

Thermal stress investigation in unidirectional composites under the hyperbolic energy model

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Abstract

The hyperbolic energy model is applied in this paper for determining transient temperature variation across a unidirectional composite plate subjected to different temperature changes at the boundaries. Thermal stresses developed are then analyzed for different material properties. Governing equations are solved numerically using implicit methods. The results are presented over a wide range of variables commonly found in most composite materials. The transient thermal stresses generated inside the plate were found to fluctuate between compressive and tensile quantities, a result that was not predicted using the classical heat model. Consequently, this will lead to an earlier crack initiation and failure of the material.
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1. Introduction

Technological advances in materials development have been motivated primarily by military and aerospace requirements. These requirements have been met with advances in new material forms known as fibre-reinforced composite structures. Composites have become common engineering materials. The prospect of producing “designer” materials for specific applications is of great interest to many engineers. However, the thermo-mechanical behaviour of these composites is more complex than that of homogeneous materials. Composite plates are extensively used due to their high specific strength and high specific stiffness. For these materials, thermal conductivity varies with direction that means they are anisotropic materials. Many natural and synthetic materials are anisotropic such as: single crystals, wood, sedimentary rocks, laminated sheets, cables, and many others.

Thermally induced stresses in anisotropic materials occur when a considerable temperature change takes place. The material anisotropy of composites has made their analysis and design considerably more difficult than for isotropic materials. In the literature, many researchers have investigated the residual stresses in aniso-

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Nomenclature

c	specific heat
h	plate thickness
k_{ij}	thermal conductivity, off-axis $i,j = x,y,s$, on-axis $i,j = 1,2$
L	plate side length
Q_{ij}	stiffness components, off-axis $i,j = x,y,s$, on-axis $i,j = 1,2$
S_i	dimensionless components of stress, $i = x,y,s$
T	temperature
T_∞	plate initial temperature (ambient temperature)
T_w	wall temperature
t	time
t_0	reference time
x	x -coordinate
X	dimensionless x -coordinate
y	y -coordinate
Y	dimensionless y -coordinate
s	parameter obtained due to changing the axis of interest
Greek symbols	
α	coefficient of thermal expansion
α_i	thermal expansion coefficient, $i = 1,2$
e_i	non-mechanical strains
ε_i	off-axis components of stress, $i = x,y,s$
η	dimensionless time
ϕ	fibre orientation
σ_i	off-axis components of stress, $i = x,y,s$
θ	dimensionless temperature
$\bar{\tau}$	thermal relaxation time, s
ρ	mass density

tropic materials and under the effect of the classical Fourier heat conduction model (Keary et al., 2001; Jin and Batra, 1998; Sib, 1965; Ganguly et al., 1975; Tangikar and Rao, 1994).

More recently, lasers and microwaves have been used as heating sources for composite manufacturing because of supplying high thermal energy with extremely short time. For such applications, the Fourier law for heat conduction fails to model the problem since the response to a temperature gradient is not immediate. To account for the phenomena involving the finite propagation velocity of the thermal wave, the hyperbolic heat conduction model should be used.

Hyperbolic heat transport has been receiving increasing attention both for theoretical motivations (analysis of thermal waves and second sound in dielectric solids, finite speed of heat transport, etc.) as for the analysis of some practical problems involving a fast supply of thermal energy (by laser pulse or chemical explosion, etc.). A number of researchers (Wu et al., 1998; Wu and Chu, 1999; Shen and Han, 2003) has investigated the numerical solution of the hyperbolic heat conduction model for isotropic materials.

In a previous work by Darabseh et al. (2007), the phase-lag concept in the wave theory of heat conduction was extended to describe the thermal and stress behaviour of anisotropic materials. In this work an orthotropic cylinder was investigated by assuming one directional hyperbolic heat transfer conduction model subjected to a constant temperature at the surface. Transient thermal stresses based on this model were estimated.

Al-Huniti and Al-Nimr (2004) investigated the transient thermoelastic response of thin composite plate composed of a dominant matrix and an insert using hyperbolic heat conduction model. The thermal stresses generated within the plate were found to be compressive, following the behaviour of the temperature, and largely affected by the heating source intensity and duration.

The aim of the present work is to determine the temperature field predicted by the hyperbolic energy equation for a simply supported unidirectional composite plate whose boundary undergoes a change in temperature. After that, the thermally induced stresses inside the anisotropic material are explored. The effect of material properties and dimensions, and other parameters on the thermal stresses of the anisotropic material is examined.

2. Thermal analysis

In the literature, many papers have been devoted to determine temperature solutions from Fourier's two-dimensional transient heat conduction equation for anisotropic materials with constant material properties. For unsteady heat conduction and using a fixed (x, y) coordinate system, the temperature (T) equation is written as:

$$k_{xx} \frac{\partial^2 T}{\partial x^2} + 2k_{xy} \frac{\partial^2 T}{\partial x \partial y} + k_{yy} \frac{\partial^2 T}{\partial y^2} = \rho c \frac{\partial T}{\partial t} \quad (1)$$

Here, ρ is the present mass density, c is the specific heat capacity and t is the time. The coefficients of thermal conductivity are: k_{xx} (in the x -direction), k_{yy} (in the y -direction), and k_{xy} (the coupling term). These conductivities are functions of the longitudinal (on-axis) k_{11} and transverse (off-axis) k_{22} conductivities, and also are functions of fibre (laminate) direction ϕ according to the equation:

$$\begin{bmatrix} k_{xx} \\ k_{xy} \\ k_{yy} \end{bmatrix} = \begin{bmatrix} \cos^2 \phi & \sin^2 \phi \\ (\cos \phi)(\sin \phi) & -(\cos \phi)(\sin \phi) \\ \sin^2 \phi & \cos^2 \phi \end{bmatrix} \begin{bmatrix} k_{11} \\ k_{22} \end{bmatrix} \quad (2)$$

However, many experiments have shown that the above classical theory of heat conduction in solids may fail when unsteady processes with rapid changes of the temperature are involved. To account for the phenomena involving the finite propagation velocity of the thermal wave, the hyperbolic heat conduction equation is employed (Darabseh et al., 2007).

$$k_{xx} \frac{\partial^2 T}{\partial x^2} + 2k_{xy} \frac{\partial^2 T}{\partial x \partial y} + k_{yy} \frac{\partial^2 T}{\partial y^2} = \rho c \bar{\tau} \frac{\partial^2 T}{\partial t^2} + \rho c \frac{\partial T}{\partial t} \quad (3)$$

where $\bar{\tau}$ is the thermal relaxation time.

Let us consider a unidirectional composite square plate with the dimensions shown in Fig. 1. The initial and boundary conditions within the plate are as follows:

$$\begin{aligned} T(x, y, 0) &= \frac{\partial T}{\partial t}(x, y, 0) = T_{\infty} \\ T(0, y, t) &= T_{\infty} \\ T(L, y, t) &= T_{\infty} \\ T(x, 0, t) &= T_{\infty} \\ T(x, L, t) &= T_w \end{aligned} \quad (4)$$

where T_{∞} is the ambient temperature and T_w is the wall temperature.

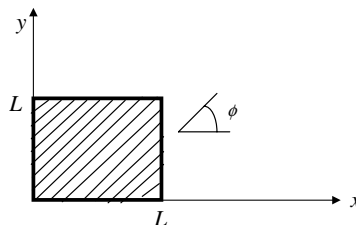


Fig. 1. Plate geometry used in the analysis.

To generalise analysis and results, Eq. (3) is transformed into a dimensionless form using the following parameters:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \eta = \frac{t}{t_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \tau = \frac{\bar{\tau}}{t_0} \quad (5)$$

Here, θ is the dimensionless temperature, η is the dimensionless time, τ is the dimensionless phase lag, X and Y are the dimensionless lengths along x - and y -axes, respectively. t_0 is a reference time given by: $t_0 = \frac{\rho c L^2}{k_{xx}}$.

Hence, substituting these parameters into Eq. (3) yields:

$$\frac{\partial^2 \theta}{\partial X^2} + 2R_{xy} \frac{\partial^2 \theta}{\partial X \partial Y} + R_{yy} \frac{\partial^2 \theta}{\partial Y^2} = \tau \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial \theta}{\partial \eta} \quad (6)$$

with the conductivity ratios: $R_{xy} = \frac{k_{xy}}{k_{xx}}$ and $R_{yy} = \frac{k_{yy}}{k_{xx}}$. The initial and boundary conditions become:

$$\begin{aligned} \theta(X, Y, 0) = \frac{\partial \theta}{\partial \eta}(X, Y, 0) &= 0 \\ \theta(0, Y, \eta) &= 0 \\ \theta(1, Y, \eta) &= 0 \\ \theta(X, 0, \eta) &= 0 \\ \theta(X, 1, \eta) &= 1 \end{aligned} \quad (7)$$

3. Stress analysis

Just like any other material, composites deform when their temperature changes (Johns, 1965). When the temperature rises in the plate, different elements of body tend to expand by different amount, an amount proportional to the local temperature raise. In the linear theory of elasticity, the resulting non-mechanical strains are simply added to the mechanical strains induced by the stress to obtain the total strain.

Because composite plies are anisotropic, the thermal expansion or contraction in the longitudinal direction is much less than that in the transverse direction. This differential contraction after cool down, and expansion after moisture absorption will give rise to macro-mechanical residual stresses among plies in a multidirectional laminate. Using laminated plate theory, the general form for these stresses is:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x - e_1 \\ \varepsilon_y - e_2 \\ \varepsilon_s \end{bmatrix} \quad (8)$$

Here, σ_i are the off-axis components of stress, ε_i are the off-axis mechanical strain components, and Q_{ij} are the off-axis stiffness components. The non-mechanical strain components e_i , or thermal strain, are defined as:

$$\begin{aligned} e_1 &= \alpha_1 \Delta T \\ e_2 &= \alpha_2 \Delta T \end{aligned} \quad (9)$$

where, α_1 and α_2 are the longitudinal and transverse thermal expansion coefficients, respectively, and ΔT is the change in temperature.

In this paper, the plate is assumed to be simply supported at the four edges with no mechanical loads applied. Hence, the stresses created inside the plate are due to temperature changes only, i.e. thermal stresses. They are given by:

$$\begin{aligned} \sigma_x &= p + q \cos 2\phi \\ \sigma_y &= p - q \cos 2\phi \\ \sigma_s &= q \sin 2\phi \end{aligned} \quad (10)$$

where

$$p, q = \frac{(Q_{11} \pm Q_{12})e_1 + (Q_{12} \pm Q_{22})e_2}{2} \quad (11)$$

Similar to temperature, stresses are normalised into dimensionless values through the equation:

$$S_x = \frac{\sigma_x}{Q_{11}}, \quad S_y = \frac{\sigma_y}{Q_{11}}, \quad S_s = \frac{\sigma_s}{Q_{11}} \quad (12)$$

4. Solution

Solution of the heat conduction equation, Eq. (6) along with the boundary and initial conditions in Eq. (7), is the first step in the evaluation of the temperature distribution, and stresses. An implicit finite difference method is used for the numerical calculation of these quantities. A reduction in the amount of calculation time may be realised by using implicit methods compared to explicit methods, which both sides of Eq. (6) have a temperature at the new time step ($n+1$), instead of (n) in explicit method, and will be solved at each time level. The implicit schemes are unconditionally stable for any size of time step, but the accuracy of the solution is only first-order in time. Hence, relatively small time steps are needed to ensure reasonable accuracy of results. A forward-difference representation is used for time derivative and central difference formulas are employed for other derivatives. Hence, the transformed equation will be as shown below:

$$\begin{aligned} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta\eta} + \tau \frac{\theta_{i,j}^{n+1} - 2\theta_{i,j}^n + \theta_{i,j}^{n-1}}{\Delta\eta^2} &= \frac{\theta_{i+1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i-1,j}^{n+1}}{\Delta X^2} + R_{yy} \frac{\theta_{i,j+1}^{n+1} - 2\theta_{i,j}^{n+1} - \theta_{i,j-1}^{n+1}}{\Delta Y^2} \\ &+ 2R_{xy} \frac{\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1}}{4\Delta X \Delta Y} \end{aligned} \quad (13)$$

Arranging the above Eq. (13) gives,

$$\begin{aligned} (2A + 2BR_{yy} + \tau + \Delta\eta)\theta_{i,j}^{n+1} - A(\theta_{i+1,j}^{n+1} + \theta_{i-1,j}^{n+1}) - BR_{yy}(\theta_{i,j+1}^{n+1} + \theta_{i,j-1}^{n+1}) \\ - R_{xy} \frac{\Delta\eta^2}{2\Delta X \Delta Y} (\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1}) = (2\tau + \Delta\eta)\theta_{i,j}^n - \tau\theta_{i,j}^{n-1} \end{aligned} \quad (14)$$

where $A = \frac{\Delta\eta^2}{\Delta X^2}$, and $B = \frac{\Delta\eta^2}{\Delta Y^2}$

The plate is discretised into equal spaces in both X and Y directions which leads to the following difference equation:

$$\begin{aligned} (2A + 2AR_{yy} + \tau + \Delta\eta)\theta_{i,j}^{n+1} - A(\theta_{i+1,j}^{n+1} + \theta_{i-1,j}^{n+1}) - AR_{yy}(\theta_{i,j+1}^{n+1} + \theta_{i,j-1}^{n+1}) \\ - A \frac{R_{xy}}{2} (\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1}) = (2\tau + \Delta\eta)\theta_{i,j}^n - \tau\theta_{i,j}^{n-1} \end{aligned} \quad (15)$$

The above system of linear algebraic equations can be written in matrix equation as $\Psi\theta = b$. The matrix inverse method is used to determine the dimensionless temperature distribution at each time step. Knowing this, the thermal stress distribution is then found from Eqs. (10)–(12).

Table 1
Properties of unidirectional graphite epoxy composite

$k_{11} = 4.62 \text{ W/(m K)}$	$\alpha_1 = 0.02 \text{ } \mu\text{m/(m K)}$
$k_{22} = 0.72 \text{ W/(m K)}$	$\alpha_2 = 22.5 \text{ } \mu\text{m/(m K)}$
$Q_{11} = 138.8 \text{ GPa}$	$\rho = 1.6 \text{ g/cm}^3$
$Q_{22} = 9.013 \text{ GPa}$	$c = 1 \text{ J/(g K)}$
$Q_{12} = 2.704 \text{ GPa}$	$\bar{\tau} = 1 \times 10^{-12} \text{ s}$

The plate used in the analysis is made of unidirectional graphite epoxy composite. The properties used in the calculations are listed in Table 1 (Tsai and Hahn, 1980).

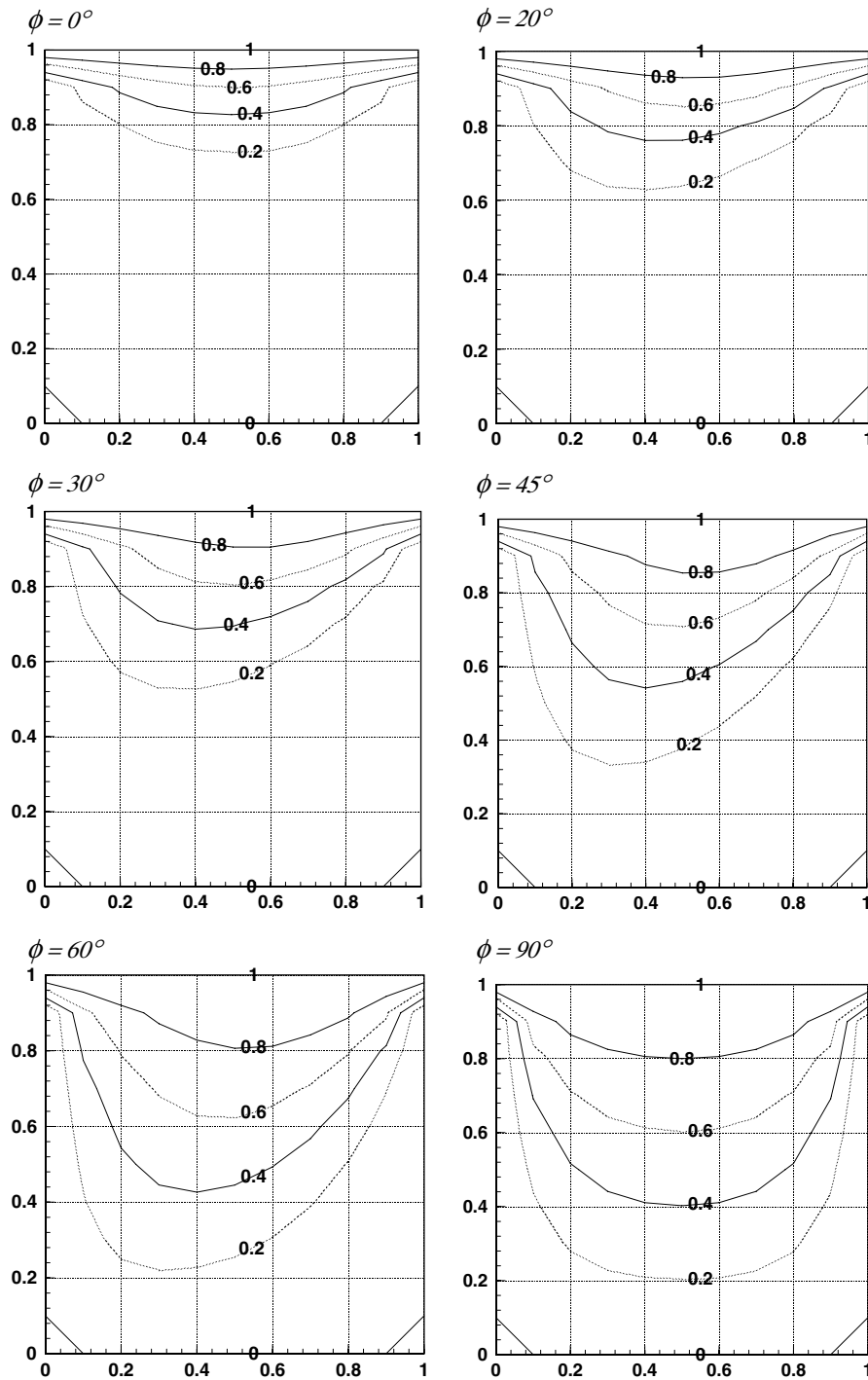


Fig. 2. Contour plots for the dimensionless temperature distributions in the plate at time $\eta = 3$ for different fibre angles.

5. Results and discussion

Transient temperature distribution across the plate is determined first after the application of the upper temperature boundary condition. In Fig. 2, the temperature contour plots are shown for different fibre angles at a specific time. This temperature penetrates deeper as fibres angle increases, which is true since thermal conductivity in the fibres direction is greater than that in the transverse direction.

Fig. 3 shows the transient temperature variation at two locations inside the plate. As noticed, the temperature fluctuates up and down till it reaches a steady value. This interesting behaviour did not appear under the classical heat model shown in Fig. 4 where $\bar{\tau} = 0$. The reason for this fluctuation in temperature may be referred to the interaction between phase lag effect and the anisotropic conductivity of the plate. It seems that neglecting the phase lag will not show the transient temperature fluctuation even though the steady values for temperature are the same at all fibre angles.

The fluctuation in temperature was reflected on the stress distribution across the plate as shown in Fig. 5. It is interesting to see here the variation of thermal stresses (at earlier times) from tensile inside the plate to compressive as we go outside to the edges of the plate. This fluctuation changes from tensile to compressive as time

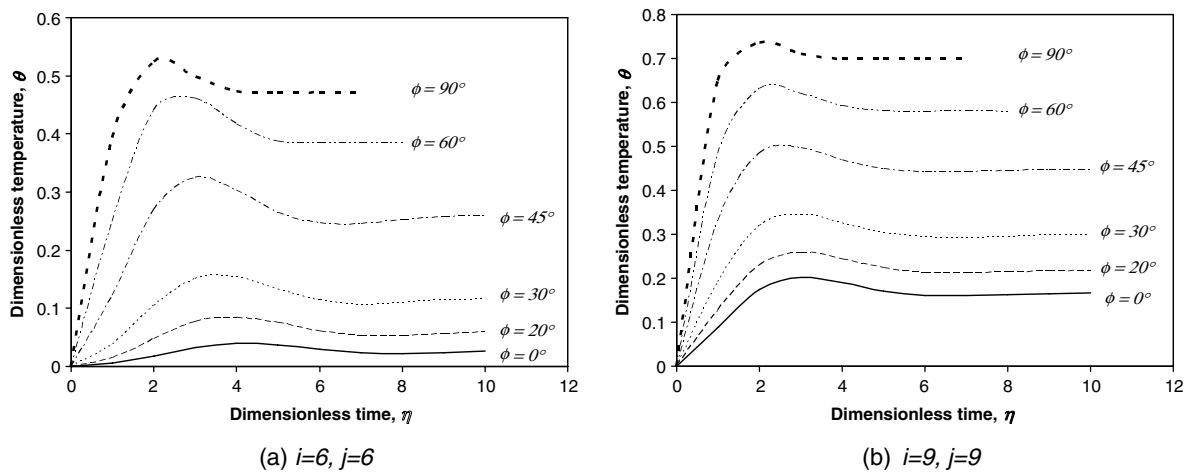


Fig. 3. Transient dimensionless temperature variation at two interior points as a function of fibre angles.

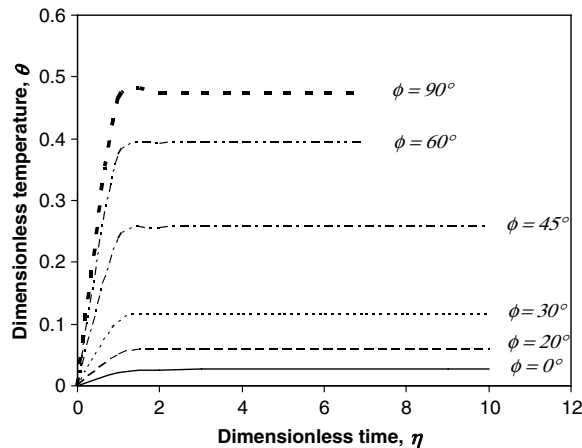


Fig. 4. Transient dimensionless temperature variation at point $i=6, j=6$ as a function of fibre angles under the classical heat model $\bar{\tau} = 0$.

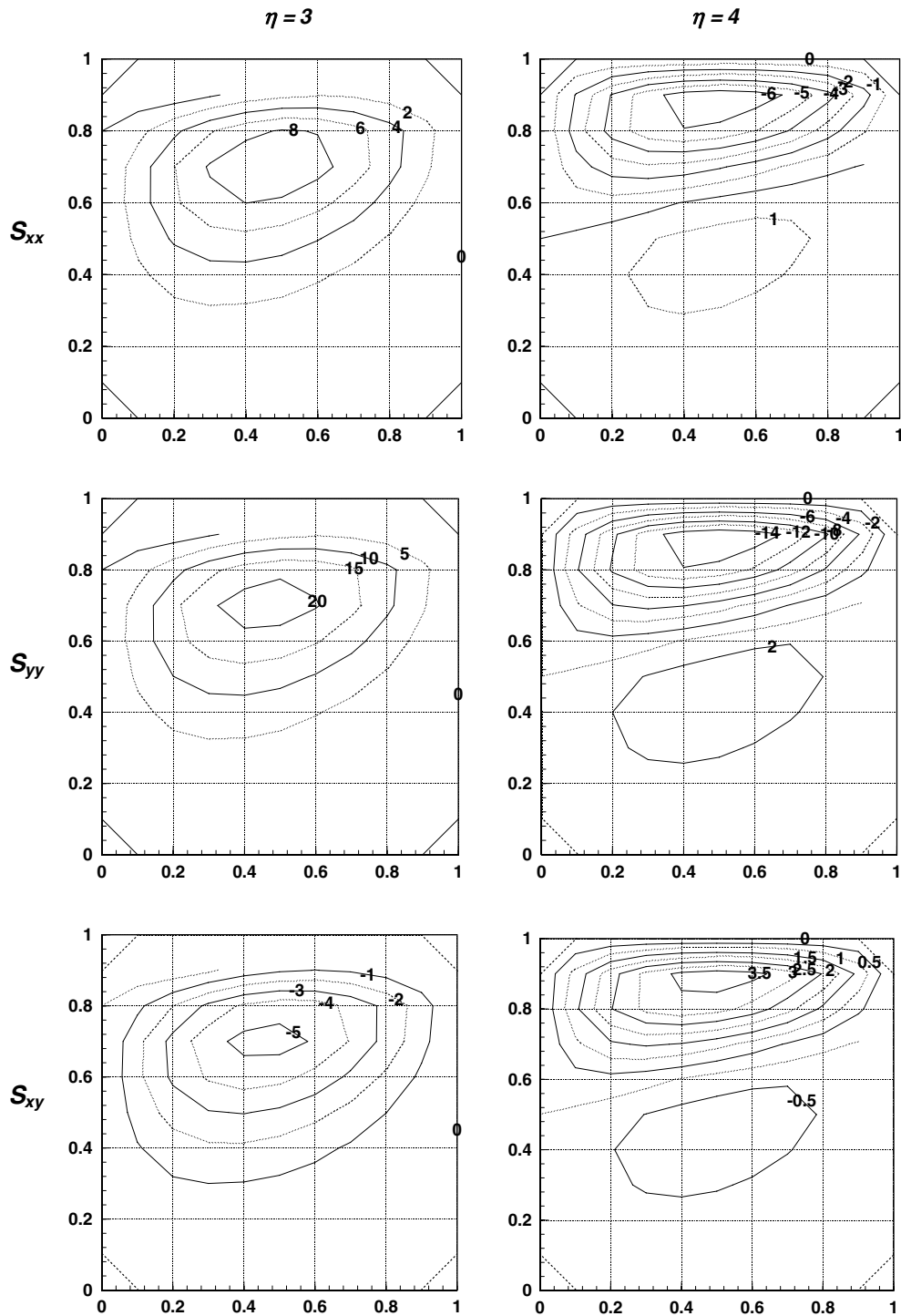


Fig. 5. Contour plots for the normalised stress distributions in the plate at time $\eta = 3$ and time $\eta = 4$ for fibre angle $\phi = 20^\circ$.

progressives. It is worth mentioning here that all stress components, normal and shear, show this fluctuation in their values. Thus, an interior crack may initiate and propagate outward to cause catastrophic failure. In some cases, an edge crack may also cause a fracture of the plate.

Fig. 6 shows the transient longitudinal stress variation at two locations inside the plate. As noticed earlier, the stress fluctuates between tensile and compressive values till it vanishes when time proceeds. Furthermore, it can be seen that this is not the case with the classical (parabolic) heat conduction model where no fluctuation in temperature occurs when $\bar{\tau} = 0$ as shown in Fig. 7. This is very dangerous for the material since failure will initiate at this stage and will propagate at a much faster rate than would be predicted by the classical heat conduction model. Neglecting thermal relaxation will also cause significant errors in thermal stress magnitude. Again, the reason for this fluctuation in stress may be referred to the interaction between phase lag effect and the anisotropic conductivity of the plate.

Fig. 8 shows the variation of longitudinal stress S_{xx} as a function of fibre angle at the same time shown in Fig. 2. Here, when only the thermal stress dependence of fibre orientation is considered, the maxima of the thermal stress moves towards the center of the plate as fibre angle increases; the magnitude becomes large when the combined effects of the temperature boundary conditions and the anisotropy in conductivity are considered.

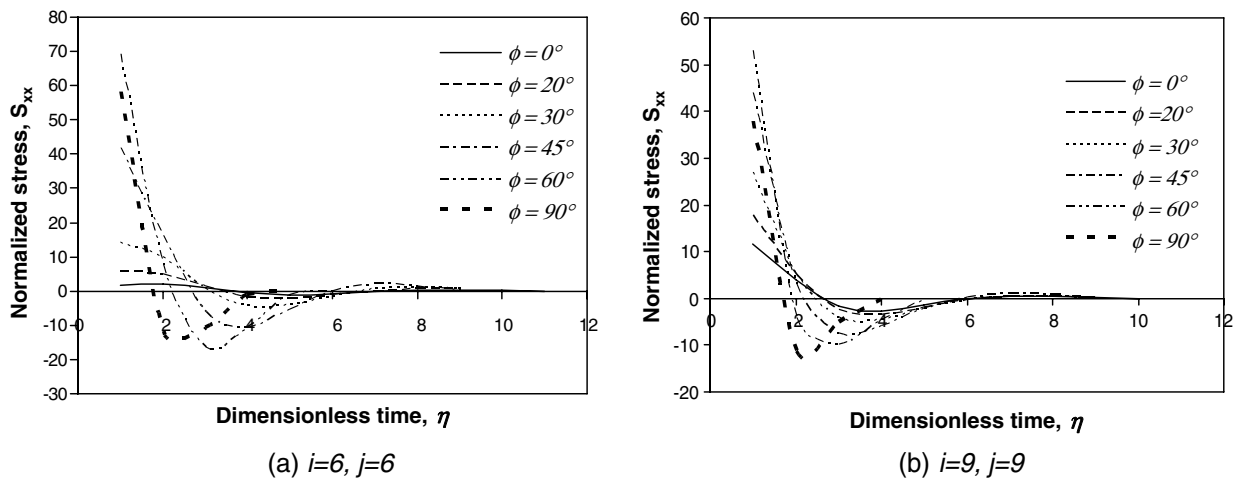


Fig. 6. Transient longitudinal stress variation at two interior points as a function of fibre angle.

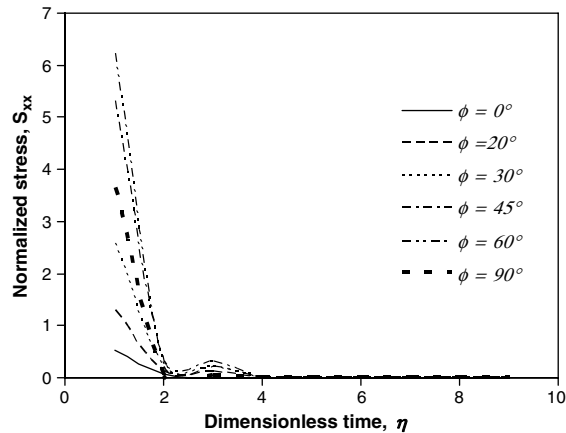


Fig. 7. Transient longitudinal stress variation at point $i = 6, j = 6$ as a function of fibre angle under the classical heat model $\bar{\tau} = 0$.

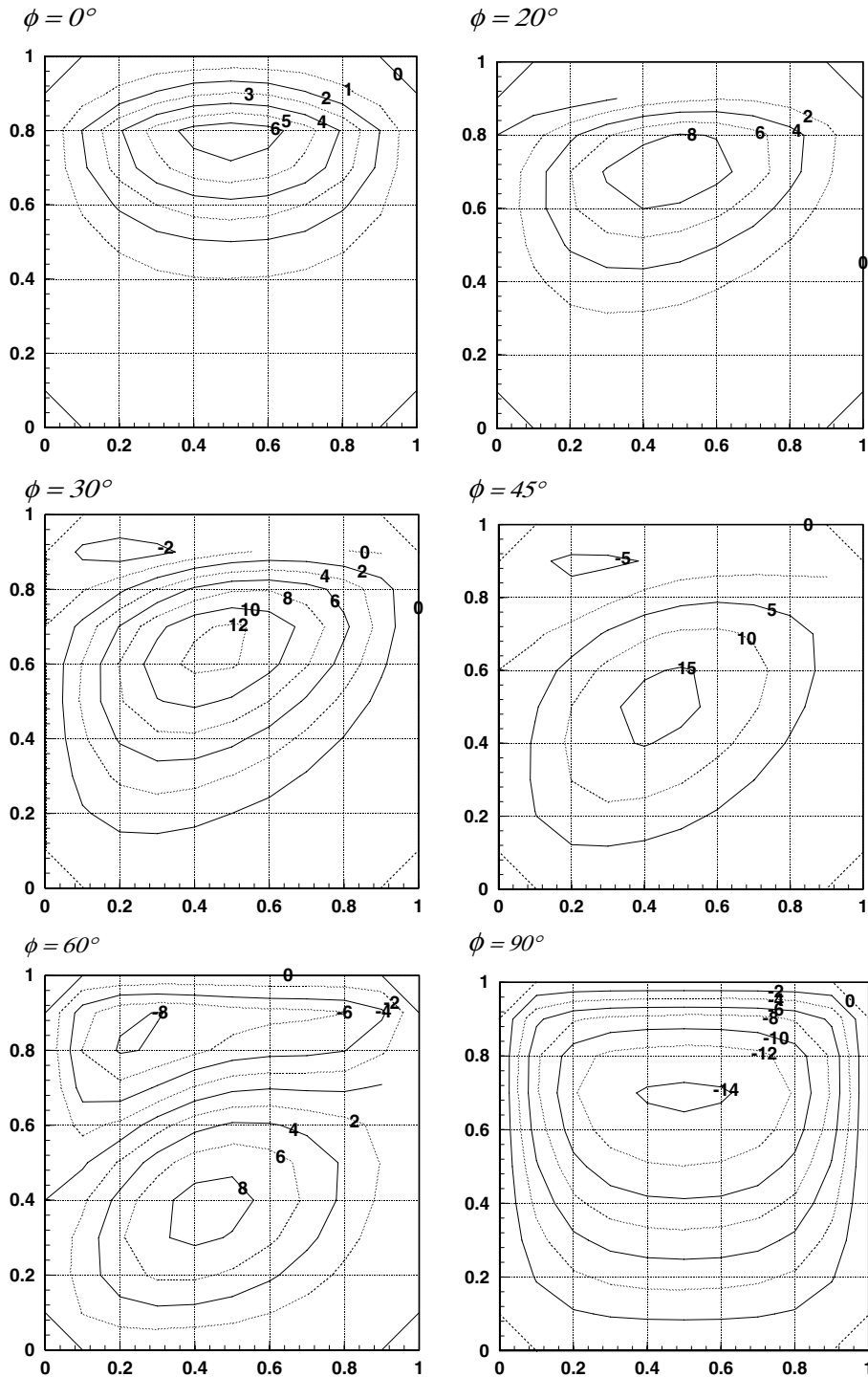


Fig. 8. Contour plots for the normalised longitudinal stress S_{xx} distribution in the plate at time $\eta = 3$ for different fibre angles.

6. Conclusions

The transient temperature and thermal stresses are investigated in a unidirectional composite solid under the hyperbolic heat conduction model. A square composite plate of uniform initial temperature is assumed

to be simply supported and is subjected to a high boundary temperature at the upper edge. The heat conduction equation for an anisotropic solid is modified to include the phase lag effect and is solved numerically using an implicit finite difference scheme. A typical composite material is chosen for the plate with numerical values to facilitate numerical analysis. The results show the significant effect of including the relaxation time where temperature fluctuates up and down as time proceeds. This fluctuation affects all thermal stress components where it fluctuates between tensile and compressive values till it vanishes when time proceeds. Thus, an interior crack at the plate interior will propagate and, in some cases, an edge crack will also cause a fracture of the plate.

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