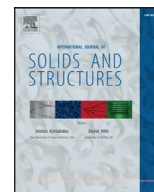




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A continuum model of close packing granular materials for the study of rock filled gabions

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ABSTRACT

We propose a model for the study of rock filled gabions, based on the theory of continua with microstructure, where the interlocking of stones through contacts figures explicitly.

In this model, each stone is partly locked by its first neighbours, appearing as a continuous deformable box. While stones are material items, as such endowed with inertia, boxes are configurational features, as e.g., crack tips or phase boundaries. Two kind of internal actions are then included in the model: (1) material actions that represent the internal forces and moments arising in the boxing process and (2) configurational actions that possibly drive boxes to change shape or make contacts within them be updated.

Following the theory of continua with microstructures, each material and configurational element of the continuum is endowed with geometrical properties, working with respect to the above mentioned internal actions. In order to reduce the complexity of this geometry (and of the ensuing mathematical problem), preserving nonetheless a sufficiently accurate description of the boxing process, we will consider stones and boxes having a one-to-one permanent connection, represented by a symmetric traceless tensor, akin to a fabric tensor.

The set of equilibrium equations that define the mechanical problem is then divided into two groups: (1) balance of material momentum and moment of momentum, expressed in the usual vectorial space of forces and (2) balance of configurational dynamical entities, expressed in the space of second order tensors.

The dissipative and non-dissipative parts of the stresses are then related to a free energy potential and to a dissipation potential, within the framework of rate independent processes.

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1. Introduction

We aim at proposing a mechanical model of a close-packing of randomly shaped stones, appropriate for the analysis of systems such as rock filled gabions or confined stone walls.

Rock filled gabions have long been used in civil engineering, especially as retaining walls. Largely diffused as such, they have rarely been operated as bearing walls in buildings. Today, some edifices, erected in developed, non-seismic, areas, include gabions as sustaining structure or cladding. However, it is the recourse to these systems in low-tech and even rural environments suffering seismic hazard that captures our attention.

The confined stone wall (CSW) technology for sustainable housing was designed and developed by the French international aid association “Architecture & Développement” since 2005 and is currently implemented in Morocco, India, Haiti and Nepal. Similar technologies were used in Haiti, where the “Gabion House project”, was carried on by the Australian Red Cross in 2012 (D’Urzo and Schneider, 2012), and Nepal, where the “Gabion band” technology was proposed by R. Langenbach for the reconstruction of a demonstration house after the 2015 earthquakes. Those low cost applications, helping fragile populations in vulnerable areas, require today a knowledge of the seismic behaviour of gabion structures that has not been developed so far.

Due to the relative resistance of cages, gabion walls are in fact normally designed as gravity walls, their safety depending basically on the stacking of gabions, seen as rigid blocks, opposing the soil’s pressure. Limit analysis is then usually applied for ULS check. As

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a consequence, very few experimental results are available on the behaviour of gabions as deformable units:

- Agostini et al. (1987) performed a compression and shear (static) test on full size gabions for determining the strength and deformation parameters of the gabion box.
- Lin et al. (2010) realised a numerical simulations regarding uniaxial compression test of a single gabion based on the results obtained by Agostini et al. (1987).
- Jiang and Wang (2011) performed a compression and direct shear test for multi-group gabions, obtaining their stress-strain behaviour.

FEM analyses made at INSA Rennes (France) concluded that, following Euro-code 8, CSW can resist earthquakes of magnitudes greater than 5.5 and a maximum ground acceleration of 3 m/s^2 (Seismic Zone 4), with a first natural period in the 0.3 s–1 s range (Guezouli and Hjiat, 2012).

These results have been obtained considering the rock filler as a continuous body and the steel wire cage as an elastic membrane. Our scope is to investigate the possibility of discontinuous failures occurring under dynamic loadings, starting as a local disarrangements of stones and propagating with little energy demand.

For this purpose, we are keen to neglect most of the elastic energy stored in the system and consider stones as rigid bodies, contact energies apart. As stones are close-packed in the aggregate, interlocking each others in such a way that their mobility is very reduced, we need to detect mechanisms that can, perhaps partly, unlock a stone using its kinetic energy and the elastic energy of the contact. A similar event will most probably be localised and require little energy to appear and propagate.

The close packing feature and the need to detect the onset of an eventually localised granular flow lead us to select a particular type of model, where the gabion's filler is seen as a bonded masonry rather than a granular fluid. For this reasons, if we rely on theories derived from or kin to the seminal paper by Goodman and Cowin (1972), as Capriz and Mullenger (1995) and, Giovine (2008), we divert from these examples, which provide tools to obtain a finer description of the granular flow than we actually set up here.

On the other hand, the concept of fabric tensor, arising in the study of sands (Oda, 1972) and widely used thence, is appropriate here to describe contact networks, even though—to simplify matters—we will not pivot on the inherent statistical character of the original definition. It can be noticed that this concept has also been used in the mechanics of porous materials (Cowin, 2004; Harigan and Mann, 1984), as an analogy exists between trabeculae—or homologous frameworks, channelling the strain energy throughout the body, and contact networks. But the dissimilarity of the two is very important here, as the former are material elements, while the latter are not.

Following an idea presented in Brocato (2018), we assume that stones are boxed by their neighbours in a way that depends on their shape and on the shape of the boxing system. Therefore, we consider stones and boxes as separate entities in the model, interacting with each others through point-wise contacts: stones force boxes to change their shape; boxes induce a confinement for the motion of stones.

A precision is needed concerning our vocabulary: the adjective 'material' is here used to denote things formed of matter, bodily geometrical entities, independently on their representation in space. It is important to avoid confusion with alternative meanings appearing in the literature, where the same adjective can be used with reference to a particular representation of the body in the Euclidian space, to denote a reference configuration, or as a synonym of 'Lagrangian'—distinguished from 'spatial' or 'Eulerian'—to denote the description of processes.

2. Geometry and kinematics

To describe a close-packing of randomly shaped stones, we start looking at stones as rigid bodies of uniform density and focus only on two characteristic features of such a body: its inertial behaviour and its contact interactions.

As quoted in the Introduction, the neighbourhood of a stone can be seen as a box confining it. For the purposes of the present analysis, such a box can be understood as a simply-connected closed deformable surface, without mass, and will be supposed to be a part of a continuous network. To focus on this network, one can think to a space-filling arrangement of polyhedral cells.

As we wish to study a particular class of processes, with very reduced rearrangements of the aggregate, we may assume that stones and boxes have a permanent identity and a permanent relation to each other, such that there is one and only one stone always in the same box. Even if the term 'cell' is etymologically appropriate to indicate such a permanent confinement of a stone, we prefer to use 'box' instead, to avoid confusion with the more familiar concept of elementary cell, related to some statistical or periodical representativity of material properties.

Still to avoid confusion with the standard terminology, we will address material elements as 'stones', though they will be the infinitesimal parts of a continuum, endowed with mass and with some geometric characteristics.

It is important to notice that, in formulating the model, we assume boxes not to be material elements, but configurational entities, such as phase boundaries or crack tips. Then an analogy exists with other cases where a configurational microstructure can be put forward, such as liquids with vapour bubbles or solids with micro-cracks (Bongué-Boma and Brocato, 2008; 2010), and a similar approach can be followed in formulating a model.

Let \mathcal{E} be the Euclidean space, $\mathcal{V} \cong \mathbb{R}^3$ its translation space, $\text{SO}(3)$ the group of rotations of \mathcal{E} , $\text{GL}(\mathcal{U})$ the general linear group over any linear space \mathcal{U} . When considering $\text{GL}(\mathcal{V})$ let us define the subspaces (reference to \mathcal{V} is omitted for shortness): $\text{Sym} \subset \text{GL}(\mathcal{V})$ the subspace of symmetric tensors, $\text{Dev} \subset \text{Sym}$ the subspace of symmetric traceless tensors (or deviators), $\text{Skw} \subset \text{GL}(\mathcal{V})$ the subspace of skew-symmetric tensors.

To implement the ideas outlined above into a Lagrangian description of motion, let us consider the following information on a reference configuration of the system $\mathcal{B}_* \subset \mathcal{E}$, (remember that each stone is included in a box and every box contains a single stone):

- each stone's centre of mass has a reference placement $x_* \in \mathcal{B}_*$, by which all values related to a given stone or to the corresponding box will be labelled;
- each stone's principal axes are given by three unit vectors $b_*^{(i)}(x_*) \in S^2$, $i = 1, 2, 3$, or by the orthogonal tensor $Q_*(x_*) \in \text{SO}(3)$ whose columns are the $b_*^{(i)}(x_*)$;
- each box' centre coincides with the centre of the stone it contains;
- each stone's boundary is in contact with the box enclosing it and the geometry of these contacts is described by a symmetric traceless tensor $M_*(x_*) \in \text{Dev}$.

To understand the nature and the role of M_* , we recall some assumptions made in Brocato (2018).

Let us consider that the stone's boundary is in contact with the box enclosing it on a set of $n_*(x_*) \in \mathbb{N}$ facets (the average of n_* on some subset of \mathcal{B}_* being the coordination number in that material region), each given by a position vector $s_*^{(j)}(x_*) \in \mathcal{V}$ relative to x_* and a surface vector $m_*^{(j)}(x_*) \in \mathcal{V}$, $j = 1, \dots, n_*(x_*)$. In the reference configuration we assume $n_*(x_*) \geq 4$, $\forall x_*$; it can be noticed that numerical experiments show the coordination number of assemblies of narrowly disperse spheres be in the range 4–6

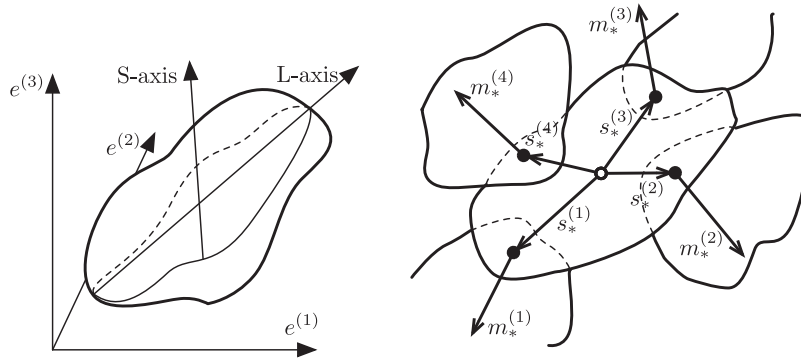


Fig. 1. Sketch of a stone with its neighbourhood, borrowed from Oda (1972) and adapted to the present notation. On the left, the longest and shortest axis of a stone are depicted to show a possible choice of the local reference frame and the information content of the orthogonal tensor Q . On the right, a neighbourhood of the stone is displayed, with the latter locked by four contacts, each represented by one couple of contact vectors, $s_*^{(j)}$ and $m_*^{(j)}$.

(Evans and Brown, 2014), while values below 4 are observed at the critical state of shearing (Rothenburg and Kruyt, 2004).

Hence the j th contact occurs on twin facets, one on the boundary of the stone and the other on the box, that have a reference point at $x_* + s_*^{(j)}(x_*)$, a normal directed as $m_*^{(j)}(x_*)$ pointing outward from the stone (i.e., such that $m_*^{(j)} \cdot s_*^{(j)} \geq 0$ for all j and x_*), and a size proportional to the norm of this vector.

The main difference between an interlocking structural system as considered in Brocato (2018) and the present case is that, even in the reference configuration, stones and boxes do not match geometrically and have random shapes. Hence the sets $\{s_*^{(j)}(x_*) \mid j = 1, \dots, n_*(x_*)\}$ and $\{m_*^{(j)}(x_*) \mid j = 1, \dots, n_*(x_*)\}$ depend on x_* (see Fig. 1).

Information on the contact vectors is paramount for the model, but a simplified description is needed. Let us tackle the question above through the assumptions:

A1 discard information on the area of the contact facets and assume that only the directions of the surface vectors play a role:

$$\|m_*^{(i)}\| = 1;$$

A2 take an embedding of the $m_*^{(i)}$ into \mathbb{R}^5 (see Ericksen, 1991; Brocato and Capriz, 2000), and consider the average:

$$M_*^{(i)} := m_*^{(i)} \otimes m_*^{(i)} - \frac{1}{3}I; \quad M_* := \frac{1}{n_*} \sum_{i=1}^{n_*} M_*^{(i)}. \quad (1)$$

(I the unit of second order tensors): the symmetric traceless tensor M_* , we call the ‘hem’ tensor—as it measures how the box hems the stone in, will thus represent any arrangement of the $m_*^{(i)}$ at x_* ;

A3 assume that one characteristic length, σ , suffices to describe the inertial behaviour of stones.

Assumption A1 is consistent with the idea that the contact stress will be transmitted through surfaces whose size is so much smaller than the square of the characteristic length of the stone that they can be modelled as points.

Assumption A2, representing all contact facets of a stone by an average tensor, means that less information is available for the analysis. The accessible information is related to the spectral decomposition of M_* : if one eigenvalue prevails, then the box confines the stone in that direction more than in the others (we need to precise the extent of this effect). If no eigenvalue prevails, the stone is equally confined from everywhere by the box. Intermediate conditions are possible, with a third limit case, where the boxing is mainly effective on a plane.

Using the nomenclature ‘hem tensor’ instead then any of the already introduced ones, as ‘fabric tensor’, ‘force-fabric tensor’,

or ‘contact density tensor’, allow us to avoid misunderstanding the statistical origin of the concept. At this stage, we do not attach probabilities to the geometrical description of contacts, even though we understand that a statistical approach is more consistent with experiments and will have to be pursued in the future.

Assumption A3 is taken here as the simplest option, generalisation to more complex cases being possible following steps given in Capriz (1989) and widely discussed in the literature (see Giovine, 2008).

Coming to the unknown fields describing the class of motions we are going to study, let $\tau \in \mathbb{R}_{\geq 0}$ be the time. We will omit, unless necessary, the explicit reference to the variables x_* and τ as it would appear in the local values of all fields entering the Lagrangian description. Let us consider what follows:

- the stone’s centre of mass has an actual placement $x \in \mathcal{B} \subset \mathcal{E}$, where \mathcal{B} is the actual image of \mathcal{B}_* given by the map $x(\cdot, \tau)$; let $F = \text{Grad} x \in \text{GL}(\mathcal{V})$ be the gradient of this map;
- the stone’s principal axes have an actual instance $b^{(i)} = Qb_*^{(i)} \in S^2$, $i = 1, 2, 3$, given by the local value of a second unknown field $Q \in \text{SO}(3)$;
- the box’ centre has an actual placement $y \in \mathcal{E}$; let us call:

$$r := y - x \quad (2)$$

- the relative position of the box’ centre;
- the hem is given by an actual tensor $M \in \text{Dev}(\mathcal{V})$.

To focus on the particular kind of problems we wish to study, where the aggregate moves with slightly sensible rearrangements, let us take some more assumptions:

A4 y has the usual differential properties of x and we can define $G := \text{Grady} \in \text{GL}(\mathcal{V})$;

A5 there are processes during which the contact facets preserve their identity; then the number of contacts of any stone with its box is always the same n_* , and the vectors $s_*^{(j)}$ and $m_*^{(j)}$, $j = 1, \dots, n_*$ have, in the actual configuration, a unique image, $s^{(j)}$ and $m^{(j)}$ respectively;

A6 in processes respecting assumption A5, the $s^{(i)}$ move according to G and r as:

$$s^{(i)} := Gs_*^{(i)} + r \quad \forall i = 1, \dots, n_*, \quad (3)$$

so that G can be understood as an ellipsoid describing the shape of the box;

A7 still in processes respecting A5, the $m^{(i)}$ rotate following a first gradient approximation of the field Q in the neighbourhood of x_* :

$$m^{(i)} := Qm_*^{(i)} + (\text{Grad} Q)(m_*^{(i)} \otimes s_*^{(i)}); \quad (4)$$

A discussion about these assumptions is given in Brocato (2018); we report here some essential issues, adapted to the present topic.

Assumption A5 is consistent with the case of interlocking square-cut blocks, as studied in Brocato (2018); here it helps coping with the randomness of the aggregate, but must be considered as a restriction to generality, as it negates the possibility to master contact creations or annihilations. Hence it represents a particular class of processes, described into further details by the two assumptions that follow.

Assumption A6 reduces the evolution of the contact facets' centre to an affine transformation from the reference configuration as if those points were linked to the centre of the box by material vectors of a continuum whose transformation is y . We understand this assumption as the simplest possible one, and deem it to be realistic within the same limits than the previous assumptions.

Assumption A7 is based on the idea that the contact faces are entrained by the motion of the stone. It can then happen that a pure entrainment of the hem tensor ensues, i.e., that the evolution of this tensor is kinematically constrained to Q and its gradient. For this reason, let us call a process of 'entrained hems' one made accessible by assumption A5.

To make the previous statement precise, we need defining the present image of the hem tensor. Let us define M as M_* in Eq. (1), replacing $m_*^{(i)}$ with $m^{(i)}$, n_* with n ; let us also introduce the third order hem tensor:

$$\mathbf{m}_* := \frac{1}{n_*} \sum_{i=1}^{n_*} m_*^{(i)} \otimes s_*^{(i)} \otimes m_*^{(i)}.$$

In a process of entrained hems, we can then express M in terms of the reference image M_* , of \mathbf{m}_* , and of Q , within the limit of the first gradient approximation accepted in Eq. (4):

$$M_e = Q M_* Q^T + 2 \text{sym}(Q \mathbf{m}_* (\text{Grad} Q)^T) \quad (5)$$

(priority is given to the right-hand side of operators, 'sym' denotes the symmetric part), where the subscript 'e' is for 'entrained'.

Let us now consider a more general case, when the actual hem tensor is M and the actual stone rotation is Q , but M is not purely entrained by Q . Let us define M_0 and \mathbf{m}_0 such that:

$$M = Q M_0 Q^T + 2 \text{sym}(Q \mathbf{m}_0 (\text{Grad} Q)^T). \quad (6)$$

As M differs from M_e , so it is for M_0 and \mathbf{m}_0 and \mathbf{m}_* .

Starting from (6), the rate of change of M can be expressed as:

$$\dot{M} = B W + \mathbf{b} \text{grad} W + Z,$$

where $W := \dot{Q} Q^T \in \text{Skw}$ and 'grad' denotes the gradient with respect to x (i.e., $\text{grad} W = (\text{Grad} W) F^{-1}$). The operators B and \mathbf{b} , are defined as:

$$B \in \text{GL}(\text{Skw}, \text{Sym}) : B W = W M - M W \quad \forall W \in \text{Skw}; \quad (7)$$

$$\mathbf{b} \in \text{GL}(\text{GL}(\mathcal{V}, \text{Skw}), \text{Sym}) : \mathbf{b} X = 2 \text{sym}(Q \mathbf{m}_0 (X F)^T) \\ \forall X \in \text{GL}(\mathcal{V}, \text{Skw});$$

and Z is :

$$Z := Q \dot{M}_0 Q^T + 2 \text{sym}(Q \dot{\mathbf{m}}_0 (\text{Grad} Q)^T).$$

Z depends on the two velocities \dot{M}_0 and $\dot{\mathbf{m}}_0$ that are defined in the reference configuration: the process leading from M_* and \mathbf{m}_* to M_0 and \mathbf{m}_0 is a configurational transformation and Z is its rate in the actual configuration.

A Lagrangian description of the motion of the close-packing aggregate can be based on the following unknowns:

- the placement of the stone's centre x , with gradient F ;
- the rotation of the stone's axes, Q ;

- the placement of the box' centre, relative to centre of the stone they enclose, r , or absolute, y ;
- the ellipsoid describing the shape of the box G , related to y by (3);
- the hem tensor M .

3. Balance laws

3.1. Conservation of mass

We have not introduced any measure of the porosity or other parameters usually adopted to describe the conservation of mass in granular flows. The reason for this disregard is that we focus on problems where we expect no sensible evolutions of the porosity of the aggregate to occur. As a consequence, we admit that the conservation of mass can be simply stated as for a standard continuum, taking the apparent density ρ_* in the reference configuration as the sole relevant information.

By definition, ρ_* is the mass of the material the stones are made of, taken per unit volume of the aggregate. If 'det' denotes the determinant, we have an actual apparent density ρ obeying the conservation law:

$$\rho \det F = \rho_*. \quad (8)$$

3.2. Conservation of momentum

The term momentum must be taken here in a broader sense than standard, including actions working with respect to microstructural velocities.

Let $f^{(s)}$ and $c^{(s)}$ be the fields of internal forces and moments acting on the stones due to the confinement effect of the boxes and expressed per unit volume. Let T and S be fields of configurational tensors, which will be further analysed later. Let $f^{(ext)}$ and $c^{(ext)}$ be the fields of external forces and moments acting on the unit volume, or on the unit surface of the body (we use the same notation, regardless of the dimension, as the distinction between actions per unit volume and actions per unit surface will be evident in the equations). Let \mathbf{e} be Ricci's third order alternating tensor, $\partial \mathcal{B}$ the boundary of \mathcal{B} and n the normal to a surface element of $\partial \mathcal{B}$. The following balance equations and boundary conditions can be stated, following methods presented in Germain (1973) (see Brocato, 2018):

- In \mathcal{B} :

$$\begin{cases} f^{(s)} + f^{(ext)} = \rho \sigma^3 \dot{u} \\ \text{div} T = f^{(s)} \\ \text{div}(\mathbf{b}^T S) - B^T S - \frac{1}{2} \mathbf{e}(c^{(ext)} + c^{(s)}) = \frac{1}{2} \rho \sigma^5 \dot{W} \end{cases} \quad (9)$$

- On $\partial \mathcal{B}$:

$$\begin{cases} f^{(ext)} = 0 \\ \text{if } \dot{x} \neq \dot{y} \text{ is allowed : } \begin{cases} T^T n = 0 \\ \mathbf{b}^T S n = -\frac{1}{2} \mathbf{e} c^{(ext)} \end{cases} \\ \text{if not : } \begin{cases} T^T n = f^{(ext)} \\ \mathbf{b}^T S n = -\frac{1}{2} \mathbf{e} c^{(ext)} \end{cases} \end{cases}$$

The boundary of a gabion is normally confined by a steel wire mesh. It can be expected that the stones of the outer layers are more restrictively confined, due to their contact with the mesh, than those in the bulk of the gabion. Hence it can be reasonably stated that the boxes of the boundary have their centre fixed to that of the stone and the second type of boundary conditions is

most probably to be adopted (as if the link between the stone and its box was kinematically perfect). If not, the surface will not admit the application of external forces and the configurational tensor T will be balanced to zero at the boundary as a self-stress.

Information on the different constitutive representations of the system, next to the boundary or in the bulk, which differentiate the boxing effect of the steel wire mesh from that of the neighbouring stones, will be given in the next section.

There is a last equilibrium condition that cannot be expressed locally unless a specific description of the evolution of the contact points is given, which expresses the fact that S is powerless if stones do not spin:

$$\int_{\mathcal{C}} S \cdot (Q\hat{A}Q^T + 2\text{sym}(Q\hat{\mathbf{a}}(\text{Grad}Q)^T)) d(\text{vol}) = 0 \quad \forall \hat{A}, \hat{\mathbf{a}}, \mathcal{C} \subseteq \mathcal{B}; \quad (10)$$

where \hat{A} and $\hat{\mathbf{a}}$ are virtual velocities of M_\circ and \mathbf{m}_\circ respectively and are thus not totally independent variables as these tensors derive from the local description of contacts. Writing a local expression derived from (10) is then a matter of constitutive analysis, which might be based on a local modelling of the geometry of the aggregate, starting, e.g., from spatial tilings as in Gabbriellini (2009).

3.3. Conservation of energy

In writing the inertial actions appearing in the equations of balance of momentum we have assumed that the kinetic energy theorem applies, so that the local equation of conservation of the total energy can be split in the balance of mechanical powers (identically satisfied by field obeying the previous equations of balance of momentum) and in that of the internal energy (Capriz, 1989).

Let ε be the internal energy per unit mass, λ the heat exchanged per unit mass and q the heat flux; the conservation of the internal energy is:

$$\rho \dot{\varepsilon} = -\pi^{(int)} + \rho \lambda - \text{div} q$$

where

$$\pi^{(int)} := -f^{(s)} \cdot (\dot{\mathbf{y}} - \dot{\mathbf{x}}) - \frac{1}{2} c^{(s)} \cdot \mathbf{e}W - T \cdot \text{grad} \dot{\mathbf{y}} - S \cdot \dot{M} \quad (11)$$

is the specific power of internal actions. The latter definition shows that T is a configurational stress working for the velocity gradient of the box' centre and S a configurational stress working for the rate of change of the hem tensor.

Notice that, when (10) leads to a local condition, we have $S \cdot Z = 0$, and thus:

$$S \cdot \dot{M} = S \cdot (BW + \mathbf{b} \text{grad} W).$$

3.4. Production of entropy

We can express the Second Principle in the form of the Clausius–Duhem inequality. Let $\psi := \varepsilon - \theta \eta$ be the Helmholtz free energy per unit mass, with θ the temperature and η the entropy per unit mass. The inequality is:

$$\rho (\dot{\psi} + \dot{\theta} \eta) \leq -\pi^{(int)} - \theta^{-1} q \cdot \text{grad} \theta;$$

and for isothermal processes (which we consider to simplify matters at a first stage of the analysis).

$$-\rho \dot{\psi} - \pi^{(int)} = \varphi \geq 0; \quad (12)$$

where φ is a purely mechanical dissipation rate.

4. Constitutive statements

4.1. Material constitutive statements

The definition (11) of the power of internal actions shows that there is no power in the model linearly related to the velocity gradient of material elements (which are rather seen as separate stones) or to the gradient of their spin. Correspondingly, there is no stress of Cauchy type neither of Cosserat type; the system we describe is simply a set of material points (the stones) endowed with special directions, each tied to a box with possibly non linear links, so that only internal forces and moments arise. The fields x and Q describe the kinematics of these points, and we have assumed them be continuous at least with their first gradients, which have been included in the analysis to model the continuity of mass (8) and of the hem tensor (5).

The boxes have, on the contrary, the kinematical structure of a classical continuum, with transformation y and transformation gradient G , though they have no mass and we do not expect them to undergo any reversible process. Actually they give the background where the stones (that are the material elements here) move, tied as they are to the boxes.

The movements of the stones must be measured with respect to those of the boxes; if totally entrained by the box they belong to, they do not modify the free energy of the system, otherwise they do, so that we have to choose a measure of their relative motion and use it as a variable in the free energy functional.

Let us consider the following objective tensor giving, in the reference configuration, the rotation of a stone relative to the deformation of the box:

$$H := Q^T G.$$

Notice that H is not a rotation, as this is—in general—the case of the relative motion of a rigid body with respect to a deformable one. A polar decomposition of H would provide a variable of reduced dimension, namely the rotation tensor of the principal axes of H , that will possibly simplify matters keeping them well representative of observations, but we will not enter into such details here.

A second variable representing the relative motion, namely translations, is r , as defined by (2). We take:

$$t := Q^T r$$

and assume:

A8 The Helmholtz free energy is a local function of the variables describing the motion of the boxes with respect to the stones, in the reference configuration:

$$\psi = \psi(t, H).$$

Let us now call upon the Clausius–Duhem inequality in its isothermal form (12) and the previous statement:

$$-\rho Q \frac{\partial \psi}{\partial t} \cdot \dot{r} + \left(\rho \frac{\partial \psi}{\partial t} \otimes t - \rho Q \frac{\partial \psi}{\partial H} G^T \right) \cdot W + \rho Q \frac{\partial \psi}{\partial H} F^T \cdot \text{grad} \dot{\mathbf{y}} + f^{(s)} \cdot \dot{r} + \left(\frac{1}{2} \mathbf{e} c^{(s)} + B^T S \right) \cdot W + T \cdot \text{grad} \dot{\mathbf{y}} + S \cdot (\mathbf{b} \text{grad} W + Z) \geq 0$$

for all processes described by the rates appearing in the equation.

Let us take $\text{grad} \dot{\mathbf{y}} = 0$, $W = 0$ (hence $\text{grad} W = 0$), and $Z = 0$; a process where \dot{r} is arbitrary set can be contrived. We can so define the irreversible part of the internal force, $\bar{f}^{(s)}$, and derive from (12) the constitutive statement that the dissipation rate associated with this force is non negative:

$$\bar{f}^{(s)} := f^{(s)} - \rho Q \frac{\partial \psi}{\partial t}; \quad \bar{f}^{(s)} \cdot \dot{r} \geq 0 \quad \forall \dot{r}. \quad (13)$$

Take then $\dot{r} = 0$, $\text{grad}\dot{y} = 0$, $\text{grad}W = 0$, and $Z = 0$, and leave W as an arbitrary variable to derive the definition of the irreversible part of the internal moment and the corresponding constitutive statement of non negative dissipation rate:

$$\begin{aligned} \bar{c}^{(s)} &:= c^{(s)} + \rho \mathbf{e} \frac{\partial \psi}{\partial t} \otimes t - \rho \mathbf{e} Q \frac{\partial \psi}{\partial H} G^T; \\ \left(\frac{1}{2} \mathbf{e} \bar{c}^{(s)} + B^T S \right) \cdot W &\geq 0 \forall W; \end{aligned} \quad (14)$$

notice that we have left the term $B^T S$ out of the definition of $\bar{c}^{(s)}$.

Finally, let us define:

$$\bar{T} := T + \rho Q \frac{\partial \psi}{\partial H} F^T; \quad (15)$$

which entails the constitutive equation of balance:

$$GF^{-1} \mathbf{e}(T - \bar{T}) + Q^T (f^{(s)} - \bar{f}^{(s)}) \otimes t + c^{(s)} - \bar{c}^{(s)} = 0; \quad (16)$$

which is of a higher order than the equations of balance of momentum (see the factor $GF^{-1} = \text{grad}y$). This equation is a necessary condition for the given form of ψ to apply, but it is not written in terms of dissipation rates: it expresses a rotational equilibrium where configurational tensors and material moments are balanced. It shows that, apart from the kinematic factor $\text{grad}y$, the skew symmetric part of the reversible part of the stress T (working for the rotation rate of the box) has the same constitutive information content of the reversible part of the internal moment and of the moment of the reversible part of the internal forces (both working for the rotation rate of the stones); possibly (if $x = y$) the sum of the last two is the axial vector of the former.

Notice that $Z = 0$ is not necessarily compatible with $\text{grad}y$ arbitrary variable. As already quoted, there is the need of a finer constitutive description of Z , regarding the evolution of contacts, and there can be physical circumstances where M_o , \mathbf{m}_o , and G , are correlated, so that a relationship between Z and $\text{grad}y$ ensues. To simplify matters, let us exclude in the following the occurrence of such conditions.

The Clausius–Duhem inequality is now written under the form:

$$\bar{f}^{(s)} \cdot \dot{r} + \frac{1}{2} \mathbf{e} \bar{c}^{(s)} \cdot W + \bar{T} \cdot \text{grad}\dot{y} + S \cdot \dot{M} \geq 0;$$

where only the first two terms are intrinsically dissipative, the last two being due to a rearrangement of the microstructure and possibly demand an energy release rate (from the two others) to occur.

4.1.1. A simple case

If the processes described by \dot{r} and W can be conducted reversibly (i.e., with arbitrary sign of these time rate), then the inequalities (13) and (14) entail that the dissipative internal forces $\bar{f}^{(s)}$ and moments $\bar{c}^{(s)}$ are null, the free energy containing all information on the constitutive behaviour of the stone-box links:

$$f^{(s)} = \rho \frac{\partial \psi}{\partial r}; \quad c^{(s)} + \mathbf{e} B^T S = \rho \mathbf{e} \left(Q \frac{\partial \psi}{\partial H} G^T - \frac{\partial \psi}{\partial t} \otimes t \right);$$

and the dissipation rate is

$$\varphi = \bar{T} \cdot \text{grad}\dot{y} + S \cdot \dot{M}.$$

The case described by these equations is one where the stones are elastically constrained by the boxes; whatever relative motion they take modifies the free energy of the system (reversibly). We can refer to this system as one with perfectly masoned bonds, as it could be the case when each stone is placed by an expert mason in building a dry-jointed wall.

The condition $\text{grad}\dot{y} = 0$ & $\dot{M} = 0$ is that of a system where the stones are tied with elastic links to the boxes and the latter do not change. If $\text{grad}\dot{y} \neq 0$, some boxes change shape, though all contacts are maintained. If $\dot{M} \neq 0$, some contact of some stone change, i.e.,

the above mentioned links are rearranged, annihilated, or created. These two are the configurational kind of changes that we are going to analyse in the next subsection.

4.1.2. Constitutive description of the steel wire mesh

As already quoted, the boundary conditions to be normally adopted are those of the second type in the set of equilibrium equations (9), because all sides of the gabion are usually confined by the steel wire mesh. When modelling the boxing effect made by this mesh at the boundary, a constitutive description of T can be assumed in a layer next to the boundary that incorporates the material behaviour of the mesh. For this purpose, Eq. (15) can be specialised, writing an expression of ψ that takes the elasticity of the mesh into account and introducing rules for \bar{T} that reckon with the plasticity of the mesh.

4.2. Configurational constitutive statements

Four fields have been introduced to describe the evolution of the system (x , y , Q , and M); two of them represent the absolute motion of stones (x and Q), one, with its first gradient, represents the absolute motion of the boxes (y , with G), and one is a representation of the stone-box connection, named the hem tensor (M). The free energy of the system has been written in terms of relative transformations of the box with respect to the stone (t and H), that can be directly computed from the previous fields (x , y , Q , and G), excluding the hem tensor (M).

A first kind of configurational transformation is one where the shape of the box changes, so that a different space is left available for the stone: when the latter rotates by Q , the effect on H , and thus on ψ , is different because of G .

A second type of configurational transformation is the emergence of a mismatch between the actual contacts between stone and box and those that would have occurred under the sole effect of some rotations of the stones or deformation of the boxes. Such a difference testifies of radical changes in the link between box and stone, as those due to annihilation or creation of contacts, and is to be included in the analysis by means of the hem tensor M .

It is useful to introduce two fictive intermediate configurations, set in-between the reference and the actual ones. Any process can be split into three ideal steps (Fig. 2):

- (i) Rearrangements of contacts: stones and boxes do not move, but their connections change until they reach the effect that they have in the actual configuration (contacts are created or annihilated); this part of the evolution leads the body from the reference to the first intermediate placement, which is equal to the reference one except for the hem tensor field; this first intermediate configuration can be named, after the fact that it describes the actual geometry of contacts, the contact configuration.
- (ii) Reshaping of the boxes: the dissipative part of the stress T acts on its associated flux, though all contacts preserve their identity, so that the system takes the (possibly non compatible) configuration it would have had if the free energy were totally released; this second intermediate configuration is a collection of unloaded elements and can thus be named the released configuration.
- (iii) Material deformation (with entrained hem tensors): stones and boxes do move reversibly from the unloaded configuration to the actual configuration ($\varphi = 0$ in this process).

Notice that both intermediate configurations are, by definition, pure abstractions. There is no need of contriving a physical process that allows one to get them from any real configuration, though sometimes in special circumstances one can imagine this be accomplished by unloading the reference configuration.

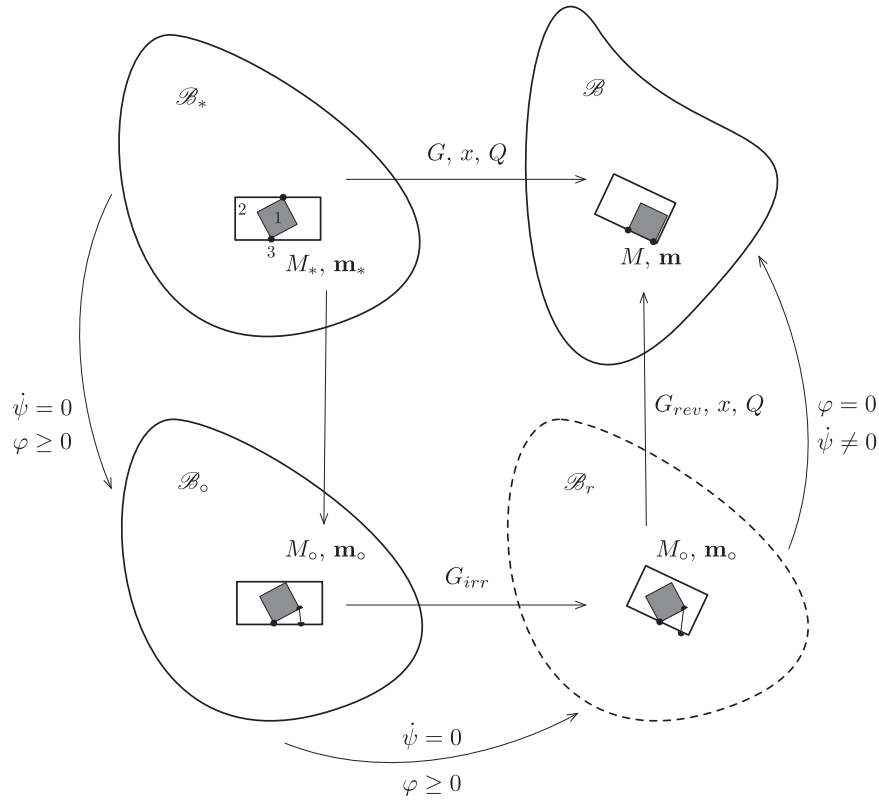


Fig. 2. Sketch of the adopted decomposition of the transformations.

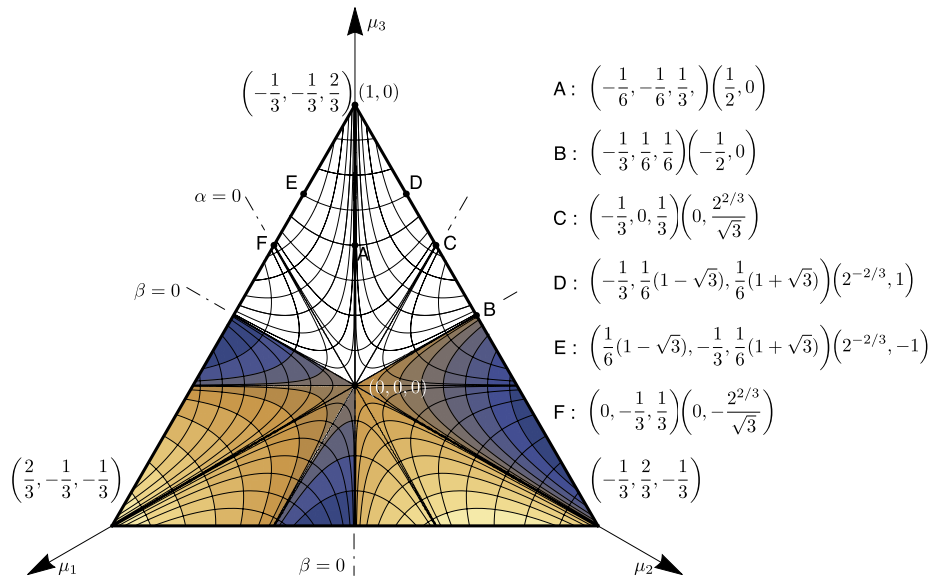


Fig. 3. The space of traceless symmetric tensors, represented as a triangle and its subdivision in equivalence classes obtained by the choice of the parameters α and β of Eq. (18). The vertices of the triangle correspond to tensors with one maximum eigenvalue; its center corresponds to the null tensor. Triples are coordinates in the three dimensional space of real eigenvalues, couples are coordinates in the α - β map. The coloured plot on the left shows contour levels of the function $\alpha(\mu_1, \mu_2, \mu_3)$, that on the right of the function $\beta(\mu_1, \mu_2, \mu_3)$.

4.2.1. Measuring the configurational transformation related to M

Let us consider a spectral decomposition of any symmetric traceless tensor M :

$$M = P[\mu_i]P^T; \quad P \in \text{SO}(3); \quad \mu_i \in \mathbb{R}; \quad i \in [1, \dots, 3]$$

where $[\mu_i]$ is the diagonal matrix of the eigenvalues of M , respecting the condition of null trace:

$$\sum_{i=1}^3 \mu_i = 0; \quad (17)$$

and P the orthogonal tensor whose columns are the eigenvectors of M .

Let us recall the analyses of orientational order made for the study of liquid crystals by Ericksen (1991) and borrow the defini-

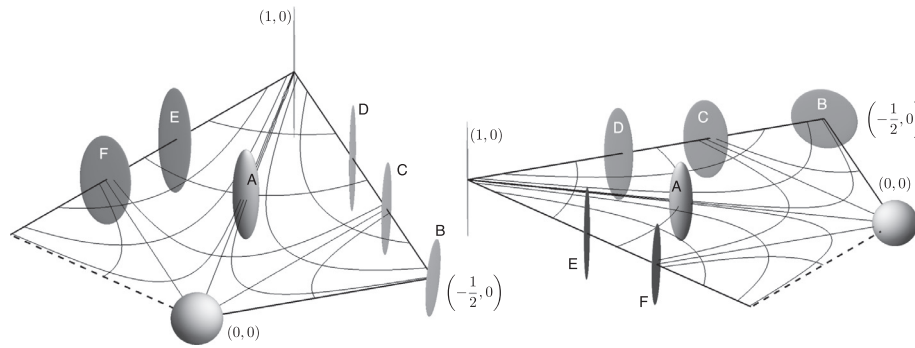


Fig. 4. The quotient of the space of traceless symmetric tensors given by the α - β map of Fig. 3, with some elements represented as ellipsoids (letters correspond to those in Fig. 3). Two views are given to better discern prolate ellipsoids. The dashed line does not belong to the map.

tions of degree of prolation and of degree of triaxiality introduced by Biscari and Capriz (1993) (assume indices cycling modulo 3 in the next formula, so that $\mu_{3+1} = \mu_1$; notice that we adapt the definition of β to our scopes, removing the absolute value given in Biscari and Capriz, 1993):

$$\alpha = 3 \left(\frac{1}{2} \prod_{i=1}^3 \mu_i \right)^{1/3} \in \left[-\frac{1}{2}, 1 \right];$$

$$\beta = \sqrt{3} \left(2 \prod_{i=1}^3 (\mu_i - \mu_{i+1}) \right)^{1/3} \in [-1, 1]. \quad (18)$$

Solving the system of Eqs. (17) and (18) allows us to represent in terms of the two parameters α and β an equivalence class of the space of traceless symmetric tensors, covering 1/3 of it, as shown in Fig. 3 and 4. Any two tensors whose eigenvalues have two by two the same value, independently of their eigenvectors, are equivalent and thus represented by a single element in the α - β map.

The physically important information given by the degree of prolation, α , is the distance from the condition of isotropy (the origin of the space), when $\alpha > 0$ (prolate ellipsoids), or from any condition of planar isotropy, when $\alpha < 0$ (oblate ellipsoids; see the contour plot at the left-hand side of Fig. 3). On the other side, the degree of triaxiality, β , takes a maximum value when there is a maximum difference between any couple of eigenvalues, whence its name (see the contour plot at the right-hand side of Fig. 3).

Numerous alternative maps have been proposed, mainly in the field of image processing (Schultz and Kindlmann, 2010; Vilanova et al., 2006). Some particularly simple ones are based on the ordering of eigenvalues, i.e., taking $\mu_{\max} = \mu_1 \geq \mu_{\text{mid}} = \mu_2 \geq \mu_3 = \mu_{\min}$, thus preserving their metric, with derivative jumps.

For example we quote:

$$\alpha' = \frac{3}{2} \mu_{\max}; \beta' = \frac{1}{2} \mu_{\max} - \mu_{\min}; \quad (19)$$

giving rise to the map shown in Fig. 5, and:

$$\alpha'' = \mu_{\max} - \mu_{\text{mid}}; \beta'' = 2(\mu_{\text{mid}} - \mu_{\min}); \quad (20)$$

giving the map of Fig. 6. Combinations are possible, as e.g., in Chang and Liu (2013), where parameters proportional to α' and β'' appear.

The choice of the most appropriate map is a matter of convenience, which will have to be decided on the basis of more information on the constitutive description of the system engendered by these parameters that we presently have.

Formally independently on that choice, we can focus on any element M in the thus defined quotient space of the hem tensors and call:

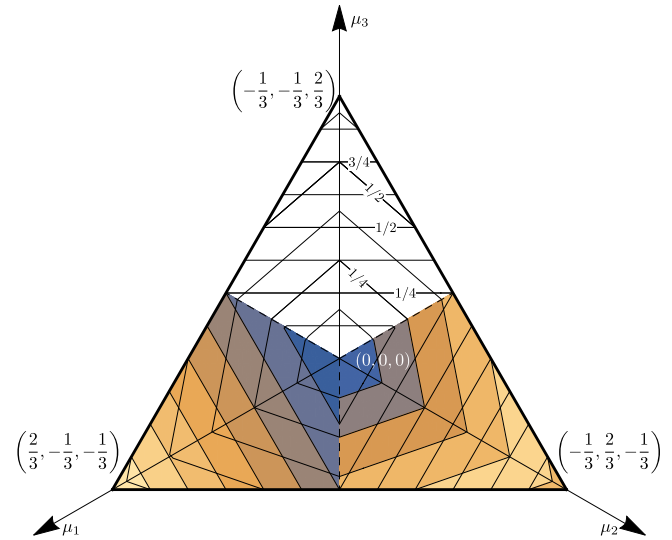


Fig. 5. Map of the space of symmetric traceless tensors given by the parameters of Eq. (19). The colour maps represent the values of the functions α' (on the left) and β' (on the right) defining the map.

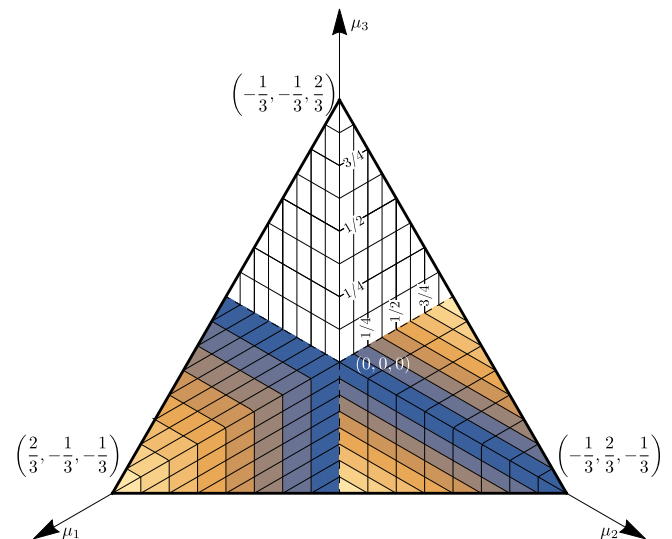


Fig. 6. Map of the space of symmetric traceless tensors given by the parameters of Eq. (20). The colour maps represent the values of the functions α'' (on the left) and β'' (on the right) defining the map.

$$\eta_{i\alpha} = \frac{\partial \mu_i}{\partial \alpha}; \quad \eta_{i\beta} = \frac{\partial \mu_i}{\partial \beta};$$

we can then represent \dot{M} as:

$$\dot{M} = BV + P[\eta_{i\alpha}]P^T\dot{\alpha} + P[\eta_{i\beta}]P^T\dot{\beta}$$

where B was introduced in Eq. (7),

$$V = \dot{P}P^T$$

is the spin of the eigenvectors of M , and P is an orthogonal tensor belonging to the subgroup of $SO(3)$ defined by the equivalence.

Coming to the description of irreversible processes, let us assume:

- A9 The dissipation rate is, locally, a positive homogeneous function of degree one of $\text{grad}\dot{y}$, $\dot{\alpha}$, and $\dot{\beta}$:

$$\varphi = C \cdot \text{grad}\dot{y} + \chi_\alpha \dot{\alpha} + \chi_\beta \dot{\beta}$$

with coefficients related to the energy release rates engendered, respectively, by a change of shape of the box, by a change of prolation and by a change of triaxiality of the hem tensor.

- A10 These rates do not depend on the physical scale of time, so that a rate-independent irreversible behaviour of the system ensues.

- A11 The equivalence classes defined by α and β are physically significative and sufficient for the study.

From the definition of φ and in the simple case of Section 4.1.1, we get:

$$(\tilde{T} - C) \cdot \text{grad}\dot{y} + B^T S \cdot V +$$

$$+ (P^T SP \cdot [\eta_{i\alpha}] - \chi_\alpha) \dot{\alpha} + (P^T SP \cdot [\eta_{i\beta}] - \chi_\beta) \dot{\beta} = 0.$$

We can assume that a convex set \mathcal{T} exists in the space of second order tensors, such that $\tilde{T} - C \in \mathcal{T}$ entails no changes for G (i.e., $\text{grad}\dot{y} = 0$), while $\tilde{T} - C \in \partial\mathcal{T}$ ($\partial\mathcal{T}$ the boundary of \mathcal{T}) entails the possibility of such changes, with the normality condition:

$$(\tilde{T} - C) \cdot \text{grad}\dot{y} = 0.$$

The remaining equation of dissipation is one related to the configurational stress S and associated with the rates V , $\dot{\alpha}$, and $\dot{\beta}$. In cases where these rates can be considered as mutually independent of each other and if we accept the existence of a convex set \mathcal{S} in the space of symmetric tensors, enclosing all S that does not tend to modify M , we have three separate normality conditions for the rates at issue:

$$B^T S \cdot V = 0;$$

$$P^T SP \cdot [\eta_{i\alpha}] = \chi_\alpha \text{ or } \dot{\alpha} = 0;$$

$$P^T SP \cdot [\eta_{i\beta}] = \chi_\beta \text{ or } \dot{\beta} = 0.$$

We observe that when $B^T S$ is symmetric, V is undetermined by the flow rule, but powerless. This condition corresponds to the idea that only the degrees of prolation and triaxiality count for the local description of contacts, which is certainly the simplest conjecture one can formulate.

5. Conclusion

A continuum model of rock filled gabions has been proposed, where the confinement induced by the steel cage or wire mesh on the filler has a special role. Stones are described as rigid bodies whose movements are confined by their neighbours; any process

is described starting from the knowledge of a reference confinement state. The evolution of this confinement, which can occur in processes, is an irreversible phenomenon, described, in the simplest case, by parameters borrowed from the geometric analysis of symmetric traceless tensors or from the theory of nematic liquid crystals.

The model is based on the theory of continua with microstructure and deals with the minimal number of fields helping for the above outlined description of confinements. Doing so, we disregarded—willing to reduce complexity in exploring new options—some common features of granular materials, such as the scalar microstructure related to porosity and the effects of granular inertia, being confident that a more precise description of the system can be put forward in the future.

The proposed model at its present stage makes possible to follow, in calculations, the local state of confinement of stones and to detect the occurrence of failures starting from the unlocking of some stones. If it deals with information appropriate for this purpose at the geometric and constitutive levels, it includes only a very simplified description of the inertial actions, fit for clusters of inertially isotropic stones. Having the seismic analyses of CSW buildings in mind, we plan to improve the model through a description of inertia fit for more complex shapes than at present. At the same time, we plan to run numerical and physical tests, in statics, to gain some insight into the special form that shall be given to the presently adopted constitutive description in some particular cases.

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References

- Agostini, R., Cesario, L., Conte, A., 1987. Flexible gabion structures in earth retaining works. Officine Maccaferri.
- Biscari, P., Capriz, G., 1993. Optical and statistical anisotropy in nematics (anisotropia ottica e statistica nei nematici). Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti Lincei. Matematica e Applicazioni Serie 9 4 (4), 307–313.
- Bongué-Boma, M., Brocato, M., 2008. Liquids with vapour bubbles. Comput. Math. Appl. 55, 268–284.
- Bongué-Boma, M., Brocato, M., 2010. A continuum model of micro-cracks in concrete. Continuum Mech. Thermodyn. 22 (2), 137–161.
- Brocato, M., 2018. A continuum model of interlocking structural systems. Rendiconti Lincei - Matematica e Applicazioni 1.
- Brocato, M., Capriz, G., 2000. Polycrystalline microstructure. Rend. Sem. Mat. Univ. Pol. Torino 58 (1), 49–56.
- Capriz, G., 1989. Continua with Microstructure. Springer-Verlag, New York.
- Capriz, G., Mullenger, G., 1995. Extended continuum mechanics for the study of granular flows. Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti Lincei. Matematica e Applicazioni Serie 9 6 (4), 275–284.
- Chang, C.S., Liu, Y., 2013. Stress and fabric in granular material. Theor. Appl. Mech. Lett. 3 (021002), 1–6.
- Cowin, S.C., 2004. Anisotropic poroelasticity: fabric tensor formulation. Mech. Mater. 36, 665–677.
- D'Urzo, S., Schneider, C. (Eds.), 2012. Sustainable Reconstruction in Urban Areas. A Handbook. Skat Swiss Resource Centre and Consultancies for Development and IFRC International Federation of Red Cross and Red Crescent Societies.
- Ericksen, J.L., 1991. Liquid crystals with variable degree of orientation. Arch. Rat. Mech. Anal. 113, 97–120.
- Evans, T.M., Brown, C.B., 2014. Microstates and macrostructures for granular assemblies. In: Abu-Farsakh, M., Yu, X., Hoyos, L.R. (Eds.), Geo-Congress 2014 Technical Papers. American Society of Civil Engineers, pp. 2858–2866. doi:10.1061/9780784413272.
- Gabbriellini, R., 2009. Foam Geometry and Structural Design of Porous Material. University of Bath. Department of Mechanical Engineering Ph.D. thesis.
- Germain, P., 1973. The method of virtual power in continuum mechanics. ii: microstructure. SIAM J. Appl. Math. 25, 556–575. doi:10.1137/0125053.
- Giovine, P., 2008. An extended continuum theory for granular media. In: Capriz, G., Giovine, P., Mariano, P.M. (Eds.), Mathematical Models of Granular Matter. Springer Verlag, pp. 167–192.

- Goodman, M.A., Cowin, S.C., 1972. A continuum theory for granular materials. *Arch. Ration. Mech. Anal.* 44 (4), 249–266. doi:10.1007/BF00284326.
- Guezouli, S., Hjiab, M., 2012. *Memoire Technique Projet Gabions Méditerranée. Technical Report. Laboratoire de Génie Civil et Génie Mécanique - INSA Rennes.*
- Harrigan, T.P., Mann, R.W., 1984. Characterization of microstructural anisotropy in orthotropic materials using a second rank tensor. *J. Mater. Sci.* 19 (3), 761–767. doi:10.1007/BF00540446.
- Jiang, Y., Wang, X., 2011. Stress-strain behavior of gabion in compression test and direct shear test. In: Peng, Q. (Ed.), *Third International Conference on Transportation Engineering (ICTE)*. American Society of Civil Engineers (ASCE), pp. 1457–1462.
- Lin, D.G., Lin, Y.H., Yu, F.C., 2010. Deformation analyses of gabion structures. In: Chen, S. (Ed.), *Interpraevent 2010 : International Symposium in Pacific Rim. Taipei : International Research Society*, pp. 512–526.
- Oda, M., 1972. Initial fabrics and their relations to mechanical properties of granular material. *Soils Found.* 12 (1), 17–36. doi:10.3208/sandf1960.12.17.
- Rothenburg, L., Kruyt, N., 2004. Critical state and evolution of coordination number in simulated granular materials. *Int. J. Solids Struct.* 41, 5763–5774.
- Schultz, T., Kindlmann, G.L., 2010. Superquadric glyphs for symmetric second-order tensors. *IEEE Trans. Vis. Comput. Graph.* 16 (6), 1595–1604.
- Vilanova, A., Zhang, S., Kindlmann, G., Laidlaw, D., 2006. An introduction to visualization of diffusion tensor imaging and its applications. In: Weickert, J., Hagen, H. (Eds.), *Visualization and Processing of Tensor Fields*. Springer, Berlin, Heidelberg, pp. 121–153.