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Abstract:

We examine experimentally and theoretically the effect of frictional shakedown of a three-dimensional elastic rolling contact. Small oscillations of the local normal forces lead to incremental sliding processes within the area of contact. Consequently, this causes a macroscopic slip motion of the two contacting bodies. If the oscillation amplitude is sufficiently small, the frictional slip ceases after the first few loading periods and a safe shakedown occurs. Otherwise the slip motion is continued and the contact fails.

Using the method of dimensionality reduction, we derive analytical shakedown limits on the parameter plane tangential loading-oscillation amplitude and compare them to results of numerical simulations with Kalker's program CONTACT. Both models show very good agreement with experimental data and allow an accurate prediction of the shakedown displacement and the maximum tangential load capacity in the shakedown state.

Keywords:

oscillating rolling contact; Coulomb friction; shakedown; micro-slip; ratcheting; method of dimensionality reduction

1. Introduction:

Frictional contacts are crucial for the generation of solid detachable and non-detachable connections between technical components. Examples are bolted connections, interference fits and machining fixtures. The load capacity and thus reliability of these systems mainly depends on the properties of the tangential contact (Booker et al., 2004; Chung and Ip, 2000; Law et al., 2006; Li et al., 2000; McCarthy et al., 2005). Its capacity is in turn determined primarily by the macroscopic normal force F_N and the friction coefficient μ. According to Coulomb's law, such a connection fails if the applied tangential force F_T exceeds the maximum holding force:

$$F_{T,\max} = \mu F_N. \tag{1}$$

In many technical systems, the normal force either consists of a static part superposed with small oscillations, or the overall normal force is constant and there are only local oscillations of normal pressure. Both scenarios lead to a periodic incremental slip of the contact interface, even if the tangential force is far below the maximum holding force of Eq.(1). Some consequences of this effect are micro-slip (Hartwigsen et al., 2004) and fretting fatigue (Huq and Celis, 2002; Nowell et al., 2006) of the relevant components. However, it is also possible that the slip ceases after the first few loading periods (Antoni et al., 2007; Churchman and Hills, 2006). This is the case, if the initial displacement produces a residual force in the interface, which is sufficiently strong to prevent any further slip. Subsequently, the entire contact area will finally remain in a state of stick, even if the oscillation of the force is continued.

Two practical possibilities exist, to prevent cyclic slip. One is a simple increase of the acting normal force. The other one is to enable a system response of the former described type, which raises the question of the necessary prerequisites for this to occur (Ahn, 2009). There is a strong analogy to the shakedown case in plasticity problems, where the deformed bodies only show plastic strain in the first few loading cycles and elastic response afterwards. Consequently, the well-known Melan theorem for plastic shakedown (Melan, 1936) was transferred to both discrete (Klarbring et al., 2007) and continuous systems (Barber et al., 2008) with Coulomb friction as in Eq. (1).

Thus, they demonstrated that a frictional system in which the contact is complete, what means that the contact area must not change during the loading cycle, will shakedown and monotonically reach a safe shakedown displacement if subjected to oscillating loads. One important requirement for this, is an uncoupled system, meaning that relative displacements in the interface do not influence the local normal forces. In case of coupled two dimensional discrete systems shakedown is also possible, if the friction coefficient in each node is less than a critical value, which depends on the coupling between adjacent nodes (Klarbring et al., 2007).

The intention of this study is to formulate an analytical expression for the shakedown limits of an uncoupled elastic rolling contact in which the contact area changes, due to the oscillation. In addition, we determine the shakedown displacement and examine the effect of the shakedown process on the maximum tangential load capacity. Finally we give an experimental proof for the findings.

2. Model and Methods:

Our starting point is a tangentially loaded Hertzian contact of an elastic sphere and a rigid flat substrate for which the theoretical foundations can be found, for example, in Johnson (1987) or Popov (2010). The elastic properties of the sphere E^* and G^* as well as its radius

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R are chosen as effective quantities of a contact consisting of two elastic spheres with particular radii R_i, shear moduli G_i and Poisson-ratios ν_i:

$$E^* = \left(\frac{1-\nu_1}{G_1} + \frac{1-\nu_2}{G_2} \right)^{-1}, \; G^* = \left(\frac{2-\nu_1}{4G_1} + \frac{2-\nu_2}{4G_2} \right)^{-1}, \; R = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}. \tag{2}$$

The normal force F_N leads to the indentation depth:

$$d = \left(\frac{3}{4} \frac{F_N}{E^* R^2} \right)^{2/3} \tag{3}$$

and the initial area of contact is delimited by the contact radius $a = \sqrt{Rd}$. Assuming Coulomb friction with constant coefficient μ, a tangential loading F_T less than the maximum value of μF_N, will lead to a slight rigid body displacement of the substrate, called the static displacement (Johnson, 1987; Popov, 2010):
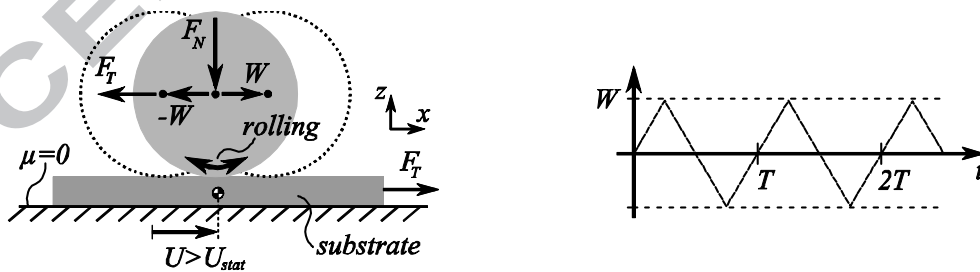
$$U_{stat} = \mu \frac{E^*}{G^*} d \left(1 - \left(1 - \frac{F_T}{\mu F_N} \right)^{2/3} \right). \tag{4}$$

Slipping will only occur at the boundary region of the contact area, whereas the center region remains in a state of stick and is limited by the stick-radius (Johnson, 1987; Popov, 2010):

$$c = a \left(1 - \frac{F_T}{\mu F_N} \right)^{1/3}. \tag{5}$$

For axially symmetric three-dimensional contacts, that satisfy the half-space approximation, these quantities can be determined using the principle of Ciavarella (1998) and Jäger (1998). We will restrict ourselves to uncoupled systems, meaning that variations in normal forces will not induce any tangential displacement and vice versa. This requires Dundurs' constant $\beta = 0$ as it is the case for frictionless contacts, similar materials, incompressible materials or if one body is rigid and the other one is incompressible(Ahn, 2009).

In the next step, the static tangential contact is superposed by a slight oscillatory rolling of the sphere with amplitude W, being the lateral movement of the sphere's center, as depicted in Fig. 1. This will lead to an increase of the displacement of the substrate U in relation to U_{stat}.

Fig. 1: oscillating, elastic rolling contact with lateral movement of the center W and displacement of the substrate U (left). Oscillation amplitude W as a function of time with oscillation period T (right).

The overall macroscopic normal and tangential forces will both be kept constant. Thus, the pure rolling does not lead to any additional friction force or momentum, but changes both the contact area and the local normal forces, and this leads to changes in stick and slip areas. According to this, the problem setting is equivalent to a frictional contact with constant macroscopic forces, which is exposed to a seesaw movement or rocking of the contacting

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bodies. In addition, the system is assumed to be quasi-static, meaning that we assume a constant μ and neglect inertia effects. This is valid as long as the rolling is slower than the propagation speed of elastic waves within the body.

2.1. Quasi-static incremental approach using MDR

The method of dimensionality reduction (MDR) enables an exact mapping of uncoupled, rotationally symmetric tangential contacts with Coulomb friction without loss of essential properties (Heß, 2011; Popov et. al., 2013). Hence, we model the initial three-dimensional problem by introducing an equivalent one-dimensional elastic foundation of independent springs, as described in Popov and Psakhie (2007) and Heß (2011). Both, the radius of the foundation $R_{1D} = \frac{1}{2}R$ and its normal and tangential spring stiffness $k_z = E^*\Delta x$ and $k_x = G^*\Delta x$ are chosen according to the rules of Popov (Popov, 2012) with Δx being the distance between adjacent springs. The physical background of these rules lies in the proportionality of the stiffness of a three-dimensional contact to the associated contact length instead of its contact area (Geike and Popov, 2007). Using this mapping, the influence of the oscillatory rolling on the tangential displacement of the substrate is simulated with a quasi-static incremental approach as described in Wetter (2012). Here an incremental rolling ΔW changes the normal deflection u_z of a spring at position x:

$$u_z = d - \frac{(x \pm \Delta W)^2}{R}. \tag{6}$$

Through a case distinction of the spring forces $f_z = k_z u_z$ and $f_x = k_x u_x$, the distribution of stick and slip can be identified. In turn this gives U and u_x in the next time step:

$$\text{stick-region: } f_x < \mu f_z \Rightarrow u_x = U, \tag{7}$$

$$\text{slip-region: } f_x \geq \mu f_z \Rightarrow u_x = \mu \frac{k_z}{k_x} u_z. \tag{8}$$

Apart from the numerical simulation, the model enables the derivation of the analytical shakedown limits as described in section 4.1.

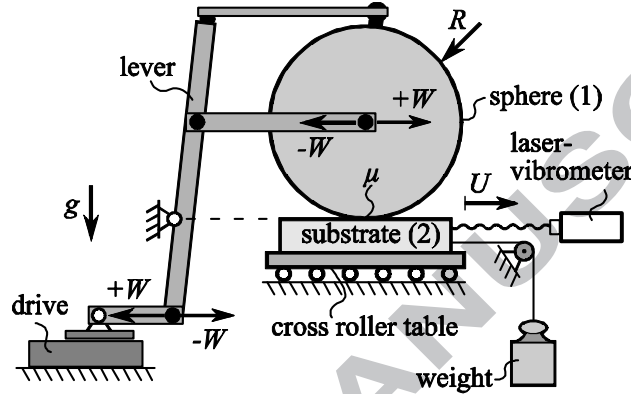## 2.2. Three dimensional simulation

As an alternative to the MDR model, we use the well-known CONTACT software package, based on the Kalker theory of rolling contacts (Kalker, 1990) to conduct a three dimensional quasi static simulation of the problem. This enables the determination and presentation of the two-dimensional contact properties, e.g. the distribution of stick- and slip, and the tangential displacement. In this case the exact parameters of the experimental setup as described in section 2.3 are being used for a mutual verification. We use 108×89 quadratic discretization elements with a side length of $\delta x, \delta y = 0.11 \ mm$ and an incremental step size of $\Delta W = 0.11 \ mm$ to simulate the transient rolling of the sphere.

2.3. Experimental setting

We use the experimental setting depicted in Fig. 2. The sphere (1) is made of ST-52 steel, whereas silicone rubber serves as the substrate (2). Thus, the system is almost uncoupled as $\beta \approx 0$. Other important parameters are listed in Table1. The weight of the sphere acts as the normal force $F_N = m_1 g$ and the tangential load is controlled by a single weight $F_W = m_W g$

which is connected to the substrate through a string. For minimization of external influences, the substrate is put on a low friction cross roller table. Its resistance force of $F_R = 0.1\,\text{N}$ lowers the actual tangential force, which results to $F_T = F_W - F_R$.

To maintain rolling of the sphere, a lever arm construction is used, which main bearing is located exactly on the same level as the contact point between sphere and substrate. As this point corresponds to the instantaneous center of motion, the oscillations of the lever-arm result in a pure rolling of the sphere. A high-precision linear drive is used for the back and forth motion of the lever arm and the displacement of the substrate U is measured using a high resolution laser-vibrometer.

Fig. 2: experimental setting: steel sphere (1), silicone rubber substrate (2), weight, drive (PI-M 405-DG) and laser-vibrometer (Polytec OFV-5000).

Table 1: properties of the experimental setting.

characteristic properties		
R	=	40 mm
μ	=	0.58
E_1/E_2	=	$206\cdot10^3/5$ MPa
v_1/v_2	=	0.3/0.5
F_N	=	21.1 N
a	=	4.53 mm
d	=	0.51 mm

With the drive's maximum speed of $\dot{W} = 1\ mm/s$, the highest excitation frequency is lower than 0.21 Hz. Since this is two orders of magnitude less than the lowest natural frequency of the system, which has been calculated as 22 Hz, the dynamical influences of the experimental setup are neglected.

3. Experiments and Analysis:

In the following, we normalize the tangential force F_T, the oscillation amplitude W and the displacement U with the maximum holding force, the maximum tangential displacement (Hess, 2011) and the contact radius and get the following dimensionless counterparts:

$$f_T = F_T/\mu F_N,\ u = U/\mu\tfrac{E^*}{G^*}d,\ w = W/a. \tag{9}$$

Two important assumptions are that the tangential forces are below the maximum holding force, and that the oscillation amplitudes are smaller than the contact radius. In normalized variables this reads:
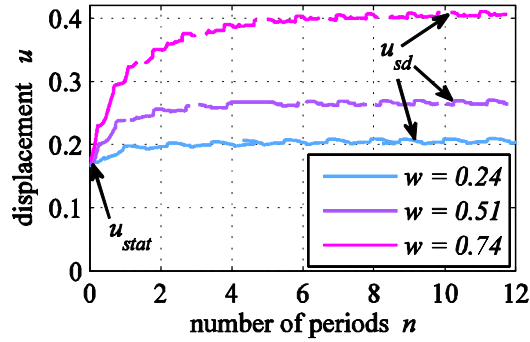
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$$f_T \leq 1, \; w \leq 1. \tag{10}$$

In other words, without the oscillatory rolling, no complete sliding will occur and the center of the sphere will not be moved beyond the initial area of contact at any time. Thus, taken by itself, neither of the two factors leads to a failure of the contact.
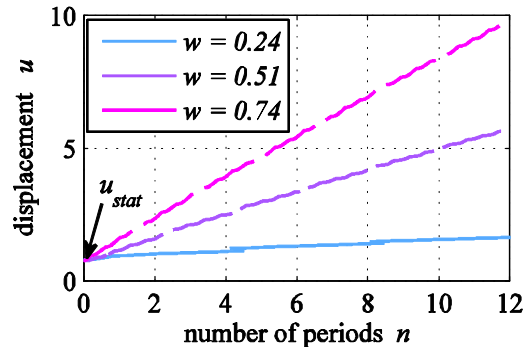
3.1. Shakedown and induced micro-slip

Experimental and numerical results show that the oscillatory rolling causes incremental sliding processes in the area of contact. Depending on the actual direction of the rolling, the boundary area is released and the local maximum holding force of Eq. (1) is reduced on one side of the contact. As a result, partial slip occurs, which increases both the slip area of the contact and the displacement of the substrate, as depicted in Fig. 3 and Fig. 4. We only report the experimental results here (For specific details on the numerical results see Wetter (2012)). If f_T and w are below the shakedown limits, the system reaches a constant mean displacement, even if the oscillatory rolling is continued, as shown in Fig. 3. This refers to the constant time independent shakedown displacement u_{sd} (Klarbring et al., 2007). As one can see, some slight oscillations occur, that are caused by geometrical deviations of the experimental setting.

In contrast, when the shakedown limits are exceeded, the oscillatory rolling causes a complete sliding of the contact. After a transient process, the accumulated slip in every period leads to a continuing displacement of the substrate as depicted in Fig. 4. This effect is referred to as ratcheting or induced micro-slip (Wetter, 2012). In the steady state, the mean velocity increases in line with both, increasing tangential force and oscillation amplitude.

Fig. 3: displacement u for different oscillation amplitudes w and $f_T = 0.24$ in case of shakedown.
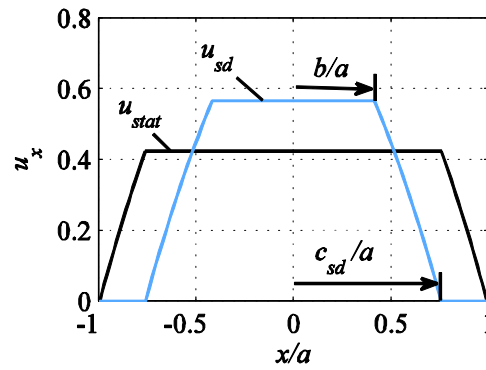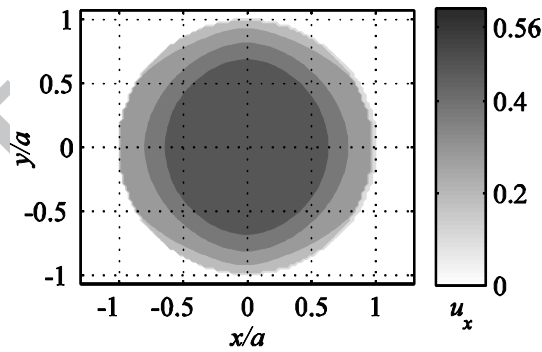

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Fig. 4: displacement u for different oscillation amplitudes w and $f_T = 0.88$ in case of induced micro-slip.

3.2. Contact region after shakedown

Both, the numerical model based on the MDR and the three-dimensional simulation using the CONTACT software package show, that the contact remains constant and sticks, after the system has reached the new equilibrium state. This corresponds to the shakedown theory, stating that the entire contact region must remain in a state of stick, even if the oscillation continues (Klarbring, 2007). Fig. 5 shows the normalized tangential spring deflection u_x before (black line) and after shakedown for the MDR-model with the characteristic slip-radius b and the stick-radius c_{sd}.

Fig. 5: displacement u_x after shakedown for $w = 0.24$ and $f_t = 0.56$ (MDR-model).

Fig. 6: displacement u_x after shakedown for $w = 0.24$ and $f_T = 0.88$ (3-D CONTACT model).

On basis of the MDR model, the radii can be derived directly by simple kinematic considerations. This is due to the independence of the degrees of freedom of the elastic foundation. The slip-radius $b/a = 1 - w$ delimits the region, outside of which the periodical release causes the tangential deflection to be zero, as shown in Fig. 5. In comparison to the static case, the stick radius after shakedown $c_{sd}/a = \sqrt{1 - u_{sd}} - w$ is decreased. Altogether, the tangential deflections in the contact region $0 \leq |x/a| \leq 1$ become:

$$0 \leq |x/a| \leq c_{sd}/a : u_x = \sqrt{1 - u_{sd}} - w, \tag{11}$$

$$c_{sd}/a < |x/a| \leq b/a : u_x = 1 - (x/a - w)^2. \tag{12}$$

As the macroscopic tangential force must match the overall local tangential force in the new equilibrium state, we get the following analytical relation by a simple integration of all the tangential spring forces in the contact region (Wetter, 2012):

$$f_T = 1 - \tfrac{3}{2} w u_{sd} - (1 - u_{sd})^{\frac{3}{2}} . \tag{13}$$

This expression is the result of the MDR, which is based on the assumption of rotational symmetry of the contact region (Hess, 2011). Fig. 6 shows the tangential deflections in the contact region computed with the three-dimensional simulation. It turns out, that the distribution of tangential deflections is slightly elliptic or spindle-shaped. Consequently, we expect a small deviation in the relation between loading and oscillation amplitude of Eq.(13). Considering this, we introduce the mapping-parameter κ, such that:

$$f_T = 1 - \kappa \cdot w u_{sd} - (1 - u_{sd})^{\frac{3}{2}} . \tag{14}$$

4. Results and Discussion

Given the experimental and analytical models of the rolling contact, we will identify the analytical shakedown limits as described in section 4.1. Additionally, in section 4.2, we will describe the supercritical system response.

4.1. Shakedown limits for the oscillating rolling contact

Expression (14) enables the prediction of the shakedown displacement u_{sd} for a combination of f_T and w below the shakedown limits. The unknown mapping parameter is gained via comparison with the results of the experiments and the three-dimensional simulations. This indicates $\kappa = 1$ which finally yields the following expression for the relation of shakedown displacement, tangential force and oscillation amplitude:

$$f_T = 1 - w u_{sd} - (1 - u_{sd})^{\frac{3}{2}} . \tag{15}$$

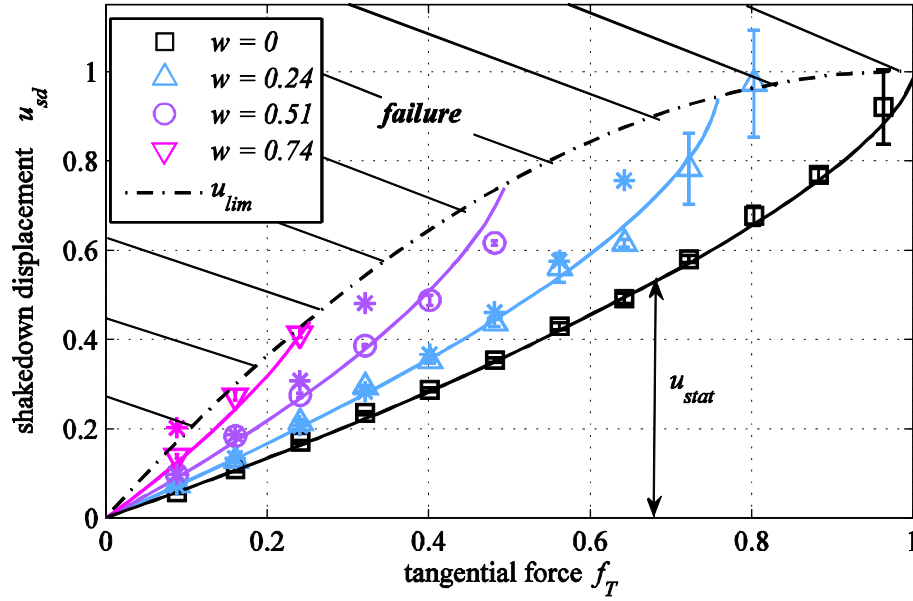Fig. 7 illustrates u_{sd} as a function of f_T for different w. The solid lines show the analytical results of (15), whereas the experimental results are indicated by the error bars and marks. The asterisks depict the results for the three-dimensional simulation, where values for tangential forces close to the limit are not given, because the iterative solution procedure of Contact$^{\copyright}$ lacks robustness in this case (Vollebregt, 2012). As depicted in Fig. 7, u_{sd} is increased compared with the static value $u_{stat} = 1 - (1 - f_T)^{\frac{2}{3}}$. It is also known that in the case that the oscillation stops sooner, the final displacement might differ from this theoretical shakedown value(Klarbring et al., 2007). In the experiments, the shakedown state was already reached after $n \approx 10$ rolling periods.

The dotted line in Fig. 7 indicates the maximum displacement for different amplitudes which can be achieved, before the oscillation leads to a failure of the contact. In this case, the stick radius c_{sd} is zero, which in combination with Eq.(15) gives the maximum tangential force:

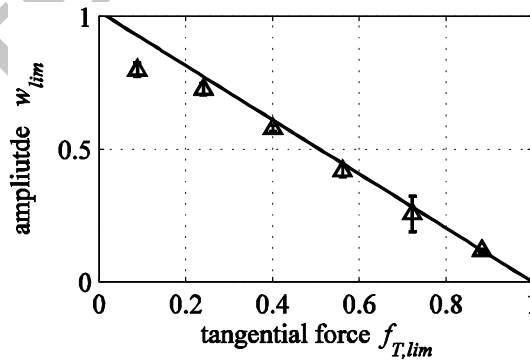$$f_{T,\lim} = 1 - w_{\lim}, \tag{16}$$

and the maximum displacement:
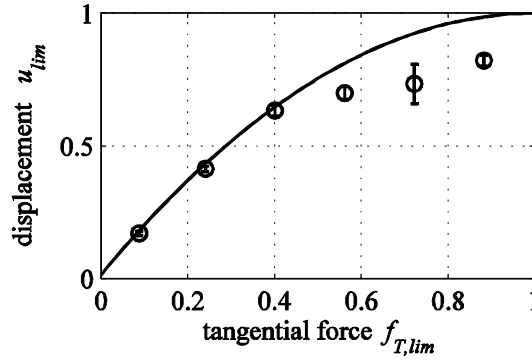
$$u_{\lim} = 2 f_{T,\lim} - f_{T,\lim}^2 . \tag{17}$$

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Fig. 7: shakedown displacement of the substrate u_{sd} as a function of the tangential force f_T for different oscillation amplitudes w. The oscillatory rolling causes an increase of the displacement by comparison with its static value u_{stat}. Analytical (solid lines), experimental (error bars and marks) and three-dimensional simulation (asterisks) results.

Both, analytical (solid lines) and experimental (error bars and marks) are depicted in Fig. 8 and Fig. 9. Since in the experiments, the maximum amplitude was identified by increasing it stepwise while f_T was kept constant, Fig. 8 shows $w_{lim} = 1 - f_{T,lim}$. For medium tangential forces, both w_{lim} and u_{lim} show strong agreement with the theory.

Fig. 8: maximum oscillation amplitude w_{lim} as a function of the tangential force $f_{T,lim}$.

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Fig. 9: maximum displacement $u_{\lim}$ as a function of the tangential force $f_{T,\lim}$.

At this point, it should be emphasized, that these maximum values correspond to the exact analytical shakedown limits for the oscillating rolling contact. Thus, for a given oscillation amplitude, the maximum tangential force to achieve a safe shakedown can be identified and vice versa. Additionally, since $f_{T,\lim} \leq 1$, it turns out that in the case of the oscillatory elastic rolling contact, shakedown is accompanied with a significant reduction of the tangential loading capacity. This effect must be considered in the design and construction of frictional contact systems under the influence of vibrations.

4.2. Induced micro-slip of the rolling contact

Induced micro-slip, also known as frictional ratcheting, occurs, if the actual tangential force or oscillation amplitude exceeds the shakedown limits given in Eq.(16). In this case, one side of the contact alternately sticks, while the other slips. This leads to an accumulated displacement of the substrate, referred to as walking (Mugadu et al., 2004). Due to the analogy to ratcheting in plasticity, this effect is also called frictional ratcheting.

The displacement per period or incremental displacement Δu, is an increasing function of f_T and w, as depicted in Fig. 4. Numerical experiments using the model described in section 2.1 show, that Δu is proportional to the supercritical portion of the oscillation amplitude $\Delta w = w - w_{\lim}$:
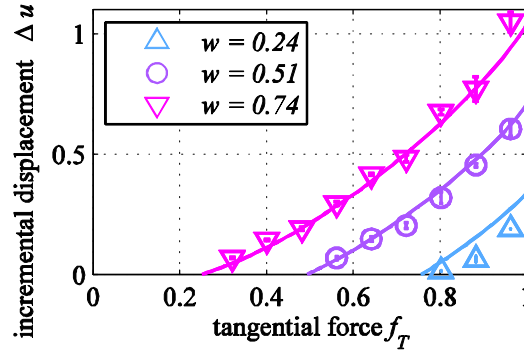
$$\Delta u = \lambda \cdot \left(w - w_{\lim} \right). \tag{18}$$

Here the constant of proportionality λ is a function of f_T. Using the experimental results and a linear regression analysis with u_{stat} as the regressor (Wetter, 2012) we approximate the incremental displacement as follows:

$$\Delta u \approx \sqrt{2u_{stat}} \cdot \left(w - w_{\lim} \right) = \sqrt{2\left(1 - \left(1 - f_T\right)^{2/3}\right)} \cdot \left(w - w_{\lim} \right). \tag{19}$$

It must be noted that $w_{\lim}$ also depends on f_T, as stated in equation (16). Fig. 10 shows Δu as a function of f_T for different oscillation amplitudes. Again, the solid lines depict the analytical results whereas the error bars and marks indicate the experimental values. The results show qualitatively good agreement with those for the walking of a rocking punch, as examined by Mugadu et al. (2004).

The micro-slip effect must not only have a negative impact, but can also be used for the generation of small displacements in case that an increase of the tangential loading is impossible or if high accuracy is needed as in MEMS-devices. Using Eq. (19) we can calculate this supercritical system response.

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Fig. 10: incremental displacement Δu as a function of the tangential force f_T for different oscillation amplitudes w in case of induced micro-slip. Analytical (solid lines) and experimental (error bars and marks) results.

5. Conclusions

An oscillating rolling contact between a sphere and a flat substrate with constant normal and tangential loading has been considered. We assumed Coulomb type friction with a constant coefficient μ and linear elastic material behavior. In addition, the system was assumed to be quasi-static and uncoupled, meaning that a variation in the normal force will not induce a displacement in the tangential direction and vice versa.

It was shown, that slight oscillatory rolling of the sphere leads to an increased rigid body displacement of the substrate, due to oscillations of the normal pressure and contact area. Depending on both, the oscillation amplitude and tangential force, the displacement stops after a few periods or continues. The former case is referred to as shakedown and the latter as induced micro-slip or ratcheting.

The results show, that shakedown also occurs in systems, where the contact is not known a priori and changes during the loading cycle. We derived the exact limits for both, the tangential force and the oscillation amplitude necessary to reach a safe shakedown of the system. It turned out, that the new equilibrium state after shakedown is accompanied with a reduced maximum tangential load capacity. Besides these shakedown limits, we can predict the rigid body displacement for the shakedown case and the incremental displacement in case of frictional ratcheting. Additionally, the comparison of experiment and theory shows, that the method of dimensional reduction (MDR) has proven to be a suitable instrument for the modeling of the oscillating rolling contact.

One objective for further research in this area should be dynamic influences. For this purpose the inertia properties of the system and possibly visco-elastic material behavior must be taken into account. Especially for technical applications it would also be important to investigate the interaction of various parameters. For example, the oscillating rolling may be superimposed by varying normal and tangential forces. Additionally, contact geometries, different from the one considered in this work, should be examined.

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Vitae:

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Figure captions:

Fig.1: oscillating, elastic rolling contact with lateral movement of the center W and displacement of the substrate U (left). Oscillation amplitude W as a function of time with oscillation period T (right).

Fig. 2: experimental setting: steel sphere (1), silicone rubber substrate (2), weight, drive PI-M 405-DG and laser-vibrometerPolytec OFV-5000.

Fig.3: displacement u for different oscillation amplitudes w and $f_T = 0.24$ in case of shakedown.

Fig. 4: displacement u for different oscillation amplitudes w and $f_T = 0.88$ in case of induced micro-slip.

Fig. 5: displacement u_x after shakedown for $w = 0.24$ and $f_T = 0.56$ (MDR-model).

Fig. 6: displacement u_x after shakedown for $w = 0.24$ and $f_T = 0.56$ (3-D Contact$^\copyright$-model).

Fig. 7: shakedown displacement of the substrate u_{sd} as a function of the tangential force f_T for different oscillation amplitudes w. The oscillatory rolling causes an increase of the displacement by comparison with its static value u_{stat}. Analytical (solid lines), experimental (error bars and marks) and three-dimensional simulation (asterisks) results.

Fig. 8: maximum oscillation amplitude w_{lim} as a function of the tangential force $f_{T,\text{lim}}$.

Fig. 9: maximum displacement u_{lim} as a function of the tangential force $f_{T,\text{lim}}$.

Fig. 10: incremental displacement Δu as a function of the tangential force f_T for different oscillation amplitudes w in case of induced micro-slip. Analytical (solid lines) and experimental (error bars and marks) results.

Table captions:

Table 1: properties of the experimental setting.

Highlights of the arcticle:

- We examine a three dimensional elastic rolling contact with Coulomb friction
- Slight oscillatory rolling leads to frictional shakedown or ratcheting
- We derive the analytical shakedown limits and the shakedown displacement
- Shakedown leads to a significant reduction of the tangential load capacity