International Journal of Solids and Structures 46 (2009) 2309–2319

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Elasto-thermodiffusive (ETNP) surface waves in semiconductor materials

J.N. Sharma[a,*], Naveen Thakur[a], Surinder Singh[b]

[a] Department of Applied Sciences, National Institute of Technology, Hamirpur, Himachal Pradesh 177 005, India
[b] Department of Physics, Guru Nanak Dev University, Amritsar 143 005, India

ARTICLE INFO

Article history:
Received 14 July 2008
Received in revised form 29 December 2008
Available online 29 January 2009

Keywords:
Semiconductor
Relaxation time
Electrons and holes
Surface waves
Germanium

ABSTRACT

The present article is devoted to investigate the propagation of elasto-thermodiffusive (ETNP) surface waves in a homogeneous isotropic, thermally conducting semiconductor material of half-space with relaxation of heat and charge carrier fields. The secular equation, a more general functional relation, that governs the propagation of elasto-thermodiffusive (ETNP) surface waves in homogeneous isotropic, thermoelastic semiconductor material halfspace with relaxation of heat and charge carrier fields has been derived by solving a system of coupled partial differential equations. A hybrid numerical technique consisting of Descartes algorithm for solving complex polynomial characteristic equation along with functional iteration scheme has been successfully used to solve the secular equation in order to obtain dispersion curves, attenuation coefficient and specific loss factor of energy dissipation for p-type germanium (Ge) semiconductor. Some particular forms of the general secular equation governing the propagation of elasto-thermodiffusive (ETN/ETP), thermoelastic (ET), elastodiffusive (EP/EN) and thermodiffusive (TP/TN) surface waves have been also deduced and discussed. In order to illustrate the analytical development, the numerical solution of the secular equation and other relevant relations under different situations is also carried out for Ge semiconductor materials to characterize the elasto-thermodiffusive (ETP) and thermodiffusive (TP) surface waves. The computer simulated results have been presented graphically in respect of the dispersion curves, attenuation coefficient and specific loss factor.

1. Introduction

The existence of Rayleigh waves was predicted by Lord Rayleigh (1885) in connection with the earth quake spectrum analysis. Rayleigh waves travel along the surface of the earth at about 10 times the speed of sound in air. It is well known that surface waves are not only of a pure acoustic or elastic character but they can also be coupled with other physical fields such as piezoelectric, magnetic, electric, diffusion, etc. Lockett (1958), Chadwick and Windle (1964) and Atkin and Chadwick (1981) studied the propagation of thermoelastic Rayleigh waves in the context of coupled thermoelasticity.

In order to eliminate the paradox of infinite velocity of heat propagation in classical theory of thermoelasticity different theories of non-classical thermoelasticity have been evolved, see Hetnarski and Ignaczak (1999). Some researchers namely Lord and Shulman (1967), Green and Lindsay (1972), Dhaliwal and Sherief (1980) and Chandrasekharaiah (1986) modified the Fourier law of heat conduction and constitutive relations so as to obtain a hyperbolic equation for heat conduction. These models include the time needed for the acceleration of heat flow and take into account the coupling between temperature and strain fields. These theories are also supported (Ackerman et al., 1966; Guyer and Krumhansal, 1966; Ackerman and Overtone, 1969) by experimental exhibition of the actual occurrence of finite velocity of heat propagation so-called 'second sound' though the frequency window of its existence is extremely small. Banerjee and Pao (1974) investigated the propagation of plane harmonic waves in infinitely extended anisotropic solids, taking into account the thermal relaxation time. It is also pertinent to mention here that the non-classical theories of thermoelasticity are inconsistent with the second law of thermodynamics as stated by, Ignaczak (2006) that the entropy inequality does not always hold in the 'generalized thermoelasticity'. An extensive study of wave propagation in heat conducting elastic solids has been carried out by some authors namely Nayfeh and Nasser (1971), Sharma (1986), Sharma and Singh (1985), Scott (1989), and Sharma et al. (2000) under the influence of thermal relaxation time in "infinite velocity" and "finite velocity" descriptions.

The interaction of elastic, thermal and diffusion of charge carrier's fields in semiconductors has been investigated after formulating the problem mathematically by Maruszewski (1986a,b, 1987a,b, 1989) and Many et al. (1965). The theory developed in these researches is phenomenological by its nature and its

* Corresponding author. Tel.: +91 1972 223296; fax: +91 1972 223834.
E-mail addresses: jns@nitham.ac.in (J.N. Sharma), rinkalthakur@yahoo.com (N. Thakur), surinder_s1951@yahoo.co.in (S. Singh).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2009.01.019

application led to phonons of mixed nature, which cannot be considered of a pure transversal, longitudinal or interface character. This provides a description of optical phonons in semiconductors of different kinds, achieving a relatively good coincidence with both experimental data and calculation base on microscopic (atomistic) approaches. In case of semiconductors the presence of coupled oscillations together with uncoupled one of pure mechanical nature was also found. It is also mentioned that in the uncoupled cases, the velocities for thermal fields are observed at very low temperatures, then, for diffusion, the velocities are observed in semiconductors at room temperatures dealing with charge carriers, see Maruszewski (1989). In order to explore the simultaneous interactions of elastic, thermal and diffusion of charge carrier's fields Maruszewski (1989) studied the propagation of thermodiffusive surface waves in semiconductor materials based on the phenomenological model developed by him that includes relaxation times of heat and charge carriers in addition to life times of the carriers. He also presented numerical solutions of his model under these specific situations. But his investigations finally limited to some special and particular situations and remained departed from the general solution of the said model thereby ignoring the presence of some of the interacting fields included in the basic governing equations at a time. Recently, Sharma and Thakur (2006) simplified the Maruszewski (1989) model of governing equations by introducing non-dimensional quantities and studied the propagation of plane harmonic elasto-thermodiffusive (ETNP) waves in semiconductor materials. Four coupled longitudinal waves, namely, the quasi-thermoelastic (QTE), quasi-elastodiffusive (QEN/QEP), quasi-thermodiffusive (QTN/QTP) and quasi-thermal (T-mode) in addition to decoupled shear waves are found to exit in an infinite semiconductor. Sharma et al. (2007) investigated the propagation characteristics of elasto-thermodiffusive (ETN) surface waves in semiconductor material half-space.

The engineering literature on heat waves suffer from a lack of observational data which could establish the applications in which they are important and the theoretical approximations appropriate to these applications. According to Achenbach (2005) unlike the hyperbolic solutions, the classical solution show no distinct wave front and temperature increase starts at the initial time, as expected. However, the difference in the predicted temperature between two theories is small and only apparent for a very small time scale (of the order of 100 ps). The selection of the theory for the time scales of interest can be done for convenience with no practical effect on calculated results. Likewise, the choice of a specific value for the heat propagation speed in the hyperbolic equation does not affect the results. However, from the practical point of view, the choice of a value of heat propagation speed equal to the speed of longitudinal waves in the hyperbolic formulation, presents some numerical advantages. Therefore, in case of applications to common materials it is necessary to do experiments with process times in the window 10^{-13}–10^{-8} s where hyperbolic phenomena and relaxation effects can be important. Ning et al. (2004) studied the characteristics of temperature field due to pulsed heat input based on non-Fourier heat conduction hypothesis. Noting that the relaxation time for ordinary materials is recognized to be very small, they observed that the non-Fourier effect would give some influence, especially for such phenomena with high rate of temperature change as picosecond or femtosecond pulsed laser heating. The temperature distribution in such cases is to be predicted not by the classical Fourier law but by the non-Fourier heat conduction theory. Although in engineering applications Fourier's law and diffusion give an easier and better description, but in cases where the relaxation and process times are comparable it would be desirable to allow for both diffusion and relaxation by adopting constitutive models which have both thermal conductivity and a relaxation time or relaxation spectra.

Keeping in view the above stated facts in the present communication, we propose to solve a boundary value problem dealing with surface wave propagation in thermoelastic semiconductor materials based on the governing equations derived by Maruszewski (1989) which includes both relaxation and diffusion processes. It pertinent to mention here that the Maruszewski's model did not receive much attention during the last one and half decade because of complex mathematical nature and non-availability of its solution governing the simultaneous interaction of all the involved coupled fields in compact and isolated mathematical conditions. A more general secular equation that governs the propagation of ETNP-surface waves in thermoelastic semiconductors has been obtained in closed and compact form through analytical treatment and method. Keeping in view the complicacies involved in this secular equation due to various interacting fields, a numerical technique has also been developed for its solution in order to extract information regarding the influence of coupling of various fields on the wave characteristics. Some special cases of surface wave propagation have also been deduced and discussed. In order to illustrate the analytical development, the numerical solution of secular equation governing ETP-surface waves in a relaxation type germanium (Ge) semiconductor is carried out by writing a FORTRAN code. The computer simulated results so obtained have been presented graphically.

2. Formulation of the problem

We consider a homogeneous isotropic, thermally conducting, elastic semiconductor medium initially under undeformed state and at uniform temperature T_0. We take the origin of coordinate system O x y z at any point 'O' on the top plane surface and z-axis pointing normally into the halfspace, which is thus represented by $z \geqslant 0$. We assume that the surface $z = 0$ is stress free, thermally insulated or isothermal and in addition there is no flow of electrons/holes across it or equilibrium state of charge carrier fields is set up on the surface. We choose x-axis along the direction of wave propagation in such a way so that all the particles on a line parallel to y-axis are equally displaced and hence all the field quantities are independent of y-coordinate. Further, the disturbance is assumed to be confined to the neighbourhood of the free surface and hence vanishes as $z \to \infty$. In linear theory of thermoelasticity for semiconductors, the governing field equations for temperature $T(x,z,t)$, displacement vector $\vec{u}(x,z,t) = (u, 0, w)$, electron and hole charge carrier fields $N(x,z,t)$ and $P(x,z,t)$, respectively, in the absence of body forces and heat sources, are given by Maruszewski (1989).

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} - \lambda^n\nabla N - \lambda^p\nabla P - \lambda^T\nabla T = \rho\ddot{\vec{u}}, \tag{1.1}$$

$$\begin{aligned} &K\nabla^2T + m^{nq}\nabla^2N + m^{pq}\nabla^2P \\ &\quad - \left(1 + t^Q\frac{\partial}{\partial t}\right)\left(\rho C_e\dot{T} + \rho T_0\alpha^n\dot{N} + \rho T_0\alpha^p\dot{P} + T_0\lambda^T\nabla\cdot\vec{u}\right) \\ &\quad - \rho\left(a_1^n\dot{N} + a_1^p\dot{P}\right) = \left[a_1^n\left(\frac{\rho}{t_n^+}\right)N + a_1^p\left(\frac{\rho}{t_p^+}\right)P\right], \end{aligned} \tag{1.2}$$

$$\begin{aligned} &\rho D^n\nabla^2N + m^{qn}\nabla^2T - \rho\left(1 - a_2^nT_0\alpha^n + t^n\frac{\partial}{\partial t}\right)\dot{N} \\ &\quad - a_2^n\left(\rho C_e\dot{T} + \rho T_0\alpha^p\dot{P} + T_0\lambda^T\nabla\cdot\vec{u}\right) \\ &\quad = -\left(1 + t^n\frac{\partial}{\partial t}\right)\left(\frac{\rho}{t_n^+}\right)N, \end{aligned} \tag{1.3}$$

$$\rho D^p\nabla^2 P + m^{qp}\nabla^2 T - \rho\left(1 - a_2^p T_0\alpha^p + t^p\frac{\partial}{\partial t}\right)\dot{P} - a_2^p\left(\rho C_e\dot{T} + \rho T_0\alpha^n\dot{N} + T_0\lambda^T\nabla\cdot\vec{u}\right) = -\left(1 + t^p\frac{\partial}{\partial t}\right)\left(\frac{\rho}{t_p^+}\right)P, \quad (1.4)$$

where the notations

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2},\quad a_1^n = \frac{a^{Qn}}{a^Q},\quad a_1^p = \frac{a^{Qp}}{a^Q},$$
$$a_2^n = \frac{a^{Qn}}{a^n},\quad a_2^p = \frac{a^{Qp}}{a^p},\quad P = p - p_0,\quad N = n - n_0,\quad \lambda^T = (3\lambda + 2\mu)\alpha_T,$$
$$T = T_1 - T_0 \quad (2)$$

are used. Here, λ, μ are Lame parameters; ρ is the density of the semiconductor; λ^n, λ^p are the elastodiffusive constants of electrons and holes; α_T is the coefficient of linear thermal expansion of the material; K is the thermal conductivity; α^p, α^n are thermodiffusive constants of holes and electrons; a^{Qn}, a^{Qp}, a^Q, a^n, a^p are the flux like constant; D^n, D^p are the diffusion coefficients of electron and holes. The quantities m^{nq}, m^{pq}, m^{qn}, m^{qp} are the Peltier–Seebeck–Dufour–Soret like constants; t^Q, $t^n t^p$ are, respectively, the relaxation times of heat, electron and hole fields; C_e is the specific heat; t_n^+, t_p^+ denote the life times of the carriers' fields; n, p and n_0, p_0 are the non-equilibrium, equilibrium values of electrons and holes concentrations, respectively. The comma notation is used for spatial derivatives and a superposed dot represents differentiation with respect to time. Here, in addition the field variables are also assumed to satisfy all restrictions as described by Maruszewski (1989).

2.1. Boundary conditions

The surface $z = 0$ of the half space is assumed to satisfy the following boundary conditions:

$$\lambda u_{,x} + (\lambda + 2\mu)w_{,z} - \lambda^T T - \lambda^n N - \lambda^p P = 0, \quad (3.1)$$

$$u_{,z} + w_{,x} = 0, \quad (3.2)$$

$$KT_{,z} + m^{nq}N_{,z} + m^{pq}P_{,z} + K^S T + \rho a_1^n s^n N + \rho a_1^p s^p P = 0, \quad (3.3)$$

$$m^{qn}T_{,z} + \rho D^n N_{,z} + a_2^n K^S T + \rho s^n\left(1 + t^n\frac{\partial}{\partial t}\right)N = 0, \quad (3.4)$$

$$m^{qp}T_{,z} + \rho D^p P_{,z} + a_2^p K^S T + \rho s^p\left(1 + t^p\frac{\partial}{\partial t}\right)P = 0. \quad (3.5)$$

Here, K^S and s^n, s^p are, respectively, the surface heat conduction coefficient and surface recombination velocities.

We define the quantities

$$x' = \frac{\omega^* x}{c_1},\quad z' = \frac{\omega^* z}{c_1},\quad t' = \omega^* t,\quad T' = \frac{T}{T_0},\quad P' = \frac{P}{p_0},$$
$$N' = \frac{N}{n_0},\quad u' = \frac{\rho\omega^* c_1}{\lambda^T T_0}u,\quad w' = \frac{\rho\omega^* c_1}{\lambda^T T_0}w\quad t^{Q'} = t^Q\omega*,$$
$$t^{p'} = t^p\omega^*,\quad t^{n'} = t^n\omega^*,\quad t_n^{+'} = t_n^+\omega*,\quad t_p^{+'} = t_p^+\omega^*,$$
$$\delta^2 = \frac{c_2^2}{c_1^2},\quad \epsilon_T = \frac{\lambda^{T^2}T_0}{\rho C_e(\lambda + 2\mu)}\quad \omega^* = \frac{C_e(\lambda + 2\mu)}{K},\quad c_1^2 = \frac{\lambda + 2\mu}{\rho},$$
$$c_2^2 = \frac{\mu}{\rho},\quad \chi = \frac{K}{\rho C_e},\quad \bar{\lambda}_n = \frac{\lambda^n n_0}{\lambda^T T_0},\quad \bar{\lambda}_p = \frac{\lambda^P p_0}{\lambda^T T_0},\quad \epsilon^{qn} = \frac{m^{qn}T_0}{\rho D^n n_0},$$
$$\epsilon^{qp} = \frac{m^{qp}T_0}{\rho D^p p_0},\quad \epsilon_n = \frac{a_2^n K T_0}{\rho n_0 D^n},\quad \epsilon_p = \frac{a_2^p K T_0}{\rho p_0 D^p},\quad \epsilon^{pq} = \frac{m^{pq}p_0}{K T_0},$$
$$\epsilon^{nq} = \frac{m^{nq}n_0}{K T_0},\quad a_0^n = \frac{a_1^n n_0}{C_e T_0},\quad a_0^p = \frac{a_1^p p_0}{C_e T_0},\quad \alpha_0^n = \frac{\alpha^n n_0}{C_e},\quad \alpha_0^p = \frac{\alpha^p p_0}{C_e} \quad (4)$$

Here, ε_T is thermoelastic coupling parameter, χ is the thermal diffusivity. Introducing the quantities (4) in Eq. (1), we obtain

$$\delta^2\nabla^2\vec{u} + (1 - \delta^2)\nabla\nabla.\vec{u} - \ddot{\vec{u}} - \bar{\lambda}_n\nabla N - \bar{\lambda}_p\nabla P - \nabla T = 0, \quad (5.1)$$

$$\nabla^2 T - \left(\dot{T} + t^Q\ddot{T}\right) + \varepsilon^{nq}\nabla^2 N - \left((a_0^n + \alpha_0^n)\dot{N} + t^Q\alpha_0^n\ddot{N} + \frac{a_0^n}{t_n^+}N\right) + \varepsilon^{pq}\nabla^2 P_{-}\left((a_0^p + \alpha_0^p)\dot{P} + t^Q\alpha_0^p\ddot{P} + \frac{a_0^p}{t_p^+}P\right) - \epsilon_T\nabla\left(\dot{\vec{u}} + t^Q\ddot{\vec{u}}\right) = 0, \quad (5.2)$$

$$\nabla^2 N + \frac{\chi}{D^n}\left[\frac{1}{t_n^+}N - \left(1 - \frac{\epsilon_n\alpha_0^n D^n}{\chi} - \frac{t^n}{t_n^+}\right)\dot{N} - t^n\ddot{N}\right] - \epsilon_n\dot{T} + \varepsilon^{qn}\nabla^2 T - \epsilon_n\alpha_0^p\dot{P} - \epsilon_n\epsilon_T\nabla\cdot\dot{\vec{u}} = 0, \quad (5.3)$$

$$\nabla^2 P + \frac{\chi}{D^p}\left[\frac{1}{t_p^+}P - \left(1 - \frac{\epsilon_p\alpha_0^p D^p}{\chi} - \frac{t^p}{t_p^+}\right)\dot{P} - t^p\ddot{P}\right] - \varepsilon_p\dot{T} + \varepsilon^{qp}\nabla^2 T - \varepsilon_p\alpha_0^n\dot{N} - \varepsilon_p\varepsilon_T\nabla\cdot\dot{\vec{u}} = 0, \quad (5.4)$$

where $\vec{u} = (u, 0, w)$ is the non-dimensional displacement vector. The boundary conditions (3) at the surface $z = 0$, takes the following non-dimensional form:

$$(1 - 2\delta^2)u_{,x} + w_{,z} - \overline{\lambda_n}N - \overline{\lambda_p}P - T = 0, \quad (6.1)$$

$$u_{,z} + w_{,x} = 0, \quad (6.2)$$

$$T_{,3} + h_T T + \varepsilon^{nq}(N_{,3} + h_n N) + \varepsilon^{pq}\left(P_{,3} + h_p P\right) = 0, \quad (6.3)$$

$$\varepsilon^{qn}\left(T_{,3} + h_T\frac{\varepsilon_n}{\varepsilon^{qn}}T\right) + N_{,3} + h_n\bar{\varepsilon}_{nq}(N + t^n\dot{N}) = 0, \quad (6.4)$$

$$\varepsilon^{qp}\left(T_{,3} + h_T\frac{\varepsilon_p}{\varepsilon^{qp}}T\right) + P_{,3} + h_p\bar{\varepsilon}_{pq}(P + t^p\dot{P}) = 0, \quad (6.5)$$

where $h_T = \frac{K^S c_1}{K\omega*}$, $h_n = \frac{a_0^n s^n}{\varepsilon^{nq}c_1}$, $h_p = \frac{a_0^p s^p}{\varepsilon^{pq}c_1}$, $\bar{\varepsilon}_{nq} = \frac{m^{nq}}{\rho D^n a_1^n}$, and $\bar{\varepsilon}_{pq} = \frac{m^{pq}}{\rho D^p a_1^p}$.

3. Solution of the problem

In order to solve the problem, we use Helmholtz decomposition theorem to express the displacement vector as $\vec{u} = \nabla\phi + \nabla\times\vec{\psi}$, $\nabla\cdot\vec{\psi} = 0$ so that the displacement components are written as

$$u = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial z},\quad w = \frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}, \quad (7)$$

where the vector point potential function is defined as $\vec{\psi} = (0, -\psi, 0)$.

Upon introducing Eq. (7) in Eqs. (5.1)–(5.4), we obtain

$$\nabla^2\phi - \ddot{\phi} - \overline{\lambda_n}N - \overline{\lambda_p}P - T = 0, \quad (8.1)$$

$$-\varepsilon_T\nabla^2\left(\dot{\phi} + t^Q\ddot{\phi}\right) + \varepsilon^{nq}\nabla^2 N - \left\{\frac{\alpha_0^n t^Q\partial^2}{\partial t^2} + \left(a_0^n + \alpha_0^n\right)\frac{\partial}{\partial t} + \frac{a_0^n}{t_n^+}\right\}N + \varepsilon^{pq}\nabla^2 P - \left\{\frac{\alpha_0^p t^Q\partial^2}{\partial t^2} + \left(a_0^p + \alpha_0^p\right)\frac{\partial}{\partial t} + \frac{a_0^p}{t_p^+}\right\}P + \nabla^2 T - \left(\dot{T} + t^Q\ddot{T}\right) = 0, \quad (8.2)$$

$$- \varepsilon_n \varepsilon_T \nabla^2 \dot{\phi} + \nabla^2 N - \frac{\chi}{D^n}\left[-\frac{1}{t_n^+} + \left(1 - \frac{\in_n \alpha_0^n D^n}{\chi} - \frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} + \frac{t^n \partial^2}{\partial t^2}\right] N - \varepsilon_n \alpha_0^p \dot{P} - \varepsilon_n \dot{T} + \varepsilon^{qn} \nabla^2 T = 0, \quad (8.3)$$

$$- \varepsilon_p \varepsilon_T \nabla^2 \dot{\phi} + \nabla^2 P - \frac{\chi}{D^p}\left[-\frac{1}{t_p^+} + \left(1 - \frac{\in_p \alpha_0^p D^p}{\chi} - \frac{t^p}{t_p^+}\right)\frac{\partial}{\partial t} + \frac{t^p \partial^2}{\partial t^2}\right] P - \varepsilon_p \alpha_0^n \dot{N} - \varepsilon_p \dot{T} + \varepsilon^{qp} \nabla^2 T = 0, \quad (8.4)$$

$$\nabla^2 \psi = \frac{1}{\delta^2}\ddot{\psi}. \quad (8.5)$$

Eqs. (8.1) and (8.4) in the above system can be simplified under the assumption that the considered semiconductor is of relaxation type because for such materials the diffusion approximation of the physical process ceases to be obligatory and the diffusion/life times $t^n, t_n^+ (t^p, t_p^+)$, become comparable to each other in their values $(t^n = t_n^+, t^p = t_p^+)$, see Maruszewski (1989).

We consider the case of time harmonic waves so that the solutions ϕ, T, N, P and ψ, of Eq. (8) take the form of

$$(\phi, T, N, P, \psi) = [\bar{\phi}(z), \overline{T}(z), \bar{N}(z), \overline{P}(z), \bar{\psi}(z)] \exp\{ik(x - ct)\} \quad (9)$$

where $c = \frac{\omega}{k}$ is the phase velocity, k and ω are wave number and angular frequency of the waves, respectively. Substitution of solutions (9) in Eqs. (8.1)–(8.4) led to a coupled system of four equations in terms of $(\bar{\phi}, \overline{T}, \overline{N}, \overline{P})$. The requirement of the existence of non-trivial solution of this system provides us a quartic polynomial characteristic equation in m^2, which give us four pairs of the characteristic roots $\pm m_i$ $(i = 1,2,3,4)$. In general, the characteristic roots $\pm m_i$ $(i = 1,2,3,4)$ are complex and therefore, the solution is a superposition of the plane waves attenuating with depth. As we are considering surface waves only, so without loss of generality, we choose only that form of m_i which satisfies the radiation condition viz. $R_e(m_i) \geqslant 0$. After lengthy but straight forward algebraic reductions and simplifications, we obtain the following formal solution for the functions (ϕ, N, P, T, ψ) that satisfy the radiation condition $R_e(m_i) \geqslant 0$. We have

$$(\phi, T, N, P) = \sum_{i=1}^{4}(1, W_i, S_i, V_i,)A_i \exp\{-m_i z + ik(x - ct)\}, \quad (10.1)$$

$$\psi = A_5 \exp\{-\beta z + ik(x - ct)\}, \quad (10.2)$$

where

$$S_i = \frac{(m_i^2 - \alpha^2)F_1^p - \bar{\lambda}_p F_2^n + F_2^p}{\bar{\lambda}_n F_1^p - \bar{\lambda}_p F_1^n + F_1^{nq}}, \quad (11.1)$$

$$V_i = \frac{\bar{\lambda}_n F_2^n - (m_i^2 - \alpha^2)F_1^n + F_3^n}{\bar{\lambda}_n F_1^p - \bar{\lambda}_p F_1^n + F_1^{nq}}, \quad (11.2)$$

$$W_i = \frac{\bar{\lambda}_n F_3^p - \bar{\lambda}_p F_3^n + (m_i^2 - \alpha^2)F_1^{nq}}{\bar{\lambda}_n F_1^p - \bar{\lambda}_p F_1^n + F_1^{nq}}, \quad (11.3)$$

$$F_1^n = \varepsilon^{nq}\varepsilon^{qn}(m_i^2 - \alpha_n^2)(m_i^2 - \beta_1^2) - \left(m_i^2 - \alpha_Q^2\right)(m_i^2 - \alpha_n^{*2}), \quad (12.1)$$

$$F_1^p = \varepsilon^{pq}\varepsilon^{qn}(m_i^2 - \alpha_p^2)(m_i^2 - \beta_1^2) - \left(m_i^2 - \alpha_Q^2\right)(i\omega\varepsilon_n\alpha_0^p), \quad (12.2)$$

$$F_2^n = i\omega\varepsilon_T(m_i^2 - k^2)\left[\varepsilon_n\left(m_i^2 - \alpha_Q^2\right) - i\omega\tau^Q\varepsilon^{qn}(m_i^2 - \beta_1^2)\right], \quad (12.3)$$

$$F_2^p = i\omega\varepsilon_n\varepsilon_T(m_i^2 - k^2)\left[\varepsilon^{pq}(m_i^2 - \alpha_p^2) - \omega^2\tau^Q\alpha_0^p\right], \quad (12.4)$$

$$F_1^{nq} = \varepsilon^{nq}(m_i^2 - \alpha_n^2)(i\omega\varepsilon_n\alpha_0^p) - \varepsilon^{pq}(m_i^2 - \alpha_n^{*2})(m_i^2 - \alpha_p^2) \quad (12.5)$$

$$F_3^n = -i\omega\varepsilon_T(m_i^2 - k^2)\left[i\omega\tau^Q(m_i^2 - \alpha_n^{*2}) + \varepsilon_n\varepsilon^{nq}(m_i^2 - \alpha_n^2)\right], \quad (12.6)$$

$$F_3^p = -i\omega\varepsilon_n\varepsilon_T(m_i^2 - k^2)\left[\varepsilon^{pq}(m_i^2 - \alpha_p^2) - \omega^2\tau^Q\alpha_0^p\right]. \quad (12.7)$$

Here, we have defined the quantities

$$\alpha^2 = k^2(1 - c^2), \quad \alpha_n^2 = k^2\left(1 - \frac{\tau_n' c^2}{\varepsilon^{nq}}\right),$$
$$\alpha_p^2 = k^2\left(1 - \frac{\tau_p' c^2}{\varepsilon^{pq}}\right), \quad \alpha_n^{*2} = k^2(1 - \tau_n^* c^2),$$
$$\alpha_p^{*2} = k^2(1 - \tau_p^* c^2), \quad \alpha_Q^2 = k^2(1 - \tau^Q c^2),$$
$$\beta^2 = k^2\left(1 - \frac{c^2}{\delta^2}\right), \quad \beta_1^2 = k^2\left(1 - \frac{i\omega^{-1}\varepsilon_n c^2}{\varepsilon^{qn}}\right),$$
$$m_i^2 = k^2(1 - a_i^2 c^2), \qquad i = 1,2,3,4, \quad (13)$$

and $\tau_n', \tau_n^*, \tau_p^*$ and τ^Q are defined in Appendix A.

The quantities a_i^2, $i = 1,2,3,4$ are the roots of complex biquadratic equation

$$a^8 - Aa^6 + Ba^4 - Ca^2 + D = 0, \quad (14)$$

where

$$A = \sum_{i=1}^{3}\frac{J_i}{J}, \quad B = \sum_{i=1}^{4}\frac{J_i'}{J}, \quad C = \sum_{i=1}^{3}\frac{J_i''}{J}, \quad D = \frac{J^*}{J}. \quad (15)$$

The quantities J, J^*, J_i, J_i' $(i = 1,2,3,4)$ and J_i'' $(i = 1,2,3)$ are defined in Appendix A. Upon inserting solutions for ϕ and φ from (10) in Eq. (7), the displacement components are obtained as

$$u = \left(\sum_{j=1}^{4} ikA_j e^{-m_j z} - \beta A_5 e^{-\beta z}\right)e^{ik(x-ct)}, \quad (16)$$

$$w = -\left(\sum_{j=1}^{4} m_j A_j e^{-m_j z} + ikA_5 e^{-\beta z}\right)e^{ik(x-ct)}. \quad (17)$$

Clearly, the displacements get modified due to the characteristic roots corresponding to thermal and diffusion field equations because of coupling among interacting fields in addition to relaxation and life time effects. The stresses can also be obtained in similar manner.

4. Derivation of secular equation

Upon invoking the boundary conditions (6) at the surface $z = 0$, we obtain a system of five simultaneous linear equations in amplitudes A_j, $j = 1,2,3,4,5$. This system of equations provides us a non-trivial solution, if the determinant of the coefficients of unknowns A_j, $j = 1,2,3,4,5$ vanishes. This, after applying lengthy algebraic reductions and simplifications, led to the following secular equation for the propagation of Rayleigh type surface waves in the thermoelastic semiconductor material half space. We obtain

$$(\beta^2 + k^2)^2[L_1 - L_2 + L_3 - L_4] = 4k^2\beta(m_1L_1 - m_2L_2 + m_3L_3 - m_4L_4), \quad (18)$$

where

$$L_1 = P_2(Q_3R_4 - Q_4R_3) - P_3(Q_2R_4 - Q_4R_2) + P_4(Q_2R_3 - Q_3R_2), \quad (19)$$

$$P_i = \left[(h_T - m_i)W_i + \varepsilon^{nq}(h_n - m_i)S_i + \varepsilon^{pq}(h_p - m_i)V_i\right], \quad i = 1,2,3,4, \quad (20.1)$$

$$Q_i = \left[(h_T\varepsilon_n - m_i\varepsilon^{qn})W_i + \left(h_n\bar{\varepsilon}'_{nq} - m_i\right)S_i\right], \quad i = 1,2,3,4, \tag{20.2}$$

$$R_i = \left[(h_T\varepsilon_p - m_i\varepsilon^{qp})W_i + \left(h_p\bar{\varepsilon}'_{pq} - m_i\right)V_i\right], \quad i = 1,2,3,4, \tag{20.3}$$

$$\begin{aligned}\bar{\varepsilon}'_{nq} &= -i\omega\bar{\varepsilon}_{nq}\tau_1^n,\\ \bar{\varepsilon}'_{pq} &= -i\omega\bar{\varepsilon}_{pq}\tau_1^p.\end{aligned} \tag{21}$$

Here, L_2, L_3, L_4 can be obtained from L_1 defined in Eq. (19) by replacing the subscripts permutation (2,3,4) with (1,3,4), (1,2,4) and (1,2,3), respectively. The secular equation (18) contains complete information about the phase velocity, wave number and attenuation coefficient of the (ETNP) surface waves in a thermoelastic semiconductor halfspace. The secular equations in case of (i) isothermal and isoconcentrated, (ii) isothermal and impermeable, (iii) thermally insulated and isoconcentrated and (iv) thermally insulated and impermeable surface conditions prevailing at the boundary of the semiconductor half-space can be written from the secular equation (18) by setting ($h_T = 0, h_n = 0, = h_p$), ($h_T = 0, h_n, h_p \to \infty$), ($h_T \to \infty, h_n = 0, = h_p$) and ($h_T \to \infty h_n, h_p \to \infty$), respectively, in Eq. (20). The secular equations for ETN-waves in n-type semiconductor and ETP-waves in p-type semiconductor can be written from the secular equation (18) by setting ($P = 0, {\alpha_0}^p = 0 = \varepsilon_p, \varepsilon^{qp} = 0$) and ($N = 0, {\alpha_0}^n = 0 = \varepsilon_n, \varepsilon^{qn} = 0$), respectively.

5. Special cases of surface wave

This section is devoted to the reductions and deductions of the secular equation (18) in different conditions and under various situations to which semiconductor halfspace has been subjected.

5.1. ET-surface waves

Let us now consider the case of ET surface wave propagation. When complete equilibrium state of electron and hole concentration is established, the system becomes charge free. The thermoelastic (ET) waves concern with the reciprocal dynamical interactions of the elastic and thermal fields when the electron and hole fields are omitted so that we have $N = 0 = P$, $\varepsilon_n = \varepsilon_p = 0$, $\varepsilon^{qn} = 0 = \varepsilon^{qp} = \overline{\lambda_n} = 0 = \overline{\lambda_p}$.

The secular equation (18) governing the interaction in this case reduces to

$$\begin{aligned}&(\beta^2 + k^2)^2[m_1^2 + m_1m_2 + m_2^2 - \alpha^2] - 4k^2\beta m_1m_2(m_1 + m_2)\\ &\quad = h_T\left[(\beta^2 + k^2)^2(m_1 + m_2) - 4k^2\beta(m_1m_2 + \alpha^2)\right].\end{aligned} \tag{22}$$

Eq. (22) is the same as obtained and discussed by various researchers such as Chadwick and Windle (1964), Atkin and Chadwick (1981) and Lockett (1958) in case of couple thermoelasticity and Nayfeh and Nasser (1971) in the context of generalized thermoelasticity.

5.2. Elastodiffusive (EN/EP) surface waves

If we confine our discussion to the propagation of EN waves concerning the reciprocal dynamical interactions of the elastic and electron diffusion fields and omit the thermal and hole fields ($P = T = 0, \varepsilon_T = 0 = \varepsilon^{nq}, \alpha_0^n = 0 = a_0^n$), the secular equation (18) governing the interaction becomes

$$\begin{aligned}&(\beta^2 + k^2)^2[m_1^2 + m_1m_3 + m_3^2 - \alpha^2] - 4\beta k^2m_1m_3(m_1 + m_3)\\ &\quad = h_n\bar{\varepsilon}'_{nq}\left[(\beta^2 + k^2)^2(m_1 + m_3) - 4\beta k^2(m_1m_3 + \alpha^2)\right].\end{aligned} \tag{23}$$

Eq. (23) can also be discussed on parallel lines as Eq. (22) and has already been investigated by Maruszewski (1989) in an alternative form. It seems that the electron near the surface has, in principle, similar conductivity properties like that of heat when the surface of the body is covered by an infinite thin film and hence the elasto-electron coupling gives almost same type of results.

In case of EP waves concerning the reciprocal dynamical interaction of the elastic and hole-diffusion fields, the influence of thermal and electron fields is omitted ($N = T = 0, \varepsilon_T = 0 = \varepsilon^{pq}, \alpha_0^p = 0 = a_0^p$). The secular equation for EP surface wave can be obtained in Section 5.1 similar manner from Eq. (23) by replacing N with P; n with p; and m_3 with m_4.

5.3. Thermodiffusive (TN/TP) waves

If we confine our discussion to the propagation of TN-waves concerning the reciprocal dynamical interactions of the thermal and electron diffusion fields and omit the elastic and hole fields ($\phi = 0 = P, \alpha_0^p = 0 = \varepsilon_p, \varepsilon^{qp} = 0 = \varepsilon_T, \psi = 0$). Then the secular equation (18) governing the interaction in this case becomes

$$\begin{aligned}&\left[\varepsilon^{nq}(m_2 - h_T)(m_2^2 - \alpha_n^2) - \varepsilon^{nq}(m_2^2 - \alpha_Q^2)(m_2 - h_n)\right]\\ &\quad\left[(\varepsilon^{qn}m_3 - h_T\varepsilon_n)(m_3^2 - \alpha_n^{*2}) - \varepsilon^{qn}\left(m_3 - h_n\bar{\varepsilon}'_{nq}\right)(m_3^2 - \beta_1^2)\right]\\ &\quad - \left[(m_3^2 - \alpha_n^{*2})(m_3 - h_T) - (m_3 - h_n)(m_3^2 - \beta_1^2)\varepsilon^{nq}\varepsilon^{qn}\right]\\ &\quad\left[(m_2^2 - \alpha_n^2)(\varepsilon^{qn}m_2 - h_T\varepsilon_n)\varepsilon^{nq} - \left(m_2 - h_n\bar{\varepsilon}'_{nq}\right)\left(m_2^2 - \alpha_Q^2\right)\right] = 0.\end{aligned} \tag{24}$$

Eq. (24) has been obtained and discussed by Maruszewski (1989) in the dimensional form. Similarly, in case of TP-waves, the influence of elastic and electron fields is omitted ($\phi = 0 = N, {\alpha_0}^n = 0 = \varepsilon_n, \varepsilon^{qn} = 0 = \varepsilon_T, \psi = 0$). The secular equation can be written from Eq. (24) by replacing N with P; n with p; and m_3 with m_4.

6. Solution of the secular equation

In general, wave number and hence the phase velocities of the waves are complex quantities, therefore the waves are attenuated in space. In order to solve the secular equation, we take

$$c^{-1} = V^{-1} + i\omega^{-1}Q, \tag{25}$$

where $k = R + iQ$, $R = \frac{\omega}{V}$ and R, Q are real numbers. Here, it may be noted that V and Q, respectively, represent the phase velocity and attenuation coefficient of the waves. Upon using representation (25) in secular equation (18) and various relevant relations, the complex roots a_i^2 $(i = 1,2,3,4)$ of quadratic equation (14) can be computed with the help of Descartes procedure. These are then used to obtain the complex characteristic roots m_i^2 $(i = 1,2,3,4)$ from Eq. (13). The characteristic roots m_i^2 $(i = 1,2,3,4)$ are further used to solve the secular equation (18) to obtain phase velocity (V) and attenuation coefficient (Q) of the surface waves by using function iteration numerical technique outlined below.

The secular equation (18) is, in general, of the form $F(c) = 0$ which upon using representation (25) leads to a system of two real equations $f(V,Q) = 0$ and $g(V,Q) = 0$. In order to apply functional iteration method we write $V = f^*(V,Q)$ and $Q = g^*(V,Q)$, where the functions f^* and g^* are selected in such a way that they satisfy the conditions

$$\left|\frac{\partial f^*}{\partial V}\right| + \left|\frac{\partial f^*}{\partial Q}\right| < 1, \quad \left|\frac{\partial g^*}{\partial V}\right| + \left|\frac{\partial g^*}{\partial Q}\right| < 1 \tag{26}$$

for all V, Q in the neighbourhood of the root. If (V_0, Q_0) be an initial approximation to the root, then we can construct the successive approximations according to the formulae

$$
\begin{array}{ll}
V_1 = f^*(V_0, Q_0) & Q_1 = g^*(V_1, Q_0) \\
V_2 = f^*(V_1, Q_1) & Q_2 = g^*(V_2, Q_1) \\
\vdots \quad \vdots \quad \vdots & \\
V_{n+1} = f^*(V_n, Q_n) & Q_{n+1} = g^*(V_{n+1}, Q_n)
\end{array} \tag{27}
$$

The sequence $\{V_n, Q_n\}$ of approximations to the root will converge to the actual value (V_0, Q_0) of the root provided (V_0, Q_0) lies in the neighbourhood of the actual root. For the initial value of $c = c_0 = (V_0, Q_0)$, the roots m_j $(j = 1,2,3,4)$ are computed from Eqs. (13) and (14) by using Descartes' procedure for each value of the wave number R for assigned frequency. The values of m_j $(j = 1,2,3,4)$ so obtained are then used in secular equation (18) to obtain the current values of V and Q each time which are further used to generate the sequence (27). The process is terminated as and when the condition $|V_{n+1} - V_n| < \varepsilon$, ε being arbitrarily small number to be selected at random to achieve the accuracy level, is satisfied. The procedure is continuously repeated for different values of the wave number (R) to obtain corresponding values of the phase velocity (V) and attenuation coefficient (Q). Thus the real phase velocity and attenuation coefficient during the propagation of Rayleigh type disturbance in the semiconductor thermoelastic half space can be computed from dispersion relation (18).

6.1. Specific loss

The energy dissipated (ΔW) in a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is a maximum, is called specific loss. According to Kolsky (1963) in case of sinusoidal plane wave of small amplitude, the specific loss $\frac{\Delta W}{W}$ equals to times the absolute value of the imaginary part of k to the real part of k, i.e. $\frac{\Delta W}{W} = 4\pi\left|\frac{Im(k)}{Re(k)}\right|$, where k is a complex number such that $Im(k) > 0$. Here,

$$
\frac{\Delta W}{W} = 4\pi\left|\frac{Im(k)}{Re(k)}\right| = 4\pi\left|\frac{Q}{R}\right| = 4\pi\left|\frac{VQ}{\omega}\right|. \tag{28}
$$

7. Numerical results and discussion

In order to illustrate the analytical development in the previous section, we now perform some numerical computations and simulations. Here, we make assumption that the semiconductor is of the relaxation type so that the diffusion approximation of the physical processes ceases to be obligatory and t_n, t_n^+, t_p, t_p^+ turn comparable to each other in their values meaning that $t_n = t_n^+, t_p = t_p^+$. Here, we confine ourselves to discuss the phase velocity, attenuation, and specific loss profiles for ETP surface waves at various situations. The semiconductor material for the numerical purpose is taken as Ge whose physical data as reported by Maruszewski (1989) and Sharma and Thakur (2006) is given as under

$$
\begin{aligned}
&\lambda = 0.48 \times 10^{11}\ \mathrm{N\,m^{-2}}, \quad \mu = 0.53 \times 10^{11}\ \mathrm{N\,m^{-2}}, \\
&\rho = 5.3 \times 10^3\ \mathrm{kg\,m^{-3}}, \quad t_p^+ =< 10^{-5}\ \mathrm{s}, \quad D^p = 0.5 \times 10^{-2}\ \mathrm{m^2\,s^{-1}}, \\
&m^{qp} = -0.004 \times 10^{-6}\ \mathrm{v\,k^{-1}}, \quad K = 60\ \mathrm{W\,m^{-1}\,K^{-1}}, \\
&C_e = 310\ \mathrm{J\,kg^{-1}\,K^{-1}}, \quad \alpha^p = 1.3 \times 10^{-3}\ \mathrm{m^2/s}, \\
&\alpha_T = 5.8 \times 10^{-6}\ \mathrm{K^{-1}}, \quad n_0 = 10^{20}\ \mathrm{m^{-3}}, \\
&m^{pq} = -0.004 \times 10^{-6}\ \mathrm{v\,k^{-1}}, \quad T_0 = 298\ \mathrm{K}.
\end{aligned}
$$

The numerical calculations have been done for different nondimensional values of the life time $t_p^+ = 0.796, 0.0796, 0.00796$ which correspond to their respective dimensional values $t_p^+ = 10^{-12}$ s, 10^{-13} s, 10^{-14} s.

Fig. 1 shows the plots of variations of the phase velocity (V) of ETP surface waves with wave number (R) for different values of life times of hole carrier fields in Ge semiconductor material halfspace under isothermal surface conditions. The phase velocity profiles are observed to increase with decreasing values of life time of hole carrier field. The magnitude of phase velocity profile corresponding to $t_p^+ = 10^{-14}$ s has the high large magnitude as compared to that of other considered cases. The phase velocity profiles are noticed to be dispersive in character with decreasing life time of charge carrier fields, however, the variation of phase velocity becomes stable for higher values of the wave number. Fig. 2 concerns with the variations of attenuation (Q) with wave number (R) in Ge halfspace under isothermal conditions. The attenuation profile corresponding to the life time $(t_p^+ = 10^{-13}$ s$)$ of charge carriers observes Gaussian behaviour and the profiles corresponding to life times $(t_p^+ = 10^{-14}$ s and $t_p^+ = 10^{-15}$ s$)$ follow platykurtic trends with increasing wave number. The attenuation profiles suffer significant dispersion with decreasing life time of hole carrier fields. This behaviour of phase velocity and attenuation profiles in Figs. 1 and 2 shows that ETP surface waves are quite sensitive to life time of charge carriers' field. Fig. 3 concerns with the variation of non-dimensional phase velocity (V) of ETP surface waves with wave number (R) for different life times for insulated surface conditions prevailing at the surface of the halfspace. It is observed that the non-dimensional phase

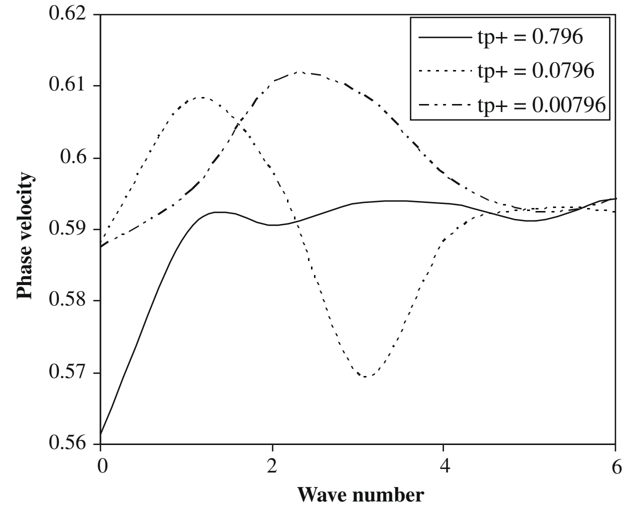

Fig. 1. Phase velocity profile of ETP isothermal waves with life time effect of charge carriers.

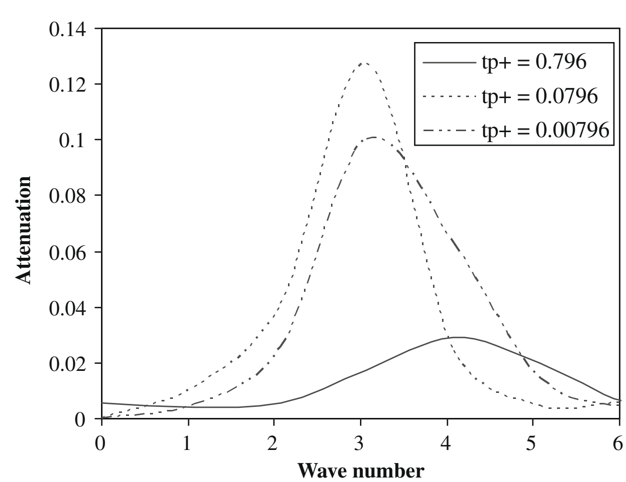

Fig. 2. Attenuation coefficient profile of ETP isothermal wave with life time effect of charge carriers.

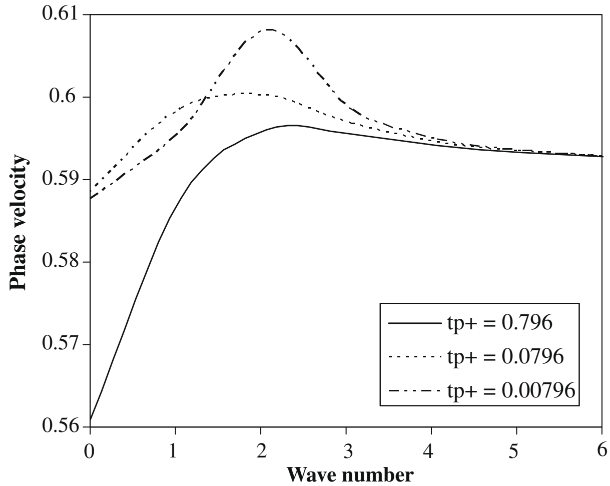

Fig. 3. Phase velocity profile of ETP insulated waves with life time effect of charge carriers.

velocity increases in wave number range $R \leqslant 2$ and then decreases. After wave number range $R \leqslant 4$, all the profiles corresponding to different life times asymptotically converge to a single value. It is noticed that at low wave number ranges the effect of life time is prominent whereas there is no effect at higher wave numbers. Fig. 4 shows the variation of non-dimensional attenuation (Q) of ETP surface waves with non-dimensional wave number (R) under insulated surface conditions for hole field prevailing at the surface of the material. The attenuation profiles corresponding to life times $t_p^+ = 10^{-14}$ s and $t_p^+ = 10^{-13}$ s, show minimum variation with respect to wave number. Initially, these profiles increase steadily in order to ultimately become constant. The attenuation profile corresponding to life time $t_p^+ = 10^{-12}$ s follows Gaussian distribution in the wave number range $1 \leqslant R \leqslant 4$ with slight dispersion outside this interval. These profiles established the sensitivity of phase velocity and attenuation towards life time of charge carrier fields.

From Figs. 1–4, it is inferred that the phase velocity and attenuation profiles are noticed to be distinctive and significantly affected due to life times at long wave lengths as compared to that at short wave lengths. This is attributed to the fact that long wavelength waves are capable of penetrating deep into the halfspace due to which the interaction between various interacting fields is set up and coupling between them becomes operative which results in significant changes in the wave characteristics. However, at the short wavelengths the waves follow the surface of the semiconductor halfspace with least disturbance to the medium and consequently coupling of various interacting fields has negligible small effect on wave characteristics at all values of life times of carrier fields. Moreover, the significant effect of thermally insulated and isothermal boundaries of the semiconductor halfspace on these wave characteristics is also visible from their plots.

It is observed from Figs. 5 and 6 that the thermal relaxation time has significant effect on the phase velocity and attenuation coefficient of ETP surface waves in case of isothermal and insulated surface conditions. The effect of thermal relaxation is observed to be comparatively low at large wave numbers rather than at small one on these quantities. The phase velocity increases due to thermal relaxation time in the wave number range $0 \leqslant R \leqslant 2$ and suffers a decrease in amplitude for $3 \leqslant R \leqslant 7$ before its trend gets reversed and ultimately zigzag type behaviour is observed by it. The attenuation profile achieves peak value at wave number $R = 3$ in case of insulated surface conditions and at wave number value $R = 6$ for isothermal condition before it starts decreasing in magnitude. These profiles exhibit a prominent effect of heat relaxation time.

Fig. 7 presents the variations of specific loss factor of energy dissipation in a stress cycle when the specimen is subjected to maximum strain with respect to wave number for two considered life times $t_p^+ = 10^{-12}$ s,10^{-13} s at fixed value of thermal relaxation time. The magnitude of specific loss factor is noticed to be significantly large at vanishing wave numbers and it sharply decreases in the range $0 \leqslant R \leqslant 1.5$ before it starts decreasing steadily for $R \geqslant 2$ after suffering a slight increase for $1.5 \leqslant R \leqslant 2$ in order to become asymptotically convergent for$R \geqslant 6$. Thus the material exhibit maximum internal friction at long wavelength limits $0 \leqslant R \leqslant 1$ as observed from the profiles. The variations of specific loss factor with respect to wave number for two considered values of non-dimensional thermal relaxation time $t^Q = 0.2, 0.4$ at fixed value of life time of charge carries are plotted in Fig. 8. It is noticed that the profiles of this quantity follow Gaussian behaviour with mean at $R = 2$ and $R = 3$ in case of $t^Q = 0.2$ and $t^Q = 0.4$ in the wave number ranges $1 \leqslant R \leqslant 3.5$ and $1 \leqslant R \leqslant 5.5$, respectively. These profiles follow sharply decreasing trend in the range $0 \leqslant R \leqslant 1$ but decrease steadily in the wave number range $R \geqslant 5.5$. The comparison of profiles in Figs. 7 and 8 suggests that at long wavelengths $(0 \leqslant R \leqslant 1)$

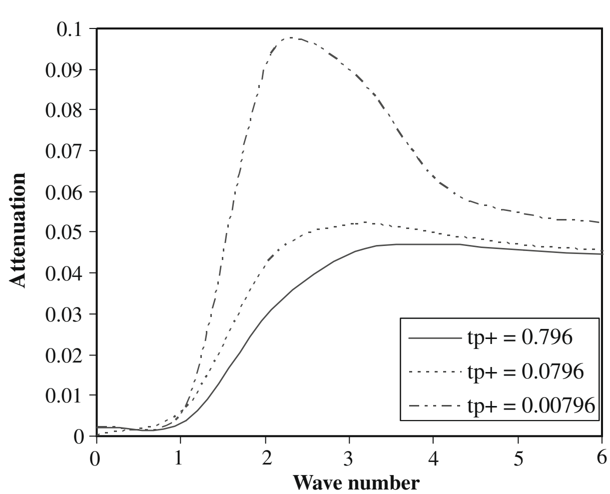

Fig. 4. Attenuation coefficient profile of ETP insulated wave with life time effect of charge carriers.

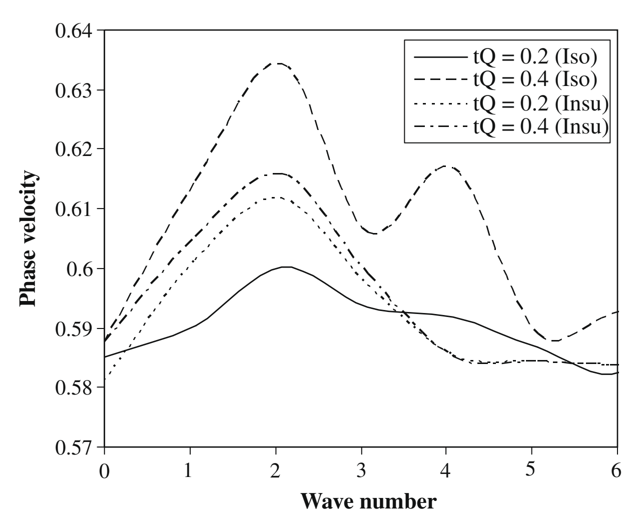

Fig. 5. Phase velocity profile of ETP isothermal waves/insulated wave with heat relaxation effect.

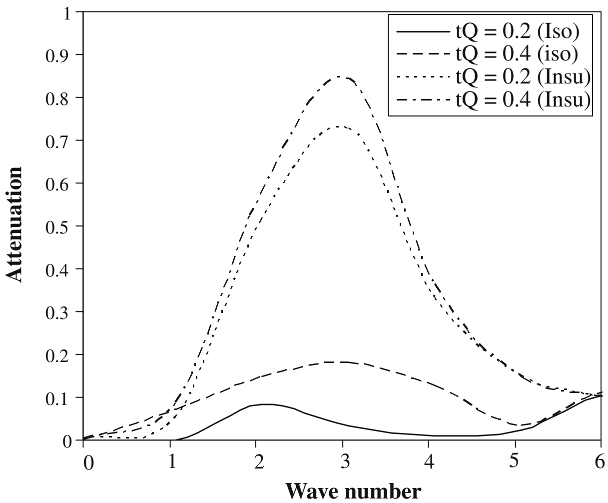

Fig. 6. Attenuation coefficient profile of ETP isothermal wave/insulated wave with heat relaxation effect.

the material exhibit more internal friction with increasing life time of charge carrier field than that in case of thermal relaxation time. However, this trend gets reversed at short wavelengths ($R \geqslant 1$).

Fig. 9 depicts the variations of phase velocity with wave number for insulated thermodiffusive (TP) waves with wave number on linear log scale. It is found that the effect of life time on the phase velocity is quite large at small wave number values as compared to that at large wave numbers. Fig. 10 represents variations of non-dimensional attenuation of TP surface waves for insulated surface conditions with respect to wave number. The attenuation profile corresponding to $t_p^+ = 10^{-12}$ s starts from zero value and increases in a fluctuating manner with increasing wave number to ultimately become steady. However, attenuation profiles for $t_p^+ = 10^{-13}$ s and $t_p^+ = 10^{-14}$ s show Gaussian behaviour with mean at $R = 2$ in the wave number range $0 \leqslant R \leqslant 4$ and then ultimately become steady for $R \geqslant 5$. The ups and downs in particle motion show the sensitivity of TP waves towards the life times of charge carriers.

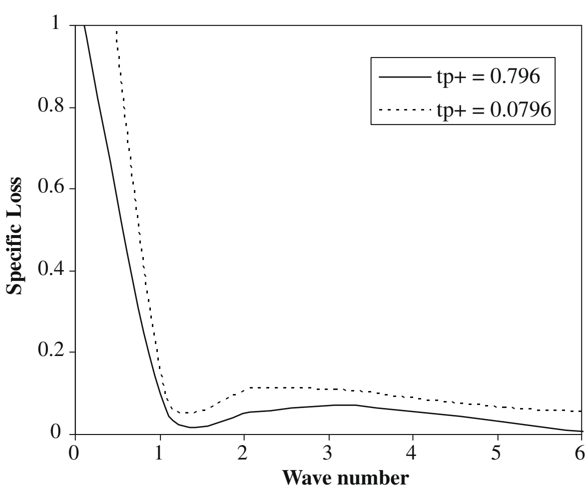

Fig. 7. Specific loss profile of ETP insulated waves with life time effect of charge carriers.

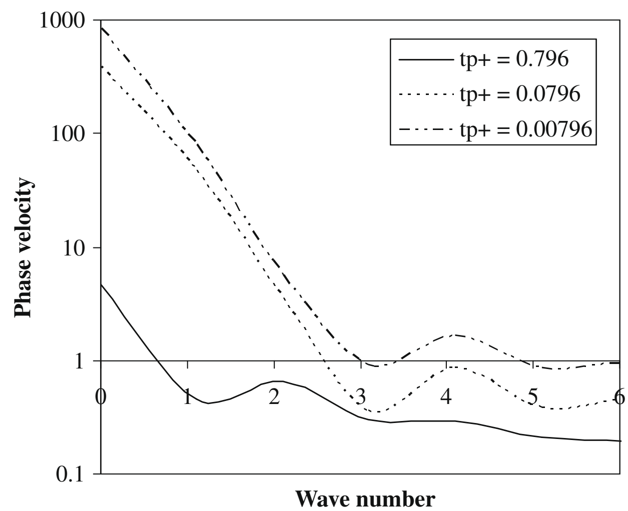

Fig. 9. Phase velocity profiles of TP insulated waves with life time effect of charge carriers.

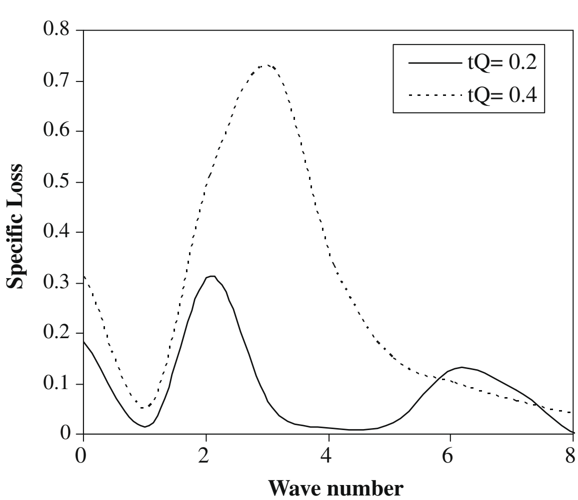

Fig. 8. Specific loss profile of ETP insulated waves with heat relaxation effect.

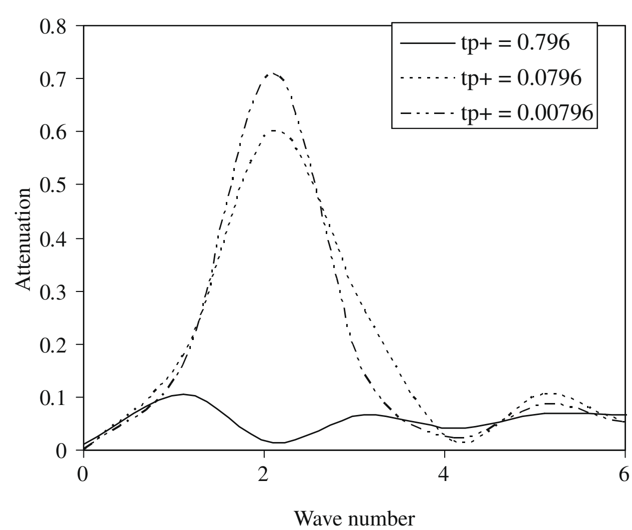

Fig. 10. Attenuation coefficient profiles of TP insulated waves with life time effect of charge carriers.

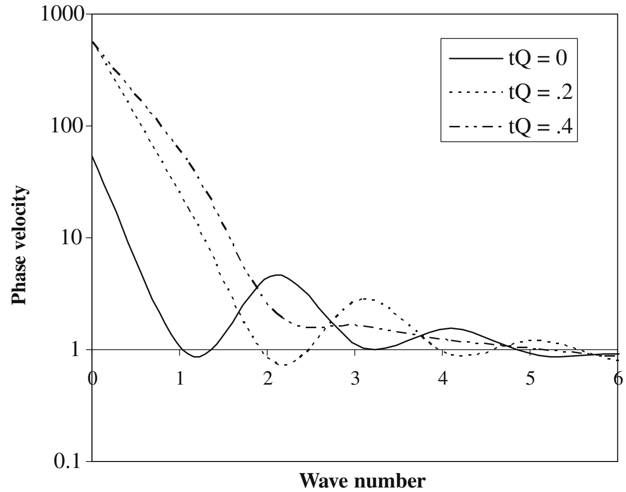

Fig. 11. Phase velocity profiles of TP insulated waves with heat relaxation effect.

The profiles in Fig. 11 show almost similar trend of variations of phase velocity as that in Fig. 9 except that here the magnitude of variations of phase velocity is comparatively small and the profiles are subjected to periodic fluctuations at small values of thermal relaxation time which disappear with increasing values of this parameter at all wave numbers. The profiles in Fig. 12 present almost similar behaviour as that in Fig. 10 in the whole wave number range with the exception that the maximum value of this quantity occur at $R = 1$ in case of $t^Q = 0, 0.2$ and at $R = 2$ for $t^Q = 0.4$ in addition to having high magnitude of variations.

Figs. 13 and 14 present the variations of specific loss factor with wave number with different life times of charge carriers and relaxation time of heat effect, respectively. It is observed that the profiles of specific loss observe almost Gaussian behaviour in the wave number range $0 \leqslant R \leqslant 3$ before these ultimately become almost steady in nature after observing fluctuating trend at large wave number values in both the considered figures. The effect of life and thermal relaxation times on the specific loss factor is noticed to be significantly large at small wave numbers (long wave lengths) as compared to that at large wave numbers (short wave lengths). This is attributed to the fact that the long wave length waves penetrate deep into the medium and thus the coupling effects become operative thereby leading significant to modifications while short wave length wave follows the surface with little disturbance to the solid.

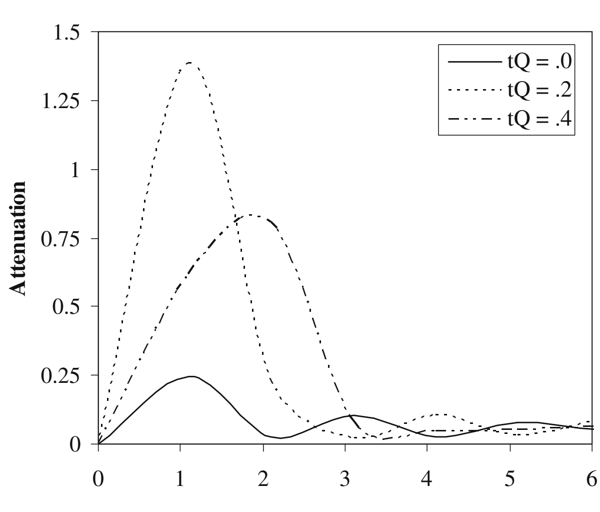

Fig. 12. Attenuation coefficient profiles of TP insulated waves with heat relaxation effect.

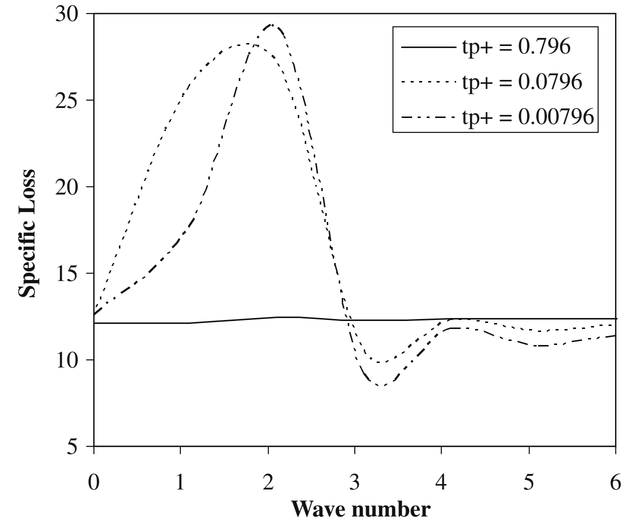

Fig. 13. Specific loss profile of TP insulated waves with life time effect of charge carriers.

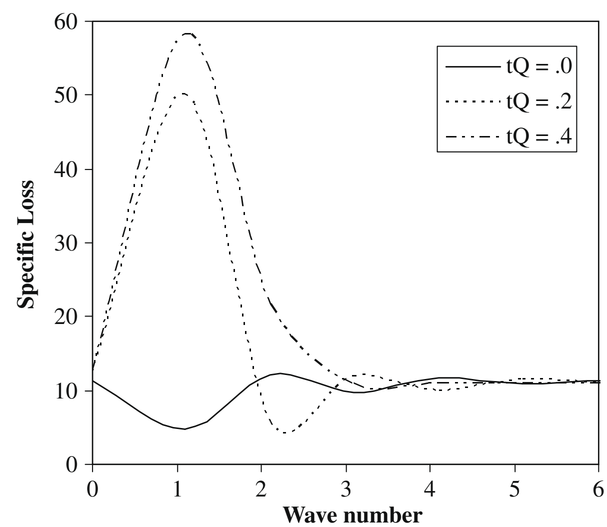

Fig. 14. Specific loss profile of TP insulated waves with heat relaxation effect.

8. Conclusions

It is observed that the life time of charge carriers and thermal relaxation time significantly affect the characteristics of ETP surface waves in Ge semiconductor. The effect of life time of holes and relaxation of heat is found to be quite large at long wavelengths rather than on short wavelengths. This is attributed to the fact that the former is capable of deep penetration into the medium thereby making coupling of various interaction fields operative but the latter follows the surface of the semiconductor with least disturbance to the medium. The considered waves are observed to be significantly affected due to the presence of thermally insulated and isothermal boundaries of the p-type semiconductor (germanium) halfspace. It is observed that at long wavelengths, the material exhibit more internal friction with increasing life time of charge carrier field than that in case of thermal relaxation time. However, this trend gets reversed at short wavelengths.

Though the transportation of heat in the solids at room temperature is normally diffusion dominant, however, the inclusion of thermal relaxation and life times in the energy as well charge carriers equations also facilitates the numerical computations, see Achenbach (2005). Usually in engineering applications Fourier's law and diffusion give an easier and better description, however, in case the relaxation and process times are comparable it would be desirable to allow for both diffusion and relaxation by adopting constitutive models which have both thermal conductivity and relaxation spectra. The treatment of thermal and diffusion fields together at room temperature in semiconductor materials may be useful in many therapies. This study may give potential impact on the fabrication quality of semiconductor and other engineering applications such as non-destructive testing (NDT), detection of cracks and other surface imperfections in the materials in addition to design and construction of SAW devices.

Acknowledgments

The authors are extremely thankful to the reviewers for their deep interest and their valuable suggestions for the improvement of this work. We also appreciate the cooperation and opportunity provided by Professor David A. Hills to carry out the improvements.

Appendix A

The quantities used in Eq. (15) are given by

$$\begin{aligned}
j &= 1-\varepsilon^{qp}\varepsilon^{pq}-\varepsilon^{qn}\varepsilon^{nq}-\varepsilon^{qn}\varepsilon^{pq}\bar{\lambda}_n(\varepsilon^{qn}-\varepsilon^{qp}),\\
j_1 &= J_p\left(1-\varepsilon^{qn}\bar{\lambda}_n\right)+K_n\bar{\lambda}_n\varepsilon^{qp}-\varepsilon^{qn}\left[J_p\left(\varepsilon^{nq}-\bar{\lambda}_n\right)+I_p\bar{\lambda}_n\varepsilon^{qp}\right]\\
&\quad+\varepsilon^{qp}\left[K_n\left(\varepsilon^{nq}-\bar{\lambda}_n\right)-I_p\left(1-\varepsilon^{qn}\bar{\lambda}_n\right)\right],\\
j_2 &= J_n\left(1-\varepsilon^{qp}\bar{\lambda}_p\right)+K_p\bar{\lambda}_p\varepsilon^{qn}-\varepsilon^{qn}\left[I_n\left(1-\varepsilon^{qp}\bar{\lambda}_p\right)\right.\\
&\quad\left.-K_p\left(\varepsilon^{pq}-\bar{\lambda}_p\right)\right]-\varepsilon^{qp}\left[I_n\varepsilon^{qn}\bar{\lambda}_p-J_n\left(\varepsilon^{pq}-\bar{\lambda}_p\right)\right],\\
j_3 &= (\varepsilon^{pq}-\bar{\lambda}_p)F_1+\varepsilon^{qn}\bar{\lambda}_pF_2+(1-\bar{\lambda}_p\varepsilon^{qp})F_3,
\end{aligned}\tag{A.1}$$

$$\begin{aligned}
j_1' &= (J_nJ_p-K_nK_p)-\varepsilon^{qn}(I_nJ_p-I_pK_p)+\varepsilon^{qp}(I_nK_n-J_nI_p),\\
j_2' &= (\tau_p'-\bar{\lambda}_p\tau^Q)F_1-i\omega^{-1}\varepsilon_n(\alpha_0^p-\bar{\lambda}_p)F_2+\left(\tau_p^*-i\omega^{-1}\varepsilon_p\bar{\lambda}_p\right)F_3,\\
j_3' &= (\varepsilon^{pq}-\bar{\lambda}_p)G_1+\varepsilon^{qn}\bar{\lambda}_pG_2+(1-\bar{\lambda}_p\varepsilon^{qp})G_3,\\
j_4' &= \tau^Q\left[(1-\bar{\lambda}_p\varepsilon^{qp})(1-\bar{\lambda}_n\varepsilon^{qn})-\bar{\lambda}_n\bar{\lambda}_p\varepsilon^{qn}\varepsilon^{qp}\right]\\
&\quad-i\omega^{-1}\varepsilon_n\left[(1-\bar{\lambda}_p\varepsilon^{qp})\left(\varepsilon^{nq}-\bar{\lambda}_n\right)+\bar{\lambda}_n\varepsilon^{qp}\left(\varepsilon^{pq}-\bar{\lambda}_p\right)\right]\\
&\quad-i\omega^{-1}\varepsilon_p\left[\bar{\lambda}_p\varepsilon^{qn}\left(\varepsilon^{nq}-\bar{\lambda}_n\right)+\left(1-\varepsilon^{qn}\bar{\lambda}_n\right)\left(\varepsilon^{pq}-\bar{\lambda}_p\right)\right],
\end{aligned}\tag{A.2}$$

$$\begin{aligned}
j_1'' &= \left(\tau_p'-\bar{\lambda}_p\tau^Q\right)G_1-i\omega^{-1}\varepsilon_n\left(\alpha_0^p-\bar{\lambda}_p\right)G_2+\left(\tau_p^*-i\omega^{-1}\varepsilon_p\bar{\lambda}_p\right)G_3,\\
j_2'' &= \tau^Q\left[J_p\left(1-\bar{\lambda}_n\varepsilon^{qn}\right)+K_n\bar{\lambda}_n\varepsilon^{qp}\right]-i\omega^{-1}\varepsilon_n\left[J_p\left(\varepsilon^{np}-\bar{\lambda}_n\right)\right.\\
&\quad\left.+I_p\bar{\lambda}_n\varepsilon^{qp}\right]+i\omega^{-1}\varepsilon_p\left[K_n\left(\varepsilon^{nq}-\bar{\lambda}_n\right)-I_p\left(1-\bar{\lambda}_n\varepsilon^{qn}\right)\right],\\
j_3'' &= \tau^Q\left[J_n\left(1-\bar{\lambda}_p\varepsilon^{qp}\right)+K_p\bar{\lambda}_p\varepsilon^{qn}\right]-i\omega^{-1}\varepsilon_n\left[I_n\left(1-\bar{\lambda}_p\varepsilon^{qp}\right)\right.\\
&\quad\left.-K_p\left(\varepsilon^{pq}-\bar{\lambda}_p\right)\right]-i\omega^{-1}\varepsilon_p\left[I_n\bar{\lambda}_p\varepsilon^{qn}-J_n\left(\varepsilon^{pq}-\bar{\lambda}_p\right)\right],\\
j^* &= \tau^Q[J_nJ_n-K_nK_p]-i\omega^{-1}\varepsilon_n[I_nJ_p-I_pK_p]\\
&\quad+i\omega^{-1}\varepsilon_p[I_nK_n-J_nI_p],
\end{aligned}\tag{A.3}$$

where

$$\begin{aligned}
&I_n=\tau_n'-\bar{\lambda}_n\tau^Q,\quad I_p=\tau_p'-\bar{\lambda}_p\tau^Q,\quad J_n=\tau_n^*-i\omega^{-1}\bar{\lambda}_n\varepsilon_n,\\
&J_p=\tau_p^*-i\omega^{-1}\bar{\lambda}_p\varepsilon_p,\quad K_n=i\omega^{-1}\varepsilon_n\left(\alpha_0^p-\bar{\lambda}_p\right),\\
&K_p=i\omega^{-1}\varepsilon_p\left(\alpha_0^n-\bar{\lambda}_n\right),
\end{aligned}\tag{A.4}$$

$$\begin{aligned}
F_1 &= g^{qp}\left(\varepsilon^{qn}\bar{\lambda}_n-1\right)-g^{qn}\varepsilon^{qp}\bar{\lambda}_n,\\
F_2 &= g^{qp}\left(\bar{\lambda}_n-\varepsilon^{nq}\right)-f_Q\varepsilon^{qp}\bar{\lambda}_n,\\
F_3 &= f_Q\left(1-\varepsilon^{qn}\bar{\lambda}_n\right)-g^{qn}\left(\varepsilon^{nq}-\bar{\lambda}_n\right),
\end{aligned}\tag{A.5}$$

$$F=\frac{\left(\beta^2+k^2\right)^2}{4k^2\beta},$$

$$\begin{aligned}
G_1 &= g^{qn}i\omega^{-1}\varepsilon_p\left(\alpha_0^n-\bar{\lambda}_n\right)-g^{qp}\left(\tau_n^*-i\omega^{-1}\varepsilon_n\bar{\lambda}_n\right),\\
G_2 &= f_Qi\omega^{-1}\varepsilon_p\left(\alpha_0^n-\bar{\lambda}_n\right)-g^{qp}\left(\tau_n'-\bar{\lambda}_n\tau^Q\right),\\
G_3 &= f_Q\left(\tau_n^*-i\omega^{-1}\varepsilon_n\bar{\lambda}_n\right)-g^{qn}\left(\tau_n'-\bar{\lambda}_n\tau^Q\right),
\end{aligned}\tag{A.6}$$

$$\begin{aligned}
&f_Q=1+(1+\varepsilon_T)\tau^Q,\quad g^{qn}=\varepsilon^{qn}+i\omega^{-1}\varepsilon_n(1+\varepsilon_T),\\
&g^{qp}=\varepsilon^{qp}+i\omega^{-1}\varepsilon_p(1+\varepsilon_T),
\end{aligned}\tag{A.7}$$

$$\begin{aligned}
&\tau^Q=t^Q+i\omega^{-1},\quad \varepsilon_n'=\frac{\varepsilon_n}{\varepsilon^{qn}},\quad \varepsilon_p'=\frac{\varepsilon_p}{\varepsilon^{qp}},\\
&\tau_n'=t^Q\alpha_0^n+i\omega^{-1}(\alpha_0^n+a_0^n)-a_0^n\omega^{-2}/t_n^+,\\
&\tau_p'=t^Q\alpha_0^p+i\omega^{-1}(\alpha_0^p+a_0^p)-a_0^p\omega^{-2}/t_p^+,\\
&\tau_n^*=\frac{\chi}{D^n}\left[t^n+i\omega^{-1}\left(1-\frac{\in_n\alpha_0^nD^n}{\chi}-\frac{t^n}{t_n^+}\right)+\frac{1}{\omega^2t_n^+}\right],\\
&\tau_p^*=\frac{\chi}{D^p}\left[t^p+i\omega^{-1}\left(1-\frac{\in_p\alpha_0^pD^p}{\chi}-\frac{t^p}{t_p^+}\right)+\frac{1}{\omega^2t_p^+}\right].
\end{aligned}\tag{A.8}$$

References

Achenbach, J.D., 2005. The thermoelasticity of laser based ultrasonics. J. Therm. Stresses 28, 713–728.

Ackerman, C.C., Overtone, W.C., 1969. Second sound in helium-3. Phys. Rev. Lett. 22, 764–766.

Ackerman, C.C., Bartman, B., Fairbank, H.A., Guyer, R.A., 1966. Second sound in helium. Phys. Rev. Lett. 16, 789–791.

Atkin, R.J., Chadwick, P., 1981. Surface waves in heat conducting elastic body – correction and extension of paper of Chadwick and Windle. J. Therm. Stresses 4, 509–521.

Banerjee, D.K., Pao, Y.H., 1974. Thermoelastic waves in anisotropic solids. J. Acoust. Soc. Am. 56, 1444–1453.

Chadwick, P., Windle, D.W., 1964. Propagation of Rayleigh waves along isothermal and insulated boundaries. Proc. Roy. Soc. Am. 280, 47–71.

Chandrasekharaiah, D.S., 1986. Thermoelasticity with second sound – a review. Appl. Mech. Rev. 39, 355–376.

Dhaliwal, R.S., Sherief, H.H., 1980. Generalized thermoelasticity for anisotropic media. Q. Appl. Math. 38, 1–8.

Green, A.E., Lindsay, K.A., 1972. Thermoelasticity. J. Elasticity 2, 1–7.

Guyer, R.A., Krumhansal, J.A., 1966. Thermal conductivity, second sound and phonon, hydrodynamic phenomenon in non-metallic crystals. Phys. Rev. 148, 778–788.

Hetnarski, R.B., Ignaczak, J., 1999. Generalized thermoelasticity. J. Therm. Stresses 22, 451–476.

Ignaczak, J., 2006. Nonlinear hyperbolic heat conduction problem: closed form solutions. J. Therm. Stresses 29, 999–1018.

Kolsky, H., 1963. Stress Waves in Solids, Dover Press, New York.

Lockett, F.J., 1958. Effect of thermal properties of a solid on the velocity of Rayleigh waves. J. Mech. Phys. Solids 7, 71–75.

Lord, H.W., Shulman, Y., 1967. The generalized dynamical theory of thermoelasticity. J. Mech. Phys. Solids 15, 299–309.

Many, A., Goldstein, J., Grover, N.B., 1965. Semiconductor Surfaces. North-Holland, Amsterdam.

Maruszewski, B., 1986a. Electro-magneto-thermo-elasticity of extrinsic semiconductors: classical irreversible thermodynamic approach. Arch. Mech. 38, 71–82.

Maruszewski, B., 1986b. Electro-magneto-thermo-elasticity of extrinsic semiconductors. Extended irreversible thermodynamic approach. Arch. Mech. 38, 83–95.
Maruszewski, B., 1987a. Coupled evolution equations of deformable semiconductors. Int. J. Eng. Sci. 25, 145–153.
Maruszewski, B., 1987b. Heat and charge carrier relaxation in deformable semiconductors. In: Yamamoto, Y., Miya, K. (Eds.), Electro-Magneto-Mechanical Interactions in Deformable Solids and Structures. North-Holland, Amsterdam, pp. 395–400.
Maruszewski, B., 1989. Thermodiffusive surface waves in semiconductors. J. Acoust. Soc. Am. 85, 1967–1977.
Nayfeh, A., Nasser, S.N., 1971. Thermoelastic waves in solid with thermal relaxation. Acta Mech. 12, 53–69.
Ning, Y.U., Shoji, I., Tatsuo, I., 2004. Characteristics of temperature field due to pulsed heat input calculated by non-Fourier heat conduction hypothesis. JSME Int. J. Ser. A 47, 574–580.
Rayleigh, L., 1885. On waves propagation along the plain surface of an elastic solid. Proc. Lond. Math. Soc. 17, 4–11.
Scott, N.H., 1989. The stability of plane waves in generalized thermoelasticity. In: McCarthy, M.F., Hayes, M.A. (Eds.), Elastic Wave Propagation. North-Holland, Amsterdam, pp. 623–628.
Sharma, J.N., 1986. On the low and high-frequency behavior of generalized thermoelastic waves. Arch. Mech. 38, 665–673.
Sharma, J.N., Singh, H., 1985. Generalized thermoelastic waves in anisotropic media. J. Acoust. Soc. Am. 85, 1407–1413.
Sharma, J.N., Thakur, N., 2006. Plane harmonic elasto-thermodiffusive waves in semiconductor materials. J. Mech. Mater. Struct. 1, 813–835.
Sharma, J.N., Kumar, V., Sud, S.P., 2000. Plane harmonic waves in orthorhombic thermoelastic materials. J. Acoust. Soc. Am. 107, 293–305.
Sharma, J.N., Thakur, N., Singh, S., 2007. Propagation characteristics of elasto-thermodiffusive surface waves in semiconductor material half-space. J. Therm. Stresses 30, 357–380.