



Inverse determination of liquid viscosity by means of the Bleustein–Gulyaev wave

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ARTICLE INFO

Article history:

Received 9 January 2012

Received in revised form 19 March 2012

Available online 27 April 2012

Keywords:

Bleustein–Gulyaev wave

Dispersion relation

Liquid sensing

Inverse problem

ABSTRACT

The Bleustein–Gulyaev (B–G) wave in piezoelectric materials of orthorhombic mm2 crystal class is reported to be a promising candidate for application to liquid sensing. In this work we present a rigorous quantitative investigation of the propagation of B–G wave in mm2 crystals in contact with a viscous liquid. An inversion algorithm is formulated to determine the liquid viscosity from the wave speed and attenuation data. Numerical results and discussions are given for potassium niobate (KNbO₃). The inversion results demonstrate that the liquid viscosity can be successfully determined from wave propagation characteristics.

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1. Introduction

Surface acoustic wave (SAW) based sensors have been applied successfully within many technological fields, such as NDE (non-destructive examination) of materials, chemistry, biology and environment science. This is largely due to their superb sensitivity, speed and reliability (Hoummady et al., 1997; McMullan et al., 2000; Vellekoop, 1998; Lee et al., 2009). The development of micro-acoustic wave sensors in bio-sensing has created the need for further investigation of surface wave propagation in viscous liquid loaded piezoelectric structures (Wu and Wu, 2000). A number of acoustic wave modes have therefore been utilized for investigating various sensor applications. The influence of a viscous liquid on acoustic waves propagating in elastic or piezoelectric materials has been studied by several researchers; this is of particular interest for the development of liquid viscosity sensors (Zaitsev et al., 2001; Lee and Kuo, 2006; Zhang et al., 2001; Yang and Wang, 2008).

Zhang et al. (2001) proposed that the Bleustein–Gulyaev (B–G) wave in mm2 crystal class piezoelectric materials is a promising candidate for elucidating various characteristics in liquid sensing applications. The B–G wave does not radiate energy into the contacting liquid and is sensitive to changes in the liquid density and viscosity. However, Zhang et al. (2001) did not give a detailed quantitative analysis of the characteristics of the B–G wave propagating in piezoelectric materials loaded with viscous liquid. Guo and Sun (2008) derived the exact dispersion relation for a B–G wave propagating in a half-space composed of 6 mm piezoelectric material loaded with viscous liquid. Their results show that the so-

called electrically shorted boundary condition is more suitable for liquid sensing applications than the open-circuit boundary condition. Later, Qian et al. (2010) studied the effect of thickness of the liquid layer on B–G wave propagation. We also remark that Du et al. (2010) investigated the properties of shear horizontal surface acoustic wave propagation in layered functionally graded piezoelectric structures loaded with viscous liquid.

Kielczyński and Plowiec, 1989 and Kielczyński et al., 2004 proposed the method of measurement of rheological properties of viscoelastic liquids using B–G wave and gave theoretical analyses of the influence of viscoelastic fluids on propagation of the B–G wave. They obtained the relations between the change in the complex propagation constant of the B–G wave and the shear acoustic impedance of the liquid by applying the theory of perturbation, which assumes that the liquid does not significantly modify the properties of the acoustic waves. Later, they also applied their method to measure liquid viscosity at high pressure for various temperatures (Kielczyński et al., 2011).

Piezoelectric mm2 crystals support B–G waves and offer promising substrates for liquid sensing applications (Zhang et al., 2001; Royer and Dieulesaint, 2000). In particular, one example of the mm2 crystal piezoelectric material KNbO₃, which exhibits a higher electro-mechanical coupling factor, offers particularly good potential for liquid sensing applications. Although a lot of work has been done in respect of the application of surface waves in piezoelectric media to liquid sensing for various scenarios, little work is available in the literature on the inversion algorithm of determining liquid properties from wave propagation parameters. This is of utmost importance for implementation of liquid sensing by means of the surface acoustic wave. In this present study, we will present a rigorous quantitative investigation of the propagation of B–G waves

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in mm2 crystals in contact with viscous liquid. The dispersion relation for metalized surface boundary conditions is obtained. Also, we study the inversion algorithm for determination of liquid viscosity from wave propagation characteristics. Numerical results of attenuation and phase velocity against viscosity, density of the liquid and wave frequency are given for potassium niobate (KNbO₃). Results for different kinds of error functions are also compared. The results of this study are expected to provide useful data and guidelines for liquid sensor design and development.

2. Description of the problem

The problem considered is concerned with a shear type surface wave propagating in mm2 piezoelectric material in contact with viscous liquid, as shown in Fig. 1. The piezoelectric material occupies the half-space $x_2 < 0$ and the liquid covers the half-space $x_2 > 0$, with x_3 the axis parallel to the twofold axis of symmetry. The x_1 and x_2 axes are parallel to the X and Y axes of the crystallographic coordinates (XYZ), respectively. This configuration is called Y cut- X propagation, which exhibits the maximum electromechanical coupling factor (Nakamura and Oshiki, 1997).

In the absence of body force and free electric charge, the coupled electroelastic governing equations for piezoelectric media can be written as (Zhang et al., 2001)

$$\begin{cases} C_{ijkl}u_{k,jl} + e_{kij}\phi_{,kj} = \rho \frac{\partial^2 u_i}{\partial t^2} \\ e_{jkl}u_{k,jl} - \varepsilon_{ik}\phi_{,ki} = 0 \end{cases} \quad (i, j, k, l = 1, 2, 3). \quad (1)$$

The elastic constants C_{ijkl} can be written into contracted form $C_{\alpha\beta}$ by the following rule

$$\begin{aligned} \alpha &= 9 - i - j, \quad \beta = 9 - k - l \quad \text{if } i \neq j, \quad k \neq l, \\ \alpha &= i = j, \quad \beta = k = l \quad \text{if } i = j, \quad k = l. \end{aligned} \quad (2)$$

Similarly, the piezoelectric constants e_{ijk} can also be put into contracted form $e_{i\alpha}$ with the index α observing the rule outlined in Eq. (2).

For mm2 piezoelectric materials, with x_3 direction being the 2-fold symmetry axis, Eq. (1) takes the following form

$$\begin{aligned} &C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + C_{55}\frac{\partial^2 u_1}{\partial x_3^2} + (C_{12} + C_{66})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} \\ &+ (C_{13} + C_{55})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + (e_{31} + e_{15})\frac{\partial^2 \phi}{\partial x_1 \partial x_3} = \rho \frac{\partial^2 u_1}{\partial t^2}, \\ &(C_{11} + C_{66})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} + C_{44}\frac{\partial^2 u_2}{\partial x_3^2} \\ &+ (C_{23} + C_{44})\frac{\partial^2 u_3}{\partial x_2 \partial x_3} + (e_{32} + e_{24})\frac{\partial^2 \phi}{\partial x_2 \partial x_3} = \rho \frac{\partial^2 u_2}{\partial t^2}, \\ &(C_{13} + C_{55})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (C_{23} + C_{44})\frac{\partial^2 u_2}{\partial x_2 \partial x_3} + C_{55}\frac{\partial^2 u_3}{\partial x_1^2} \\ &+ C_{44}\frac{\partial^2 u_3}{\partial x_2^2} + C_{33}\frac{\partial^2 u_3}{\partial x_3^2} + e_{15}\frac{\partial^2 \phi}{\partial x_1^2} + e_{24}\frac{\partial^2 \phi}{\partial x_2^2} + e_{33}\frac{\partial^2 \phi}{\partial x_3^2} = \rho \frac{\partial^2 u_3}{\partial t^2}, \\ &(e_{15} + e_{31})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (e_{24} + e_{32})\frac{\partial^2 u_2}{\partial x_2 \partial x_3} + e_{15}\frac{\partial^2 u_3}{\partial x_1^2} + e_{24}\frac{\partial^2 u_3}{\partial x_2^2} \\ &+ e_{24}\frac{\partial^2 u_3}{\partial x_3^2} - \varepsilon_{11}\frac{\partial^2 \phi}{\partial x_1^2} - \varepsilon_{22}\frac{\partial^2 \phi}{\partial x_2^2} - \varepsilon_{33}\frac{\partial^2 \phi}{\partial x_3^2} = 0. \end{aligned} \quad (3)$$

Now consider a harmonic wave propagating in the x_1 direction, with all physical quantities only dependent on the in-plane variables (x_1, x_2), and independent of x_3 . This case is an example of a generalized plane strain problem. In this situation, Eq. (3) is further simplified into

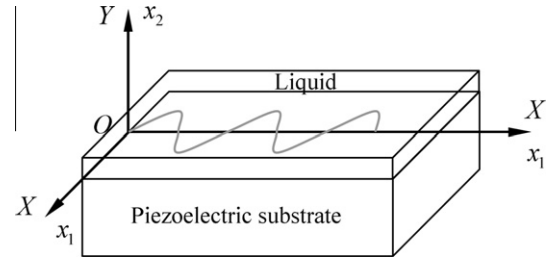


Fig. 1. A piezoelectric substrate in contact with viscous liquid.

$$\begin{aligned} &C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + (C_{12} + C_{66})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \\ &(C_{12} + C_{66})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} = \rho \frac{\partial^2 u_2}{\partial t^2}, \\ &C_{55}\frac{\partial^2 u_3}{\partial x_1^2} + C_{44}\frac{\partial^2 u_3}{\partial x_2^2} + e_{15}\frac{\partial^2 \phi}{\partial x_1^2} + e_{24}\frac{\partial^2 \phi}{\partial x_2^2} = \rho \frac{\partial^2 u_3}{\partial t^2}, \\ &e_{15}\frac{\partial^2 u_3}{\partial x_1^2} + e_{24}\frac{\partial^2 u_3}{\partial x_2^2} - \varepsilon_{11}\frac{\partial^2 \phi}{\partial x_1^2} - \varepsilon_{22}\frac{\partial^2 \phi}{\partial x_2^2} = 0. \end{aligned} \quad (4)$$

Similar to the case of piezoelectric materials with 6 mm symmetry (Guo and Sun), it can be seen from Eq. (4) that (u_1, u_2) is decoupled from (u_3, ϕ) . The first two equations of Eq. (4) show that (u_1, u_2) may constitute a purely elastic Rayleigh wave, whereas the last two equations indicate that (u_3, ϕ) could comprise a shear type surface electroelastic wave, a wave more commonly known as the Bleustein-Gulyaev wave.

The liquid is assumed to be a viscous Newtonian fluid. Suppose the motion of the liquid is induced only by wave propagation in the piezoelectric material and also propagates in the form of a harmonic wave. In regard to this problem, the embroil inertial term in the Navier–Stokes equation can be omitted. Moreover, the pressure gradient can also be ignored since only shear deformation occurs during wave propagation (Guo and Sun, 2008). Therefore, the governing equation for the liquid is simplified to

$$\frac{\partial v_3}{\partial t} - \frac{\mu_l}{\rho_l} \nabla^2 v_3 = 0, \quad (5)$$

where ρ_l is the mass density of the liquid, μ_l the dynamic viscous coefficient of the liquid and v_3 is the liquid particle velocity in the x_3 direction.

3. Dispersion relation

For this wave propagation problem, the displacement component u_3 and electric potential ϕ can be assumed to take the following form

$$\begin{aligned} u_3 &= W(x_2)e^{ik(x_1 - vt)} = W(x_2)e^{i(kx_1 - \omega t)}, \\ \phi &= \Phi(x_2)e^{ik(x_1 - vt)} = \Phi(x_2)e^{i(kx_1 - \omega t)}, \end{aligned} \quad (6)$$

where k is wave number, v phase velocity of the wave, ω angular frequency, i is the imaginary unit, and $W(x_2)$ and $\Phi(x_2)$ are unknown functions of x_2 .

Substituting the above expressions for u_3 and ϕ into the last two equations of Eq. (4) leads to

$$\begin{cases} C_{44}W''(x_2) + k^2(\rho v^2 - C_{55})W(x_2) - k^2 e_{15}\Phi(x_2) + e_{24}\Phi''(x_2) = 0, \\ e_{24}W''(x_2) - k^2 e_{15}W(x_2) + k^2 \varepsilon_{11}\Phi(x_2) - \varepsilon_{22}\Phi''(x_2) = 0, \end{cases} \quad (7)$$

where the superscript prime denotes differentiation with respect to the variable x_2 .

Suppose the solution of $W(x_2)$ and $\Phi(x_2)$ are $W(x_2) = Ae^{\lambda x_2}$, $\Phi(x_2) = Be^{\lambda x_2}$ where λ is the eigenvalue and A, B are unknown constants. Substitution of the solutions of $W(x_2)$ and $\Phi(x_2)$ into Eq. (7) reveals that

$$\begin{pmatrix} \lambda^2 C_{44} + (\rho v^2 - C_{55}) & e_{24}\lambda^2 - e_{15} \\ e_{24}\lambda^2 - e_{15} & \varepsilon_{11} - \varepsilon_{22}\lambda^2 \end{pmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = 0. \quad (8)$$

In order to get a nontrivial solution, the constants A and B cannot vanish identically. That indicates that the determinant of the coefficient matrix of A and B must be zero, thus

$$\begin{vmatrix} \lambda^2 C_{44} + (\rho v^2 - C_{55}) & e_{24}\lambda^2 - e_{15} \\ e_{24}\lambda^2 - e_{15} & \varepsilon_{11} - \varepsilon_{22}\lambda^2 \end{vmatrix} = 0, \quad (9)$$

or

$$(e_{24}^2 + \varepsilon_{22}C_{44})\lambda^4 + [\varepsilon_{22}(\rho v^2 - C_{55}) - \varepsilon_{11}C_{44} - 2e_{15}e_{24}]\lambda^2 + e_{15}^2 - \varepsilon_{11}(\rho v^2 - C_{55}) = 0. \quad (10)$$

If we assume that λ_1 and λ_2 are the two roots of Eq. (10) satisfying $\text{Re}(\lambda_1) > 0$ and $\text{Re}(\lambda_2) > 0$, the solutions of $W(x_2)$ and $\Phi(x_2)$ can then be expressed as

$$\begin{aligned} W(x_2) &= A_1 e^{k\lambda_1 x_2} + A_2 e^{k\lambda_2 x_2}, \\ \Phi(x_2) &= B_1 e^{k\lambda_1 x_2} + B_2 e^{k\lambda_2 x_2}, \end{aligned} \quad (11)$$

where A_1, A_2 and B_1, B_2 are not independent, but related by

$$B_i = \frac{e_{24}\lambda_i^2 - e_{15}}{\varepsilon_{22}\lambda_i^2 - \varepsilon_{11}} A_i \quad (i = 1, 2).$$

The Constitutive relations of mm2 piezoelectric materials are given by Royer and Dieulesaint (2000)

$$\begin{aligned} \tau_{23}^p &= C_{44} \frac{\partial u_3}{\partial x_2} + e_{24} \frac{\partial \phi}{\partial x_2}, \\ D_2^p &= e_{24} \frac{\partial u_3}{\partial x_2} - \varepsilon_{22} \frac{\partial \phi}{\partial x_2}. \end{aligned} \quad (12)$$

Substitution of Eq. (6) into the above constitutive relations yields

$$\begin{aligned} \tau_{23}^{(p)} &= k[(A_1 C_{44} + B_1 e_{24})\lambda_1 e^{k\lambda_1 x_2} + (A_2 C_{44} + B_2 e_{24})\lambda_2 e^{k\lambda_2 x_2}] e^{ik(x_1 - vt)}, \\ D_2^{(p)} &= k[(A_1 e_{24} - B_1 \varepsilon_{22})\lambda_1 e^{k\lambda_1 x_2} + (A_2 e_{24} - B_2 \varepsilon_{22})\lambda_2 e^{k\lambda_2 x_2}] e^{ik(x_1 - vt)}, \end{aligned} \quad (13)$$

or

$$\begin{aligned} \tau_{23}^{(p)} &= k[A_1(C_{44} + be_{24} + ae_{24}\lambda_1^2)\lambda_1 e^{k\lambda_1 x_2} \\ &\quad + A_2(C_{44} + be_{24} + ae_{24}\lambda_2^2)\lambda_2 e^{k\lambda_2 x_2}] e^{ik(x_1 - vt)}, \\ D_2^{(p)} &= k[A_1(e_{24} - b\varepsilon_{22} - ae_{22}\lambda_1^2)\lambda_1 e^{k\lambda_1 x_2} \\ &\quad + A_2(e_{24} - b\varepsilon_{22} - ae_{22}\lambda_2^2)\lambda_2 e^{k\lambda_2 x_2}] e^{ik(x_1 - vt)}, \end{aligned} \quad (14)$$

where the superscript (p) means quantities in the piezoelectric material, and

$$\begin{aligned} a &= \frac{\alpha}{\beta}, \quad b = \frac{(\varepsilon_{22}\rho v^2 - \gamma)}{\beta}, \quad \alpha = (C_{44}\varepsilon_{22} + e_{24}^2), \\ \beta &= (e_{15}\varepsilon_{22} - e_{24}\varepsilon_{11}), \quad \gamma = \varepsilon_{22}C_{55} + e_{15}e_{24}. \end{aligned} \quad (15)$$

In the liquid, the particle velocity in the x_3 direction is assumed to be $v_3 = V_3(x_2)e^{ik(x_1 - vt)}$. Substitution of the expression of v_3 into Eq. (5) leads to the solution of $V_3(x_2) = A_0 e^{\lambda_0 x_2}$, where $\lambda_0^2 = k^2 - ikv\frac{\rho_l}{\mu_l}$, $\text{Re}(\lambda_0) < 0$ and A_0 is a constant. The shear stress is then given by

$$\tau_{23}^{(l)} = 2\mu_l \dot{v}_3 = \mu_l \frac{\partial v_3}{\partial x_2} = A_0 \mu_l \lambda_0 e^{\lambda_0 x_2} e^{ik(x_1 - vt)}, \quad (16)$$

where the superimposed dot indicates differentiation with respect to time, the superscript (l) indicates quantities in the liquid.

From the results of previous studies (Guo and Sun, 2008; Du et al., 2010; Qian et al., 2010), it is known that the electrically shorted condition is more sensitive to surrounding disturbances and is more suitable for liquid sensing applications. Thus, motivated by this, we here only consider the electrically shorted condition. The mechanical conditions at the interface between the piezoelectric substrate and the liquid are continuity of displacement, particle velocity and stress components, that is

$$\tau_{23}^{(p)} \Big|_{x_2=0} = \tau_{23}^{(l)} \Big|_{x_2=0}, \quad \frac{\partial u_3^{(p)}}{\partial t} \Big|_{x_2=0} = v_3^{(l)} \Big|_{x_2=0}. \quad (17)$$

The electrically shorted condition means the electric potential vanishes at the interface, that is $\phi|_{x_2=0} = 0$.

The following dispersion relation can be derived by imposing the interfacial conditions at the interface between the piezoelectric substrate and the viscous liquid,

$$\begin{vmatrix} ikv & ikv & 1 \\ \lambda_1 k(ae_{24}\lambda_1^2 + C_{44} + be_{24}) & \lambda_2 k(ae_{24}\lambda_2^2 + C_{44} + be_{24}) & -\lambda_0 \mu_l \\ a\lambda_1^2 + b & a\lambda_2^2 + b & 0 \end{vmatrix} = 0, \quad (18)$$

Once the wave number is obtained, the phase velocity is calculated by $v = \omega/\text{Re}(k)$. The imaginary part of wave number k represents the attenuation per unit length in the propagation direction.

4. Inverse determination of liquid properties

If the properties of the piezoelectric material and liquid are known, attenuation and wave speed can be calculated from the dispersion relation Eq. (18). On the other hand, attenuation and wave speed are measurable in experiments. Therefore, in principle, the properties of the liquid in contact with the piezoelectric material can be determined inversely from measured data of attenuation and wave speed. However, measurement errors always exist and are unavoidable. Thus, measured attenuation and wave speed cannot be expected to satisfy the dispersion relation Eq. (18) exactly. We therefore need to construct an error function so as to determine liquid properties by making the error function reach its minimum value. One straightforward way to formulate an error function is to make use of the dispersion relation. Suppose the dispersion relation Eq. (18) can be expressed as this form

$$\text{dpn}(\rho_l, \mu_l, v, \text{att}) = 0, \quad (19)$$

where att is attenuation given by $\text{Im}(k)$. Then, an error function can be formulated as follows

$$\text{ErrI} = \sum_{i=1}^n |\text{dpn}(\rho_l, \mu_l, \omega_i, v_i, \text{att}_i)|^2, \quad (20)$$

where $i = 1 \dots n$ represents the number of measured data points, and $|\cdot|$ indicates the complex modulus.

Another type of error function is of the following form

$$\text{ErrII} = \sum_{i=1}^n \left[\left(\frac{v_i^m - v_i^c}{v_i^m} \right)^2 + \left(\frac{\text{att}_i^m - \text{att}_i^c}{\text{att}_i^m} \right)^2 \right], \quad (21)$$

where v_i^m and v_i^c are measured wave speed and calculated wave speed, respectively; att_i^m and att_i^c are measured and calculated attenuation, respectively.

Wu and Liu (1999) gave an inversion algorithm to determine the thickness and the elastic properties of the bonded layer in a layered medium from the measured wave speed dispersion of laser-generated surface waves. The second type of error function is similar to that employed in Wu and Liu (1999). Since the mass density is easily measured, we only consider a one-parameter (viscos-

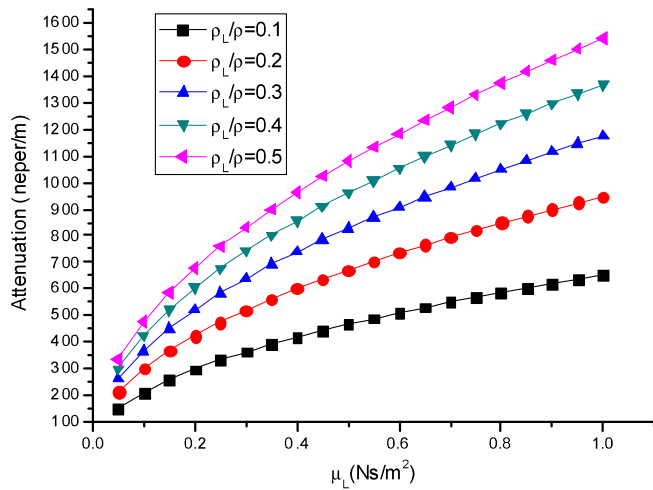


Fig. 2. Attenuation vs. viscosity (frequency = 50 MHz).

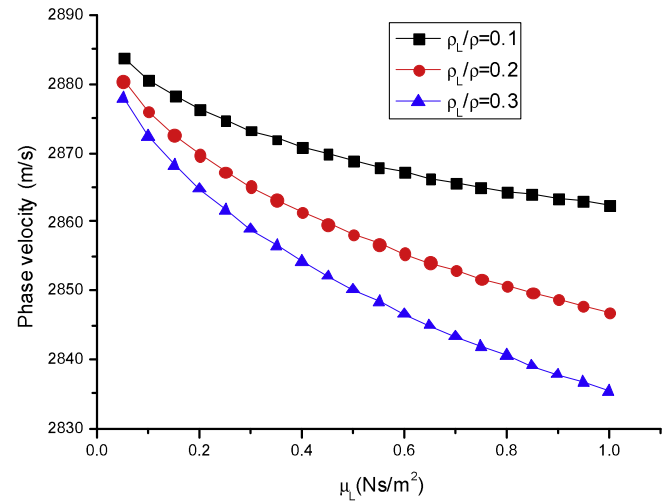


Fig. 5. Phase speed vs. viscosity (200 MHz).

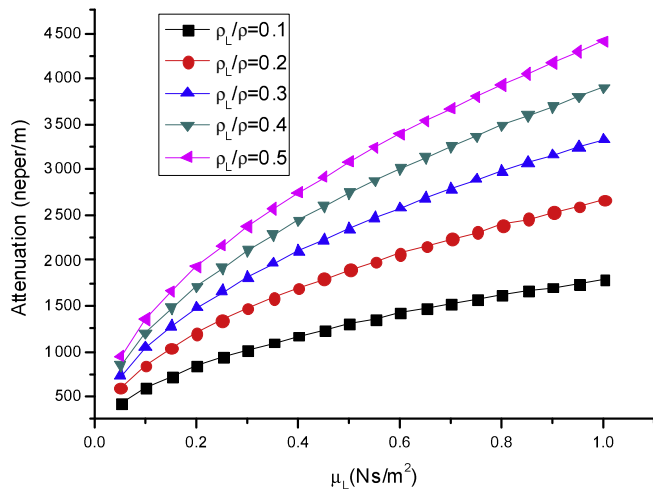


Fig. 3. Attenuation vs. viscosity (frequency = 100 MHz).

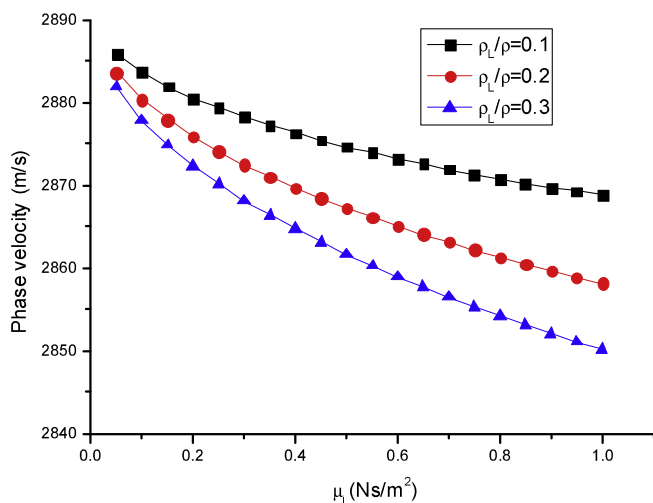


Fig. 4. Phase speed vs. viscosity (100 MHz).

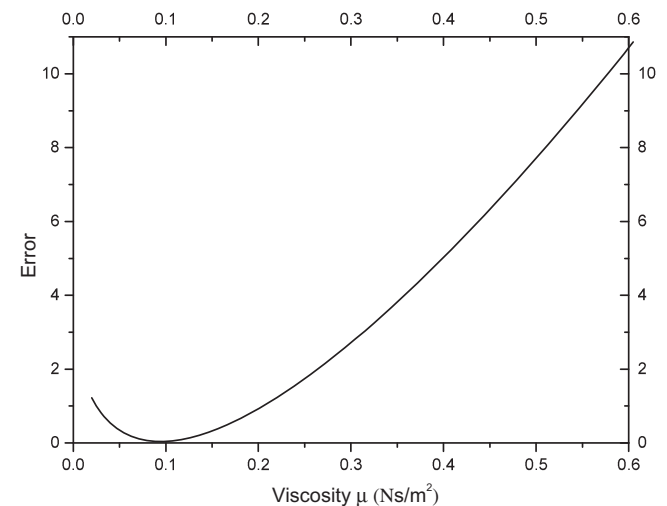


Fig. 6. Dependence of the error function on liquid viscosity (second type of error function with maximum 10% random errors and four data points at 50 MHz, 100 MHz, 150 MHz and 200 MHz).

5. Numerical results and discussion

Here in our study we consider a potassium niobate (KNbO₃) piezoelectric half-space loaded with viscous liquid. This piezoelectric material has very high electro-mechanical coupling factor and is regarded as a good candidate for liquid sensing applications. Material properties of potassium niobate are taken from Zgonik et al. (1993) and listed below

$$\begin{aligned} C_{11} &= 2.26 \times 10^{11} \text{ Pa}, \quad C_{12} = 0.96 \times 10^{11} \text{ Pa}, \quad C_{22} = 2.70 \times 10^{11} \text{ Pa}, \\ C_{66} &= 0.955 \times 10^{11} \text{ Pa}, \quad C_{44} = 0.743 \times 10^{11} \text{ Pa}, \quad C_{55} = 0.25 \times 10^{11} \text{ Pa}, \\ e_{15} &= 5.16 \text{ C/m}, \quad e_{24} = 11.7 \text{ C/m}, \quad \varepsilon_{11} = 37.0\varepsilon_0, \quad \varepsilon_{22} = 780.0\varepsilon_0, \\ \varepsilon_0 &= 8.854 \times 10^{-12} \text{ F/m}, \quad \rho = 4630.0 \text{ kg/m}^3. \end{aligned}$$

The numerical results of wave speed and attenuation against liquid viscosity are shown in Figs. 2–5. Figs. 2 and 3 show the change of attenuation with liquid viscosity at different values of liquid density under the electrically shorted condition. Figs. 4 and 5 illustrate the phase speed as a function of liquid viscosity at different values of liquid density. Fig. 6 shows the dependence of the error

ity) inversion scheme. Comparison between the results of these two types of error functions will be given in the next section.

Table 1Phase speed and attenuation at different frequencies ($\mu = 0.1 \text{ Ns/m}^2$, $\rho_l = 0.2\rho$).

Frequency (MHz)	Phase speed (m/s)	Attenuation (neper/m)
50	2883.65	300.26
100	2880.42	854.18
150	2877.97	1576.91
200	2875.93	2438.54

function on the viscosity of the liquid. The relationship implies that only one global minimum exists around the possible viscosity values. This property still holds when even 10% random errors are added to the phase speed and attenuation data.

The results in Table 1 are obtained from the dispersion relation Eq. (18). Since errors are unavoidable in experiments, we add random errors to the values of phase speed and attenuation. Then, using data containing random errors as measured data, the viscosity of liquid can be deduced by making the error functions attain their minimum. The optimization procedures of finding the minimum of error functions are implemented by the nonlinear simplex method and programmed by Maple 10.

When the error function shown in Eq. (20) is employed, the average value of viscosity for 8 calculations is 0.113 Ns/m^2 , for the case with maximum 0.5% random errors as listed in Table 3. If a maximum of 0.1% random errors is added, the average value of viscosity for eight calculations is 0.104 Ns/m^2 as shown in Table 2.

If the second type of error function Eq. (21) is utilized, accuracy of the viscosity is much improved. For the case with maximum 5% errors, the average value of viscosity for 8 calculations is 0.0985 Ns/m^2 as presented in Table 4. For the case with maximum 10% errors, the average value of viscosity for eight calculations is 0.09775 Ns/m^2 as shown in Table 5.

The inversion algorithm for determining viscosity and mass density of liquid simultaneously has also been investigated. How-

ever, the results are not as conclusive as for one parameter inversion, even if the second type of error function is employed. In this case, careful investigation of the error function shows that the error function does have a minimum around the true values of viscosity and mass density. However, it becomes very flat and changes slowly around the minimum. This characteristic may cause large errors to the inversion parameters.

6. Conclusions

In this work, the dispersion relation for a shear-type surface wave traveling in a half-space substrate of crystal class mm2 piezoelectric material in contact with viscous liquid was derived. Based on this relation, wave velocity and attenuation can be calculated. From numerical results, it can be seen that attenuation increases with the increase of viscosity and mass density of the liquid. For large viscosity, the relationship between viscosity and attenuation is approximately linear. The algorithm for inverse determination of liquid properties from wave propagation characteristics is also studied. The results show that the error function Eq. (21) has better properties than Eq. (20). If the error function Eq. (21) is employed, the accuracy of the calculated viscosity is even greater than that of measured data of phase speed and attenuation. This property is of great importance for liquid sensing applications. The inversion results demonstrate that the viscosity of liquid can be successfully determined from wave propagation characteristics.

Acknowledgements

This work was supported by National Natural Science Foundation of China and Municipal Natural Science foundation of Shanghai through grant 10472068 and grant 10ZR1415200, respectively. This manuscript was partly completed when the first author visited Keele University, UK. He would also like to thank the

Table 2Viscosity determined by inversion algorithm using the first error function with maximum 0.1% random error ($\rho_l = 0.2\rho$).

Calculation no.	1	2	3	4	5	6	7	8	Average	Error
Viscosity (Ns/m^2)	0.103	0.098	0.111	0.105	0.099	0.097	0.107	0.108	0.104	4%

Table 3Viscosity determined by inversion algorithm using the first error function with maximum 0.5% random error ($\rho_l = 0.2\rho$).

Calculation no.	1	2	3	4	5	6	7	8	Average	Error
Viscosity (Ns/m^2)	0.126	0.128	0.090	0.098	0.093	0.111	0.119	0.137	0.113	13%

Table 4Viscosity determined by inversion algorithm using the second error function with maximum 5% random error ($\rho_l = 0.2\rho$).

Calculation no.	1	2	3	4	5	6	7	8	Average	Error
Viscosity (Ns/m^2)	0.102	0.099	0.096	0.100	0.097	0.096	0.099	0.096	0.098	1.5%

Table 5Viscosity determined by inversion algorithm using the second error function with maximum 10% random error ($\rho_l = 0.2\rho$).

Calculation no.	1	2	3	4	5	6	7	8	Average	Error
Viscosity (Ns/m^2)	0.088	0.098	0.102	0.104	0.095	0.094	0.102	0.099	0.097	2.3%

Chinese Scholarship Council for its support during his stay in Keele University.

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