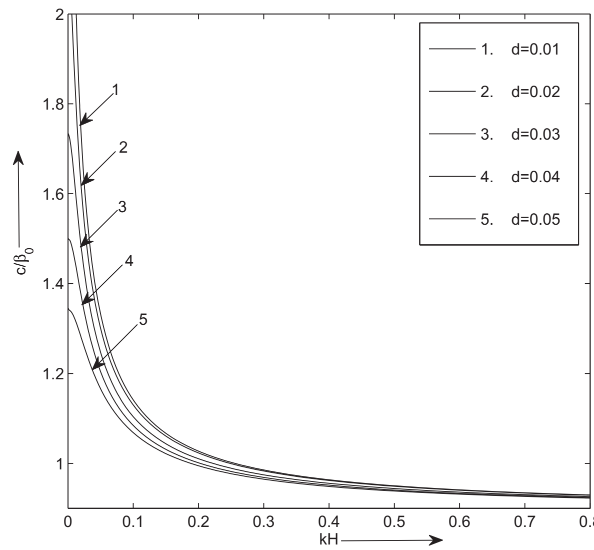

Fig. 6. Case II: variation of phase velocity $\frac{c}{\beta_0}$ with wave number kH of Love waves for different values of d.

$$\omega'_x = \frac{1}{2}\left(\frac{\partial w_2}{\partial y} - \frac{\partial v_2}{\partial z}\right), \quad \omega'_y = \frac{1}{2}\left(\frac{\partial u_2}{\partial z} - \frac{\partial w_2}{\partial x}\right),$$
$$\omega'_z = \frac{1}{2}\left(\frac{\partial v_2}{\partial x} - \frac{\partial u_2}{\partial y}\right). \tag{9}$$

The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are related to the density ρ', ρ_s and ρ_w of the layer, solid and liquid, respectively, by

$$\rho_{11} + \rho_{12} = (1-f)\rho_s, \quad \rho_{12} + \rho_{22} = f\rho_w, \tag{10}$$

the mass densities of the bulk material is

$$\rho' = \rho_{11} + 2\rho_{12} + \rho_{22} = \rho_s + f(\rho_w - \rho_s). \tag{11}$$

These mass co-efficients also obey the following inequalities

$$\rho_{11} > 0, \quad \rho_{22} > 0, \quad \rho_{12} < 0, \quad \rho_{11}\rho_{22} - \rho_{12}^2 > 0. \tag{12}$$

The stress–strain relations for the water saturated anisotropic porous layer under the normal initial stress P' are

$$\left.\begin{aligned} \tau_{11} &= (A+P')e_{xx} + (A-2N+P')e_{yy} + (F+P')e_{zz} + Q\varepsilon,\\ \tau_{22} &= (A-2N)e_{xx} + Ae_{yy} + Fe_{zz} + Q\varepsilon,\\ \tau_{33} &= Fe_{xx} + Fe_{yy} + Ce_{zz} + Q\varepsilon,\\ \tau_{12} &= 2Ne_{xy},\\ \tau_{23} &= 2Le_{yz},\\ \tau_{13} &= 2Le_{zx}, \end{aligned}\right\} \tag{13}$$

where A, F, C, N and L are elastic constants for the medium; in particular, N and L are the shear moduli of the anisotropic layer in the x and z direction respectively and $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$, $\varepsilon = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_x}{\partial z}$. The positive quantity Q is the measure of coupling between the change of volume of solid and liquid.

The dynamic Eq. (8) has been constructed by coupling the Biot's dynamic equations in an initially stressed medium (Biot, 1965) and the dynamic equation for a poro-elastic medium (Biot, 1956).

For the propagation of Love waves, we know that the direction of particle displacement is parallel to the plane of propagation. The displacement along the x axis and z axis vanishes, as well as the rate of change along y axis is also absent i.e., we have

$$\left.\begin{aligned} u_2 = 0, \quad w_2 = 0, \quad v_2 = v_2(x,z,t)\\ U_x = 0, \quad W_z = 0, \quad V_y = V(x,z,t). \end{aligned}\right\} \tag{14}$$

These conditions will produce only the e_{yz} and e_{xy} strain components and the other strain components will remain zero. Hence the stress–strain relations are

$$\tau_{23} = 2Le_{yz}, \quad \tau_{12} = 2Ne_{xy}. \tag{15}$$

Substituting relations (15) into Eq. (8), the equations of motion which are not automatically satisfied are

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} - P'\left(\frac{\partial \omega'_z}{\partial x}\right) = \frac{\partial^2}{\partial t^2}(\rho_{11}v_2 + \rho_{12}V_y),$$
$$\frac{\partial \tau'}{\partial y} = 0 = \frac{\partial^2}{\partial t^2}(\rho_{12}v_2 + \rho_{22}V_y).$$

Since $v_2 = v_2(x,z,t)$, $V_y = V(x,z,t)$ and with the help of (14) and (15), the above equations transforms into

$$\left(N - \frac{P'}{2}\right)\frac{\partial^2 v_2}{\partial x^2} + L\frac{\partial^2 v_2}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}v_2 + \rho_{12}V), \tag{16}$$

$$\frac{\partial^2}{\partial t^2}(\rho_{12}v_2 + \rho_{22}V) = 0. \tag{17}$$

From $\frac{\partial^2}{\partial t^2}(\rho_{12}v_2 + \rho_{22}V) = 0$ and $\rho_{12}v_2 + \rho_{22}V = d''$ (say)

$$V = \frac{d'' - \rho_{12}v_2}{\rho_{22}}.$$

Now, $\frac{\partial^2}{\partial t^2}(\rho_{11}v_2 + \rho_{12}V) = d'\frac{\partial^2 v_2}{\partial t^2}$ where $d' = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}$.
Hence Eq. (16) can be written as

$$\left(N - \frac{P'}{2}\right)\frac{\partial^2 v_2}{\partial x^2} + L\frac{\partial^2 v_2}{\partial z^2} = d'\frac{\partial^2 v_2}{\partial t^2}. \tag{18}$$

From Eq. (18) we found that the velocity of shear wave along x direction is $\sqrt{\frac{\left(N-\frac{P'}{2}\right)}{d'}}$ and along the z direction is $\sqrt{\frac{L}{d'}}$.

The shear wave velocity in the porous medium along the x direction can be expressed as

$$\beta = \sqrt{\frac{N - \frac{P'}{2}}{d'}} = \beta_2\sqrt{\frac{1-\zeta'}{d}}, \quad \text{where } d = \gamma_{11} - \frac{\gamma_{12}^2}{\gamma_{22}}, \quad \beta_2 = \sqrt{\frac{N}{\rho'}}, \tag{19}$$

β_2 is the velocity of shear wave in the corresponding initial stress-free, non-porous, anisotropic, elastic medium along the direction

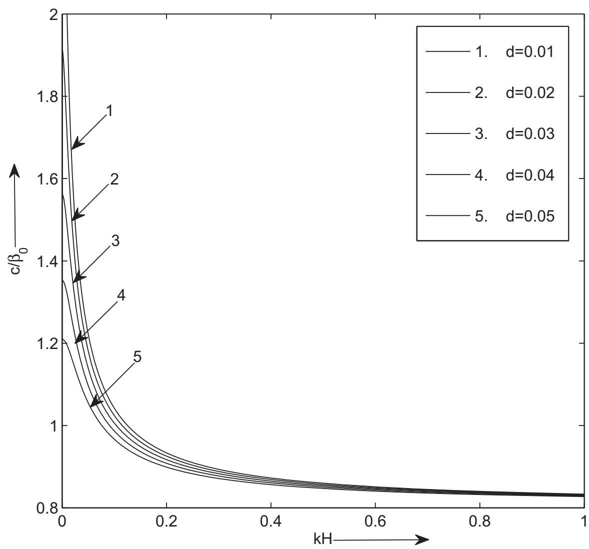

Fig. 7. Case III: variation of phase velocity $\frac{c}{\beta_0}$ with wave number kH of Love waves for different values of d.

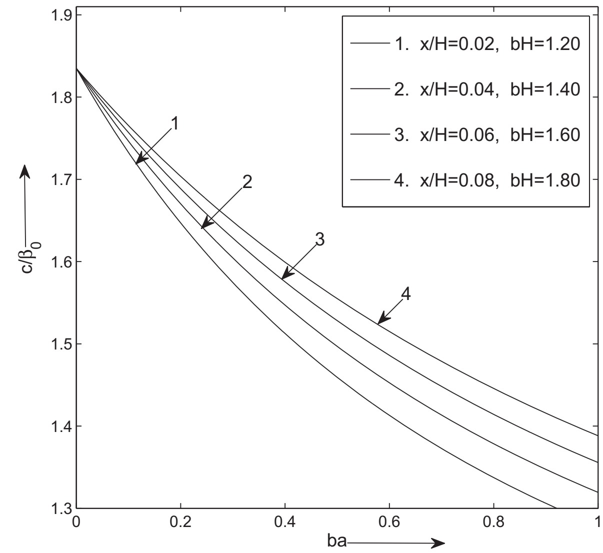

Fig. 8. Case I: variation of phase velocity $\frac{c}{\beta_0}$ with corrugation parameter ba of Love waves for different values of $(\frac{x}{H}, bH)$.

of x, $\zeta' = \frac{P'}{2N}$ is the non-dimensional parameter due to the initial stress P' and

$$\gamma_{11} = \frac{\rho_{11}}{\rho'}, \quad \gamma_{13} = \frac{\rho_{13}}{\rho'}, \quad \gamma_{23} = \frac{\rho_{23}}{\rho'} \tag{20}$$

are the non-dimensional parameters for the material of the porous layer as obtained by Biot (1956).

Thus, one gets the following:

i. $d \to 1$, when the layer is non porous solid.
ii. $d \to 0$, when the layer is fluid.
iii. $0 < d < 1$, when the layer is poro-elastic.

For the Love wave propagating along the x direction, the solution of Eq. (18) may be taken as

$$v_2(x,z,t) = V_2(z)e^{ik(x-ct)} \tag{21}$$

applying (21) in Eq. (18), we get

$$\frac{d^2V_2}{dz^2} + q'^2k^2V_2 = 0, \tag{22}$$

where

$$q' = \sqrt{\frac{1}{L}\left(c^2d' - N + \frac{P'}{2}\right)}.$$

Therefore, the solution of Eq. (22) takes the form as

$V_2(z) = B_3e^{-iq'kz} + B_4e^{iq'kz}$, where B_3 and B_4 are arbitrary constants.

We are interested in the solution of Eq. (22) which is bounded and vanishes as $z \longrightarrow \infty$. So the solution of Eq. (18) can be taken as

$$v_2 = B_3e^{-iq'kz}e^{ik(x-ct)}. \tag{23}$$

This is the displacement on an initially stressed porous half-space, where

$$q' = \sqrt{\frac{\left[c^2d' - \left(N - \frac{P'}{2}\right)\right]}{L}} = \sqrt{\gamma d\left[\frac{c^2}{\beta_2^2} - \frac{1-\zeta'}{d}\right]},$$

$\gamma = \frac{N}{L}$, $\zeta' = \frac{P'}{2N}$, $\beta_2^2 = \frac{N}{\rho'}$ and k is the wave number.

5. Boundary conditions

In the present problem we have considered a homogeneous layer under initial stress whose free surface is traction free and hence the shearing stress component vanishes there. At the common boundary of homogeneous layer and porous half-space the displacement of the particles and the tangential stress components are continuous. The boundary conditions can be written mathematically as.

(i) At $z = \lambda_1(x) - H$, $[\sigma_{23} - \lambda_1'\sigma_{12}]_{M_1} = 0$.
(ii) At $z = \lambda_2(x)$,
  (a) $v_1 = v_2$,
  (b) $[\sigma_{23} - \lambda_2'\sigma_{12}]_{M_1} = [\tau_{23} - \lambda_2'\tau_{12}]_{M_2}$,

where $\lambda_1' = \frac{\partial\lambda_1}{\partial x}$, $\lambda_2' = \frac{\partial\lambda_2}{\partial x}$.

6. Dispersion relation

Using Eqs. (7) and (23) the stress components into the above boundary conditions 5(i) and 5(ii), we get

$$(q+\lambda_1')B_1e^{ikq(\lambda_1-H)} - (q+\lambda_1')B_2e^{-ikq(\lambda_1-H)} = 0, \tag{24}$$

$$B_1e^{ikq\lambda_2} + B_2e^{-ikq\lambda_2} - B_3e^{ikq'\lambda_2} = 0, \tag{25}$$

$$\mu_1(q-\lambda_2')B_1e^{ikq\lambda_2} - \mu_1(q+\lambda_2')B_2e^{-ikq\lambda_2} + (Lq'+N\lambda_2')B_3e^{-ikq'\lambda_2} = 0. \tag{26}$$

Eliminating the arbitrary constants B_1, B_2 and B_3 from Eqs. (24)–(26), we get

$$\begin{vmatrix} (q+\lambda_1')e^{ikq(\lambda_1-H)} & -(q+\lambda_1')e^{-ikq(\lambda_1-H)} & 0 \\ e^{ikq\lambda_2} & e^{-ikq\lambda_2} & -e^{ikq'\lambda_2} \\ \mu_1(q-\lambda_2')e^{ikq\lambda_2} & -\mu_1(q+\lambda_2')e^{-ikq\lambda_2} & +(Lq'+N\lambda_2')e^{-ikq'\lambda_2} \end{vmatrix} = 0,$$

which reduces to

$$(Lq'+N\lambda_2')\left[(q+\lambda_1')e^{-ikq(\lambda_1-\lambda_2-H)} + (q-\lambda_1')e^{ikq(\lambda_1-\lambda_2-H)}\right] + \mu_1(q+\lambda_1')(q-\lambda_2')e^{ikq(\lambda_2-\lambda_1-H)} - \mu_1(q-\lambda_1')(q+\lambda_2')e^{-ikq(\lambda_2-\lambda_1-H)} = 0,$$

which takes the form

$$tan\left[k(H-\lambda_1+\lambda_2)\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}\right] = \frac{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{1-\frac{P'}{2N}}{d}-\frac{c^2}{\beta_1^2}\right)}\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}+i\left[\lambda_2'\left(\frac{N}{\mu_1}-1\right)+\lambda_1'\right]\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}}{\left(\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}\right)+\lambda_1'\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{c^2}{\beta_1^2}-\frac{1-\frac{P'}{2N}}{d}\right)}+\left(\frac{N}{\mu_1}-1\right)\lambda_1'\lambda_2'}. \quad (27)$$

It is observed from Eq. (27) that the first part of the equation is real and the second part is complex, so the wave number must be a complex value.

Equating real and imaginary parts of Eq. (27) we obtain

$$tan\left[k(H-\lambda_1+\lambda_2)\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}\right] = \frac{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{1-\frac{P'}{2N}}{d}-\frac{c^2}{\beta_1^2}\right)}\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}}{\left(\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}\right)+\lambda_1'\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{c^2}{\beta_1^2}-\frac{1-\frac{P'}{2N}}{d}\right)}+\left(\frac{N}{\mu_1}-1\right)\lambda_1'\lambda_2'}, \quad (28)$$

which is the phase velocity equation of Love-type waves in a pre-stressed homogeneous medium over a porous half-space under initial stress and

$$\frac{c^2}{\beta_1^2}=1-\frac{P}{2\mu_1}. \quad (29)$$

We described this velocity as damping phase velocity and it may be neglected.

7. Particular cases

7.1. Case I:

When the upper layer is bounded by a plane surface $z=-H$ i.e., $\lambda_1=0$ and intermediate surface is periodic nature, represented by $\lambda_2=acos(bx)$, then $\lambda_1'=0$ and $\lambda_2'=-ab\,sin(bx)$. In this case, the dispersion equation (28) becomes

$$tan\left[k(H+acos\,bx)\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}\right] = \frac{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{1-\frac{P'}{2N}}{d}-\frac{c^2}{\beta_1^2}\right)}}{\sqrt{\left(\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}\right)}}. \quad (30)$$

Eq. (30) gives the dispersion relation, depends on the elastic parameters, initial stress parameters, porosity of the half-space, the amplitude of the irregular boundary surface and the frequency of the Love wave. The velocity of the surface wave must be $\beta_1 \leqslant c \leqslant \beta_2$ for the existence of Love wave.

7.1.1. Subcase I

If the upper layer of above case is initial stress-free, homogeneous elastic layer (i.e., $\frac{P}{2\mu_1}=0$) over an initially stress-free, non-porous, homogeneous elastic half-space (i.e., $\frac{P'}{2N}=0,\ \gamma=1$, $d=1,\ N=L=\mu_2$) then Eq. (30) reduces to Singh (2011)

$$tan\left[k(H+acos\,bx)\sqrt{\frac{c^2}{\beta_1^2}-1}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{\left(1-\frac{c^2}{\beta_1^2}\right)}}{\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}}. \quad (31)$$

This is the dispersion equation of Love wave in a homogeneous layer over a homogeneous half-space.

7.1.2. Subcase II

If the amplitude of the lower periodic surface is zero, i.e., $a=0$, and both the layers are initial stress-free, non-porous, homogeneous then Eq. (30) reduces to Ewing et al. (1957)

$$tan\left[kH\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{\left(1-\frac{c^2}{\beta_2^2}\right)}}{\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}}. \quad (32)$$

Eq. (32) gives the phase speed c of Love wave in a homogeneous layer over a homogeneous half-space.

7.2. Case II:

When the surfaces of the upper layer and lower half-space are bounded by a periodic surface $z=-H+acos(bx)$ i.e., $\lambda_1=acos(bx)$ and intermediate surface is a plane surface i.e., $\lambda_2=0$ then $\lambda_1'=-ab\,sin(bx)$ and $\lambda_2'=0$. In this case dispersion equation (28) reduces to

$$tan\left[k(H-acos\,bx)\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}\right] = \frac{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{1-\frac{P'}{2N}}{d}-\frac{c^2}{\beta_1^2}\right)}\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}}{\left(\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}\right)-\frac{L}{\mu_1}ba\,sin(bx)\sqrt{\gamma d\left(\frac{c^2}{\beta_1^2}-\frac{1-\frac{P'}{2N}}{d}\right)}}. \quad (33)$$

Eq. (33) gives the frequency equation of Love wave in a homogeneous irregular boundary surface under initial stress over a plane surface initially stressed porous half-space.

7.2.1. Subcase I

In dispersion equation (33) if the upper layer is initial stress-free (i.e., $\frac{P}{2\mu_1}=0$), homogeneous elastic layer overlying an initial stress-free, non-porous, homogeneous elastic half-space (i.e., $\frac{P'}{2N}=0,\ \gamma=1, d=1,\ N=L=\mu_2$) then Eq. (33) reduces to

$$tan\left[k(H-acos\,bx)\sqrt{\frac{c^2}{\beta_1^2}-1}\right]=\frac{\frac{\mu_2}{\mu_1}\sqrt{1-\frac{c^2}{\beta_1^2}}\sqrt{\frac{c^2}{\beta_1^2}-1}}{\left(\frac{c^2}{\beta_1^2}-1\right)-\frac{\mu_2}{\mu_1}ba\,sin(bx)\sqrt{\frac{c^2}{\beta_1^2}-1}}. \quad (34)$$

This is the dispersion equation of Love wave in a homogeneous layer over a homogeneous half-space.

7.2.2. Subcase II

If the amplitude of the upper periodic surface is neglected, i.e., $a=0$, and lower half-space is initial stress-free, non-porous homogeneous medium, then Eq. (33) reduces to Eq. (32), which is the same relation as in Ewing et al. (1957).

7.3. Case III:

When the surfaces of upper and lower half-space are bounded by periodic surfaces $z=-H+a_1cos(bx)$ i.e., $\lambda_1=a_1cos(bx)$ and $\lambda_2=a_2cos(bx)$, respectively then $\lambda_1'=-a_1b\,sin(bx)$ and $\lambda_2'=-ba_2\,sin(bx)$. In this case dispersion equation (28) reduces to

$$tan\left[k\{H+(a_2-a_1)cos\,bx\}\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}\right] = \frac{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{1-\frac{P'}{2N}}{d}-\frac{c^2}{\beta_1^2}\right)}\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}}{\left(\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}\right)-ba_1\,sin(bx)\left\{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{c^2}{\beta_1^2}-\frac{1-\frac{P'}{2N}}{d}\right)}+\left(\frac{N}{\mu_1}-1\right)ba_2\,sin(bx)\right\}}. \quad (35)$$

Eq. (35) represents the dispersion equation for Love wave propagation in two layers bounded by periodic boundaries. It is clear that the phase velocity of Love wave depends on the elastic parameters and amplitudes of the irregular boundary surface. If $a_1 = a_2 = a$, then Eq. (35) reduces to

$$tan\left[kH\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}\right] = \frac{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{1-\frac{P'}{2N}}{d}-\frac{c^2}{\beta_1^2}\right)}\sqrt{\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}}}{\left(\frac{c^2}{\beta_1^2}-1+\frac{P}{2\mu_1}\right)-ba\,sin(bx)\left\{\frac{L}{\mu_1}\sqrt{\gamma d\left(\frac{c^2}{\beta_1^2}-\frac{1-\frac{P'}{2N}}{d}\right)}+\left(\frac{N}{\mu_1}-1\right)ba\,sin(bx)\right\}}. \tag{36}$$

7.3.1. Subcase I

In this case if the upper layer is without initial stress, homogeneous elastic layer and lower half-space is non-porous, homogeneous elastic layer without initial stress (i.e., $\frac{P'}{2N}=0,\ \gamma=1,\ d=1,\ N=L=\mu_2$) then Eq. (36) reduces to

$$tan\left[kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right] = \frac{\frac{\mu_2}{\mu_1}\sqrt{1-\frac{c^2}{\beta_1^2}}\sqrt{\frac{c^2}{\beta_1^2}-1}}{\left(\frac{c^2}{\beta_1^2}-1\right)-ba\ sin(bx)\left\{\frac{\mu_2}{\mu_1}\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}+\left(\frac{\mu_2}{\mu_1}-1\right)ba\ sin(bx)\right\}}, \tag{37}$$

which is the dispersion equation of Love wave in a homogeneous medium over a homogeneous half-space bounded by the periodic boundaries.

7.3.2. Subcase II

If the amplitude of the irregular boundary surfaces are neglected, i.e., $a=0$, and both the layers are initial stress-free, non-porous, homogeneous then the frequency equation (36) reduces to Ewing et al. (1957)

$$tan\left[kH\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{\left(1-\frac{c^2}{\beta_2^2}\right)}}{\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}}. \tag{38}$$

This is the classical dispersion equation of Love wave in a homogeneous layer overlying a homogeneous half-space.

8. Numerical calculation and discussion of dispersion equations

In order to show the effects of different values of initial stresses, porosity and the variation of phase velocity with the corrugation parameter on the propagation of Love waves, numerical computation of Eq. (28) were performed in three different cases with different values of parameters representing the above characteristic. For the computational purpose, we consider some numerical data, $\mu_1 = 3.23\times10^{10}\ \mathrm{N/m^2}$ and $\rho_1 = 2802\ \mathrm{kg/m^3}$ for upper layer and for lower porous half-space numerical values are taken from Chattaraj and Samal (2013) as $L=0.1387\times10^{10}\ \mathrm{N/m^2}, N=0.2774\times10^{10}\ \mathrm{N/m^2}, \rho_{11}=1.926137\times10^3\ \mathrm{kg/m^3}, \rho_{12}=-0.002137\times10^3\ \mathrm{kg/m^3}, \rho_{22}=0.215337\times10^3\ \mathrm{kg/m^3}, f=0.26$. We also have considered $ba=0.05$, $hH=1.40$ and $\frac{x}{H}=0.04$. Using above numerical data, results are presented in Figs. 2–10. Figs. 1, 5 and 8 are plotted from the dispersion equation (30). Figs. 3, 6 and 9 represents from dispersion equation (33). Figs. 4, 7 and 10 are plotted from the dispersion equation (35).

Fig. 2 shows the variation of dimensionless phase velocity against dimensionless wave number for different values of initial stress parameters in case I. Values of $\frac{P}{2\mu_1}$ and $\frac{P'}{2N}$ for curve 1, curve 2, curve 3, curve 4 and curve 5 have been taken as 0.0, 0.1, 0.2, 0.3 and 0.4. In this figure curve 1 represent the dispersion relation (31), which gives the phase velocity of Love wave when both the layers are initial stress free. Curves 2–5 represents the dispersion curve for Love wave when both the layer and half-space are under compressive initial stresses i.e., $\frac{P}{2\mu_1}, \frac{P'}{2N} > 0$. As the compressive initial stresses increase, the dimensionless phase velocity $\frac{c}{\beta_1}$ decreases in a particular wave number *kH*. At the initial stage of wave number curves are accumulating that although the compressive initial stress varies, velocity remains constant for that particular wave number. Curves being little far from each other and decreases with the increases of wave number.

Fig. 3 shows the variation of phase velocity as a function of wave number for case II. The curves are plotted for different values of initial stress parameters. The values of all parameters are taken same as Fig. 2. From the figure we have seen that the phase velocity decreases with increase of initial stress parameters. Fig. 4 represents the dispersion curve of Love wave against non-dimensional wave number for the case III when $a=a_1=a_2$. The curves are plotted for different values of initial stress parameters. The curves of the figure give the same results as Fig. 2.

In Fig. 5 (case I) study has been made to get the effect of porosity in the lower half-space. The figure has been represented for the dimensionless phase velocity $\frac{c}{\beta_1}$ against non-dimensional wave number *kH*. The curves are plotted for different values of poro-elastic constant *d* and fixed values of initial stress parameters $\frac{P}{2\mu_1}=0.35$ and $\frac{P'}{2N}=0.35$. The values of *d* for curve 1, curve 2, curve 3, curve 4 and curve 5 have been taken as 0.01, 0.02, 0.03, 0.04 and 0.05 respectively. It has been found that the phase velocity of Love wave decreases for the increasing value of poro-elastic constant *d*. The curves being very little far from the wave number 0.4.

Fig. 6 shows the study of dimensionless phase velocity as a function of non-dimensional wave number for the case II. The curves have been plotted for fixed values of initial stress parameters and different values of *d*. The values of $\frac{P}{2\mu_1}$ and $\frac{P'}{2N}$ have been taken as 0.2 and 0.2, respectively and the values of *d* have been taken same as Fig. 5. These curves show that the phase velocity

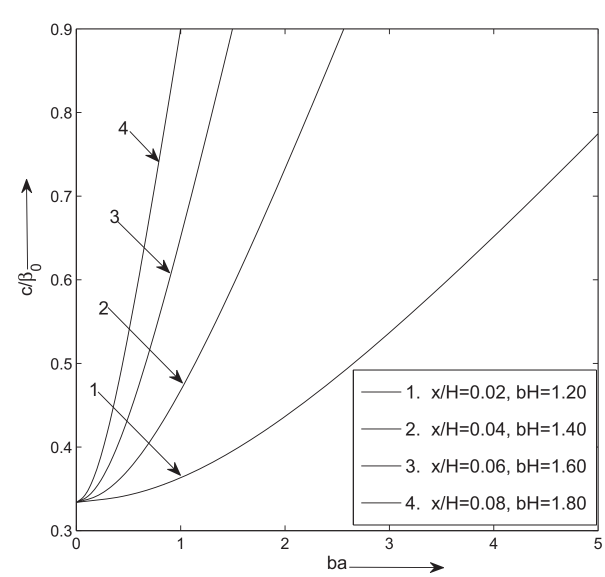

Fig. 9. Case II: variation of phase velocity $\frac{c}{\beta_0}$ with corrugation parameter *ba* of Love waves for different values of $(\frac{x}{H}, bH)$.

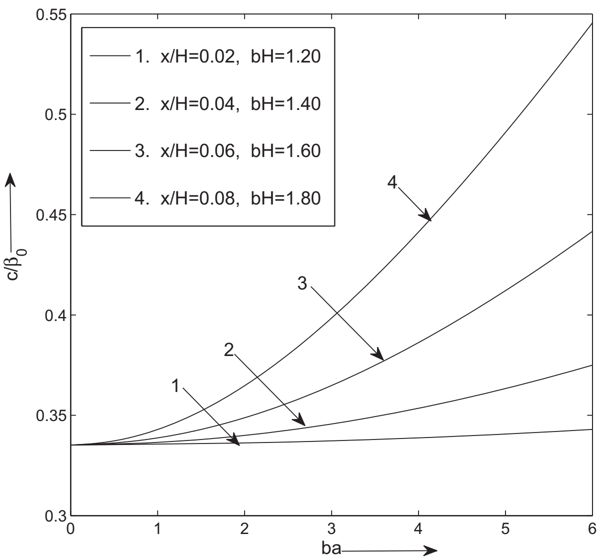

Fig. 10. Case III: variation of phase velocity $\frac{c}{\beta_0}$ with corrugation parameter *ba* of Love waves for different values of $(\frac{x}{H}, bH)$.

decreases with the increases of poro-elastic constant. Again curves in Fig. 7 represent dimensionless phase velocity against dimensionless wave number for the case III when $a = a_1 = a_2$. The curves are plotted by taking same numerical results as Fig. 5. From the curves it is found that the phase velocity decreases when poro-elastic constant of the half-space increases.

Figs. 8–10 show the variation of non-dimensional phase velocity $\frac{c}{\beta_1}$ as a function of dimensionless corrugation parameter *ba* for different values of $\frac{x}{H}$ and *bH*. In case I, Fig. 8 represents the phase velocity of Love wave for fixed values of initial stress parameters and poro-elastic constant and different values of $(\frac{x}{H}, bH)$. The values of $(\frac{x}{H}, bH)$ for curve 1, curve 2, curve 3 and curve 4 have been taken as $(0.02, 1.20)$, $(0.04, 1.40)$, $(0.06, 1.60)$ and $(0.08, 1.80)$ respectively. From this figure it is observed that phase velocity increases with the decrease of *ba*. It also shows that the curves of $\frac{c}{\beta_1}$ start from a single point at $ba = 0$ and increases with the increase of $\frac{x}{H}$ and *bH*.

In Fig. 9 which shows case II, the values of $\frac{x}{H}$ and *bH* have taken same as Fig. 8. It has been found that phase velocity $\frac{c}{\beta_1}$ decreases with the increases of *ba*. Again it is noted that the phase velocity curves start from a single point and increase with the increase of $(\frac{x}{H}, bH)$.

Fig. 10 represents case III, where a_1, a_2 both are assumed to be *a* and numerical values of the parameters are taken same as Fig. 8. It is observed from this figure that the value of phase velocity $\frac{c}{\beta_1}$ increases with the increase of corrugation parameter *ba*. Again in this case the phase velocity curves are accumulated at $ba = 0$ and it increases as the values of $(\frac{x}{H}, bH)$ increases. Thus we have observed that the phase velocity of Love wave is influenced by the roughness of the interface.

9. Conclusions

Propagation of Love wave in an initially stressed homogeneous layer overlying a porous half-space under initial stress with irregular boundary surfaces has been studied in details. The solutions for displacement in the layer and half-space have been derived separately in closed form. The dispersion equation of Love wave in a different type of irregular boundary surface layer over an initially stressed porous half-space has been derived. This dispersion equation is reduced to some particular type of irregular layer bounded by a plane surface and a periodic boundary. The phase velocity has been found as a function of wave number and corrugation parameter. The numerical computations of particular cases are performed and effect of initial stress parameters, poro-elastic constant and corrugation parameter are studied graphically. From the above figures it may conclude that:

- As depth increases the velocity of surface wave decreases which is the well-known nature of seismic wave.
- As initial stresses in the layer and lower half-space increases the velocity of surface wave decreases.
- In the above three particular cases, phase velocity of Love type wave decreases with the increase of porosity of the half-space. It is also observed that velocity curves are accumulated after a particular wave number at $kH = 0.4$ i.e., after that particular wave number the velocity of wave does not take any effect of poro-elastic constant.
- When the intermediate surface of the layers is periodic, the phase velocity decreases with the increase of ba for different values of $(\frac{x}{H}, bH)$. It is also noted that the curves started from a single point and increases with the increase of corrugation parameters.
- Again, when the surface of the upper layer is periodic the velocity of wave increases with the increase of ba for different values of $(\frac{x}{H}, bH)$. It is also noted that the velocity of wave stated from a single point and increases with the increase of corrugation parameters. Therefore, it has been realized that the phase velocity of Love wave is influenced by roughness of the interface.

Finally, when both the layer and half-space are homogeneous with plane surface boundaries then the dispersion equation reduces to the general equation of Love wave by Ewing et al. (1957).

References

Ahmed, S.M., Abd-Dahab, S.M., 2010. Propagation of Love waves in an orthotropic granular layer under initial stress overlying a semi-infinite granular medium. J. Vib. Control 16 (12), 1845–1858.
Bacigalupo, A., Gambarotta, L., 2014. Second-gradient homogenized model for wave propagation in heterogeneous periodic media. Int. J. Solids Struct. 51, 1052–1065.
Biot, M.A., 1956. Theory of deformation of a porous viscoelastic anisotropic solid. J. Appl. Phys. 27, 459–467.
Biot, M.A., 1965. Mechanics of Incremental Deformation. John Willey and Sons, New York, London.
Chattaraj, R., Samal, S.K., 2013. Love waves in the fiber-reinforced layer over a gravitating porous half-space. Acta Geophys. 61 (5), 1170–1183.
Chattaraj, R., Samal, S.K., Mahanti, N., 2013. Dispersion of Love wave propagating in irregular anisotropic porous stratum under initial stress. Int. J. Geomech. 13 (4), 402–408.
Chattopadhyay, A., 1975. On the dispersion equation for Love wave due to irregularity in the thickness of non-homogeneous crustal layer. Acta Geophys. Pol. 23, 307–317.
Chattopadhyay, A., Gupta, S., Singh, A.K., Sahu, S.A., 2010. Propagation of SH waves in an irregular non-homogeneous monoclinic crustal layer over a semi-infinite monoclinic medium. Appl. Math. Sci. 4 (44), 2157–2170.
Chattopadhyay, A., Gupta, S., Sahu, S.K., Singh, A.K., 2013. Dispersion of horizontally polarized shear waves in an irregular non homogeneous self-reinforced crustal layer over a semi-infinite self-reinforced medium. J. Vib. Control 19 (1), 109–119.
Dey, S., Gupta, S., Gupta, A.K., 1996. Propagation of Love waves in heterogeneous crust over a heterogeneous mantle. J. Acta Geophys. Pol. Poland XLIX (2), 125–137.
Dey, S., Gupta, S., Gupta, A.K., 2004. Propagation of Love waves in an elastic layer with void pores. Sādhanā 29, 355–363.
Ewing, W.M., Jardetzky, W.S., Press, F., 1957. Elastic Waves in Layered Media. McGraw-Hill, New York.
Ghorai, A.P., Samal, S.K., Mahanti, N.C., 2010. Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity. Appl. Math. Modell. 34 (7), 1873–1883.
Gubbins, D., 1990. Seismology and Plate Tectonics. Cambridge University Press, Cambridge.
Gupta, S., Chattopadhyay, A., Kundu, S., 2010a. Influence of irregularity and rigidity on the propagation of torsional wave. Appl. Math. Sci. 4 (17), 805–816.

Gupta, S., Chattopadhyay, A., Majhi, D.K., 2010b. Effect of initial stress on propagation of Love waves in an anisotropic porous layer. J. Solid Mech. 2 (1), 50–62.
Gupta, S., Majhi, D.K., Kundu, S., Vishwakarma, S.K., 2013a. Propagation of Love waves in non-homogeneous substratum over initially stressed heterogeneous half-space. Appl. Math. Mech. Engl. Ed. 34 (2), 249–258.
Gupta, S., Vishwakarma, S.K., Majhi, D.K., Kundu, S., 2013b. Possibility of Love wave propagation in a porous layer under the effect of linearly varying directional rigidities. Appl. Math. Modell. http://dx.doi.org/10.1016/j.apm.2013.01.008.
Kalyani, V.K., Sinha, A., Pallavika, Chakraborty, S.K., Mahanti, N.C., 2008. Finite difference modeling of seismic wave propagation in monoclinic media. Acta Geophys. 56 (4), 1074–1089.
Ke, L.L., Wang, J.S., Zhang, Z.M., 2006. Love waves in an inhomogeneous fluid saturated porous layered half-space with linearly varying properties. Soil Dyn. Earthquake Eng. 26, 574–581.
Kielczynski, P., Szalewski, M., Balcerzak, A., 2012. Effect of a viscous liquid loading on Love wave propagation. Int. J. Solids Struct. 49, 2314–2319.
Kundu, S., Gupta, S., Manna, S., 2013. SH-type waves dispersion in an isotropic medium sandwiched between an initially stressed orthotropic and heterogeneous semi-infinite media. Meccanica. http://dx.doi.org/10.1007/s11012-013-9825-5.
Kundu, S., Gupta, S., Manna, S., 2014. Propagation of Love wave in fiber-reinforced medium lying over an initially stressed orthotropic half-space. Int. J. Numer. Anal. Methods Geomech. http://dx.doi.org/10.1002/nag.2254.
Liu, J., He, S., 2010. Properties of Love waves in layered piezoelectric structures. Int. J. Solids Struct. 47, 169–174.
Manna, S., Kundu, S., Gupta, S., 2013. Love wave propagation in a piezoelectric layer overlying in an inhomogeneous elastic half-space. J. Vib. Control. http://dx.doi.org/10.1177/1077546313513626.
Pal, P.C., Sen, B., 2011. Disturbance of SH-type waves due to shearing-stress discontinuity in a visco-elastic layered half-space. Int. J. Mech. Solids 6 (2), 176–189.
Pradhan, A., Samal, S.K., Mahanti, N.C., 2003. The Influence of anisotropy on the Love waves in a self-reinforced medium. Tamkang J. Sci. Eng. 6 (3), 173–178.
Sharma, M.D., 2004. Wave propagation in a general anisotropic poroelastic medium with anisotropic permeability: phase velocity and attenuation. Int. J. Solids Struct. 41, 4587–4597.
Singh, S.S., 2011. Love wave at a layer medium bounded by irregular boundary surfaces. J. Vib. Control 17, 789–795.
Wolf, B., 1970. Propagation of Love waves in layers with irregular boundaries. Pure Appl. Geophys. 78 (1), 48–57.