



Dependence of stress intensity factors on elastic constants for cracks in an orthotropic bimaterial with a thin film

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ABSTRACT

An interface crack and a subinterface crack in an orthotropic bimaterial structure consisting of a thin film and a half plane substrate are analyzed. The orthotropic bimaterial structure is subjected to compressive load and bending moment per unit thickness. Complete expressions of stress intensity factors for the two cracks are obtained based on the path independence of the J integral, apart from one dimensionless parameter undetermined each. The dependence of the dimensionless parameters on material constants is examined. A reduction of the number of necessary material parameters for the parameters is made based upon the modified Stroh formalism. The explicit dependence of the dimensionless parameters on one orthotropic parameter for the film is determined by using the orthotropy rescaling technique. Variations of the dimensionless parameters with the other material parameters are also obtained through numerical computations.

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1. Introduction

Bimaterial structures consisting of a thin film and a substrate have found wide technological applications in electronic devices, semiconductors and optical electronics. Material failure frequently occurs via the propagation of cracks. Film and substrate cracking in thin film bilayers has been extensively studied for the evaluation of the mechanical reliability of the structures. Ye et al. (1992) examined cracks in thin films caused by residual tension. Xia and Hutchinson (2000) studied various crack patterns in thin films subject to equibiaxial residual stress. Wang and Qiao (2004) considered an interface crack between two shear deformable elastic layers under general edge loading conditions. They investigated the shear deformation effect on the energy release rate and interface stress intensity factor based on a split bilayer deformable beam model. Chakravarthy et al. (2005) investigated the influence of external loads on thin film and substrate cracking. Vellinga et al. (2008) employed a discrete lattice model to simulate interactions between cracking, delamination and buckling in elastic thin films. Recently, Taylor et al. (2011) assessed the effect of film thickness variations in periodic cracking.

There have been numerous attempts to find the role of elastic constants on the fracture behavior of bilayered structures. Suo and Hutchinson (1990) studied an interface crack between two infinite isotropic elastic layers with finite thicknesses under compressive load and bending moment. They explored the influence

of material parameters on stress intensity factors for the interface crack. Suo and Hutchinson (1989) analyzed a crack in a substrate parallel to the interface between two isotropic elastic layers composed of a film and a substrate. Stress intensity factors for the crack with arbitrary crack depth were obtained as functions of the elastic properties and thicknesses of film and substrate, which were used to predict the steady-state substrate cracking depth. Xu et al. (1993) were concerned with a semi-infinite interface crack between an orthotropic thin film and an orthotropic substrate. In their work, the effect of the elastic properties of film and substrate on interface stress intensity factors was investigated. The interface stress intensity factors depend on only six independent material parameters. It is difficult to consider all combinations of the six parameters, and hence their attention was restricted to special cases, such as epitaxial film and substrate, elastic thin film on a rigid substrate, and cross-ply film and substrate of the same material.

It is the purpose of this study to investigate the problem of an interface crack between a thin film and a half plane substrate. Each material is assumed to be orthotropic. Compressive load and bending moment per unit thickness are applied on the neutral axis of the film. We seek an expression of interface stress intensity factors based on the path independence of the J integral (Rice, 1968). The complete expression of interface stress intensity factors is obtained, apart from only one undetermined dimensionless parameter. According to Xu et al. (1993), the stress fields in the orthotropic bimaterial depend on six independent material parameters. These many material parameters cause the great complexity in presenting numerical results of the dimensionless parameter. We attempt

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to reduce the number of necessary material parameters involved in the dimensionless parameter. Based upon the modified Stroh formalism (Beom et al., 2012), it is shown that the parameter depends on five nondimensional material parameters. Next, we use the orthotropic rescaling technique to obtain the explicit dependence of one orthotropic parameter for the film on the dimensionless parameter. The dimensionless parameter for the original problem can be obtained from the dimensionless parameter for the transformed problem. The dimensionless parameter for the transformed problem, which depends on four material parameters, is numerically calculated. A crack in an orthotropic substrate with an adherent film is also considered. In a similar way, we obtain the stress intensity factor for the subinterface crack.

2. Formulation

Consider a deformation of a homogeneous orthotropic elastic solid. We are concerned with an inplane deformation for a state of plane strain or plane stress. The Cartesian coordinates, x_1 and x_2 , are chosen to coincide with principal axes of the orthotropic material. Strain–stress relations for a linear elastic orthotropic material can be written in the following form (Lekhnitskii, 1963):

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} S_{11}^e & S_{12}^e & 0 \\ S_{12}^e & S_{22}^e & 0 \\ 0 & 0 & S_{66}^e \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \quad (1)$$

where ε_{ij} and σ_{ij} are the strain and stress, respectively; and $S_{ij}^e = S_{ij}$ for plane stress and $S_{ij}^e = S_{ij} - S_{i3}S_{j3}/S_{33}$ ($i, j=1, 2, 6$) for plane strain in which S_{ij} is the conventional compliance component. Based upon Stroh formalism (Eshelby et al., 1953; Stroh, 1958; Suo, 1990a), a general solution to the equilibrium equation for the displacements and the corresponding stresses can be expressed in terms of complex functions as

$$\begin{aligned} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} &= 2\text{Re} \left[\mathbf{A} \begin{Bmatrix} f_1(z_1) \\ f_2(z_2) \end{Bmatrix} \right], \\ \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \end{Bmatrix} &= -2\text{Re} \left[\mathbf{B} \begin{Bmatrix} p_1 f_1'(z_1) \\ p_2 f_2'(z_2) \end{Bmatrix} \right], \\ \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \end{Bmatrix} &= 2\text{Re} \left[\mathbf{B} \begin{Bmatrix} f_1'(z_1) \\ f_2'(z_2) \end{Bmatrix} \right]. \end{aligned} \quad (2)$$

Here u_i is the displacement, Re represents the real part and $(\cdot)'$ designates the derivative with respect to the associate argument. Functions $f_j(z_j)$ ($j=1,2$) are analytic in their arguments, $z_j = x_1 + p_j x_2$ ($j=1,2$). The characteristic roots p_j ($j=1,2$), and the matrices \mathbf{A} and \mathbf{B} for the orthotropic material are

$$p_1 = i\lambda^{-\frac{1}{4}}(n+m), \quad (3)$$

$$p_2 = i\lambda^{-\frac{1}{4}}(n-m),$$

$$\mathbf{A} = \begin{bmatrix} S_{11}^e p_1^2 + S_{12}^e & S_{11}^e p_2^2 + S_{12}^e \\ S_{12}^e p_1 + \frac{S_{22}^e}{p_1} & S_{12}^e p_2 + \frac{S_{22}^e}{p_2} \end{bmatrix}, \quad (4)$$

$$\mathbf{B} = \begin{bmatrix} -p_1 & -p_2 \\ 1 & 1 \end{bmatrix},$$

where

$$\begin{aligned} \lambda &= \frac{S_{11}^e}{S_{22}^e}, \\ \rho &= \frac{2S_{12}^e + S_{66}^e}{2\sqrt{S_{11}^e S_{22}^e}}, \end{aligned} \quad (5)$$

$$\begin{aligned} n &= \sqrt{\frac{1}{2}(\rho+1)}, \\ m &= \sqrt{\frac{1}{2}(\rho-1)}. \end{aligned} \quad (6)$$

The dimensionless parameters λ and ρ measure a degree of orthotropy. In particular, $\lambda = \rho = 1$ for an isotropic material. The positive definiteness of the strain energy density leads to $\lambda > 0$ and $-1 < \rho < \infty$.

When an orthotropic material degenerates to one with $\rho = 1$, which is referred to as a degenerate orthotropic material, the Stroh formalism in Eq. (2) is well known to break down. Recently, Beom et al. (2012) modified the Stroh formalism by introducing complex functions $g_j(z)$ ($j=1,2$) defined by

$$\mathbf{g}(z) = \mathbf{B} \begin{Bmatrix} f_1(z) \\ f_2(z) \end{Bmatrix}, \quad (7)$$

where $\mathbf{g}(z) = (g_1(z)g_2(z))^T$ and $z = x_1 + px_2$ where p is a complex number with a positive imaginary part. The modified Stroh formalism in which $g_j(z)$ ($j=1,2$) are used instead of $f_j(z)$ reduces to a classical solution for a degenerate orthotropic material. The modified Stroh formalism enables boundary value problems in both orthotropic elasticity and degenerate orthotropic elasticity to reduce the determination of the complex functions $g_j(z)$ ($j=1,2$). The original Stroh formalism Eq. (2) in conjunction with Eq. (7) can be applied to degenerate orthotropic problems.

Next, consider a bimaterial composed of two dissimilar orthotropic materials with a straight interface paralleling the x_1 -axis. Materials 1 and 2 occupy regions above and below the interface, respectively. The principal axes of two orthotropic materials are assumed to be coincident with the Cartesian coordinates, x_1 and x_2 . According to Beom and Atluri (1995a) and Ting (1995), two bimaterial matrices that are needed to describe stress fields in an anisotropic bimaterial are given by

$$\begin{aligned} \boldsymbol{\alpha} &= (\mathbf{H}^{(1)} + \mathbf{H}^{(2)})^{-1}(\mathbf{H}^{(2)} - \mathbf{H}^{(1)}), \\ \boldsymbol{\beta} &= (\mathbf{H}^{(1)} + \mathbf{H}^{(2)})^{-1}(\mathbf{Q}^{(2)} - \mathbf{Q}^{(1)}), \end{aligned} \quad (8)$$

where the superscripts 1 and 2 in parentheses indicate the quantities taken for materials 1 and 2, respectively, and \mathbf{H} and \mathbf{Q} are real matrices defined by

$$i\mathbf{A}\mathbf{B}^{-1} = \mathbf{H} + i\mathbf{Q}. \quad (9)$$

Two bimaterial matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are anisotropic versions of the well known Dundurs parameters for isotropic bimaterial. The matrices \mathbf{H} and \mathbf{Q} for the orthotropic material are (Suo, 1990a)

$$\begin{aligned} \mathbf{H} &= 2n\sqrt{S_{11}^e S_{22}^e} \begin{bmatrix} \lambda^{\frac{1}{4}} & 0 \\ 0 & \lambda^{-\frac{1}{4}} \end{bmatrix}, \\ \mathbf{Q} &= \begin{bmatrix} 0 & \sqrt{S_{11}^e S_{22}^e + S_{12}^e} \\ -(\sqrt{S_{11}^e S_{22}^e + S_{12}^e}) & 0 \end{bmatrix}. \end{aligned} \quad (10)$$

The substitution of Eq. (10) into Eq. (8) yields

$$\begin{aligned} \boldsymbol{\alpha} &= \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix}, \\ \boldsymbol{\beta} &= \begin{bmatrix} 0 & \beta_{12} \\ \beta_{21} & 0 \end{bmatrix}, \end{aligned} \quad (11)$$

where

$$\begin{aligned}\alpha_{11} &= \frac{(1 + \gamma_0^{\frac{1}{4}})\alpha_0 + (1 - \gamma_0^{\frac{1}{4}})}{(1 - \gamma_0^{\frac{1}{4}})\alpha_0 + (1 + \gamma_0^{\frac{1}{4}})}, \\ \alpha_{22} &= \frac{(1 + \gamma_0^{\frac{3}{4}})\alpha_0 + (1 - \gamma_0^{\frac{3}{4}})}{(1 - \gamma_0^{\frac{3}{4}})\alpha_0 + (1 + \gamma_0^{\frac{3}{4}})}, \\ \beta_{12} &= \beta_0 \lambda_1^{-\frac{1}{4}} \gamma_0^{-\frac{1}{4}} \sqrt{\frac{(1 - \gamma_0^{\frac{3}{4}})\alpha_0 + (1 + \gamma_0^{\frac{3}{4}})}{(1 - \gamma_0^{\frac{1}{4}})\alpha_0 + (1 + \gamma_0^{\frac{1}{4}})}}, \\ \beta_{21} &= -\beta_0 \lambda_1^{\frac{1}{4}} \gamma_0^{\frac{1}{4}} \sqrt{\frac{(1 - \gamma_0^{\frac{1}{4}})\alpha_0 + (1 + \gamma_0^{\frac{1}{4}})}{(1 - \gamma_0^{\frac{3}{4}})\alpha_0 + (1 + \gamma_0^{\frac{3}{4}})}}, \\ \alpha_0 &= \frac{(nS_{11}^e)^{(2)} - (nS_{11}^e)^{(1)}}{(nS_{11}^e)^{(2)} + (nS_{11}^e)^{(1)}}, \\ \beta_0 &= \frac{(\sqrt{S_{11}^e S_{22}^e} + S_{12}^e)^{(2)} - (\sqrt{S_{11}^e S_{22}^e} + S_{12}^e)^{(1)}}{2[(\lambda_1^{-\frac{1}{4}} nS_{11}^e)^{(2)} + (\lambda_1^{-\frac{1}{4}} nS_{11}^e)^{(1)}]\{(\lambda_1^{-\frac{3}{4}} nS_{11}^e)^{(2)} + (\lambda_1^{-\frac{3}{4}} nS_{11}^e)^{(1)}\}^{\frac{1}{2}}}, \\ \gamma_0 &= \frac{\lambda_2}{\lambda_1}.\end{aligned}\quad (12)$$

The subscripts 1 and 2 attached to λ stand for the two materials 1 and 2, respectively. It is clearly seen from Eqs. (11) and (12) that the bimaterial matrix α is a function of two bimaterial parameters α_0 and γ_0 , whereas β depends on three bimaterial parameters α_0 , β_0 and γ_0 , and one orthotropy parameter λ_1 . When the orthotropic bimaterial degenerates to an isotropic one, the bimaterial parameters α_0 and β_0 reduce to the Dundurs parameters. From the definitions of α_0 and γ_0 , we have that $-1 < \alpha_0 < 1$ and $\gamma_0 > 0$. Typical values of β_0 for orthotropic bimaterials in practice are small (Suo, 1990a).

An orthotropy rescaling method for a homogeneous orthotropic material has been developed using the stress function formulation by Suo (1990b). Recently, Beom et al. (2012) presented the orthotropy rescaling technique based on the modified Stroh formalism. We employ an orthotropy rescaling technique to obtain the explicit dependence of elastic fields in the bimaterial on λ_1 . We consider the transformed bimaterial, mapped by the following linear transformation:

$$\begin{aligned}\hat{x}_1 &= x_1, \\ \hat{x}_2 &= \lambda_1^{-\frac{1}{4}} x_2,\end{aligned}\quad (14)$$

where a hat (^) over the letter indicates the quantity taken for the transformed problem. The linear transformation Eq. (14) can be considered as an orthotropy rescaling. The transformed material 1 is assumed to be a material with cubic symmetry. The material constants of the transformed solids are chosen as

$$\begin{aligned}\hat{S}_{11}^{e(1)} &= \lambda_1^{-\frac{1}{2}} S_{11}^{e(1)}, \\ \hat{S}_{22}^{e(1)} &= \lambda_1^{\frac{1}{2}} S_{22}^{e(1)}, \\ \hat{S}_{12}^{e(1)} &= S_{12}^{e(1)}, \\ \hat{S}_{66}^{e(1)} &= S_{66}^{e(1)}, \\ \hat{S}_{11}^{e(2)} &= \lambda_1^{-\frac{1}{2}} S_{11}^{e(2)}, \\ \hat{S}_{22}^{e(2)} &= \lambda_1^{\frac{1}{2}} S_{22}^{e(2)}, \\ \hat{S}_{12}^{e(2)} &= S_{12}^{e(2)}, \\ \hat{S}_{66}^{e(2)} &= S_{66}^{e(2)}.\end{aligned}\quad (15)$$

For the transformed materials 1 and 2, it is easily seen from Eqs. (5) and (15) that

$$\begin{aligned}\hat{\lambda}_1 &= 1, \\ \hat{\rho}_1 &= \rho_1, \\ \hat{\lambda}_2 &= \gamma_0, \\ \hat{\rho}_2 &= \rho_2.\end{aligned}\quad (16)$$

Using Eqs. (14) and (16), we obtain

$$\hat{p}_i^{(j)} = \lambda_1^{\frac{1}{4}} p_i^{(j)}, \quad (17)$$

$$z_i^{(j)} = \hat{z}_i^{(j)} \quad (i, j = 1, 2), \quad (18)$$

where $\hat{z}_i^{(j)} = \hat{x}_1 + \hat{p}_i^{(j)} \hat{x}_2$. Substituting Eqs. (15) and (17) into Eq. (4) gives

$$\begin{aligned}\mathbf{A}^{(j)} &= \lambda_1^{-\frac{1}{4}} \mathbf{\Lambda}^{-1} \hat{\mathbf{A}}^{(j)}, \\ \mathbf{B}^{(j)} &= \mathbf{\Lambda} \hat{\mathbf{B}}^{(j)},\end{aligned}\quad (19)$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1^{-\frac{1}{4}} & 0 \\ 0 & 1 \end{bmatrix}. \quad (20)$$

Without loss of generality, we may put

$$\hat{\mathbf{B}}^{(j)} \begin{Bmatrix} \hat{f}_1^{(j)}(\hat{z}_1) \\ \hat{f}_2^{(j)}(\hat{z}_2) \end{Bmatrix} = \hat{\mathbf{B}}^{(j)} \begin{Bmatrix} f_1^{(j)}(\hat{z}_1) \\ f_2^{(j)}(\hat{z}_2) \end{Bmatrix} \quad (j = 1, 2). \quad (21)$$

Making use of Eqs. (2), (19), and (21), it can be shown that the displacements and stresses of the orthotropic material are expressed as

$$\begin{aligned}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} &= \lambda_1^{-\frac{1}{4}} \mathbf{\Lambda}^{-1} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix}, \\ \begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \end{Bmatrix} &= \lambda_1^{-\frac{1}{4}} \mathbf{\Lambda} \begin{Bmatrix} \hat{\sigma}_{11} \\ \hat{\sigma}_{12} \end{Bmatrix}, \\ \begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \end{Bmatrix} &= \mathbf{\Lambda} \begin{Bmatrix} \hat{\sigma}_{21} \\ \hat{\sigma}_{22} \end{Bmatrix}.\end{aligned}\quad (22)$$

It is noted that Eq. (22) is also valid for a degenerate orthotropic material with multiple characteristic roots since Eq. (21) can be satisfied when $\rho \rightarrow 1$. We refer to the reader to the work of Beom et al. (2012) for the rigorous proof. The boundary and continuity conditions for the transformed problem are determined so as to satisfy Eq. (22) on the boundary and the bonded interfaces.

From Eqs. (9) and (19), it is obtained that

$$\begin{aligned}\mathbf{H}^{(j)} &= \lambda_1^{-\frac{1}{4}} \mathbf{\Lambda}^{-1} \hat{\mathbf{H}}^{(j)} \mathbf{\Lambda}^{-1}, \\ \mathbf{Q}^{(j)} &= \lambda_1^{-\frac{1}{4}} \mathbf{\Lambda}^{-1} \hat{\mathbf{Q}}^{(j)} \mathbf{\Lambda}^{-1}.\end{aligned}\quad (23)$$

The bimaterial and matrices and parameters for the transformed bimaterial are

$$\begin{aligned}\hat{\alpha} &= \mathbf{\Lambda}^{-1} \alpha \mathbf{\Lambda} = \alpha, \\ \hat{\beta} &= \mathbf{\Lambda}^{-1} \beta \mathbf{\Lambda},\end{aligned}\quad (24)$$

$$\begin{aligned}\hat{\alpha}_0 &= \alpha_0, \\ \hat{\beta}_0 &= \beta_0, \\ \hat{\gamma}_0 &= \gamma_0.\end{aligned}\quad (25)$$

Here Eqs. (8) and (23) were used in deriving Eq. (24), and Eq. (25) was obtained from Eqs. (13) and (15). The bimaterial matrix $\hat{\beta}$ is given by

$$\hat{\beta} = \begin{bmatrix} 0 & \hat{\beta}_{12} \\ \hat{\beta}_{21} & 0 \end{bmatrix}, \quad (26)$$

where

$$\begin{aligned}\hat{\beta}_{12} &= \beta_0 \gamma_0^{-\frac{1}{4}} \sqrt{\frac{(1 - \gamma_0^{\frac{3}{4}})\alpha_0 + (1 + \gamma_0^{\frac{3}{4}})}{(1 - \gamma_0^{\frac{1}{4}})\alpha_0 + (1 + \gamma_0^{\frac{1}{4}})}}, \\ \hat{\beta}_{21} &= -\beta_0 \gamma_0^{\frac{1}{4}} \sqrt{\frac{(1 - \gamma_0^{\frac{1}{4}})\alpha_0 + (1 + \gamma_0^{\frac{1}{4}})}{(1 - \gamma_0^{\frac{3}{4}})\alpha_0 + (1 + \gamma_0^{\frac{3}{4}})}}.\end{aligned}\quad (27)$$

As expected, $\hat{\beta}$ depends on only three bimaterial parameters α_0 , β_0 and γ_0 , but not λ_1 .

3. Interface crack between a thin film and a substrate

3.1. Interface stress intensity factors

Consider an interface crack between a thin film with material 1 and a substrate with material 2 as shown in Fig. 1. Each material is assumed to be orthotropic, with the principal material axes of the materials being coincident with the Cartesian coordinates x_1 and x_2 axes. The film with thickness h is bonded to the half plane substrate. The interface crack lies on the interface between the film and substrate along the negative x_1 axis. The crack surface is assumed to be traction free, and the uncracked interface is bonded perfectly. Compressive load P and bending moment M per unit thickness are applied on the neutral axis of the film at $x_1 = -\infty$. In addition, all the stresses in the substrate vanish at infinity.

We seek an expression of interface stress intensity factors based on the path independence of the J integral (Park and Earmme, 1986; Rice, 1968). The complex functions generating singular fields in the vicinity of the tip of an interface crack between two dissimilar anisotropic media are expressed as (Beom and Atluri, 1995a)

$$\begin{aligned}\mathbf{g}^{(1)}(z) &= \frac{1}{2\sqrt{2\pi z}}(\mathbf{I} + i\beta)\mathbf{Y}(z^{ie}, z^{-ie})\mathbf{k}, \\ \mathbf{g}^{(2)}(z) &= \frac{1}{2\sqrt{2\pi z}}(\mathbf{I} - i\beta)\mathbf{Y}(z^{ie}, z^{-ie})\mathbf{k}.\end{aligned}\quad (28)$$

Here \mathbf{I} is the identity matrix and $\mathbf{Y}(\zeta_1, \zeta_2)$ is the matrix function defined by

$$\mathbf{Y}(\zeta_1, \zeta_2) = \frac{1}{2}(\zeta_1 + \zeta_2)\mathbf{I} + \frac{i}{2|\beta_0|}(\zeta_1 - \zeta_2)\beta. \quad (29)$$

ε and \mathbf{k} are the oscillatory index and vector of interface stress intensity factors, respectively, defined by

$$\varepsilon = \frac{1}{2\pi} \ln \frac{1 + |\beta_0|}{1 - |\beta_0|}, \quad (30)$$

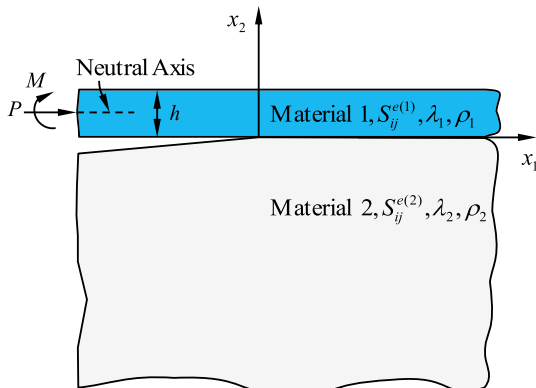


Fig. 1. Interface crack in an orthotropic bimaterial with a thin film under compressive load P and bending moment M . The normal stress in the film at $x_1 = -\infty$ is $\sigma_{11}(-\infty, x_2) = -\frac{P}{h} - \frac{12M}{h^3}(x_2 - \frac{h}{2})$ ($0 \leq x_2 \leq h$).

$$\mathbf{k} = \begin{Bmatrix} K_2 \\ K_1 \end{Bmatrix} = \lim_{x_1 \rightarrow 0^+} \sqrt{2\pi x_1} \mathbf{Y}(x_1^{-ie}, x_1^{ie}) \begin{Bmatrix} \sigma_{21}(x_1, 0) \\ \sigma_{22}(x_1, 0) \end{Bmatrix}. \quad (31)$$

The interface stress intensity factors recover the classical stress intensity factors as an orthotropic bimaterial degenerates to be a homogeneous one. The definition of interface stress intensity factors is different from that used in Suo (1990a) and Xu et al. (1993). Suo (1990a) introduced a complex stress intensity factor based on components of the traction vector in a coordinate system whose base vectors are orthogonal eigenvectors. He noted that the complex stress intensity factor for the oscillatory field does not reduce to the classical stress intensity factor for a homogeneous orthotropic material. Using Eq. (28) in evaluating the J integral, it can be shown in accordance with Beom and Atluri (1995b) that

$$J^0 = \frac{1}{4\cosh^2 \pi \varepsilon} \mathbf{k}^{*T} \mathbf{D} \mathbf{k}^* \quad (32)$$

where J^0 is the J integral over a circle enclosing the tip of the interface crack with a vanishingly small radius, and

$$\mathbf{D} = \mathbf{H}^{(1)} + \mathbf{H}^{(2)} \quad (33)$$

$$\mathbf{k}^* = \begin{Bmatrix} K_2^* \\ K_1^* \end{Bmatrix} = \mathbf{Y}(h^{ie}, h^{-ie})\mathbf{k}. \quad (34)$$

J^0 has the physical meaning of energy release rate. It is noted that \mathbf{k}^* has the same dimension of the conventional stress intensity factor with stress times the square root of length. The J integral evaluated over an outer boundary of the orthotropic composite consisting of the film and the half plane substrate, denoted by J^∞ , is given by (Xu et al., 1993)

$$J^\infty = S_{11}^{e(1)} \left(\frac{6M^2}{h^3} + \frac{P^2}{2h} \right) = S_{11}^{e(1)} \mathbf{t}^\infty T \mathbf{t}^\infty, \quad (35)$$

where

$$\mathbf{t}^\infty = \begin{Bmatrix} -\sqrt{\frac{6}{h^3}} M \\ \frac{P}{\sqrt{2h}} \end{Bmatrix}. \quad (36)$$

The path independence of the J integral, $J^0 = J^\infty$, implies

$$\|\mathbf{D}^{\frac{1}{2}} \mathbf{k}^*\| = 2 \cosh \pi \varepsilon \|\sqrt{S_{11}^{e(1)}} \mathbf{t}^\infty\| \quad (37)$$

where $\|\cdot\|$ denotes the magnitude of a vector. The two vectors, $\mathbf{D}^{\frac{1}{2}} \mathbf{k}^*$ and $2 \cosh \pi \varepsilon \sqrt{S_{11}^{e(1)}} \mathbf{t}^\infty$ have the same magnitude, but differ only by phase shift. Introducing a rotation matrix Ω , thus it can be shown from Eq. (37) that

$$\mathbf{k} = 2 \cosh \pi \varepsilon \sqrt{S_{11}^{e(1)}} \mathbf{Y}(h^{-ie}, h^{ie}) \mathbf{D}^{-\frac{1}{2}} \Omega(\omega) \mathbf{t}^\infty, \quad (38)$$

in which

$$\Omega(\omega) = \begin{bmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{bmatrix}, \quad (39)$$

where ω is an angle between two vectors of $\mathbf{D}^{\frac{1}{2}} \mathbf{k}^*$ and \mathbf{t}^∞ . Using the relations

$$\mathbf{D} = 2\mathbf{H}^{(1)}(\mathbf{I} - \alpha)^{-1}. \quad (40)$$

$$\mathbf{H}^{(1)} = 2 \left(n \sqrt{S_{11}^e S_{22}^e} \right)^{(1)} \lambda_1^{-\frac{1}{4}} \Lambda^{-2} \quad (41)$$

Eq. (38) can be rewritten as

$$\mathbf{k} = \frac{\lambda_1^{\frac{3}{4}} \cosh \pi \varepsilon}{\sqrt{n^{(1)}}} \mathbf{Y}(h^{-ie}, h^{ie}) (\mathbf{I} - \alpha)^{\frac{1}{2}} \Lambda \Omega(\omega) \mathbf{t}^\infty \quad (42)$$

The complete expression of interface stress intensity factors is obtained as above, apart from only one dimensionless function ω undetermined.

We consider a homogeneous orthotropic material and an isotropic bimaterial as special cases of an orthotropic bimaterial. For a homogeneous material case, Eq. (42) reduces to

$$\mathbf{k} = \frac{1}{\sqrt{n}} \begin{bmatrix} \lambda^{\frac{1}{8}} & 0 \\ 0 & \lambda^{\frac{3}{8}} \end{bmatrix} \mathbf{\Omega}(\omega) \mathbf{t}^\infty. \quad (43)$$

Eq. (43) can be rewritten in the component forms as

$$\begin{aligned} K_I &= \frac{\lambda^{\frac{1}{8}}}{\sqrt{n}} \left(\frac{P}{\sqrt{2h}} \cos \omega + \sqrt{\frac{6}{h^3}} M \sin \omega \right), \\ K_{II} &= \frac{\lambda^{\frac{1}{8}}}{\sqrt{n}} \left(\frac{P}{\sqrt{2h}} \sin \omega - \sqrt{\frac{6}{h^3}} M \cos \omega \right), \end{aligned} \quad (44)$$

where K_I and K_{II} are the mode I and Mode II stress intensity factors, respectively. The result Eq. (44) is identical to that obtained by Suo (1990b). After some simple algebra, it is readily shown that the interface stress factor for an isotropic bimaterial is obtained from Eq. (42) as

$$\mathbf{k} = \sqrt{\frac{1 - \alpha_0}{1 - \beta_0^2}} \mathbf{\Omega}(\omega - \varepsilon_0 \ln h) \mathbf{t}^\infty, \quad (45)$$

where

$$\varepsilon_0 = \frac{1}{2\pi} \ln \frac{1 - \beta_0}{1 + \beta_0}. \quad (46)$$

In obtaining Eq. (45), the following relation was used

$$\cosh \pi \varepsilon = \frac{1}{\sqrt{1 - \beta_0^2}}. \quad (47)$$

Eq. (45) can be recast in the complex form

$$K_1 + iK_2 = \sqrt{\frac{1 - \alpha_0}{1 - \beta_0^2}} \left(\frac{P}{\sqrt{2h}} - i \sqrt{\frac{6}{h^3}} M \right) e^{i\omega} h^{-i\varepsilon_0}. \quad (48)$$

The complex stress intensity factor in Eq. (48) is consistent with Suo and Hutchinson (1990).

3.2. Dependence of ω on material constants

According to Xu et al. (1993), the stress fields in the orthotropic bimaterial depend on six independent material parameters. These many material parameters cause the great complexity in presenting the numerical results of dimensionless function ω . We attempt to reduce the number of necessary material parameters involved in ω . Based upon the modified Stroh formalism, it can be shown that the boundary and continuity conditions for the orthotropic bimaterial structure lead to

$$\begin{aligned} 2\text{Re} \left[\sum_{j,m=1}^2 B_{ij}^{(1)} p_j^{(1)} B_{jm}^{-1(1)} g_m^{(1)}(x_1 + p_j^{(1)} x_2) \right] &= \begin{cases} \frac{P}{h} + \frac{12M}{h^3} (x_2 - \frac{h}{2}) \\ 0 \end{cases} x_1 \rightarrow -\infty, \quad 0 \leq x_2 \leq h, \\ \text{Re}[g^{(1)}(x_1)] &= \text{Re}[g^{(2)}(x_1)] = 0, \quad x_1 < 0, \\ \text{Re}[g^{(1)}(x_1)] &= \text{Re}[g^{(2)}(x_1)], \quad x_1 > 0, \\ \text{Im}[(\mathbf{I} - \boldsymbol{\alpha} - i\boldsymbol{\beta})g^{(1)}(x_1)] &= \text{Im}[(\mathbf{I} + \boldsymbol{\alpha} + i\boldsymbol{\beta})g^{(2)}(x_1)], \quad x_1 > 0, \\ \text{Re} \left[\sum_{j,m=1}^2 B_{ij}^{(1)} B_{jm}^{-1(1)} g_m^{(1)}(x_1 + p_j^{(1)} h) \right] &= 0, \quad -\infty < x_1 < \infty, \quad x_2 = h, \\ g^{(2)}(z) &= 0, \quad \sqrt{x_1^2 + x_2^2} \rightarrow \infty, \quad x_2 < 0, \end{aligned} \quad (49)$$

where Im denotes the imaginary part. In deriving Eq. (49), use of the results for the continuity conditions on the interface obtained by Beom and Atluri (1995a) has been made. All the equations in Eq.

(49) explicitly involve only $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, λ_1 and ρ_1 , but not λ_2 and ρ_2 , the solutions have the forms:

$$\mathbf{g}^{(j)}(z) = \mathbf{g}^{(j)}(z; \alpha_0, \beta_0, \gamma_0, \rho_1, \lambda_1) \quad (j = 1, 2). \quad (50)$$

Since the interface stress intensity factor can be evaluated from

$$\mathbf{k} = \lim_{x_1 \rightarrow 0^+} 2\sqrt{2\pi x_1} \mathbf{Y}(x_1^{-ie}, x_1^{ie}) \text{Re}[g^{(1)}(x_1)] \quad (51)$$

it is apparent from Eq. (50) that

$$\omega = \omega(\alpha_0, \beta_0, \gamma_0, \rho_1, \lambda_1). \quad (52)$$

The function ω depends on five nondimensional material parameters, excluding λ_2 and ρ_2 explicitly.

Next, we use the orthotropic rescaling technique to obtain the explicit dependence of λ_1 on ω . The transformed bimaterial structure consisting of a film with thickness \hat{h} and a half plane substrate is subjected to the transformed load \hat{P} and bending moment \hat{M} . The corresponding quantities for the transformed problem are given by

$$\begin{aligned} \hat{h} &= \lambda_1^{-\frac{1}{4}} h, \\ \hat{P} &= \lambda_1^{\frac{1}{4}} P, \\ \hat{M} &= M, \\ \hat{\mathbf{t}}^\infty &= \lambda_1^{\frac{3}{4}} \mathbf{t}^\infty. \end{aligned} \quad (53)$$

The interface stress intensity factor $\hat{\mathbf{k}}$ for the transformed problem, defined by

$$\hat{\mathbf{k}} = \lim_{\hat{x}_1 \rightarrow 0^+} \sqrt{2\pi \hat{x}_1} \hat{\mathbf{Y}}(\hat{x}_1^{-ie}, \hat{x}_1^{ie}) \begin{Bmatrix} \hat{\sigma}_{21}(\hat{x}_1, 0) \\ \hat{\sigma}_{22}(\hat{x}_1, 0) \end{Bmatrix} \quad (54)$$

is given from Eq. (42) by

$$\hat{\mathbf{k}} = \frac{\cosh \pi \varepsilon}{\sqrt{n^{(1)}}} \hat{\mathbf{Y}}(\hat{h}^{-ie}, \hat{h}^{ie}) (\mathbf{I} - \boldsymbol{\alpha})^{\frac{1}{2}} \mathbf{\Omega}(\hat{\omega}) \hat{\mathbf{t}}^\infty, \quad (55)$$

where

$$\hat{\mathbf{Y}}(\zeta_1, \zeta_2) = \frac{1}{2} (\zeta_1 + \zeta_2) \mathbf{I} + \frac{i}{2|\beta_0|} (\zeta_1 - \zeta_2) \hat{\boldsymbol{\beta}}. \quad (56)$$

It is clear from Eq. (52) that for the transformed problem with $\hat{\lambda}_1 = 1$

$$\hat{\omega} = \hat{\omega}(\alpha_0, \beta_0, \gamma_0, \rho_1). \quad (57)$$

Using Eqs. (22), (31), and (54), we obtain

$$\mathbf{k} = \boldsymbol{\Lambda} \hat{\mathbf{k}}. \quad (58)$$

Substituting Eq. (53) for $\hat{\mathbf{t}}^\infty$ and Eq. (55) into Eq. (58) yields

$$\mathbf{k} = \frac{\lambda_1^{\frac{3}{4}} \cosh \pi \varepsilon}{\sqrt{n^{(1)}}} \boldsymbol{\Lambda} \hat{\mathbf{Y}}(\hat{h}^{-ie}, \hat{h}^{ie}) (\mathbf{I} - \boldsymbol{\alpha})^{\frac{1}{2}} \mathbf{\Omega}(\hat{\omega}) \mathbf{t}^\infty. \quad (59)$$

Combining Eq. (42) and Eq. (59) and after some mathematical manipulations, it is readily shown that

$$\mathbf{\Omega}(\omega) = \hat{\mathbf{Y}}^0(\lambda_1^{\frac{1}{4}ie}, \lambda_1^{-\frac{1}{4}ie}) \mathbf{\Omega}(\hat{\omega}), \quad (60)$$

where

$$\begin{aligned} \hat{\mathbf{Y}}^0(\zeta_1, \zeta_2) &= \frac{1}{2} (\zeta_1 + \zeta_2) \mathbf{I} + \frac{i}{2|\beta_0|} (\zeta_1 - \zeta_2) \hat{\boldsymbol{\beta}}^0, \\ \hat{\boldsymbol{\beta}}^0 &= \beta_0 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \end{aligned} \quad (61)$$

In obtaining Eq. (60), the following relations were used

$$\begin{aligned}\Lambda^{-1}\mathbf{Y}(h^{ie}, h^{-ie})\Lambda &= \hat{\mathbf{Y}}(h^{ie}, h^{-ie}), \\ \mathbf{Y}(\xi_1, \xi_2)\mathbf{Y}(\xi_1, \xi_2) &= \mathbf{Y}(\xi_1\xi_1, \xi_2\xi_2), \\ (\mathbf{I} - \boldsymbol{\alpha})^{\frac{1}{2}}\hat{\mathbf{Y}}(h^{-ie}, h^{ie})(\mathbf{I} - \boldsymbol{\alpha})^{\frac{1}{2}} &= \hat{\mathbf{Y}}^0(h^{-ie}, h^{ie}).\end{aligned}\quad (62)$$

With the help of the relation

$$\hat{\mathbf{Y}}^0(\mathbf{x}^{ie}, \mathbf{x}^{-ie}) = \boldsymbol{\Omega}(\varepsilon_0 \ln \mathbf{x}), \quad (63)$$

we finally have from Eq. (60)

$$\omega = \hat{\omega} + \frac{1}{4}\varepsilon_0 \ln \lambda_1. \quad (64)$$

Once $\hat{\omega}$ for the transformed problem, which depends on four material parameters α_0 , β_0 , γ_0 , and ρ_1 , is determined, ω can be obtained from Eq. (64).

3.3. Numerical results

We first verify the independence of ω from ρ_2 through numerical computations. The elastic fields for the problem were obtained from finite element analysis using the commercial program, ABAQUS. The finite element mesh as shown in Fig. 2 was used in the analysis. The number of 8-node elements was about 50,000. The values used here were as follows: $P = 1$, $M = 0$, $h = 1$, $W/h = 10^5$, $h_0/h = 2000$ where W and h_0 are the half width and the height of the material 2, respectively. The close agreement between J^∞ and J^0 was observed within a 0.05% error. The interface stress intensity factors were evaluated by using the domain integral form of the mutual integral based on the J integral (Chow et al., 1995). ω then was extracted from Eq. (42). The numerical results of ω are plotted in Fig. 3 for three combinations of bimaterial parameters α_0 , β_0 and γ_0 , as ρ_2 varies. Here the orthotropic parameters for material 1 used in the numerical computations were $\lambda_1 = 2$ and $\rho_1 = 5$. Fig. 3 shows that ω does not depend on ρ_2 for all the cases of three combinations of the bimaterial parameters.

Similarly, $\hat{\omega}$ for the transformed problem was numerically calculated. The numerical results of $\hat{\omega}$ for the transformed structure with isotropic film ($\lambda_1 = 1$, $\rho_1 = 1$) are plotted as a function of α_0 in Figs. 4 and 5 for $\beta_0 = 0$ and $\beta_0 = \alpha_0/4$, respectively. Here $\gamma_0 = 0.1, 1$, and 10 were used. For the special case of $\gamma_0 = 1$, our results were observed to be in close agreement with those for the isotropic bimaterial case obtained by Suo and Hutchinson (1990). The results for the orthotropic bimaterial with $\gamma_0 = 0.1$ and 10 are slightly different from those for the isotropic bimaterial with $\gamma_0 = 1$, and the differences between them are within about 2° (0.035 radian). However, the weak dependence of $\hat{\omega}$ on γ_0 does not imply that the effect of γ_0 on the interface stress intensity factor is negligible, since the interface stress intensity factor also explicitly depends on $\boldsymbol{\alpha}$. For the film with cubic symmetry, the effect of ρ_1 on $\hat{\omega}$ is illustrated in Figs. 6–9. It is seen from Figs. 6 and 7 that as for $\gamma_0 = 0.1$, the influence of ρ_1 on $\hat{\omega}$ for $\alpha_0 > 0$ is larger

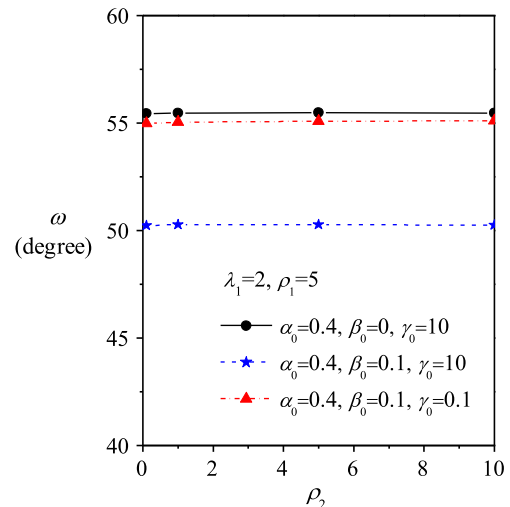


Fig. 3. Variation of dimensionless function ω with ρ_2 for three combinations of α_0 , β_0 and γ_0 . Here $\lambda_1 = 2$ and $\rho_1 = 5$.

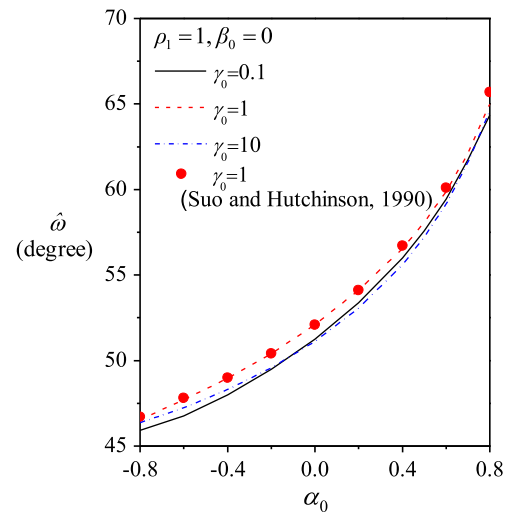


Fig. 4. Dimensionless function $\hat{\omega}$ for bimaterial structures with $\rho_1 = 1$ and $\beta_0 = 0$ as a function of α_0 for specific values of γ_0 .

than that for $\alpha_0 < 0$. Figs. 8 and 9 show the effect of ρ_1 on $\hat{\omega}$ for the bimaterial with $\gamma_0 = 10$. The effect of ρ_1 on $\hat{\omega}$ for $\alpha_0 > 0$ is smaller than that for $\alpha_0 < 0$. For the bimaterial with $\beta_0 = 0$, $\hat{\omega}$ increases monotonically with the increasing α_0 regardless of the ρ_1 values ($\rho_1 = 0.1, 1, 10$). When $\beta_0 = \alpha_0/4$, however, the change in

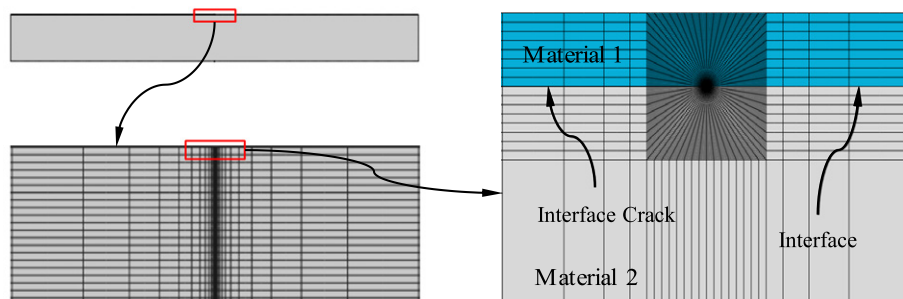


Fig. 2. Finite element mesh configuration for the interface crack problem. Details of finite element mesh near the crack tip are shown at two different levels of magnification.

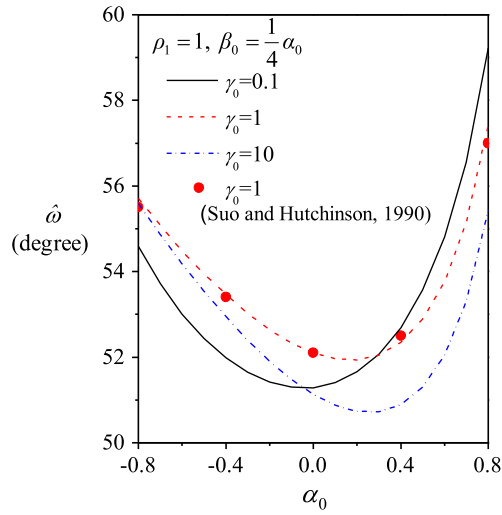


Fig. 5. Dimensionless function $\hat{\omega}$ for bimaterial structures with $\rho_1 = 1$ and $\beta_0 = \alpha_0/4$ as a function of α_0 for specific values of γ_0 .

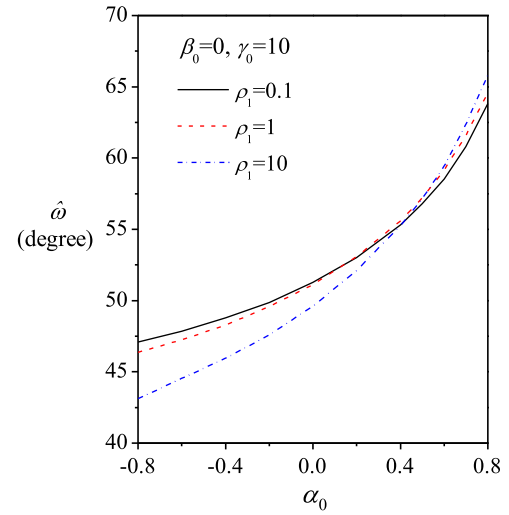


Fig. 8. Dimensionless function $\hat{\omega}$ for bimaterial structures with $\beta_0 = 0$ and $\gamma_0 = 10$ as a function of α_0 for specific values of ρ_1 .

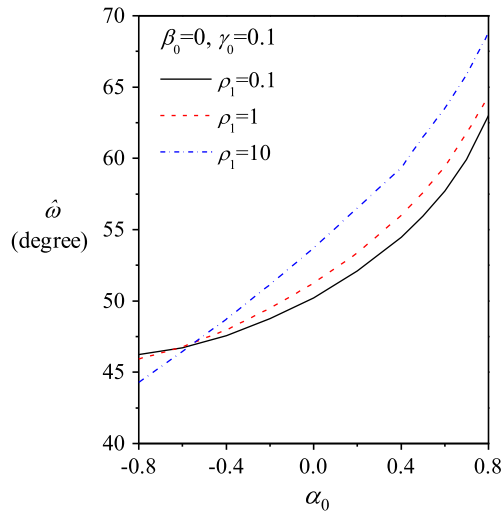


Fig. 6. Dimensionless function $\hat{\omega}$ for bimaterial structures with $\beta_0 = 0$ and $\gamma_0 = 0.1$ as a function of α_0 for specific values of ρ_1 .

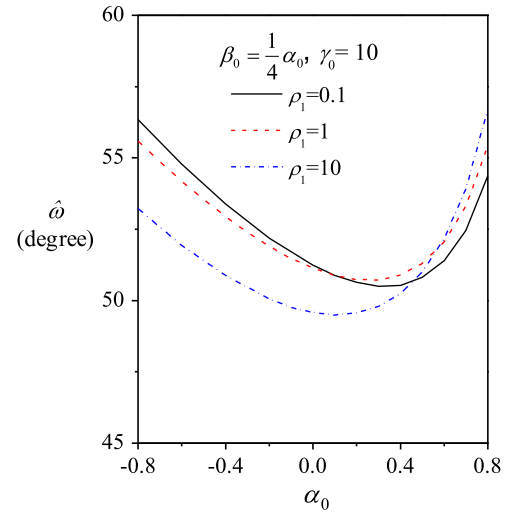


Fig. 9. Dimensionless function $\hat{\omega}$ for bimaterial structures with $\beta_0 = \alpha_0/4$ and $\gamma_0 = 10$ as a function of α_0 for specific values of ρ_1 .

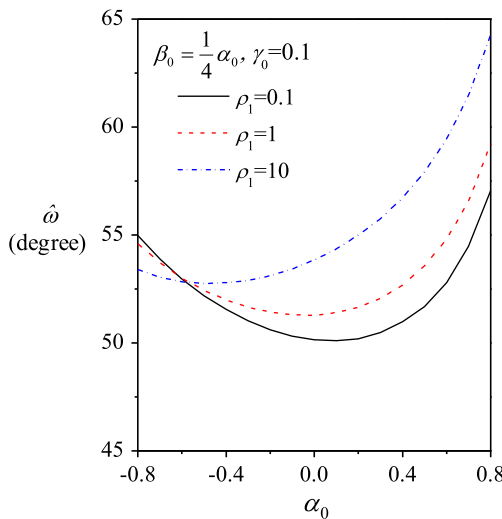


Fig. 7. Dimensionless function $\hat{\omega}$ for bimaterial structures with $\beta_0 = \alpha_0/4$ and $\gamma_0 = 0.1$ as a function of α_0 for specific values of ρ_1 .

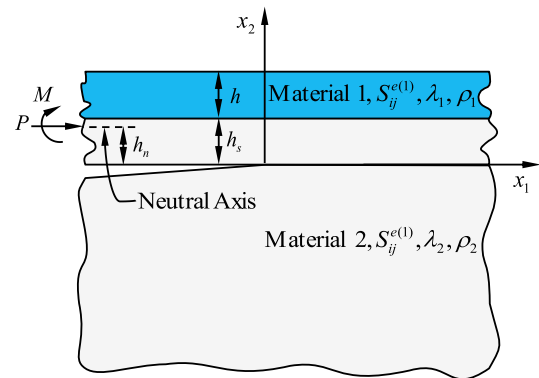


Fig. 10. Crack in an orthotropic substrate with an orthotropic film under compressive load P and bending moment M . The normal stress in the edge at $x_1 = -\infty$ is $\sigma_{11}(-\infty, x_2) = -\Gamma \left[\frac{P}{hA_b} + \frac{M}{h^2 I_b} (x_2 - h_n) \right]$ for $h_s < x_2 \leq h_s + h$, and $\sigma_{11}(-\infty, x_2) = -\left[\frac{P}{hA_b} + \frac{M}{h^2 I_b} (x_2 - h_n) \right]$ for $0 \leq x_2 < h_s$.

$\hat{\omega}$ with α_0 followed a different pattern; $\hat{\omega}$ has a minimum which depends on ρ_1 and γ_0 . For all the cases ($-0.8 \leq \alpha_0 \leq 0.8$; $\beta_0 = 0$, $\alpha_0/4$; $\gamma_0 = 0.1, 1, 10$; $\rho_1 = 0.1, 1, 10$), the difference between $\hat{\omega}$ for the orthotropic bimaterial and $\hat{\omega}$ for the isotropic bimaterial is within about 5° (0.087 radian).

4. Crack in a substrate with an adherent film

4.1. Stress intensity factor

Consider a crack in the orthotropic substrate as shown in Fig. 10. The crack parallels the interface, and the distance from the interface to the crack surface is h_s . The origin of the Cartesian coordinates x_1 and x_2 coinciding with the principal material axes of the orthotropic substrate and film is located at the crack tip. Tractions on the crack surfaces are free. Compressive load P and bending moment M per unit thickness are acted on the neutral axis of the bimaterial layer consisting of material 1 with thickness h and material 2 with thickness h_s at $x_1 = -\infty$.

In a similar manner to the case of the interface crack, we can obtain the stress intensity factor for the subinterface crack. The solution procedure and result are briefly presented. Evaluating the J integrals for the small path enclosing the crack tip and large contour consisting of the infinite boundary and the upper surface of film, we have

$$J^s = \frac{1}{2} \mathbf{k}^{sT} \mathbf{H}^{(2)} \mathbf{k}^s, \quad (65)$$

$$J^\infty = \frac{1}{2} S_{11}^{(2)} \left[\frac{M^2}{I_b h^3} + \frac{P^2}{A_b h} \right] = \frac{1}{2} S_{11}^{(2)} \mathbf{t}^{sT} \mathbf{t}^s$$

where J^s and J^∞ are the J integrals at the crack tip and evaluated over the outer boundary, respectively, and

$$\mathbf{k}^s = \begin{Bmatrix} K_{II}^s \\ K_I^s \end{Bmatrix}, \quad \mathbf{t}^s = \begin{Bmatrix} -\frac{M}{\sqrt{I_b h^3}} \\ \frac{P}{\sqrt{A_b h}} \end{Bmatrix},$$

$$A_b = \eta_s + \Gamma,$$

$$I_b = \frac{1}{3} [\Gamma \{3(\eta_n - \eta_s)^2 - 3(\eta_n - \eta_s) + 1\} + 3\eta_n \eta_s (\eta_n - \eta_s) + \eta_s^3],$$

$$\Gamma = \frac{S_{11}^{(2)}}{S_{11}^{(1)}} = \frac{n^{(1)}}{n^{(2)}} \frac{1 + \alpha_0}{1 - \alpha_0},$$

$$\eta_n = \frac{h_n}{h},$$

$$\eta_s = \frac{h_s}{h},$$

$$\eta_n = \frac{\eta_s^2 + 2\Gamma\eta_s + \Gamma}{2(\eta_s + \Gamma)}. \quad (66)$$

Here, K_I^s and K_{II}^s are the mode I and mode II stress intensity factors for the subinterface crack, respectively, and h_n is the distance from the x_1 axis to the neutral axis. Invoking the path independence of the J integral, $J^s = J^\infty$, it can be shown that

$$\mathbf{k}^s = \sqrt{S_{11}^{(2)}} \mathbf{H}^{(2)-\frac{1}{2}} \boldsymbol{\Omega}(\psi) \mathbf{t}^s, \quad (67)$$

where ψ is a dimensionless function for the crack in the substrate. We note that Eq. (67) reduces to the known results for a homogeneous orthotropic material (Suo, 1990b) and for an isotropic bimaterial (Suo and Hutchinson, 1989). Eq. (67) can be rewritten as

$$\mathbf{k}^s = \frac{1}{\sqrt{2}} \frac{\lambda_1^{\frac{3}{8}} \gamma_0^{\frac{3}{8}}}{\sqrt{n^{(2)}}} \boldsymbol{\Lambda} \boldsymbol{\Lambda}^* \boldsymbol{\Omega}(\psi) \mathbf{t}^s, \quad (68)$$

where

$$\boldsymbol{\Lambda}^* = \begin{bmatrix} \gamma_0^{-\frac{1}{4}} & 0 \\ 0 & 1 \end{bmatrix}. \quad (69)$$

Using the modified Stroh formalism and dimensional analysis, we have

$$\psi = \psi(\alpha_0, \beta_0, \gamma_0, \rho_1, \rho_2, \lambda_1, \eta_s). \quad (70)$$

Applying the orthotropy rescaling method, we obtain for the transformed problem

$$\begin{aligned} \hat{h} &= \lambda_1^{-\frac{1}{4}} h, \\ \hat{A}_b &= A_b, \\ \hat{I}_b &= I_b, \\ \hat{P} &= \lambda_1^{\frac{1}{4}} P, \\ \hat{M} &= M, \\ \hat{\mathbf{t}}^s &= \lambda_1^{\frac{3}{8}} \mathbf{t}^s. \end{aligned} \quad (71)$$

Using the relations

$$\begin{aligned} \mathbf{k}^s &= \boldsymbol{\Lambda} \hat{\mathbf{k}}^s, \\ \hat{\mathbf{k}}^s &= \frac{1}{\sqrt{2}} \frac{\gamma_0^{\frac{3}{8}}}{\sqrt{n^{(2)}}} \boldsymbol{\Lambda}^* \boldsymbol{\Omega}(\hat{\psi}) \hat{\mathbf{t}}^s \end{aligned} \quad (72)$$

it is easily seen that

$$\mathbf{k}^s = \frac{1}{\sqrt{2}} \frac{\lambda_1^{\frac{3}{8}} \gamma_0^{\frac{3}{8}}}{\sqrt{n^{(2)}}} \boldsymbol{\Lambda} \boldsymbol{\Lambda}^* \boldsymbol{\Omega}(\hat{\psi}) \mathbf{t}^s. \quad (73)$$

From Eqs. (68) and (73), we have

$$\psi = \hat{\psi}(\alpha_0, \beta_0, \gamma_0, \rho_1, \rho_2, \eta_s). \quad (74)$$

Eq. (74) predicts that ψ does not depend on λ_1 explicitly.

4.2. Dimensionless function ψ

The solution of ψ for the case of $0 < \eta_s \ll 1$ can be obtained from the asymptotic analysis of a semi-infinite subinterface crack in the orthotropic bimaterial composed of material 1 and material 2 (Hutchinson et al., 1987). The crack paralleling the interface lies in the lower material 2 along the negative x_1 axis. The depth of the crack below the interface is h_s . Tractions on the crack surfaces are free. The remote field with the form in Eq. (28) is prescribed at infinity. In the asymptotic problem, the applied interface stress intensity factor, \mathbf{k} , is given by Eq. (38). According to Beom and Atluri (1995b), the stress intensity factor for the subinterface crack is given by

$$\mathbf{k}^s = \frac{1}{\sqrt{2}} \frac{1}{\cosh \pi \varepsilon} \mathbf{H}^{(2)-\frac{1}{2}} \boldsymbol{\Omega}(\phi) \mathbf{D}^{\frac{1}{2}} \mathbf{Y}(h_s^{ie}, h_s^{-ie}) \mathbf{k}, \quad (75)$$

where ϕ is a dimensionless function given by

$$\phi = \phi(\alpha_0, \beta_0, \gamma_0, \rho_2, \lambda_1). \quad (76)$$

The numerical solution of ϕ was given in Beom and Atluri (1995b). Substituting Eq. (38) into Eq. (75) yields

$$\mathbf{k}^s = \sqrt{2S_{11}^{(1)}} \mathbf{H}^{(2)-\frac{1}{2}} \boldsymbol{\Omega}(\phi) \mathbf{D}^{\frac{1}{2}} \mathbf{Y}(\eta_s^{ie}, \eta_s^{-ie}) \mathbf{D}^{-\frac{1}{2}} \boldsymbol{\Omega}(\omega) \mathbf{t}^\infty. \quad (77)$$

From Eqs. (67) and (77), we have

$$\boldsymbol{\Omega}(\psi) = \boldsymbol{\Omega}(\phi) \mathbf{D}^{\frac{1}{2}} \mathbf{Y}(\eta_s^{ie}, \eta_s^{-ie}) \mathbf{D}^{-\frac{1}{2}} \boldsymbol{\Omega}(\omega). \quad (78)$$

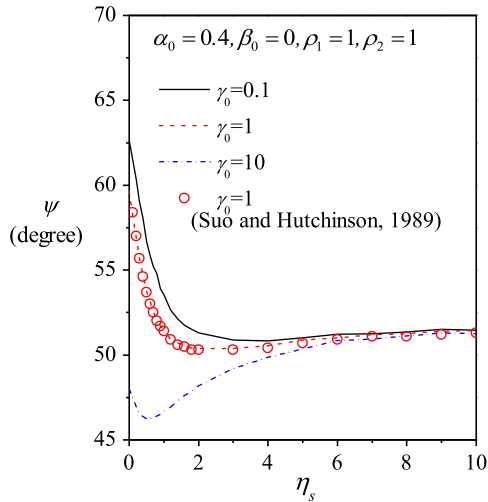


Fig. 11. Dimensionless function ψ for degenerate orthotropic bimetals with specific values of γ_0 as a function of η_s . Here $\alpha_0 = 0.4$ and $\beta_0 = 0$.

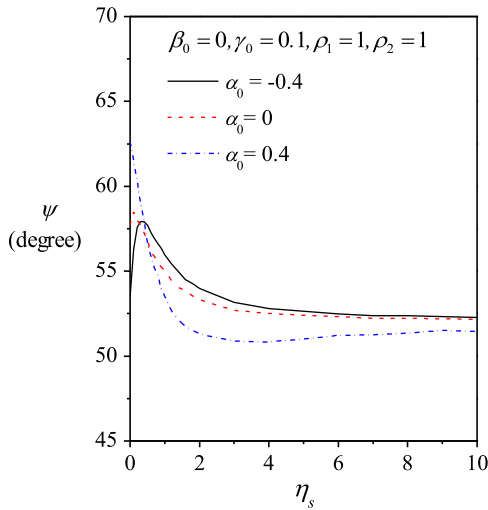


Fig. 12. Dimensionless function ψ for degenerate orthotropic bimetals with specific values of α_0 as a function of η_s . Here $\beta_0 = 0$ and $\gamma_0 = 0.1$.

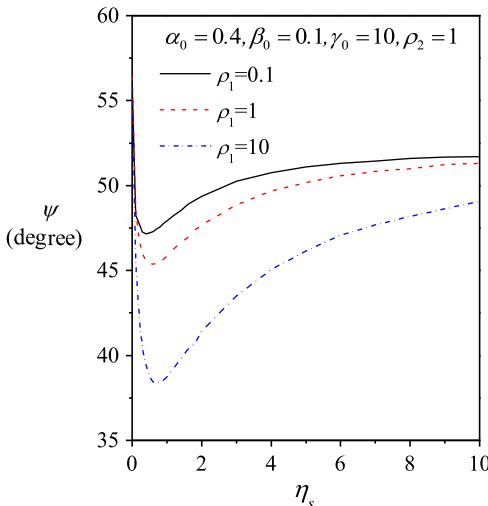


Fig. 13. Dimensionless function ψ for orthotropic bimetals with specific values of ρ_1 as a function of η_s . Here $\alpha_0 = 0.4$, $\beta_0 = 0.1$, $\gamma_0 = 10$, and $\rho_2 = 1$.

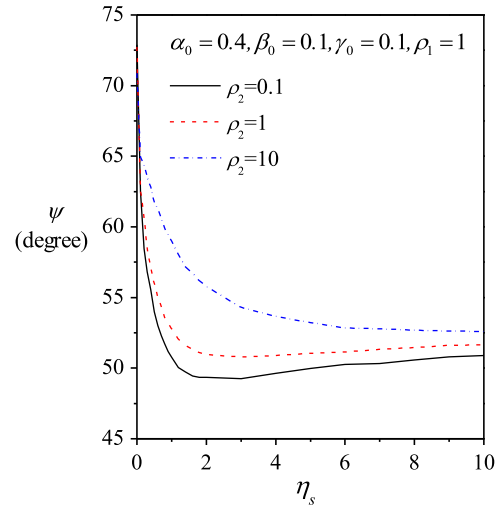


Fig. 14. Dimensionless function ψ for orthotropic bimetals with specific values of ρ_2 as a function of η_s . Here $\alpha_0 = 0.4$, $\beta_0 = 0.1$, $\gamma_0 = 0.1$, and $\rho_1 = 1$.

In obtaining Eq. (78), the following relation was used

$$\mathbf{t}^s = \sqrt{\frac{2}{\Gamma}} \mathbf{t}^\infty \quad \text{for } 0 < \eta_s \ll 1. \quad (79)$$

Using the relation

$$\mathbf{D}^{\frac{1}{2}} \mathbf{Y}(\eta_s^{ie}, \eta_s^{-ie}) \mathbf{D}^{-\frac{1}{2}} = \mathbf{\Omega}(\varepsilon_0 \ln \eta_s) \quad (80)$$

we obtain from Eq. (78)

$$\psi = \omega + \phi + \varepsilon_0 \ln \eta_s \quad \text{for } 0 < \eta_s \ll 1. \quad (81)$$

The function ψ for $0 < \eta_s \ll 1$ is obtained from the solutions to the interface crack problem and the asymptotic problem of subinterface crack. Employing the orthotropic rescaling technique, it can be shown that

$$\phi = \hat{\phi} - \frac{1}{4} \varepsilon_0 \ln \lambda_1. \quad (82)$$

We note that ω and ϕ depend on λ_1 , whereas ψ is independent of λ_1 since $\omega + \phi = \hat{\omega} + \hat{\phi}$.

Similarly, numerical computations were performed to obtain the dimensionless function ψ . The numerical results of ψ are plotted as a function of η_s in Figs. 11–14 for various combinations of material parameters α_0 , β_0 , γ_0 , ρ_1 , and ρ_2 . For the isotropic bimaterial case ($\alpha_0 = 0.4$ and $\beta_0 = 0$), as seen in Fig. 11, our results were in good agreement with those of Suo and Hutchinson (1989). As η_s approached a small value, the values of ψ converged to those given in Eq. (81). The results of ψ for the orthotropic bimaterial cases were significantly different from those for the isotropic bimaterial cases. The change of ψ was observed to be sensitive to material parameters α_0 , γ_0 , ρ_1 , and ρ_2 . However, the effect of material parameters α_0 , γ_0 , ρ_1 , and ρ_2 on ψ decreases as η_s becomes larger. When η_s has a small value, ψ for $\alpha_0 > 0$ ($\alpha_0 = 0, 0.4$) decreases in the increase of η_s regardless of β_0 , γ_0 , ρ_1 , and ρ_2 , while ψ for $\alpha_0 < 0$ ($\alpha_0 = -0.4$) increases.

4.3. Thin film under residual tension

Consider an orthotropic bimaterial with a thin film manufactured at an elevated temperature T_0 under the stress-free state. As it cools from T_0 to the room temperature T , thermal residual stresses are created within the film due to the coefficient of thermal expansion mismatch between the film and the substrate. The biaxial misfit stresses σ_{11}^T and σ_{33}^T in the film are

$$\begin{aligned}\sigma_{11}^T &= \frac{1}{S_{11}^{(1)}S_{33}^{(1)} - S_{13}^{(1)2}} [S_{33}^{(1)}(\kappa_{11}^{(1)} - \kappa_{11}^{(2)}) - S_{13}^{(1)}(\kappa_{33}^{(1)} - \kappa_{33}^{(2)})](T_0 - T), \\ \sigma_{33}^T &= \frac{1}{S_{11}^{(1)}S_{33}^{(1)} - S_{13}^{(1)2}} [S_{11}^{(1)}(\kappa_{33}^{(1)} - \kappa_{33}^{(2)}) - S_{13}^{(1)}(\kappa_{11}^{(1)} - \kappa_{11}^{(2)})](T_0 - T),\end{aligned}\quad (83)$$

where κ_{11} and κ_{33} are the thermal expansion coefficients in the directions of x_1 and x_3 axes, respectively. The misfit stress σ_{11}^T is tensile when $S_{33}^{(1)}(\kappa_{11}^{(1)} - \kappa_{11}^{(2)}) > S_{13}^{(1)}(\kappa_{33}^{(1)} - \kappa_{33}^{(2)})$. The residual stresses in the film far away from side edges under traction-free conditions are the same as the biaxial misfit stresses. Using the cut and paste technique (Suo and Hutchinson, 1989), the stress intensity factors of the subinterface crack in the orthotropic bimaterial with the tensile residual stress in the film can be evaluated using the solution to the corresponding plane strain problem of subinterface crack under equivalent compressive load and bending moment. The equivalent loads are given by

$$\begin{aligned}P &= \sigma_{11}^T h, \\ M &= \sigma_{11}^T h \left(h_s - h_n + \frac{1}{2} h \right).\end{aligned}\quad (84)$$

Substituting Eq. (84) into Eq. (68), the stress intensity factors of the subinterface crack associated with the residual stress can be obtained.

Similarly, the interface stress intensity factors of the interface crack associated with the residual stress can be obtained from the solution of the corresponding plane strain interface crack problem with the following loads:

$$P = \sigma_{11}^T h, \quad M = 0. \quad (85)$$

The mode mixity for the interface crack is defined as (Hutchinson and Suo, 1992)

$$\chi = \tan^{-1} \left(\frac{\sigma_{12}(l, 0)}{\sigma_{22}(l, 0)} \right), \quad (86)$$

where χ is the mode mixity and l is a reference length chosen within the zone of the crack tip singular fields. The choice of reference length l is arbitrary. Taking the film thickness h as the reference length scale, it is obtained from Eqs. (34) and (86) that

$$\chi = \tan^{-1} \left(\frac{K_2^*}{K_1^*} \right). \quad (87)$$

The mode mixity can be evaluated from Eqs. (34), (42), (85), and (87), which results in

$$\chi = \tan^{-1} \left[\lambda_1^{-\frac{1}{4}} \sqrt{\frac{1 - \alpha_{11}}{1 - \alpha_{22}}} \tan \omega \right]. \quad (88)$$

It is seen from Eq. (88) that the mode mixity depends only on the material parameters. The role of the orthotropic parameters in χ is significant. For the isotropic bimaterial case, $\chi = \omega$. Based on the energy criterion, if $J^0 < J_c(\chi)$, the interface cracking with a large length of crack does not occur. Here J_c is the interface fracture energy which depends on the mode mixity. Our attention is restricted to the case of $\sigma_{11}^T > 0$. When the residual stress in the film is compressive, buckling delamination of the film may occur (Hutchinson and Suo, 1992).

6. Concluding remarks

We analyzed an interface crack in an orthotropic bimaterial structure consisting of a thin film and a half plane substrate. The orthotropic bimaterial structure is subjected to compressive load and bending moment per unit thickness on the neutral axis of the film. A complete expression of interface stress intensity factors

was obtained based on the path independence of the J integral, apart from one dimensionless parameter undetermined. The interface stress intensity factors recover the classical stress intensity factors as the orthotropic bimaterial degenerates to be a homogeneous one, in contrast to the previous work. Dependence of the dimensionless parameter on material constants was explored. A reduction of the number of necessary material parameters for the parameter was made based upon the modified Stroh formalism. The explicit dependence of the dimensionless parameter on one orthotropic parameter for the film was determined by using the orthotropy rescaling technique. It is shown that the dimensionless parameter can be obtained from the corresponding parameter for the transformed problem, which depends on five nondimensional material parameters. The effect of five material parameters on the corresponding parameter was investigated through numerical computations. The results for the orthotropic bimaterial are slightly different from those for the isotropic bimaterial. We also considered a subinterface crack, paralleling the interface, in the orthotropic substrate with a thin film. In a similar manner, we derived an expression of stress intensity factors for the subinterface crack. The stress intensity factors involved one undetermined parameter, which was obtained from numerical computations. The numerical results of the undetermined parameter for the orthotropic bimaterial cases were significantly different from those for the isotropic bimaterial cases. Special attention was given to the case in which the distance from the interface to the crack surface is very small. The stress intensity factors were obtained from the solutions to the interface crack problem and the asymptotic problem of subinterface crack.

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