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Nonlocal effects of longitudinal vibration in nanorod with internal long-range interactions

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ABSTRACT

The physically-based nonlocal model is used to investigate influences of the nonlocal long-range interactions on the longitudinal vibration of nanorod. The exact solution of the vibration is determined under the condition of a uniform nonlocal kernel. Nonlocal effects in the vibration of the nanorod are examined in detail. The results show that the nanorod becomes stiffer due to the internal long-range interactions. Meanwhile, an upper bound of the material parameter characterizing the long-range interactions is found. The low-frequency insulating effect induced by the long-range interactions is predicted. This effect shows that there exists a forbidden band of basic frequency within which external excitation is not transmitted in the nanorod. The Lagrangian formulations of the physically-based nonlocal theory are established based on a new definition of nonlocal variable. By these formulations, the physically-based nonlocal model can be conveniently expanded into beam, plate and shell.

1. Introduction

The origin of nonlocality is due to the long-range interactions within material. This idea was firstly advanced by Kroner (1967), and then systematically developed into the nonlocal mechanics theory (Edelen, 1972; Edelen et al., 1976; Kunin, 1982; Eringen, 2002; Bazant and Jirasek, 2002). So far, the nonlocal mechanics theory has been applied to account for some phenomena that are not explained by the classical elasticity and plasticity, such as stress singularity at the crack tip (Eringen, 2002; Bazant and Jirasek, 2002), softening bands in tensile specimens (Bazant and Jirasek, 2002; Borino et al., 2003; Polizzotto, 2003) and dispersion of acoustic waves in solids (Kunin, 1982; Eringen, 2002), etc. With the development of nanotechnology, considerable interests to the nonlocal mechanics are once again excited (Peddieson et al., 2003; Wang, 2005; Reddy, 2007; Lim, 2010).

Peddieson et al. (2003) proposed the nonlocal Euler–Bernoulli beam model to characterize mechanical behaviors of the carbon nanotube (CNT). They concluded that nonlocal elasticity is a useful tool in analysis related to nanotechnology applications. Wang (2005) and Wang et al. (2006) studied the scale effect on the CNT wave and vibration characteristics by applying the nonlocal Euler–Bernoulli and Timoshenko beam models. The numerical simulations are qualitatively in agreement with the experimental results. The results show that the scale effect becomes more obvious with increase of the scale parameter in wave and vibration of CNT. Reddy (2007) and Reddy (2010) generalized the nonlocal models of beam. He advanced the nonlocal nonlinear model of beam and plate. These works provide a new way to describe the scale effects micro/nano structural elements.

Paola et al. (2009) and Paola et al. (2010) proposed a physically-based nonlocal model in which the long-range interactions among non-adjacent volume elements are incorporated into the balance equation. Under the one dimensional case, the balance equation is written as

$$E\frac{d^2u}{dx^2}+f+R=0, \tag{1}$$

where u and f are displacement and the external body force, respectively. E refers to the Young's modulus. R is a central body force representing the long-range interactions among non-adjacent volume elements, and it is assumed to be proportional to the volumes of interacting elements, to a proper distance-decaying function dependent on the material and to the relative displacements between the volume elements (Paola et al., 2009, 2010). So R reads

$$R=\int_0^l g(|x-y|)[u(y)-u(x)]dy, \tag{2}$$

where l is the length of rod. The distance-decaying function $g(|x-y|)$ is also called the nonlocal kernel.

The long-range body force R in crystal materials can attribute to the electrostatic interactions within materials in essence. It was reported that the electrostatic interactions between non-adjacent atoms within crystal can extend about 10 lattice parameter's distances (Edelen, 1976). For the nano-sized elements, such a acting

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extension can be considered to be of long-range. In polymer materials, the internal long-range interactions are more obvious, and their acting range can even reach to the order of micron (Arinstein et al., 2007; Arinstein and Zussman, 2011). This is because a polymer is made of the poly-molecular networks in which the molecular chains are bonded together by the random crosslink between different chains. The crosslinks not only produce between the adjacent chains but also between non-adjacent chains. The latter is the origin of the long-range body forces governing the nonlocal effects in polymers.

Applying the physically-based nonlocal model, in this paper we investigate nonlocal effects of the longitudinal vibration in nanorod. These effects maybe occur in micro/nano-electro-mechanical system (MEMS/NEMS)and bio-micro-structures (e.g. DNA). In fact, ones have observed the effect of long-range interactions within supramolecular on polymer nanofibre elasticity (Arinstein et al., 2007; Arinstein and Zussman, 2011). In theory, the physically-based nonlocal model has been used to analyze the wave and vibration of elastic bar. Cottone et al studied the fractal behaviors in the wave propagation of bar (Cottone et al., 2009). Zingales investigated the features of wave in 1-dimensional elastic solids by means of a distance-decaying exponential kernel (Zingales, 2011). Huang reported a dispersion relation of elastic wave in an infinitely long bar (Huang, 2011a). Distinguished from the existing results, here we solve the longitudinal vibration equation of nanorod under a simplified condition that the nonlocal kernel is uniform. An exact solution is given. the free and forced vibrations of the nanorod are examined in detail. Moreover, we derive the Lagrangian formulations of the physically-based nonlocal theory based on a new definition of nonlocal variable.

The paper is divided into four parts. This introductory section is Part 1. In Part 2, the longitudinal vibration of nanorod is investigated based on the physically-based nonlocal model. The nonlocal effects in the vibration are analyzed and predicted. In Part 3, the Lagrangian formulations for the physically-based nonlocal theory are established based on a new definition of nonlocal variable. Finally, this paper is closed after some conclusions are drawn.

2. Nonlocal effects in the longitudinal vibration of nanorod

2.1. Free vibration

Consider a nanorod free of external body forces. The equation governing the longitudinal vibration of the nanorod can be given by inserting Eq. (2) and inertial term in Eq. (1):

$$E\frac{\partial^2 u}{\partial x^2}-\rho\frac{\partial^2 u}{\partial t^2}=\int_0^l g(|x-y|)[u(x)-u(y)]\mathrm{d}y. \tag{3}$$

For simplicity, we assume that the long-range interactions are homogeneous within the nanorod, i.e., $g(|x-y|)$ is a constant. Let $g(|x-y|)=a/l$, where a is called the micro-force factor, being a parameter dependent of the intrinsic scale of material. Thus, Eq. (3) reduces to

$$c^2\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial t^2}=\frac{a}{\rho}\left[u-\frac{1}{l}\int_0^l u(x)\mathrm{d}x\right], \tag{4}$$

where $c=\sqrt{E/\rho}$ being the wave velocity.

It should be emphasized that kernel $g(|x-y|)$ is a function diminishing with distance $|x-y|$ in physics. So $g(|x-y|)=a/l$ is a simplification. Owe to this simplification, we can find the exact solution of Eq. (4). This solution benefits to verify the numerical algorithm solving Eq. (3) under the case that $g(|x-y|)$ takes a general form. On the other hand, it was reported that the long-range interactions between non-adjacent atoms within crystal can extend about 10 lattice parameter's distances, after becoming extremely small (Edelen, 1976). So the action extent of the long-range interactions is finite. Due to this fact, the total long-range interactions exerting on every atom are same on each other if the distance of these atoms to the ends of the nanorod is larger than the size of the action extent of the long-range interactions. When the nanorod is enough long, its end effects can be neglected. Thus, we can use a constant kernel to approximately characterize the long-range interactions in the nanorod.

Let the solution of Eq. (4) be represented as

$$u(x,t)=f(x)e^{-i(\omega_j t+\theta_j)}, \tag{5}$$

where ω_j and θ_j are the eigenfrequency and initial phase angle, respectively. Substituting Eq. (5) into (4) leads to

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}+\left(\frac{\omega_j}{c}\right)^2 f=\frac{a}{E}\left[f-\frac{1}{l}\int_0^l f(x)\mathrm{d}x\right]. \tag{6}$$

Let

$$b=-\frac{a}{El}\int_0^l f(x)\mathrm{d}x,\quad s_j^2=\left(\frac{\omega_j}{c}\right)^2-\frac{a}{E}. \tag{7}$$

By means of Eq. (7), Eq. (6) is rewritten as

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}+s_j^2 f=b. \tag{8}$$

The general solution of Eq. (8) can be represented as

$$f(x)=p\cos(s_j x)+q\sin(s_j x)+\frac{ac^2}{Els_j\omega_j^2}\{q[\cos(s_j l)-1]-p\sin(s_j l)\}, \tag{9}$$

where p and q are two arbitrary constants.

For a nanorod with one end clamped and another end free, boundary conditions of free vibration are written as

$$f(0)=0,\quad \left.\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x=l}=0. \tag{10}$$

Substituting Eq. (9) into (10) yields

$$\begin{cases} p[1-\dfrac{ac^2}{Els_j\omega_j^2}\sin(s_j l)]+q\dfrac{ac^2}{Els_j\omega_j^2}[\cos(s_j l)-1]=0. \\ -p\sin(s_j l)+q\cos(s_j l)=0. \end{cases} \tag{11}$$

Nonzero p and q exist if and only if the coefficient determinant of Eq. (11) is equal to zero. As a result, we have the characteristic equation below:

$$\cos(s_j l)-\frac{ac^2}{Els_j\omega_j^2}\sin(s_j l)=0. \tag{12}$$

Similarly, we can also determine the characteristic equation of the nanorod with two ends clamped, which reads

$$\sin(s_j l)-\frac{2ac^2}{Els_j\omega_j^2}[1-\cos(s_j l)]=0. \tag{13}$$

Inserting Eq. $(7)_2$ in (12), we have

$$\cos\left(l\sqrt{\left(\frac{\omega_j}{c}\right)^2-\frac{a}{E}}\right)-\frac{ac^2}{Els_j\omega_j^2}\sin\left(l\sqrt{\left(\frac{\omega_j}{c}\right)^2-\frac{a}{E}}\right)=0. \tag{14}$$

If $a=0$, Eq. (14) will revert to the characteristic equation of the longitudinal vibration in the classical rod model. So the magnitude of a has direct influences on the nonlocal features of the eigenfrequency. Noticing the physical dimension of a, we have $a/E\sim 1/d^2$, where d is an intrinsic size of nanorod. In general, d is at the order of nano/micrometer.

For a given a/E, a series of ω_i/c, $(i=1,2,3\ldots)$ can be determined by solving Eq. (14). Thus, we can calculate the change of

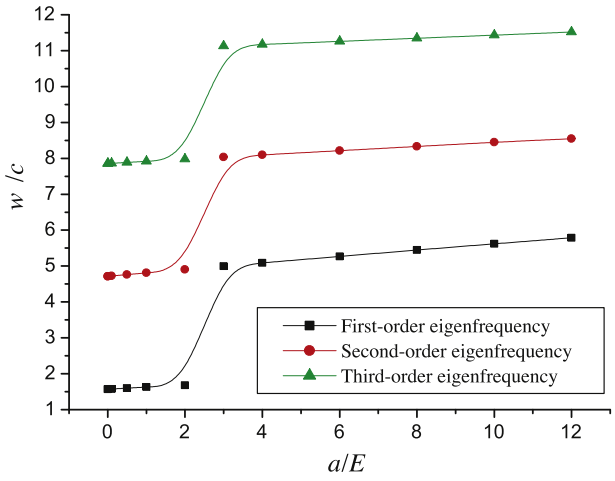

Fig. 1. The change of eigenfrequency with the micro-force factor.

ω_i/c, $(i = 1, 2, 3\ldots)$ with a/E. Fig. 1 shows $\omega_1/c, \omega_2/c$ and ω_3/c changing with a/E. It can be seen that first three eigenfrequencies increase with a rise in the micro-force factor, and a sudden change in the eigenfrequencies will appear when a/E exceeds 2. ω_1 jumps into ω_2, ω_2 into ω_3 and so on. In physics, this effect may be interpreted as the self-excitation of phonons due to the internal long-range interactions. When the phonons are excited under the condition that $a/E > 2$, the phonons in the first energy level with ω_1 will jump to the second energy level with ω_2, the phonons in the second energy level with ω_2 will jump to the third energy level with ω_3, and so on. Therefore, if $a/E > 2$, the lowest eigenfrequency will correspond to the second-order mode, and no first-order mode exists in the vibration of nanorod. This is a prediction awaiting verification. Before it is verified by experiments, a reasonable conclusion is that a/E does not exceed 2. This gives an upper bound of the micro-force factor a.

Let $a/E = 0.5$. Solving Eq. (14), we have $\omega_1/c = 1.6003$, $\omega_2/c = 4.7605$ and $\omega_3/c = 7.8847$. Corresponding to them, first three modes of the nanorod vibration are depicted in Fig. 2. Form it, we can see that the long-range interactions only have the obvious influences on the first-order vibration mode of the nanorod. Compared with results in the classical rod model, the normalized amplitude given by the physically-based nonlocal model decreases in magnitude. So the long-range interactions make the nanorod stiffer. This accords with the conclusion reported by Zingales (2011).

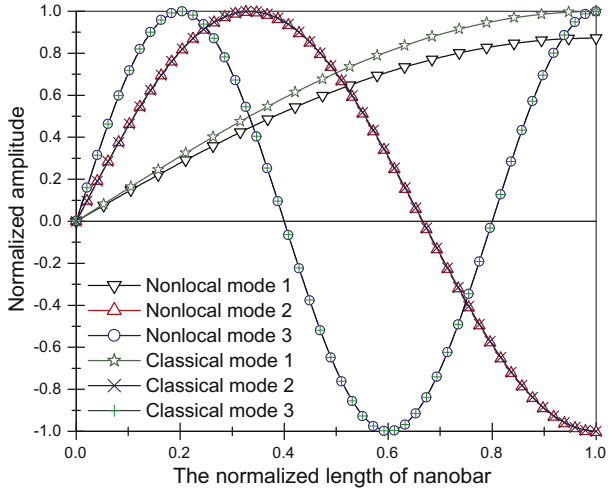

Fig. 2. First three modes of longitudinal vibration in nanorod.

2.2. Forced vibration

Let a sine stimulus $S = \sigma E \sin(nt)$ is prescribed on the free end of the nanorod.

Under this case, the solution of Eq. (4) be represented as

$$u(x,t) = g(x)\sin(nt), \tag{15}$$

where

$$g(x) = p'\cos(kx) + q'\sin(kx) + \frac{ac^2}{Elkn^2}\{q'[\cos(kl) - 1] - p'\sin(kl)\}. \tag{16}$$

Here, k denotes the wave number, which reads

$$k = \sqrt{\left(\frac{n}{c}\right)^2 - \frac{a}{E}}. \tag{17}$$

From Eq. (17), we can find an interesting result: only when $n \geqslant c\sqrt{a/E}$, can vibration be excited. This means that there exists a forbidden band of basic frequency in the nanorod with long-range interactions. When the frequency of external excitation is in the range of the forbidden band, no response will occur in the nanorod. Takin $a/E = 0.49$, we can easily see that the band-gap of n/c is [0,0.7] according to Eq. (17). Fig. 3 depicts the change of n/c with k when $a/E = 0.49$. Clearly, the greater the value of a/E is, the wider the band-gap of n/c. However, the analysis above shows that the micro-force factor a has an upper bound, i.e., $a/E < 2$. Therefore, in the longitudinal vibration of nanorod, the band-gap induced by the long-range interactions is less than $\sqrt{2}$.

In Eq. (16), the coefficients p' and q' are determined by the boundary conditions below:

$$f(0) = 0, \quad \left.\frac{df}{dx}\right|_{x=l} = \sigma. \tag{18}$$

Substituting Eq. (16) into (18) yields

$$\begin{cases} p'[1 - \dfrac{ac^2}{Elkn^2}\sin(kl)] + q'\dfrac{ac^2}{Elkn^2}[\cos(kl) - 1] = 0. \\ -p'\sin(kl) + q'\cos(kl) = \sigma. \end{cases} \tag{19}$$

Solving Eq. (19), we have

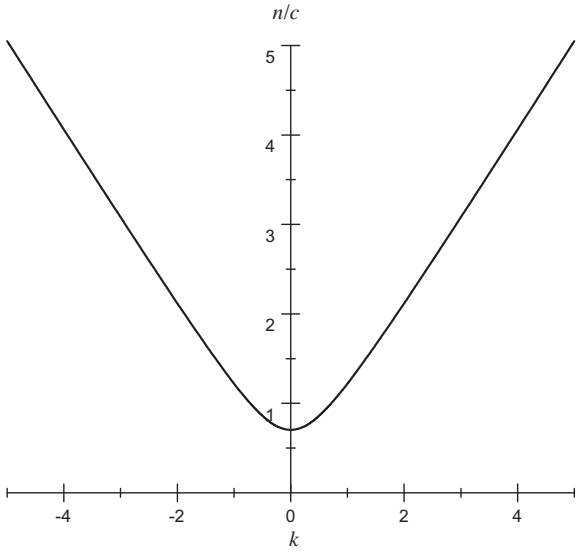

Fig. 3. Band-gap in n/c – k curve ($a/E = 0.49$).

$$p' = \frac{\sigma a c^2}{Elk^2 n^2 D}[1-\cos(kl)], \quad q' = \frac{\sigma}{kD}[1-\sin(kl)], \tag{20}$$

where

$$D = \cos(kl) - \frac{ac^2}{Elkn^2}\sin(kl). \tag{21}$$

In terms of Eq. (7)$_2$ and (17), when $n = \omega_j$, we have $k = s_j$. Therefore, comparing Eq. (12) with (21) leads to $D = 0$. If $D = 0$, then $g(x) \to \infty$ due to p' and q' approaching infinity. This shows that resonance appears in the nanorod under the condition of $n = \omega_j$, just the same as the conclusion in the classical vibration theory.

Inserting Eq. (20) in (16), we have

$$g(x) = A\sin\left(\frac{kx}{2}+\phi\right)\sin\left(\frac{kx}{2}\right), \tag{22}$$

where A is amplitude, and A and ϕ can be written as

$$A = \frac{2\sigma}{kD}\sqrt{B^2+C^2}, \quad \tan(\phi) = \frac{B}{C}. \tag{23}$$

In Eq. (23),

$$B = 1 - \frac{ac^2}{Elkn^2}\sin(kl), \quad C = \frac{ac^2}{Elkn^2}[\cos(kl)-1]. \tag{24}$$

Eq. (23)$_1$ characterizes the relation between amplitude and the frequency of external excitation in the forced vibration of nanorod.

3. Variational formulations

We find that Eq. (3) can be given by the Hamilton's principle provided the Lagrangian density function L takes the form below:

$$L = \frac{1}{2}\rho\dot{u}^2 - \frac{1}{2}E\left(\frac{\partial u}{\partial x}\right)^2 - \frac{1}{2}Ru, \tag{25}$$

where R is the internal longe-range body force represented by Eq. (2). This idea can be expanded into a general theory. For this, let a continuum occupy the domain Ω in the three-dimensional Euclidean space, and every particle in the continuum be referred to a group of the orthogonal Cartesian coordinates $\mathbf{x} = \{x^1, x^2, x^3\}$ specifying its position in Ω. $\varphi = \varphi(t, \mathbf{x})$ denotes a field variable defined on Ω. Depending on circumstances, φ is a scalar, vector or tensor. In terms of the representation of nonlocal residual (Huang, 2011b), the nonlocal variable of φ is defined as

$$\langle\varphi\rangle = \varphi(t,\mathbf{x})\int_\Omega g(|\mathbf{x}-\mathbf{y}|)\mathrm{d}v(\mathbf{y}) - \int_\Omega g(|\mathbf{x}-\mathbf{y}|)\varphi(t,\mathbf{y})\mathrm{d}v(\mathbf{y}). \tag{26}$$

Let $L = L(t, \mathbf{x}, \varphi, \dot{\varphi}, \varphi_{,k}, \langle\varphi\rangle)$, $(k = 1,2,3)$[1] Then the action functional of φ reads

$$A[\varphi] = \int_{t_0}^{t_1}\int_\Omega L(t,\mathbf{x},\varphi,\dot{\varphi},\varphi_{,k},\langle\varphi\rangle)\mathrm{d}v(\mathbf{x})\mathrm{d}t. \tag{27}$$

By means of Eq. (27), the first variation of the action functional is written as

$$\begin{aligned}\delta A[\varphi] &= \int_{t_0}^{t_1}\int_\Omega\left(\frac{\partial L}{\partial\varphi}\delta\varphi + \frac{\partial L}{\partial\dot{\varphi}}\delta\dot{\varphi} + \frac{\partial L}{\partial\varphi_{,k}}\delta\varphi_{,k} + \frac{\partial L}{\partial\langle\varphi\rangle}\delta\langle\varphi\rangle\right)\mathrm{d}v(\mathbf{x})\mathrm{d}t\\ &= \int_{t_0}^{t_1}\int_\Omega\left[\frac{\partial L}{\partial\varphi} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \left(\frac{\partial L}{\partial\varphi_{,k}}\right)_{,k}\right]\delta\varphi\mathrm{d}v(\mathbf{x})\mathrm{d}t\\ &\quad + \int_\Omega\frac{\partial L}{\partial\dot{\varphi}}\delta\varphi\Big|_{t_0}^{t_1}\mathrm{d}v(\mathbf{x}) + \int_{t_0}^{t_1}\int_{\partial\Omega}\frac{\partial L}{\partial\varphi_{,k}}n_k\delta\varphi\mathrm{d}s(\mathbf{x})\mathrm{d}t\\ &\quad + \int_{t_0}^{t_1}\int_\Omega\frac{\partial L}{\partial\langle\varphi\rangle}\delta\langle\varphi\rangle\mathrm{d}v(\mathbf{x})\mathrm{d}t,\end{aligned} \tag{28}$$

[1] If necessary, the nonlocal variables $\langle\dot{\varphi}\rangle$ and $\langle\varphi_{,k}\rangle$ may be conveniently inserted into L. But in this case, the boundary conditions will become complicated.

where $\partial\Omega$ is the boundary surface of Ω and n_k denotes the unit normal vector on $\partial\Omega$. Using the symmetry of kernel, we have

$$\begin{aligned}\int_\Omega\frac{\partial L}{\partial\langle\varphi\rangle}\delta\langle\varphi\rangle\mathrm{d}v(\mathbf{x}) &= \int_\Omega\frac{\partial L}{\partial\langle\varphi\rangle}\left[\delta\varphi - \int_\Omega g(|\mathbf{x}-\mathbf{y}|)\delta\varphi\mathrm{d}v(\mathbf{y})\right]\mathrm{d}v(\mathbf{x})\\ &= \int_\Omega\left[\frac{\partial L}{\partial\langle\varphi\rangle} - \int_\Omega g(|\mathbf{x}-\mathbf{y}|)\frac{\partial L}{\partial\langle\varphi\rangle}\mathrm{d}v(\mathbf{y})\right]\delta\varphi\mathrm{d}v(\mathbf{x})\\ &= \int_\Omega\langle\frac{\partial L}{\partial\langle\varphi\rangle}\rangle\delta\varphi\mathrm{d}v(\mathbf{x}).\end{aligned} \tag{29}$$

Here, a shortened form similar to Eq. (26) is used,

$$\left\langle\frac{\partial L}{\partial\varphi}\right\rangle = \frac{\partial L}{\partial\langle\varphi\rangle}\int_\Omega g(|\mathbf{x}-\mathbf{y}|)\mathrm{d}v(\mathbf{y}) - \int_\Omega g(|\mathbf{x}-\mathbf{y}|)\frac{\partial L}{\partial\langle\varphi\rangle}\mathrm{d}v(\mathbf{y}). \tag{30}$$

Substituting Eq. (29) into (28) leads to

$$\begin{aligned}\delta A[\varphi] &= \int_{t_0}^{t_1}\int_\Omega\left[\frac{\partial L}{\partial\varphi} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \left(\frac{\partial L}{\partial\varphi_{,k}}\right)_{,k} + \langle\frac{\partial L}{\partial\langle\varphi\rangle}\rangle\right]\delta\varphi\mathrm{d}v(\mathbf{x})\mathrm{d}t\\ &\quad + \int_\Omega\frac{\partial L}{\partial\dot{\varphi}}\delta\varphi\Big|_{t_0}^{t_1}\mathrm{d}v(\mathbf{x}) + \int_{t_0}^{t_1}\int_{\partial\Omega}\frac{\partial L}{\partial\varphi_{,k}}n_k\delta\varphi\mathrm{d}s(\mathbf{x})\mathrm{d}t.\end{aligned} \tag{31}$$

Let $\partial\Omega = \partial\Omega_1 \cup \partial\Omega_2$, $\partial\Omega_1 \cap \partial\Omega_2 = \emptyset$. On $\partial\Omega_1$, φ takes a given value $\bar{\varphi}$. So the boundary condition on $\partial\Omega_1$ reads

$$\varphi|_{\partial\Omega_1} = \bar{\varphi}. \tag{32}$$

Similarly, at the initial and terminal time, we have

$$\varphi|_{t_0} = \bar{\varphi}_0, \quad \varphi|_{t_1} = \bar{\varphi}_1. \tag{33}$$

Due to Eq. (32) and (33), $\delta\varphi = 0$ on $\partial\Omega_1$, and at the initial and terminal time. Thus, Eq. (31) reduces to

$$\begin{aligned}\delta A[\varphi] &= \int_{t_0}^{t_1}\int_\Omega\left[\frac{\partial L}{\partial\varphi} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \left(\frac{\partial L}{\partial\varphi_{,k}}\right)_{,k} + \left\langle\frac{\partial L}{\partial\langle\varphi\rangle}\right\rangle\right]\delta\varphi\mathrm{d}v(\mathbf{x})\mathrm{d}t\\ &\quad + \int_{t_0}^{t_1}\int_{\partial\Omega_2}\frac{\partial L}{\partial\varphi_{,k}}n_k\delta\varphi\mathrm{d}s(\mathbf{x})\mathrm{d}t.\end{aligned} \tag{34}$$

In terms of the Hamilton's principle, we have $\delta A[\varphi] = 0$. So in Eq. (34), the fundamental lemma of variation yields the below results:

Euler–Lagrangian equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) + \left(\frac{\partial L}{\partial\varphi_{,k}}\right)_{,k} - \frac{\partial L}{\partial\varphi} = \left\langle\frac{\partial L}{\partial\langle\varphi\rangle}\right\rangle. \tag{35}$$

Natural boundary condition:

$$\frac{\partial L}{\partial\varphi_{,k}}n_k\Big|_{\partial\Omega_2} = 0. \tag{36}$$

Eq. (35) is also called the nonlocal Euler–Lagrangian equation. Its right-side term is the so-called nonlocal traction.

According to Eq. (30), it is easy to verify that the nonlocal traction satisfies the zero mean condition below:

$$\int_\Omega\langle\frac{\partial L}{\partial\langle\varphi\rangle}\rangle\mathrm{d}v(\mathbf{x}) = 0. \tag{37}$$

This result presents a notable difference between Eq. (35) and the existing nonlocal Lagrangian formulation (Edelen, 1972). Due to Eq. (37), The integral of Eq. (35) over Ω has the same expression as the ordinary Euler–Lagrangian equation.

4. Conclusions

Applying the physically-based nonlocal model, we investigate influences of the nonlocal long-range interactions on the longitudinal vibration of nanorod. Under the condition that the nonlocal

kernel is uniform, we solve the physically-based nonlocal equation for the longitudinal vibration of nanorod. An exact solution is given. By means of this solution, the nonlocal effects in the vibration of nanorod are examined in detail. The results show that the normalized amplitude given by the physically-based nonlocal model decreases in magnitude, compared with results in the classical elastic model of rod. This means that the nanorod becomes stiffer due to the internal long-range interactions.

The analysis for the forced vibration of nanorod shows there exists a forbidden band of basic frequency due to the nonlocal long-range interactions. When the frequency of external excitation is in the range of the forbidden band, no response will occur in the nanorod. If this prediction were experimentally verified in laboratory, it would be used to insulate low-frequency vibration.

The micro-force factor has a direct influence on the band-gap in the vibration of nanorod. The greater the value of micro-force factor is, the wider the band-gap of vibration. Since the micro-force factor has an upper bound, the band-gap is finite. In the longitudinal vibration of nanorod, the band-gap induced by the long-range interactions is less than $\sqrt{2}$.

As a generalization, the Lagrangian formulations of the physically-based nonlocal theory are established based on a new definition of nonlocal variable. In these formulations, the nonlocal traction characterizes the long-range interactions within material, and satisfies the zero mean condition automatically. Using these results, we can conveniently determine the physically-based nonlocal model of beam, plate, shell and bio-micro-structures.

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