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Elastodynamic analysis of a plane weakened by several cracks

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ABSTRACT

The stress fields in an infinite plane containing Volterra type climb and glide edge dislocations under time-harmonic excitation are derived. The dislocation solutions are utilized to formulate integral equations for dislocation density functions on the surfaces of smooth cracks. The integral equations are of Cauchy singular type which are solved numerically for several different cases of crack configurations and arrangements. The results are used to evaluate modes I and II stress intensity factors for multiple smooth cracks.

1. Introduction

Stress analysis under dynamic loading is a prerequisite for fatigue estimation of a mechanical component. Defects in the form of cracks and cavities nucleate in the course of the manufacturing process and/or service life of mechanical components generating regions with severe stress gradient. The interaction among defects under dynamic conditions is a very complicated problem which may be tackled analytically only in idealized regions such as infinite- and half-planes. Mal (1970), studied the diffraction of normally incident longitudinal waves by a Griffith crack in an infinite plane. The diffraction of normally incident longitudinal harmonic waves by two coplanar identical Griffith cracks situated in an isotropic infinite medium was investigated by Jain and Kanwal (1972). The same problem was solved by Itou (1978), employing the Schmidt method. Three coplanar cracks and a crack parallel with two coplanar cracks were solved, respectively, by Itou (1996) and Itou and Haliding (1997) using the Schmidt procedure. Steady state interaction between an arbitrarily located microcrack and a main crack under harmonic longitudinal excitation was studied by Meguid and Wang (1995). They also investigated the effects of microcrack orientation on the stress intensity factors of the main crack.

In this article, a procedure is devised for the analysis of multiple curved cracks in planes under in-plane time-harmonic loads. The stress fields in an infinite plane caused by climb and glide Volterra-type edge dislocations are obtained. The stress components exhibit the familiar Cauchy-type singularity at dislocation location. The dislocation solution is then used to derive singular integral equations for a plane with multiple cracks under tensile and shear tractions. These equations are then solved numerically for dislocation density on the crack surfaces. The results are utilized to determine stress intensity factors at the crack tips.

2. Formulation of the problem

The distributed dislocation technique is an efficient means for treating multiple cracks with smooth geometry. The major obstacle in the utilization of the method is the knowledge of stress fields caused by a single dislocation in the region. This task for an infinite plane containing climb and a glide edge Volterra-type dislocations under time-harmonic excitation is taken up here. The equations of motion in terms of displacements may be written as

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = \rho\frac{\partial^2 u}{\partial t^2}$$
$$\mu\nabla^2 v + (\lambda+\mu)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = \rho\frac{\partial^2 v}{\partial t^2} \quad (1)$$

where λ and μ are the Lame elastic constants and ρ is the mass density of material. The Helmholtz representation of displacement components in terms of potentials Φ and Ψ are

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y}$$
$$v = \frac{\partial \Phi}{\partial y} - \frac{\partial \Psi}{\partial x} \quad (2)$$

Consequently, Eq. (1) are satisfied provided that

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$$\nabla^2\Phi = \frac{1}{c_L^2}\frac{\partial^2\Phi}{\partial t^2}$$
$$\nabla^2\Psi = \frac{1}{c_T^2}\frac{\partial^2\Psi}{\partial t^2} \tag{3}$$

In Eq. (3), $c_L = \sqrt{(\lambda+2\mu)}/\rho$ and $c_T = \sqrt{\mu/\rho}$ are the characteristic dilatational and shear wave velocities, respectively. Under the assumption of time-harmonic excitation with angular frequency ω, the potentials may be expressed as

$$\Phi(x,y,t) = \varphi(x,y)e^{i\omega t}$$
$$\Psi(x,y,t) = \psi(x,y)e^{i\omega t} \tag{4}$$

where $i = \sqrt{-1}$ and $\varphi(x,y)$, $\psi(x,y)$ are unknown complex functions. The substitution of Eqs. (4) into (3) yields

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + k_L^2\varphi = 0$$
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + k_T^2\psi = 0 \tag{5}$$

where $k_L = \omega/c_L$ and $k_T = \omega/c_T$. From Eq. (2), the strain-displacement relationships and Hooke's law, the stress components may be written as

$$\sigma_{xx} = \mu\left[k_L^2\left(\frac{1+\kappa}{1-\kappa}\right)\varphi - 2\frac{\partial^2\varphi}{\partial y^2} + 2\frac{\partial^2\psi}{\partial x\partial y}\right]$$
$$\sigma_{yy} = \mu\left[k_L^2\left(\frac{1+\kappa}{1-\kappa}\right)\varphi - 2\frac{\partial^2\varphi}{\partial x^2} - 2\frac{\partial^2\psi}{\partial x\partial y}\right]$$
$$\sigma_{xy} = \mu\left[2\frac{\partial^2\varphi}{\partial x\partial y} - k_T^2\psi - 2\frac{\partial^2\psi}{\partial x^2}\right] \tag{6}$$

where the Kolosov constant $\kappa = 3 - 4\upsilon$ for plane strain and $\kappa = (3-\upsilon)/(1+\upsilon)$ for plane stress cases and υ signifies the Poisson's ratio of the medium. In Eq. (6) and henceforth, the coefficient exp($i\omega t$) which is common to all field variables is omitted. A climb dislocation with Burgers vector B_y situated at the origin with the line of dislocation along the positive part of the x-axis is represented by the following condition

$$v(x,0^+) - v(x,0^-) = B_yH(x) \tag{7}$$

whereas a glide dislocation, at the above mentioned location with Burgers vector B_x requires that

$$u(x,0^+) - u(x,0^-) = B_xH(x) \tag{8}$$

where $H(x)$ is the Heaviside step function. Moreover, for Volterra-type edge dislocations the continuity of stress components along the dislocation line implies that

$$\sigma_{yy}(x,0^+) = \sigma_{yy}(x,0^-)$$
$$\sigma_{xy}(x,0^+) = \sigma_{xy}(x,0^-) \tag{9}$$

In the case of the presence of only a climb dislocation the problem is symmetric with respect to the x-axis. Therefore, the half-plane, $y > 0$, subjected to the following boundary conditions may be considered

$$v(x,0^+) = \frac{B_y}{2}H(x)$$
$$\sigma_{xy}(x,0) = 0 \tag{10}$$

For a glide dislocation the problem is anti-symmetric with respect to the x-axis and the boundary conditions for the upper half-plane, $y > 0$, yield

$$u(x,0^+) = \frac{B_x}{2}H(x)$$
$$\sigma_{yy}(x,0) = 0 \tag{11}$$

The solution to Eq. (5) is accomplished by means of the complex Fourier transformation defined as

$$f^*(\xi) = \int_{-\infty}^{\infty} e^{i\xi x}f(x)dx \tag{12}$$

The inversion of (12) is

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}f^*(\xi)d\xi \tag{13}$$

Application of the Fourier transform to Eq. (5) in conjunction with the requirement that $\lim_{r\to\infty}\varphi(x,y) = 0$ and $\lim_{r\to\infty}\psi(x,y) = 0$, where $r = \sqrt{x^2+y^2}$, leads to

$$\frac{d^2\varphi^*}{dy^2} - (\xi^2 - k_L^2)\varphi^* = 0$$
$$\frac{d^2\psi^*}{dy^2} - (\xi^2 - k_T^2)\psi^* = 0 \tag{14}$$

Letting $\mathrm{Re}\left(\sqrt{\xi^2-k_L^2}\right) > 0$ and $\mathrm{Re}\left(\sqrt{\xi^2-k_T^2}\right) > 0$, where Re() designates the real part of the expressions, the solution to Eq. (14) which are finite as $y \to \infty$, yield

$$\varphi^*(\xi,y) = A(\xi)e^{-y\sqrt{\xi^2-k_L^2}}$$
$$\psi^*(\xi,y) = B(\xi)e^{-y\sqrt{\xi^2-k_T^2}} \tag{15}$$

The unknowns $A(\xi)$ and $B(\xi)$ may be obtained by utilizing boundary conditions (9) and (10), respectively, for the climb and glide dislocations. The expressions for these coefficients in a plane containing both dislocations are

$$A(\xi) = \frac{B_x}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right) + \frac{B_y}{2\sqrt{\xi^2-k_L^2}}\left[\frac{(2\xi^2-k_T^2)i}{k_T^2\xi} - \pi\delta(\xi)\right]$$
$$B(\xi) = \frac{-B_x}{2\sqrt{\xi^2-k_T^2}}\left[\frac{2i\xi}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right) + \frac{i}{\xi} + \pi\delta(\xi)\right] + \frac{B_y}{k_T^2} \tag{16}$$

where $\delta(\xi)$ is the Dirac delta function. From Eqs. (15), (13) and (16), the stress components become

$$\begin{aligned}\sigma_{xx}(x,y) = \mu\Bigg\{&\frac{B_x}{\pi}\Bigg[\left(\frac{1}{2}+\frac{1-\kappa}{1+\kappa}\right)\int_{-\infty}^{\infty} e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}d\xi \\ &+\frac{1}{2}\int_{-\infty}^{\infty} e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}d\xi - \frac{1}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right) \\ &\times\int_{-\infty}^{\infty}\xi^2\left(e^{-i\xi x-y\sqrt{\xi^2-k_L^2}} - e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}\right)d\xi\Bigg] \\ &+\frac{iB_y}{\pi}\Bigg[\frac{k_L^2}{2k_T^2}\left(\frac{1+\kappa}{1-\kappa}\right)\int_{-\infty}^{\infty}\frac{\xi e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}}{\sqrt{\xi^2-k_L^2}}d\xi \\ &-\frac{k_L^2}{4}\left(\frac{1+\kappa}{1-\kappa}\right)\int_{-\infty}^{\infty}\frac{e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}}{\xi\sqrt{\xi^2-k_L^2}}d\xi \\ &+\frac{1}{2}\int_{-\infty}^{\infty}\frac{\sqrt{\xi^2-k_L^2}}{\xi}e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}d\xi \\ &-\frac{1}{k_T^2}\int_{-\infty}^{\infty}\xi\left(\sqrt{\xi^2-k_L^2}e^{-i\xi x-y\sqrt{\xi^2-k_L^2}} - \sqrt{\xi^2-k_T^2}e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}\right)d\xi\Bigg]\Bigg\} \\ &+\frac{i\mu B_yk_L}{2}e^{-iyk_L} + \frac{i\mu B_yk_L}{4}\left(\frac{1+\kappa}{1-\kappa}\right)e^{-iyk_L}\end{aligned}$$

$$\begin{aligned}\sigma_{yy}(x,y) = \frac{\mu B_x}{\pi}\Bigg\{&\Bigg[\frac{1}{2}\int_{-\infty}^{\infty} e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}d\xi - \frac{1}{2}\int_{-\infty}^{\infty} e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}d\xi \\ &+\frac{1}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right)\int_{-\infty}^{\infty}\xi^2\left(e^{-i\xi x-y\sqrt{\xi^2-k_L^2}} - e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}\right)d\xi\Bigg]\end{aligned}$$

$$
\begin{aligned}
&-\frac{iB_y}{\pi}\left[\left(\frac{1}{2}-\frac{k_L^2}{2k_T^2}\left(\frac{1+\kappa}{1-\kappa}\right)\right)\int_{-\infty}^{\infty}\frac{\xi e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}}{\sqrt{\xi^2-k_L^2}}d\xi\right.\\
&-\frac{k_L^2}{4}\left(\frac{1+\kappa}{1-\kappa}\right)\int_{-\infty}^{\infty}\frac{e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}}{\xi\sqrt{\xi^2-k_L^2}}d\xi\\
&\left.-\frac{1}{k_T^2}\int_{-\infty}^{\infty}\frac{\xi^3 e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}-\xi\sqrt{\xi^2-k_L^2}\sqrt{\xi^2-k_T^2}\,e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}}{\sqrt{\xi^2-k_L^2}}d\xi\right]\Bigg\}\\
&-\frac{i\mu B_y k_L}{4}\left(\frac{1+\kappa}{1-\kappa}\right)e^{-iyk_L}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{xy}(x,y)=&\frac{-i\mu B_x}{\pi}\left[\left(\frac{1}{2}-\frac{k_T^2}{2k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right)\right)\int_{-\infty}^{\infty}\frac{\xi e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}}{\sqrt{\xi^2-k_T^2}}d\xi\right.\\
&-\frac{k_T^2}{4}\int_{-\infty}^{\infty}\frac{e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}}{\xi\sqrt{\xi^2-k_T^2}}d\xi-\frac{1}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right)\\
&\left.\times\int_{-\infty}^{\infty}\frac{\xi^3 e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}-\xi\sqrt{\xi^2-k_L^2}\sqrt{\xi^2-k_T^2}\,e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}}{\sqrt{\xi^2-k_T^2}}d\xi\right]\\
&-\frac{\mu B_y}{\pi}\left[\frac{1}{k_T^2}\int_{-\infty}^{\infty}\xi^2\left(e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}-e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}\right)d\xi\right.\\
&\left.-\frac{1}{2}\int_{-\infty}^{\infty}e^{-i\xi x-y\sqrt{\xi^2-k_L^2}}d\xi+\frac{1}{2}\int_{-\infty}^{\infty}e^{-i\xi x-y\sqrt{\xi^2-k_T^2}}d\xi\right] \qquad (17)
\end{aligned}
$$

The integrals in Eq. (17) may be simplified by employing the asymptotic expansion of integrands as $\xi\to\infty$, together with the procedure described in Achenbach (1976). For the sake of brevity, the details of manipulation are not given here. The final results are

$$
\begin{aligned}
\sigma_{xx}(x,y)=&\mu B_x\Bigg\{\frac{(3-\kappa)k_L y}{2(\kappa+1)r}[Y_1(k_Lr)+iJ_1(k_Lr)]-\frac{k_Ty}{2r}[Y_1(k_Tr)\\
&+iJ_1(k_Tr)]-\frac{2(1-\kappa)}{\pi k_L^2(1+\kappa)}\left[\int_0^{k_L}u^2\cos(ux)e^{-iy\sqrt{k_L^2-u^2}}du\right.\\
&+\int_{k_L}^{\infty}u^2\cos(ux)e^{-y\sqrt{u^2-k_L^2}}du-\int_{k_T}^{\infty}u^2\cos(ux)e^{-y\sqrt{u^2-k_T^2}}du\\
&\left.-\int_0^{k_T}u^2\cos(ux)e^{-iy\sqrt{k_T^2-u^2}}du\right]\Bigg\}\\
&-\mu B_y\Bigg\{\left(1-\frac{k_L^2(1+\kappa)}{k_T^2(1-\kappa)}\right)\frac{k_Lx}{2r}[Y_1(k_Lr)+iJ_1(k_Lr)]\\
&+\frac{k_L^2x}{2\pi}\left(\frac{3-\kappa}{1-\kappa}\right)\left[Ci(k_Lr)-iSi(k_Lr)+\frac{i\pi}{2}\right.\\
&\left.-\int_1^{\infty}\frac{\left(u\sqrt{u^2-1}-u^2+y^2/r^2\right)e^{-ik_Lur}}{u(u^2-y^2/r^2)}du\right]\\
&+\left(\frac{3-\kappa}{1-\kappa}\right)\frac{ik_Lx}{\pi r}\int_1^{\infty}\frac{ue^{-ik_Lur}\sqrt{u^2-1}}{(u^2-y^2/r^2)^2}du\\
&+\frac{2i}{\pi k_T^2}\left[\int_0^{k_L}u\sin(ux)e^{-iy\sqrt{k_L^2-u^2}}\sqrt{k_L^2-u^2}du\right.\\
&-\int_0^{k_T}u\sin(ux)e^{-iy\sqrt{k_T^2-u^2}}\sqrt{k_T^2-u^2}du\\
&-i\int_{k_L}^{\infty}u\sin(ux)e^{-y\sqrt{u^2-k_L^2}}\sqrt{u^2-k_L^2}du\\
&\left.+i\int_{k_L}^{\infty}u\sin(ux)e^{-y\sqrt{u^2-k_T^2}}\sqrt{u^2-k_T^2}du\right]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{yy}(x,y)=&\mu B_x\Bigg\{\frac{k_Ly}{2r}[Y_1(k_Lr)+iJ_1(k_Lr)]-\frac{k_Ty}{2r}[Y_1(k_Tr)\\
&+iJ_1(k_Tr)]+\frac{2(1-\kappa)}{\pi k_L^2(1+\kappa)}\left[\int_0^{k_L}u^2\cos(ux)e^{-iy\sqrt{k_L^2-u^2}}du\right.\\
&+\int_{k_L}^{\infty}u^2\cos(ux)e^{-y\sqrt{u^2-k_L^2}}du-\int_0^{k_T}u^2\cos(ux)e^{-iy\sqrt{k_T^2-u^2}}du\\
&\left.-\int_{k_T}^{\infty}u^2\cos(ux)e^{-y\sqrt{u^2-k_T^2}}du\right]\Bigg\}\\
&+\mu B_y\{\left(1-\frac{k_L^2}{k_T^2}\left(\frac{1+\kappa}{1-\kappa}\right)\right)\frac{k_Lx}{2r}[Y_1(k_Lr)+iJ_1(k_Lr)]\\
&+\frac{k_L^2x}{2\pi}\left(\frac{1+\kappa}{1-\kappa}\right)\left[Ci(k_Lr)-iSi(k_Lr)+\frac{i\pi}{2}\right.\\
&\left.-\int_1^{\infty}\frac{\left(u\sqrt{u^2-1}-u^2+y^2/r^2\right)e^{-ik_Lur}}{u(u^2-y^2/r^2)}du\right]\\
&+\frac{ik_Lx}{\pi r}\left(\frac{1+\kappa}{1-\kappa}\right)\int_1^{\infty}\frac{ue^{-ik_Lur}\sqrt{u^2-1}}{(u^2-y^2/r^2)^2}du\\
&+\frac{2i}{\pi k_T^2}\left(\int_0^{k_L}e^{-iy\sqrt{k_T^2-u^2}}\left[iyu^3\sin(ux)+2u\sin(ux)\sqrt{k_T^2-u^2}\right.\right.\\
&\left.+xu^2\cos(ux)\sqrt{k_T^2-u^2}\right]du\\
&+\int_0^{k_L}u\sin(ux)\sqrt{k_L^2-u^2}e^{-iy\sqrt{k_L^2-u^2}}du\\
&+i\int_{k_L}^{k_T}u\sin(ux)\sqrt{k_T^2-u^2}e^{-y\sqrt{k_T^2-u^2}}du\\
&-\int_{k_L}^{k_T}e^{-iy\sqrt{k_T^2-u^2}}\left[iyu^3\sin(ux)+2u\sin(ux)\sqrt{k_T^2-u^2}\right.\\
&\left.\left.+xu^2\cos(ux)\sqrt{k_T^2-u^2}\right]du\right)\\
&+\frac{2}{\pi k_T^2}\left\{\int_{k_T}^{\infty}\left(e^{-y\sqrt{u^2-k_T^2}}\left[yu^3\sin(ux)-2u\sin(ux)\sqrt{u^2-k_T^2}\right.\right.\right.\\
&\left.\left.-xu^2\cos(ux)\sqrt{u^2-k_T^2}\right]-\frac{k_L^2+k_T^2}{2}e^{-yu}\sin(ux)\right)du\\
&\left.-\int_{k_T}^{\infty}u\sin(ux)\sqrt{u^2-k_L^2}e^{-y\sqrt{u^2-k_L^2}}du\right\}\\
&+\left(\frac{k_L^2+k_T^2}{\pi k_T^2}\right)\frac{y\sin(k_Tx)+x\cos(k_Tx)}{r^2}e^{-yk_T}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{xy}(x,y)=&\mu B_x\Bigg\{\frac{k_Tx}{2r}\left(1-\frac{k_T^2}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right)\right)[Y_1(k_Tr)+iJ_1(k_Tr)]\\
&+\frac{k_T^2x}{2\pi}\left[Ci(k_Tr)-iSi(k_Tr)+\frac{i\pi}{2}\right.\\
&\left.-\int_1^{\infty}\frac{(u\sqrt{u^2-1}-u^2+y^2/r^2)e^{-ik_Tur}}{u(u^2-y^2/r^2)}du\right]\\
&+\frac{ik_Tx}{\pi r}\int_1^{\infty}\frac{ue^{-ik_Tur}\sqrt{u^2-1}}{(u^2-y^2/r^2)^2}du\\
&+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\left(\int_0^{k_L}e^{-iy\sqrt{k_L^2-u^2}}\left[iyu^3\sin(ux)+2u\sin(ux)\right.\right.\\
&\left.\times\sqrt{k_L^2-u^2}+xu^2\cos(ux)\sqrt{k_L^2-u^2}\right]du\\
&+\int_0^{k_L}u\sin(ux)\sqrt{k_T^2-u^2}e^{-iy\sqrt{k_T^2-u^2}}du\\
&+i\int_{k_L}^{k_T}u\sin(ux)\sqrt{u^2-k_L^2}e^{-y\sqrt{u^2-k_L^2}}du
\end{aligned}
$$

$$
\begin{aligned}
&+i\int_{k_L}^{k_T} e^{-y\sqrt{u^2-k_L^2}}\Big[yu^3\sin(ux)-2u\sin(ux)\sqrt{u^2-k_L^2}\\
&-xu^2\cos(ux)\sqrt{u^2-k_L^2}\Big]du\Big)\\
&+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\Big(\int_{k_T}^{\infty}\Big\{e^{-y\sqrt{u^2-k_L^2}}\Big[yu^3\sin(ux)-2u\sin(ux)\sqrt{u^2-k_L^2}\\
&-xu^2\cos(ux)\sqrt{u^2-k_L^2}\Big]\\
&-\frac{k_L^2+k_T^2}{2}\sin(ux)e^{-yu}\Big\}du-\int_{k_T}^{\infty}u\sin(ux)\sqrt{u^2-k_L^2}e^{-y\sqrt{u^2-k_L^2}}du\Big)\\
&+\left(\frac{(k_L^2+k_T^2)(1-\kappa)}{\pi k_L^2(1+\kappa)}\right)\frac{y\sin(k_Tx)+x\cos(k_Tx)}{r^2}e^{-yk_T}\Big\}\\
&+\mu B_y\Big\{\frac{k_Ly}{2r}[Y_1(k_Lr)+iJ_1(k_Lr)]+\frac{k_Ty}{2r}[Y_1(k_Tr)+iJ_1(k_Tr)]\\
&-\frac{2}{\pi k_T^2}\Big[\int_0^{k_L}u^2\cos(ux)e^{-iy\sqrt{k_L^2-u^2}}du\\
&+\int_{k_L}^{\infty}u^2\cos(ux)e^{-y\sqrt{u^2-k_L^2}}du-\int_0^{k_T}u^2\cos(ux)e^{-iy\sqrt{k_T^2-u^2}}du\\
&-\int_{k_T}^{\infty}u^2\cos(ux)e^{-y\sqrt{u^2-k_T^2}}du\Big]\Big\}
\end{aligned}
\tag{18}
$$

where $r=\sqrt{x^2+y^2}$, $Ci(x)$ and $Si(x)$ are, respectively, the sine and cosine integral functions, Abramowitz and Stegun (1965). Utilizing the asymptotic representation of the Bessel function of second kind

$$
Y_1(r)\sim\frac{2}{\pi}r^{-1}\quad \text{as}\quad r\to 0 \tag{19}
$$

we observe that stress components are Cauchy singular at the dislocation position which is a well-known feature of stress fields due to Volterra dislocation. Moreover, the integrands in Eq. (18) are bounded and decay sufficiently rapidly as $u\to\infty$. Thus, the integrals may be evaluated numerically employing a suitable quadrature formula.

To derive the integral equations for the crack problem, the distributed dislocations technique is employed. Let climb and glide edge dislocations with densities b_x and b_y, respectively, be distributed on a curved crack with parametric equations, $x=x(p)$, $y=y(p)$, $-1\leqslant p\leqslant 1$ in the plane. By virtue of Eq. (18), the stress fields caused at a point by the above mentioned distribution of dislocations become

$$
\begin{aligned}
\sigma_{xx}(x,y)=\mu&\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\Big\{\frac{(3-\kappa)k_L(y-\beta)}{2(\kappa+1)r_1}[Y_1(k_Lr_1)+iJ_1(k_Lr_1)]\\
&-\frac{k_T(y-\beta)}{2r_1}[Y_1(k_Tr_1)+iJ_1(k_Tr_1)]\\
&-\frac{2(1-\kappa)}{\pi k_L^2(1+\kappa)}\Big[\int_0^{k_L}u^2\cos[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_L^2-u^2}}du\\
&+\int_{k_L}^{\infty}u^2\cos[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_L^2}}du\\
&-\int_{k_T}^{\infty}u^2\cos[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_T^2}}du\\
&-\int_0^{k_T}u^2\cos[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_T^2-u^2}}du\Big]\Big\}b_x(p)dp\\
&-\mu\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\Big\{\left(1-\frac{k_L^2(1+\kappa)}{k_T^2(1-\kappa)}\right)\frac{k_L(x-\alpha)}{2r_1}\\
&\times[Y_1(k_Lr_1)+iJ_1(k_Lr_1)]+\frac{k_L^2(x-\alpha)}{2\pi}\left(\frac{3-\kappa}{1-\kappa}\right)[Ci(k_Lr_1)\\
&-iSi(k_Lr_1)+\frac{i\pi}{2}-\int_1^{\infty}\frac{\left(u\sqrt{u^2-1}-u^2+(y-\beta)^2/r_1^2\right)e^{-ik_Lur_1}}{u(u^2-(y-\beta)^2/r_1^2)}du]\\
&+\left(\frac{3-\kappa}{1-\kappa}\right)\frac{ik_L(x-\alpha)}{\pi r_1}\int_1^{\infty}\frac{ue^{-ik_Lur_1}\sqrt{u^2-1}}{(u^2-(y-\beta)^2/r_1^2)^2}du\\
&+\frac{2i}{\pi k_T^2}\Big[\int_0^{k_L}u\sin[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_L^2-u^2}}\sqrt{k_L^2-u^2}du\\
&-\int_0^{k_T}u\sin[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_T^2-u^2}}\sqrt{k_T^2-u^2}du\\
&-i\int_{k_L}^{\infty}u\sin[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_L^2}}\sqrt{u^2-k_L^2}du\\
&+i\int_{k_L}^{\infty}u\sin[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_T^2}}\sqrt{u^2-k_T^2}du\Big]\Big\}b_y(p)dp
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{yy}(x,y)=\mu&\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\Big\{\frac{k_L(y-\beta)}{2r_1}[Y_1(k_Lr_1)+iJ_1(k_Lr_1)]\\
&-\frac{k_T(y-\beta)}{2r_1}[Y_1(k_Tr_1)+iJ_1(k_Tr_1)]\\
&+\frac{2(1-\kappa)}{\pi k_L^2(1+\kappa)}\Big[\int_0^{k_L}u^2\cos[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_L^2-u^2}}du\\
&+\int_{k_L}^{\infty}u^2\cos[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_L^2}}du\\
&-\int_0^{k_T}u^2\cos[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_L^2-u^2}}du\\
&-\int_{k_T}^{\infty}u^2\cos[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_T^2}}du\Big]\Big\}b_x(p)dp\\
&+\mu\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\Big\{\left(1-\frac{k_L^2}{k_T^2}\left(\frac{1+\kappa}{1-\kappa}\right)\right)\frac{k_L(x-\alpha)}{2r_1}\\
&\times[Y_1(k_Lr_1)+iJ_1(k_Lr_1)]+\frac{k_L^2(x-\alpha)}{2\pi}\left(\frac{1+\kappa}{1-\kappa}\right)[Ci(k_Lr_1)\\
&-iSi(k_Lr_1)+\frac{i\pi}{2}-\int_1^{\infty}\frac{\left(u\sqrt{u^2-1}-u^2+(y-\beta)^2/r_1^2\right)e^{-ik_Lur_1}}{u(u^2-(y-\beta)^2/r_1^2)}du\Big]\\
&+\frac{ik_L(x-\alpha)}{\pi r_1}\left(\frac{1+\kappa}{1-\kappa}\right)\int_1^{\infty}\frac{ue^{-ik_Lur_1}\sqrt{u^2-1}}{(u^2-(y-\beta)^2/r_1^2)^2}du\\
&+\frac{2i}{\pi k_T^2}\Big(\int_0^{k_L}e^{-i(y-\beta)\sqrt{k_T^2-u^2}}[i(y-\beta)u^3\sin[u(x-\alpha)]\\
&+2u\sin[u(x-\alpha)]\sqrt{k_T^2-u^2}+(x-\alpha)u^2\cos[u(x-\alpha)]\sqrt{k_T^2-u^2}]du\\
&+\int_0^{k_L}u\sin[u(x-\alpha)]\sqrt{k_L^2-u^2}e^{-i(y-\beta)\sqrt{k_L^2-u^2}}du\\
&+i\int_{k_L}^{k_T}u\sin[u(x-\alpha)]\sqrt{k_T^2-u^2}e^{-(y-\beta)\sqrt{k_T^2-u^2}}du\\
&-\int_{k_L}^{k_T}e^{-i(y-\beta)\sqrt{k_T^2-u^2}}[i(y-\beta)u^3\sin[u(x-\alpha)]\\
&+2u\sin[u(x-\alpha)]\sqrt{k_T^2-u^2}+(x-\alpha)u^2\cos[u(x-\alpha)]\sqrt{k_T^2-u^2}]du\Big)\\
&+\frac{2}{\pi k_T^2}\Big\{\int_{k_T}^{\infty}\Big(e^{-(y-\beta)\sqrt{u^2-k_T^2}}[(y-\beta)u^3\sin[u(x-\alpha)]-2u\sin[u(x-\alpha)]\\
&\times\sqrt{u^2-k_T^2}-(x-\alpha)u^2\cos[u(x-\alpha)]\sqrt{u^2-k_T^2}\Big]\\
&-\frac{k_L^2+k_T^2}{2}\sin[u(x-\alpha)]e^{-(y-\beta)u}\Big)du-\int_{k_T}^{\infty}u\sin[u(x-\alpha)]\sqrt{u^2-k_L^2}\\
&e^{-(y-\beta)\sqrt{u^2-k_L^2}}du\Big\}+\left(\frac{k_L^2+k_T^2}{\pi k_T^2}\right)\\
&\times\frac{(y-\beta)\sin(k_T(x-\alpha))+(x-\alpha)\cos(k_T(x-\alpha))}{r_1^2}e^{-(y-\beta)k_T}\Big\}b_y(p)dp
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{xy}(x,y)=\mu&\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\Big\{\frac{k_T(x-\alpha)}{2r_1}\left(1-\frac{k_T^2}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right)\right)\\
&\times[Y_1(k_Tr_1)+iJ_1(k_Tr_1)]+\frac{k_T^2(x-\alpha)}{2\pi}[Ci(k_Tr_1)\\
&-iSi(k_Tr_1)+\frac{i\pi}{2}-\int_1^{\infty}\frac{\left(u\sqrt{u^2-1}-u^2+(y-\beta)^2/r_1^2\right)e^{-ik_Tur_1}}{u(u^2-(y-\beta)^2/r_1^2)}du\Big]\\
&+\frac{ik_T(x-\alpha)}{\pi r_1}\int_1^{\infty}\frac{ue^{-ik_Tur_1}\sqrt{u^2-1}}{(u^2-(y-\beta)^2/r_1^2)^2}du\\
&+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\Big(\int_0^{k_L}e^{-i(y-\beta)\sqrt{k_L^2-u^2}}[i(y-\beta)u^3\sin[u(x-\alpha)]\\
&+2u\sin[u(x-\alpha)]\sqrt{k_L^2-u^2}\\
&+(x-\alpha)u^2\cos[u(x-\alpha)]\sqrt{k_L^2-u^2}]du\\
&+\int_0^{k_L}u\sin[u(x-\alpha)]\sqrt{k_T^2-u^2}e^{-i(y-\beta)\sqrt{k_T^2-u^2}}du
\end{aligned}
$$

$$
\begin{aligned}
&+i\int_{k_L}^{k_T} u\sin[u(x-\alpha)]\sqrt{u^2-k_L^2}e^{-(y-\beta)\sqrt{u^2-k_L^2}}du\Big)\\
&+i\int_{k_L}^{k_T}(y-\beta)u^3\sin[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_L^2}}-2u\sin[u(x-\alpha)]\sqrt{u^2-k_L^2}\\
&+(x-\alpha)u^2\cos[u(x-\alpha)]\sqrt{u^2-k_L^2}]du)\\
&+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\Big(\int_{k_T}^{\infty}\Big\{e^{-(y-\beta)\sqrt{u^2-k_L^2}}[(y-\beta)u^3\sin[u(x-\alpha)]\\
&-2u\sin[u(x-\alpha)]\sqrt{u^2-k_L^2}-(x-\alpha)u^2\cos[u(x-\alpha)]\sqrt{u^2-k_L^2}\Big]\\
&-\frac{k_L^2+k_T^2}{2}\sin[u(x-\alpha)]e^{-(y-\beta)u}\Big\}du\\
&-\int_{k_T}^{\infty}u\sin[u(x-\alpha)]\sqrt{u^2-k_L^2}e^{-(y-\beta)\sqrt{u^2-k_L^2}}du\Big)\\
&+\frac{(k_L^2+k_T^2)(1-\kappa)}{\pi k_L^2(1+\kappa)}\frac{(y-\beta)\sin[k_T(x-\alpha)]+(x-\alpha)\cos[k_T(x-\alpha)]}{r_1^2}e^{-(y-\beta)k_T}\Big\}b_x dp\\
&+\mu\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\Big\{\frac{k_L(y-\beta)}{2r_1}[Y_1(k_Lr_1)\\
&+iJ_1(k_Lr_1)]+\frac{k_T(y-\beta)}{2r_1}[Y_1(k_Tr_1)+iJ_1(k_Tr_1)]-\frac{2}{\pi k_T^2}\Big[\int_0^{k_L}u^2\cos[u(x-\alpha)\\
&\times e^{-i(y-\beta)\sqrt{k_L^2-u^2}}du+\int_{k_L}^{\infty}u^2\cos[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_L^2}}du\\
&-\int_0^{k_T}u^2\cos[u(x-\alpha)]e^{-i(y-\beta)\sqrt{k_T^2-u^2}}du\\
&-\int_{k_T}^{\infty}u^2\cos[u(x-\alpha)]e^{-(y-\beta)\sqrt{u^2-k_T^2}}du\Big]\Big\}b_y(p)dp \qquad (20)
\end{aligned}
$$

In Eq. (20), $r_1=\sqrt{[x-\alpha(p)]^2+[y-\beta(p)]^2}$ and prime denotes differentiation with respect to the relevant argument. A moveable orthogonal coordinate system (s,n) is chosen such that the origin may move on the crack while the s-axis remains tangent to the crack surface. The stress components (20) and dislocation densities b_x and b_y in the (s,n) coordinates are

$$
\begin{aligned}
\sigma_n &= \frac{\sigma_x+\sigma_y}{2}-\frac{\sigma_x-\sigma_y}{2}\cos(2\psi)-\sigma_{xy}\sin(2\psi)\\
\sigma_s &= -\frac{\sigma_x-\sigma_y}{2}\sin(2\psi)+\sigma_{xy}\cos(2\psi)\\
b_x &= b_s\cos(\psi)-b_n\sin(\psi)\\
b_y &= b_s\sin(\psi)+b_n\cos(\psi) \qquad (21)
\end{aligned}
$$

where $\psi(p)=\tan^{-1}(\beta'(p)/\alpha'(p))$ is the angle between s- and x-axes. Employing the principle of superposition, the components of traction vector at a point with coordinates $(\alpha_j(s),\beta_j(s))$, where parameter $-1\leqslant s\leqslant 1$, on the surface of the j-th crack for a plane weakened by N curved cracks result in

$$
\begin{aligned}
\sigma_n(\alpha_j(s),\beta_j(s)) &= \sum_{i=1}^{N}\int_{-1}^{1}k_{11ij}(s,p)b_{si}(p)dp\\
&\quad+\sum_{i=1}^{N}\int_{-1}^{1}k_{12ij}(s,p)b_{ni}(p)dp\\
\sigma_{ns}(\alpha_j(s),\beta_j(s)) &= \sum_{i=1}^{N}\int_{-1}^{1}k_{21ij}(s,p)b_{si}(p)dp\\
&\quad+\sum_{i=1}^{N}\int_{-1}^{1}k_{22ij}(s,p)b_{ni}(p)dp,\quad j=1,2,\dots,N \qquad (22)
\end{aligned}
$$

where the kernels in Eq. (22) are

$$
\begin{aligned}
k_{11ij}(s,p) &= K_{11ij}(s,p)\cos\psi_i(p)+K_{12ij}(s,p)\sin\psi_i(p)\\
k_{12ij}(s,p) &= K_{12ij}(s,p)\cos\psi_i(p)-K_{11ij}(s,p)\sin\psi_i(p)\\
k_{21ij}(s,p) &= K_{21ij}(s,p)\cos\psi_i(p)+K_{22ij}(s,p)\sin\psi_i(p)\\
k_{22ij}(s,p) &= K_{22ij}(s,p)\cos\psi_i(p)-K_{21ij}(s,p)\sin\psi_i(p)
\end{aligned} \qquad (23)
$$

The functions in the above equalities are given in Appendix. It is worth mentioning that kernels in integral Eq. (22) are Cauchy singular for $i=j$ as $s\to p$. Employing the definition of dislocation density function, the equations for the crack opening displacement across the ith crack yield

$$
\begin{aligned}
u_{si}^+(s)-u_{si}^-(s) &= \int_{-1}^{s}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\\
&\quad\times[\cos(\psi_i(s)-\psi_i(p))b_{si}(p)+\sin(\psi_i(s)-\psi_i(p))b_{ni}(p)]dp\\
u_{ni}^+(s)-u_{ni}^-(s) &= \int_{-1}^{s}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}\\
&\quad\times[\cos(\psi_i(s)-\psi_i(p))b_{ni}(p)-\sin(\psi_i(s)-\psi_i(p))b_{si}(p)]dp.
\end{aligned} \qquad (24)
$$

For embedded cracks the displacement field is single-valued out of crack surfaces. Thus, the dislocation densities are subjected to the following closure requirements

$$
\begin{aligned}
&\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}[\cos(\psi_i(1)-\psi_i(p))b_{si}(p)\\
&\quad+\sin(\psi_i(1)-\psi_i(p))b_{ni}(p)]dp=0\\
&\int_{-1}^{1}\sqrt{[\alpha'(p)]^2+[\beta'(p)]^2}[\cos(\psi_i(1)-\psi_i(p))b_{ni}(p)\\
&\quad-\sin(\psi_i(1)-\psi_i(p))b_{si}(p)]dp=0
\end{aligned} \qquad (25)
$$

3. Solution of integral equations

To evaluate the dislocation density on the crack surfaces, the complex Cauchy singular integral Eqs. (22) and (25) ought to be solved simultaneously. This is accomplished by means of the Gauss-Chebyshev quadrature scheme developed by Erdogan et al. (1973). The stress fields in the neighborhood of crack tips behave like $1/\sqrt{r}$ where r is the distance from the crack tip. Therefore, the dislocation densities are taken as

$$
\begin{aligned}
b_{si}(p) &= \frac{g_{si}(p)}{\sqrt{1-p^2}}\\
b_{ni}(p) &= \frac{g_{ni}(p)}{\sqrt{1-p^2}},\quad -1\leqslant p\leqslant 1,\ \ i=1,2,\dots,N
\end{aligned} \qquad (26)
$$

In the above equations $g_{si}=g_{1si}+ig_{2si}$ and $g_{ni}=g_{1ni}+ig_{2ni}$. Substitution of Eq. (26) into Eqs. (22) and (25) and discretizing the domain, $-1<p<1$ by $m+1$ segments, we arrive at the following system of $2N\times m$ algebraic equations:

$$
\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N}\\ A_{21} & A_{22} & \cdots & A_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ A_{N1} & A_{N2} & \cdots & A_{NN}\end{bmatrix}\begin{bmatrix} g_{z1}(p_n)\\ g_{z2}(p_n)\\ \vdots\\ g_{zN}(p_n)\end{bmatrix}=\begin{bmatrix} q_1(s_r)\\ q_2(s_r)\\ \vdots\\ q_N(s_r)\end{bmatrix} \qquad (27)
$$

where the collocation points are

$$
\begin{aligned}
s_r &= \cos\left(\frac{\pi r}{m}\right)\quad r=1,2,\dots,m-1\\
p_n &= \cos\left(\frac{\pi(2n-1)}{2m}\right)\quad n=1,2,\dots,m
\end{aligned} \qquad (28)
$$

The components of the matrix in (27) are

$$
A_{ij}=\begin{bmatrix} C_{11ij} & C_{12ij}\\ C_{21ij} & C_{22ij}\end{bmatrix}
$$

and

$$
C_{mnij}=\begin{bmatrix} D_{mnij} & -E_{mnij}\\ E_{mnij} & D_{mnij}\end{bmatrix}\quad m,n=1,2 \qquad (29)
$$

where

$$D_{mnij} = \frac{\pi}{m}\begin{bmatrix} \mathrm{Re}[k_{mnij}(s_1,p_1)]\Delta_i(p_1) & \mathrm{Re}[k_{mnij}(s_1,p_2)]\Delta_i(p_2) & \cdots & \mathrm{Re}[k_{mnij}(s_1,p_m)]\Delta_i(p_m) \\ \mathrm{Re}[k_{mnij}(s_2,p_1)]\Delta_i(p_1) & \mathrm{Re}[k_{mnij}(s_2,p_2)]\Delta_i(p_2) & \cdots & \mathrm{Re}[k_{mnij}(s_2,p_m)]\Delta_i(p_m) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Re}[k_{mnij}(s_{m-1},p_1)]\Delta_i(p_1) & \mathrm{Re}[k_{mnij}(s_{m-1},p_2)]\Delta_i(p_2) & \cdots & \mathrm{Re}[k_{mnij}(s_{m-1},p_m)]\Delta_i(p_m) \\ \delta_{ij}\Delta_i(p_1) & \delta_{ij}\Delta_i(p_2) & \cdots & \delta_{ij}\Delta_i(p_m) \end{bmatrix}$$

$$E_{mnij} = \frac{\pi}{m}\begin{bmatrix} \mathrm{Im}[k_{mnij}(s_1,p_1)]\Delta_i(p_1) & \mathrm{Im}[k_{mnij}(s_1,p_2)]\Delta_i(p_2) & \cdots & \mathrm{Im}[k_{mnij}(s_1,p_m)]\Delta_i(p_m) \\ \mathrm{Im}[k_{mnij}(s_2,p_1)]\Delta_i(p_1) & \mathrm{Im}[k_{mnij}(s_2,p_2)]\Delta_i(p_2) & \cdots & \mathrm{Im}[k_{mnij}(s_2,p_m)]\Delta_i(p_m) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Im}[k_{mnij}(s_{m-1},p_1)]\Delta_i(p_1) & \mathrm{Im}[k_{mnij}(s_{m-1},p_2)]\Delta_i(p_2) & \cdots & \mathrm{Im}[k_{mnij}(s_{m-1},p_m)]\Delta_i(p_m) \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{30}$$

δ_{ij} in the last row of D_{mnij} designates the Kronecker delta, and $\Delta_i(p_j) = \sqrt{[\alpha_i'(p_j)]^2 + [\beta_i'(p_j)]^2}$. The components of vectors in (27) are

$$\begin{aligned} g_{zj}(p_n) = {} & [g_{1sj}(p_1) \quad g_{1sj}(p_2) \quad \cdots \quad g_{1sj}(p_m) \quad g_{2sj}(p_1) \quad g_{2sj}(p_2) \quad \ldots \quad g_{2sj}(p_m) \\ & g_{1nj}(p_1) \quad g_{1nj}(p_2) \quad \cdots \quad g_{1nj}(p_m) \quad g_{2nj}(p_1) \quad g_{2nj}(p_2) \quad \ldots \quad g_{2nj}(p_m)]^T \\ q_j(s_r) = {} & [\sigma_{1n}(x_j(s_1),y_j(s_1)) \quad \sigma_{1n}(x_j(s_2),y_j(s_2)) \quad \ldots \quad \sigma_{1n}(x_j(s_{m-1}),y_j(s_{m-1})) \quad 0 \\ & \sigma_{2n}(x_j(s_1),y_j(s_1)) \quad \sigma_{2n}(x_j(s_2),y_j(s_2)) \quad \ldots \quad \sigma_{2n}(x_j(s_{m-1}),y_j(s_{m-1})) \quad 0 \\ & \sigma_{1s}(x_j(s_1),y_j(s_1)) \quad \sigma_{1s}(x_j(s_2),y_j(s_2)) \quad \ldots \quad \sigma_{1s}(x_j(s_{m-1}),y_j(s_{m-1})) \quad 0 \\ & \sigma_{2s}(x_j(s_1),y_j(s_1)) \quad \sigma_{2s}(x_j(s_2),y_j(s_2)) \quad \ldots \quad \sigma_{2s}(x_j(s_{m-1}),y_j(s_{m-1})) \quad 0]^T \end{aligned} \tag{31}$$

where σ_{1n},σ_{1s} and σ_{2n},σ_{2s} are the real and imaginary parts of traction components (22), respectively, and superscript T stands for the transpose of a vector. Stress intensity factors at the tips of ith crack in terms of the crack opening displacement reduce to

$$\begin{Bmatrix} k_{IL} \\ k_{IIL} \end{Bmatrix} = \frac{2\mu}{1+\kappa}\lim_{r_{Li}\to 0}\frac{1}{\sqrt{2r_{Li}}}\begin{Bmatrix} u_{ni}^+ - u_{ni}^- \\ u_{si}^+ - u_{si}^- \end{Bmatrix}$$
$$\begin{Bmatrix} k_{IR} \\ k_{IIR} \end{Bmatrix} = \frac{2\mu}{1+\kappa}\lim_{r_{Ri}\to 0}\frac{1}{\sqrt{2r_{Ri}}}\begin{Bmatrix} u_{ni}^+ - u_{ni}^- \\ u_{si}^+ - u_{si}^- \end{Bmatrix} \tag{32}$$

where the superscript L and R designate the left and right tips of a crack, respectively. The geometry of a crack implies

$$r_{Li} = [(\alpha_i(p) - \alpha_i(-1))^2 + (\beta_i(p) - \beta_i(-1))^2]^{\frac{1}{2}},$$
$$r_{Ri} = [(\alpha_i(p) - \alpha_i(1))^2 + (\beta_i(p) - \beta_i(1))^2]^{\frac{1}{2}}. \tag{33}$$

Substituting Eqs. (24) and (26) into Eq. (32) and using the Taylor series expansion of functions $\alpha_i(p)$ and $\beta_i(p)$ around the crack tips, $p = \pm 1$, leads to

$$\begin{Bmatrix} k_{ILj} \\ k_{IILj} \end{Bmatrix} = \frac{2\mu}{1+\kappa}([\alpha_j'(-1)]^2 + [\beta_j'(-1)]^2)^{\frac{1}{4}}\begin{Bmatrix} g_{1nj}(-1) + ig_{2nj}(-1) \\ g_{1sj}(-1) + ig_{2sj}(-1) \end{Bmatrix}$$
$$\begin{Bmatrix} k_{IRj} \\ k_{IIRj} \end{Bmatrix} = \frac{2\mu}{1+\kappa}([\alpha_j'(1)]^2 + [\beta_j'(1)]^2)^{\frac{1}{4}}\begin{Bmatrix} g_{1nj}(1) + ig_{2nj}(1) \\ g_{1sj}(1) + ig_{2sj}(1) \end{Bmatrix},$$
$$j = 1,2,\ldots,N \tag{34}$$

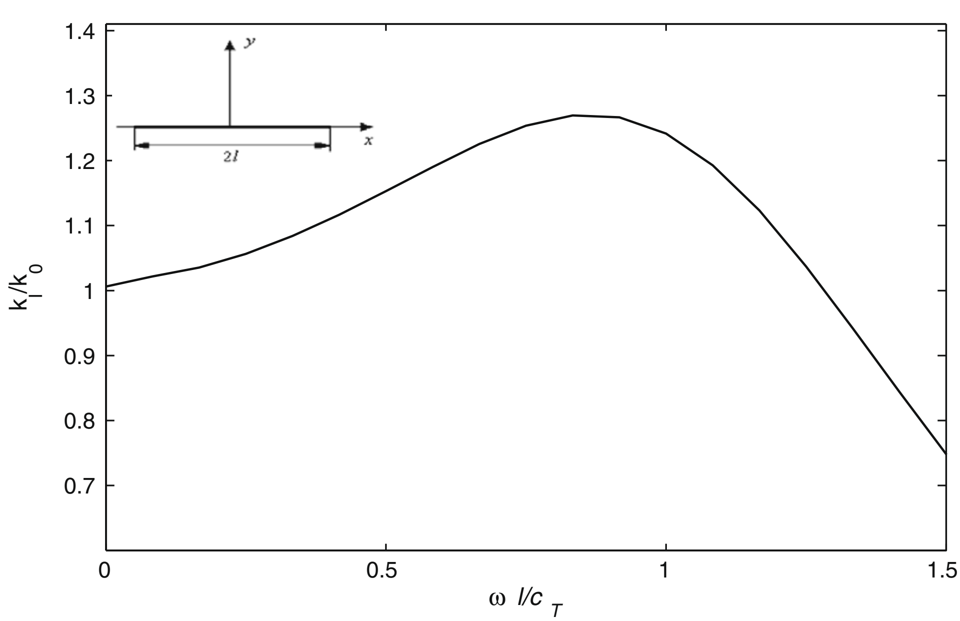

Fig. 1. Stress intensity factor versus load frequency.

The solution of Eq. (27) are plugged into (34) thereby obtaining the complex stress intensity factors.

4. Numerical examples

In what follows, the Poisson's ratio of the medium is $\upsilon = 0.25$, the elastic longitudinal and shear wave velocities of the medium are $c_L = 5100\,(\mathrm{m/s})$, $c_T = 3000\,(\mathrm{m/s})$, respectively. The quantities under consideration are the absolute value of modes I and II stress intensity factors, k_I/k_0 and k_{II}/k_0, where $k_0 = \sigma_0\sqrt{l}$ is the stress intensity factor of a crack with length $2l$ under static normal traction σ_0. Moreover, unless otherwise stated, plane strain condition prevails. The verification of formulation is accomplished in the first three examples, where the surface of straight cracks with fixed lengths are subjected to uniform normal traction $\sigma_0 e^{i\omega t}$. The variation of stress intensity factor at a crack tip versus dimensionless excitation frequency $\omega\ l/c_T$, is depicted in Fig. 1. The largest value of k_I/k_0 occurs at $\omega l/c_T = 0.8$. The comparison of results with those presented by Mal (1970) confirms the validity of the analysis. Two collinear cracks with two different distances between crack centers are considered. The problem is symmetric with respect to the y-axis. The plots of stress intensity factors against $\omega b/c_L$ are shown in Fig. 2. By adopting the definition of stress intensity factors used by Itou (1978), i.e., dividing k_I/k_0 by $\sqrt{2b}$, a close similarity between the results of the two analyses may be observed. As the last check of results, three collinear cracks in a thin plate, plain stress condition, where $l = 1\,(\mathrm{m})$, $a = 1.1\,(\mathrm{m})$, $b = 1.2\,(\mathrm{m})$, Fig. 3, are considered. The plot of k_I/k_0, versus $\omega l/c_L$ is very close to the results rendered by Itou (1996). The three aforementioned problems were symmetric with respect to *x-axis*. Consequently, only mode I stress intensity factor occurred.

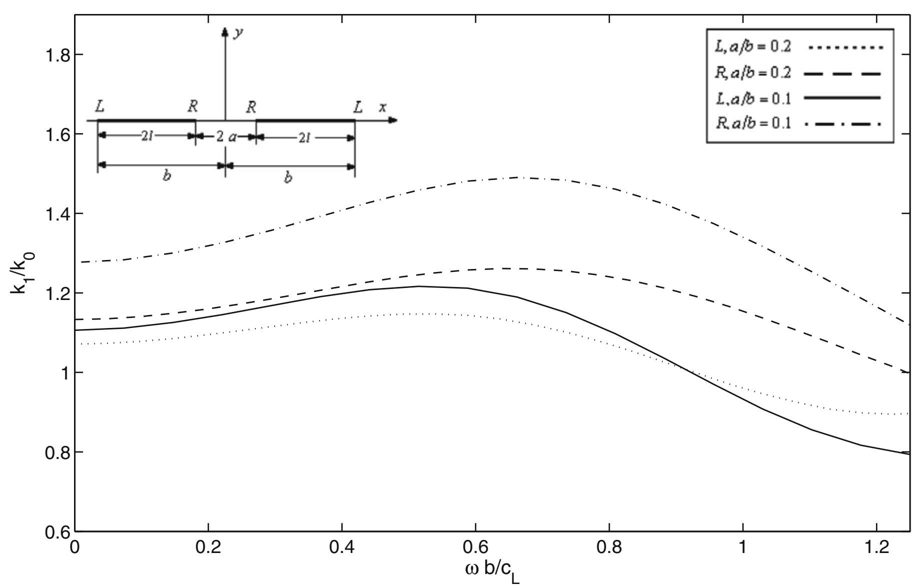

Fig. 2. Interaction of two collinear-cracks versus load frequency.

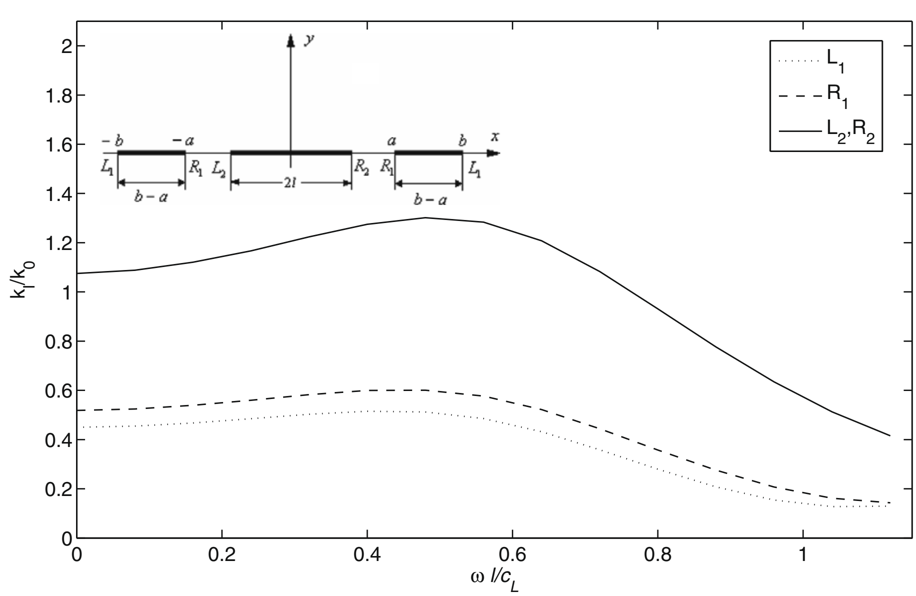

Fig. 3. Variations of stress intensity factors of three collinear cracks.

The applicability of the procedure is illustrated by solving two more examples therein the interactions of two cracks are studied. To ensure the opening of cracks with any configuration, the plan is subjected to remote uniform biaxial traction $\sigma_x(r,\theta) = \sigma_y(r,\theta) = \sigma_0 e^{i\omega t}$ as $r \to \infty$. The value of angular frequency is taken $\omega = 1500$ (rad/s). Obviously, the values of stress intensity factors are under the influence of excitation frequency but the trend of variations remain the same by changing the frequency. A stationary and a rotating crack with equal lengths $2l = 2$ (m) and distance between the centers $2a = 2.5$ (m) are considered. Crack L_1R_1 is fixed, whereas L_2R_2 is rotating around its center. The dimensionless modes I and II stress intensity factors versus angle of rotation are shown in Figs. 4 and 5, respectively. As it may be observed the crack tips with smaller variation of k_I/k_0 experience higher variation of k_{II}/k_0. Moreover the variation of k_{II}/k_0 for different cracks are similar but with different magnitude and as it was expected $k_{II}/k_0 = 0$ at $\theta = 0, \pi/2$. It is worth mentioning that for low frequency under axial traction $\sigma_y(r,\theta) = \sigma_0 e^{i\omega t}$ our results for both stress intensity factors are identical with those depicted in Fig. 6 of an article by Chen (1992).

The last example deals with the interaction of a straight and a curved crack, Figs. 6 and 7. The parametric representations of straight and curved cracks are, respectively,

$$\begin{aligned} \alpha_1(t) &= -a + lt \\ \beta_1(t) &= b \\ \alpha_2(t) &= a\cos\left[\frac{\pi}{4}(1-t)\right] \\ \beta_2(t) &= b\sin\left[\frac{\pi}{4}(1-t)\right], \quad -1 \leqslant t \leqslant 1 \end{aligned} \tag{35}$$

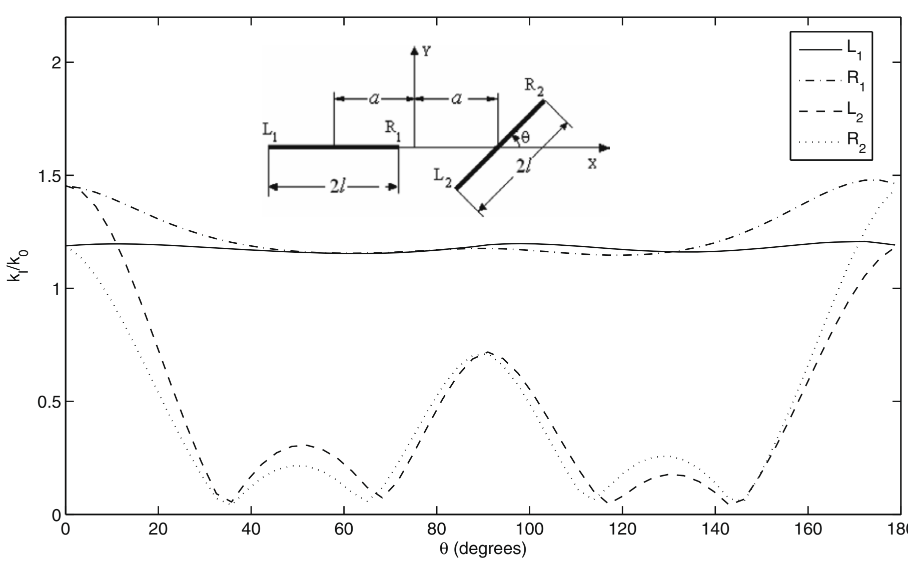

Fig. 4. Mode I stress intensity factors of a fixed and a rotating crack.

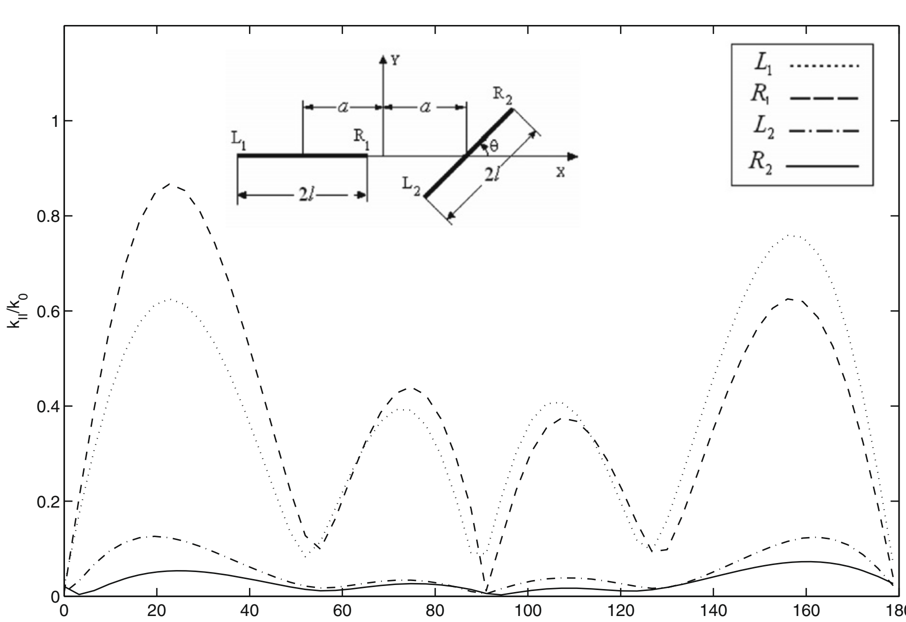

Fig. 5. Mode II stress intensity factors of a fixed and a rotating crack.

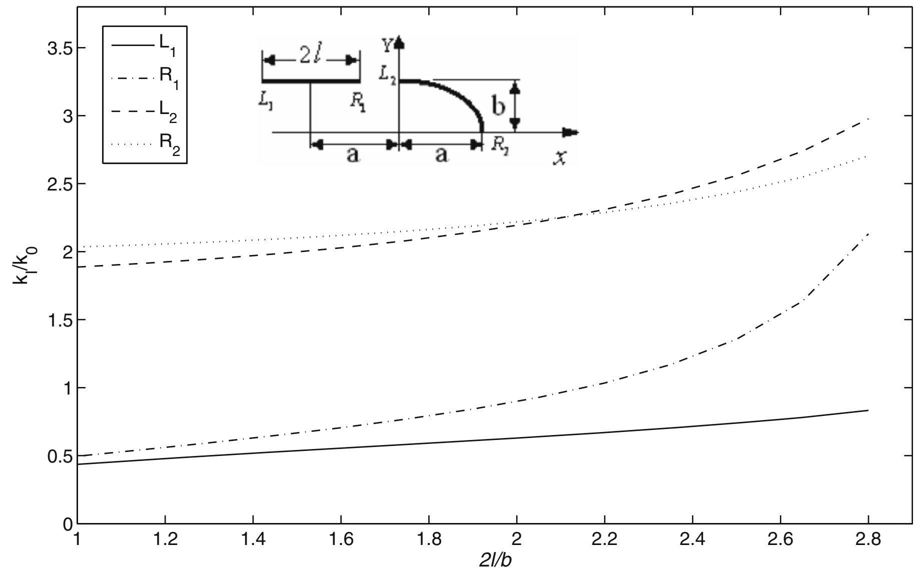

Fig. 6. Mode I stress intensity factors of a curved and a straight interacting cracks.

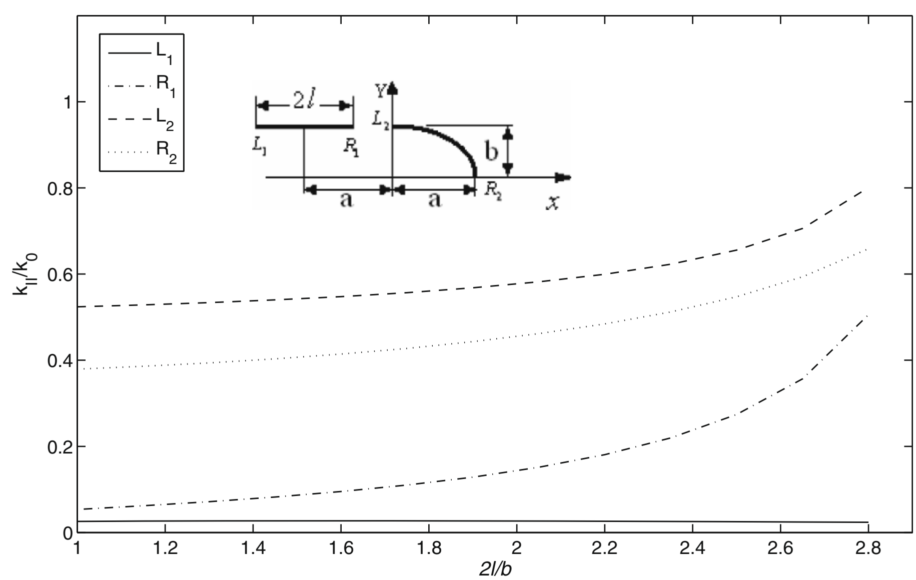

Fig. 7. Mode II stress intensity factors of a curved and a straight interacting cracks.

where $a = 0.3\,(\mathrm{m})$ and $b = 0.2\,(\mathrm{m})$. By increasing the length of the straight crack the distance between the tips R_1 and L_2 diminishes. The variation of stress intensity factor of crack tip L_1 is negligible whereas those for approaching crack tips increase rapidly. The behavior of the curved crack is rather peculiar. The variations of stress intensity factors at both tips of the curved crack, regardless of their distance from the straight cracks, are comparable.

5. Conclusion

The solution of edge dislocations with time-harmonic excitation is obtained by means of Fourier transformation. The distributed dislocation technique is used to derive integral equations for dislocation density on the cracks surfaces. By solving several examples, the capability of the technique in handling multiple smooth cracks with any arrangement under dynamic excitation is illustrated. It is observed that for a single and multiple collinear cracks under normal traction, stress intensity factor increases with increasing load frequency reaching a maximum value then decreases with a faster rate. This phenomenon was previously reported by other investigators. The analysis of interaction of a fixed and a rotating crack under biaxial load reveals that cracks with higher variation of k_I/k_0 experience lower variation of k_{II}/k_0. From the last example, it is apparent that the rates of change of stress intensity factors at the two tips of the curved crack interacting with the straight crack are almost equal. The behavior of curved cracks is complicated and can not fully be analyzed in this article.

Appendix

The functions in Eq. (23) are as follows

$$K_{11ij}=\frac{k_L(\beta_j-\beta i)}{(\kappa+1)r_{ji}}[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]-\frac{k_T(\beta_j-\beta i)}{2r_{ji}}[Y_1(k_Tr_{ji})+iJ_1(k_Tr_{ji})]+\left\{\frac{(\kappa-1)k_L(\beta_j-\beta i)}{2(\kappa+1)r_{ji}}[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]\right.$$

$$+\frac{2(1-\kappa)}{\pi k_L^2(1+\kappa)}\left[\int_0^{k_L}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}du+\int_{k_L}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du-\int_{k_T}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}du\right.$$

$$\left.\left.-\int_0^{k_T}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du\right]\right\}\cos(2\psi_j)-\left\{\frac{k_T(\alpha_j-\alpha_i)}{2r_{ji}}\left(1-\frac{k_T^2}{k_L^2}\left(\frac{1-\kappa}{1+\kappa}\right)\right)[Y_1(k_Tr_{ji})+iJ_1(k_Tr_{ji})]\frac{k_T^2(\alpha_j-\alpha_i)}{2\pi}\left[Ci(k_Tr_{ji})-\right.\right.$$

$$\left.+iSi(k_Tr_{ji})+\frac{i\pi}{2}-\int_1^{\infty}\frac{(u\sqrt{u^2-1}-u^2+(\beta_j-\beta_i)^2/r_{ji}^2)e^{-ik_Tur}}{u(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)}du\right]+\frac{ik_T(\alpha_j-\alpha_i)}{\pi r_{ji}}\int_1^{\infty}\frac{ue^{-ik_Tur_{ji}}\sqrt{u^2-1}}{(u^2-y^2/r_{ji}^2)^2}du$$

$$+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\left(\int_0^{k_L}e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}\left[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}\right]du\right.$$

$$+\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du+i\int_{k_L}^{k_T}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du$$

$$\left.+i\int_{k_L}^{k_T}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}\left[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}-[u(\alpha_j-\alpha_i)]u^2\cos(ux)\sqrt{u^2-k_L^2}\right]du\right)$$

$$+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\left(\int_{k_T}^{\infty}\left\{e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}.-(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}]\right.\right.$$

$$\left.\left.-\frac{k_L^2+k_T^2}{2}\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)u}\right\}du-\int_{k_T}^{\infty}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du\right)$$

$$\left.+\frac{(k_L^2+k_T^2)(1-\kappa)}{\pi k_L^2(1+\kappa)}\frac{(\beta_j-\beta_i)\sin[k_T(\alpha_j-\alpha_i)]+(\alpha_j-\alpha_i)\cos[k_T(\alpha_j-\alpha_i)]}{r_{ij}^2}e^{-(\beta_j-\beta_i)k_T}\right\}\sin(2\psi_j)K_{12ij}$$

$$=\left\{\frac{-k_L^2(\alpha_j-\alpha_i)}{2\pi}\left[Ci(k_Lr_{ji})-iSi(k_Lr_{ji})+\frac{i\pi}{2}-\int_1^{\infty}\frac{(u\sqrt{u^2-1}-u^2+(\beta_j-\beta_i)^2/r_{ji}^2)e^{-ik_Lur_{ji}}}{u(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)}du\right]-\frac{ik_L(\alpha_j-\alpha_i)}{\pi r}\int_1^{\infty}\frac{ue^{-ik_Lur_{ji}}\sqrt{u^2-1}}{(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)^2}d\right.$$

$$-\frac{i}{\pi k_T^2}\left[\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}\sqrt{k_L^2-u^2}du-\int_0^{k_T}u\sin[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\sqrt{k_T^2-u^2}du\right.$$

$$-i\int_{k_L}^{\infty}u\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}\sqrt{u^2-k_L^2}du+i\int_{k_L}^{\infty}u\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}\sqrt{u^2-k_T^2}du$$

$$+\frac{i}{\pi k_T^2}\left(\int_0^{k_L}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\left[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}\right]du\right.$$

$$+\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}du+i\int_{k_L}^{k_T}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}e^{-(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du$$

$$-\int_{k_L}^{k_T}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\left[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}\right]du$$

$$+\frac{1}{\pi k_T^2}\left\{\int_{k_T}^{\infty}\left(e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_T^2}-(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_T^2}]\right.\right.$$

$$\left.-\frac{k_L^2+k_T^2}{2}e^{-(\beta_j-\beta_i)u}\sin[u(\alpha_j-\alpha_i)]\right)du-\int_{k_T}^{\infty}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du$$

$$+\frac{k_L^2+k_T^2}{\pi k_T^2}\frac{(\beta_j-\beta_i)\sin[k_T(\alpha_j-\alpha_i)]+(\alpha_j-\alpha_i)\cos[k_T(\alpha_j-\alpha_i)]}{r_{ji}^2}e^{-(\beta_j-\beta_i)k_T}+\left\{\left(1-\frac{k_L^2(1+\kappa)}{k_T^2(1-\kappa)}\right)\frac{k_L(\alpha_j-\alpha_i)}{2r_{ji}^2}[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]\right.$$

$$+\frac{k_L^2(\alpha_j-\alpha_i)}{\pi}\left(\frac{1}{1-\kappa}\right)\left[Ci(k_Lr_{ji})-iSi(k_Lr_{ji})+\frac{i\pi}{2}-\int_1^{\infty}\frac{(u\sqrt{u^2-1}-u^2+(\beta_j-\beta_i)^2/r_{ji}^2)e^{-ik_Lur_{ji}}}{u(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)}du\right.$$

$$+\left(\frac{2}{1-\kappa}\right)\frac{ik_L(\alpha_j-\alpha_i)}{\pi r_{ji}}\int_1^{\infty}\frac{ue^{-ik_Lur_{ji}}\sqrt{u^2-1}}{(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)^2}du+\frac{i}{\pi k_T^2}\left[\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}\sqrt{k_L^2-u^2}du\right.$$

$$-\int_0^{k_T}u\sin[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\sqrt{k_T^2-u^2}du-i\int_{k_L}^{\infty}u\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}\sqrt{u^2-k_L^2}du$$

$$\left.\left.+i\int_{k_L}^{\infty}u\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}\sqrt{u^2-k_T^2}du\right]\right\}+\frac{i}{\pi k_T^2}\left(\int_0^{k_L}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\right.$$

$$\left.\times\sqrt{k_T^2-u^2}+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}\right]du+\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}e^{-i[\beta_j-\beta_i]\sqrt{k_L^2-u^2}}du$$

$$+i\int_{k_L}^{k_T}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}e^{-[\beta_j-\beta_i]\sqrt{k_T^2-u^2}}du-\int_{k_L}^{k_T}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\Big[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}$$

$$+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}\Big]du\Big)+\frac{1}{\pi k_T^2}\Big\{\int_{k_T}^{\infty}\Big(e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_T^2}$$

$$-(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_T^2}]-\frac{k_L^2+k_T^2}{2}e^{-(\beta_j-\beta_i)u}\sin[u(\alpha_j-\alpha_i)]\Big)du-\int_{k_T}^{\infty}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du\Big\}$$

$$+\frac{k_L^2+k_T^2}{\pi k_T^2}\frac{(\beta_j-\beta_i)\sin(k_T[u(\alpha_j-\alpha_i)])+[u(\alpha_j-\alpha_i)]\cos(k_T[u(\alpha_j-\alpha_i)])}{r_{ji}^2}e^{-(\beta_j-\beta_i)k_T}\Big\}\cos(2\psi_j)-\Big\{\frac{k_L(\beta_j-\beta_i)}{2r_{ji}}[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]$$

$$+\frac{k_T(\beta_j-\beta_i)}{2r_{ji}}[Y_1(k_Tr_{ji})+iJ_1(k_Tr_{ji})]-\frac{2}{\pi k_T^2}\Big[\int_0^{k_L}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}du+\int_{k_L}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du$$

$$-\int_0^{k_T}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du-\int_{k_T}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}du\Big]\Big\}\sin(2\psi_j)K_{21ij}=\Big\{\frac{(\kappa-1)k_L(\beta_j-\beta_i)}{2(\kappa+1)r_{ji}}[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]$$

$$+\frac{2(1-\kappa)}{\pi k_L^2(1+\kappa)}\Big[\int_0^{k_L}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}du+\int_{k_L}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du-\int_{k_T}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}du$$

$$-\int_0^{k_T}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du\Big]\Big\}\sin(2\psi_j)+\Big\{\frac{k_T(\alpha_j-\alpha_i)}{2r_{ji}}\Big(1-\frac{k_T^2}{k_L^2}\Big(\frac{1-\kappa}{1+\kappa}\Big)\Big)[Y_1(k_Tr_{ji})+iJ_1(k_Tr_{ji})]+\frac{k_T^2(\alpha_j-\alpha_i)}{2\pi}\Big[Ci(k_Tr_{ji})-+iSi(k_Tr_{ji})$$

$$+\frac{i\pi}{2}-\int_1^{\infty}\frac{(u\sqrt{u^2-1}-u^2+(\beta_j-\beta_i)^2/r_{ji}^2)e^{-ik_Tur}}{u(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)}du\Big]+\frac{ik_T(\alpha_j-\alpha_i)}{\pi r_{ji}}\int_1^{\infty}\frac{ue^{-ik_Tur_{ji}}\sqrt{u^2-1}}{(u^2-y^2/r_{ji}^2)^2}du$$

$$+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\Big(\int_0^{k_L}e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}]du$$

$$+\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du+i\int_{k_L}^{k_T}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du$$

$$+i\int_{k_L}^{k_T}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}\,[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}-[(\alpha_j-\alpha_i)]u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}]du\Big)$$

$$+\frac{2i}{\pi k_L^2}\frac{1-\kappa}{1+\kappa}\Big(\int_{k_T}^{\infty}\Big\{e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}-(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}]$$

$$-\frac{k_L^2+k_T^2}{2}\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)u}\Big\}du-\int_{k_T}^{\infty}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du\Big)$$

$$+\frac{(k_L^2+k_T^2)(1-\kappa)}{\pi k_L^2(1+\kappa)}\frac{(\beta_j-\beta_i)\sin[k_T(\alpha_j-\alpha_i)]+(\alpha_j-\alpha_i)\cos[k_T(\alpha_j-\alpha_i)]}{r_{ij}^2}e^{-(\beta_j-\beta_i)k_T}\Big\}\cos(2\psi_j)K_{22ij}$$

$$=\Big\{\frac{k_L(\alpha_j-\alpha_i)}{2r_{ji}}\Big(1-\frac{k_L^2}{k_T^2}\Big(\frac{1+\kappa}{1-\kappa}\Big)\Big)[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]+\frac{k_L^2(\alpha_j-\alpha_i)}{2\pi}\Big[Ci(k_Lr_{ji})-iSi(k_Lr_{ji})-\frac{i\pi}{2}+\int_1^{\infty}\frac{\big(u\sqrt{u^2-1}-u^2+(\beta_j-\beta_i)^2/r_{ji}^2\big)e^{-ik_Lur_{ji}}}{u(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)}du\Big]$$

$$-\frac{ik_L(\alpha_j-\alpha_i)}{\pi r}\int_1^{\infty}\frac{ue^{-ik_Lur_{ji}}\sqrt{u^2-1}}{(u^2-(\beta_j-\beta_i)^2/r_{ji}^2)^2}du-\frac{i}{\pi k_T^2}\Big[\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}\sqrt{k_L^2-u^2}du$$

$$-\int_0^{k_T}u\sin[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\sqrt{k_T^2-u^2}du-i\int_{k_L}^{\infty}u\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}\sqrt{u^2-k_L^2}du$$

$$+i\int_{k_L}^{\infty}u\sin[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}\sqrt{u^2-k_T^2}du\Big]+\frac{i}{\pi k_T^2}\Big(\Big(\int_0^{k_L}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}$$

$$+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}]du+\int_0^{k_L}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_L^2-u^2}e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}du+i\int_{k_L}^{k_T}u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}e^{-(\beta_j-\beta_i)\ sqrtk_T^2-u^2}du$$

$$-\int_{k_L}^{k_T}e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}\Big[i(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]+2u\sin[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}+(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{k_T^2-u^2}\Big]du\Big)$$

$$+\frac{1}{\pi k_T^2}\Big\{\int_{k_T}^{\infty}(e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}[(\beta_j-\beta_i)u^3\sin[u(\alpha_j-\alpha_i)]-2u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_T^2}-(\alpha_j-\alpha_i)u^2\cos[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_T^2}]$$

$$-\frac{k_L^2+k_T^2}{2}e^{-(\beta_j-\beta_i)u}\sin[u(\alpha_j-\alpha_i)])du.-\int_{k_T}^{\infty}u\sin[u(\alpha_j-\alpha_i)]\sqrt{u^2-k_L^2}e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du\Big\}$$

$$
+\frac{k_L^2+k_T^2}{\pi k_T^2}\frac{(\beta_j-\beta_i)\sin[k_T(\alpha_j-\alpha_i)]+(\alpha_j-\alpha_i)\cos[k_T(\alpha_j-\alpha_i)]}{r_{ji}^2}e^{-(\beta_j-\beta_i)k_T}\Bigg\}\sin(2\psi_j)+\Bigg\{\frac{k_L(\beta_j-\beta_i)}{2r_{ji}}[Y_1(k_Lr_{ji})+iJ_1(k_Lr_{ji})]+\frac{k_T(\beta_j-\beta_i)}{2r_{ji}}[Y_1(k_Tr_{ji})
$$

$$
+iJ_1(k_Tr_{ji})]-\frac{2}{\pi k_T^2}\left[\int_0^{k_L}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_L^2-u^2}}du+\int_{k_L}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_L^2}}du-\int_0^{k_T}u^2\cos[u(\alpha_j-\alpha_i)]e^{-i(\beta_j-\beta_i)\sqrt{k_T^2-u^2}}du\right.
$$

$$
\left.-\int_{k_T}^{\infty}u^2\cos[u(\alpha_j-\alpha_i)]e^{-(\beta_j-\beta_i)\sqrt{u^2-k_T^2}}du\right]\Bigg\}\cos(2\psi_j)
$$

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