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Propagation of Bleustein–Gulyaev wave in 6 mm piezoelectric materials loaded with viscous liquid

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Abstract

The influence of a viscous liquid on acoustic waves propagating in elastic or piezoelectric materials is of particular significance for development of liquid sensors. Bleustein–Gulyaev wave is a shear-type surface acoustic wave and has the advantage of not radiating energy into the adjacent liquid. These features make the B–G wave sensitive to changes in both mechanical and electrical properties of the surrounding environment. The Bleustein–Gulyaev wave has been reported to be a good candidate for liquid sensing application. In this paper, we investigate the potential application of B–G wave in 6 mm crystals for liquid sensing. The explicit dispersion relations for both open circuit and metalized surface boundary conditions are given. A numerical example of PZT-5H piezoelectric ceramic in contact with viscous liquid is calculated and discussed. Numerical results of attenuation and phase velocity versus viscosity, density of the liquid and wave frequency are presented. The paper is intended to provide essential data for liquid senor design and development.

Keywords: Bleustein–Gulyaev wave; 6 mm piezoelectric materials; Viscous liquid; Liquid sensing

1. Introduction

Surface waves have been applied successfully in many technological fields, such as NDE of materials, resonators, filters, sensors, etc. (Hoummady et al., 1997; McMullan et al., 2000; Vellekoop, 1998). The development of mico-acoustic wave sensor in biosensing created the need for further investigations of the surface wave propagation in a viscous liquid loaded layered medium (Wu and Wu, 2000). A number of acoustic wave modes have been utilized for various sensor applications. The influence of a viscous liquid on acoustic waves propagating in elastic or piezoelectric materials has been studied by several researchers, which is of particular interest for development of liquid viscosity sensors (Wu and Wu, 2000; Zaitsev et al., 2001; Lee and Kuo, 2006).

Zhang et al. (2001) proposed that B–G wave is a promising candidate for liquid sensing applications. The B–G wave does not radiate energy into the contacting liquid and is sensitive to the changes of

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the liquid density and viscosity. However, Zhang et al. (2001) did not give a detailed quantitative investigation of characteristics of B–G wave propagating in piezoelectric materials loaded with viscous liquid.

Kielczyński et al. (2004) presented a method for measuring the rheological properties of viscoelastic liquids using the Bleustein–Gulyaev wave. They applied the perturbation theory to obtain the relations between the change in the complex propagation constant of the B–G wave and the shear acoustic impedance of the liquid. Nevertheless, as Zaitsev et al. (2001) pointed out, the perturbation analysis may not be valid if the propagating acoustic wave has high electromechanical coupling, which is the case for acoustic waves in strong piezoelectric materials.

In this paper, we conducted a rigorous investigation on the propagation of B–G wave in 6 mm crystals in contact with viscous liquid by employing the exact theory of continuum mechanics. The explicit dispersion relations for both open circuit and metalized surface boundary conditions are given. A numerical example of PZT-5H piezoelectric ceramic in contact with viscous liquid is calculated and discussed. Numerical results of attenuation and phase velocity versus viscosity, density of the liquid and wave frequency are presented. The paper is intended to provide essential data for liquid senor design and development.

2. Bleustein–Gulyaev wave

The Bleustein–Gulyaev wave may exist in piezoelectric materials with higher symmetry. It is an electromechanical coupled shear type surface wave, in which the direction of material particle motion is perpendicular to the propagating direction and parallel to the surface of half-space. If there is no piezoelectric effect, B–G wave degenerates to the shear bulk wave. It is a peculiar surface wave existing in piezoelectric media with 6 mm or mm2 symmetry and it has no elastic counterpart (Dieulesaint and Royer, 1980, p. 282; Zhang et al., 2001).

The equation of motion and Gauss equation of piezoelectric materials without body force and free charge are $\sigma_{ij,j} = \rho_p \frac{\partial^2 u_i}{\partial t^2}\,(i,j = 1,2,3)$ and $D_{i,i} = 0\ (i = 1,2,3)$, respectively. In these two equations, σ_{ij}, u_i and D_i are stress, mechanical displacement and electric displacement components, respectively, and ρ_p is the mass density. The indices preceded by a comma denote space-coordinate differentiation. Also, a repeated index in the subscript implies summation with respect to that index. The constitutive relations of piezoelectric materials can be expressed as

$$\begin{cases} \sigma_{ij} = C_{ijkl}s_{kl} - e_{kij}E_k \\ D_i = e_{ikl}s_{kl} + \epsilon_{ik}E_k \end{cases} \quad (i,j,k = 1,2,3), \tag{1}$$

where $s_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the components of the infinitesimal strain, C_{ijkl}, e_{kij} and ϵ_{ik} are the elastic constants, piezoelectric and dielectric constants, respectively; the electric field is related to the electric potential by $E_i =- \phi_i$, $(i = 1,2,3)$.

By inserting the constitutive relations into the equation of motion and Gaussian equation, one arrives at

$$\begin{cases} C_{ijkl}u_{k,il} + e_{kij}\phi_{,ki} = \rho_p \frac{\partial^2 u_j}{\partial t^2} \\ e_{ikl}u_{k,il} - \epsilon_{ik}\phi_{,ki} = 0 \end{cases} \quad (i,j,k,l = 1,2,3). \tag{2}$$

The elastic constants C_{ijkl} can be written into contracted form $C_{\alpha\beta}$ by the following rule

$$\begin{aligned} &\alpha = 9 - i - j, \qquad \beta = 9 - k - l \quad \text{if} \quad i \neq j,\ k \neq l \\ &\alpha = i = j, \qquad \beta = k = l \quad \text{if} \quad i = j,\ k = l. \end{aligned} \tag{3}$$

Similarly, the piezoelectric constants e_{ijk} can also be put into contracted form $e_{i\alpha}$ with the index α observing the same rule as above.

For 6 mm piezoelectric materials with x_3 direction being the 6-fold symmetry axis, Eq. (2) takes the following form

$$
\begin{aligned}
&\left(C_{11}\frac{\partial^2}{\partial x_1^2}+C_{66}\frac{\partial^2}{\partial x_2^2}+C_{44}\frac{\partial^2}{\partial x_3^2}-\rho_p\frac{\partial^2}{\partial t^2}\right)u_1+(C_{12}+C_{66})\frac{\partial^2 u_2}{\partial x_1\partial x_2}+(C_{13}+C_{44})\frac{\partial^2 u_3}{\partial x_1\partial x_3}+(e_{31}+e_{15})\frac{\partial^2\phi}{\partial x_1\partial x_3}=0\\
&(C_{11}+C_{66})\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\left(C_{66}\frac{\partial^2}{\partial x_1^2}+C_{11}\frac{\partial^2}{\partial x_2^2}+C_{44}\frac{\partial^2}{\partial x_3^2}-\rho_p\frac{\partial^2}{\partial t^2}\right)u_2+(C_{13}+C_{44})\frac{\partial^2 u_3}{\partial x_2\partial x_3}+(e_{31}+e_{15})\frac{\partial^2\phi}{\partial x_2\partial x_3}=0\\
&(C_{13}+C_{44})\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}\right)+\left(C_{44}\frac{\partial^2}{\partial x_1^2}+C_{44}\frac{\partial^2}{\partial x_2^2}+C_{33}\frac{\partial^2}{\partial x_3^2}-\rho_p\frac{\partial^2}{\partial t^2}\right)u_3+e_{15}\frac{\partial^2\phi}{\partial x_1^2}+e_{15}\frac{\partial^2\phi}{\partial x_2^2}+e_{33}\frac{\partial^2\phi}{\partial x_3^2}=0\\
&(e_{15}+e_{31})\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}\right)+e_{15}\frac{\partial^2 u_3}{\partial x_1^2}+e_{15}\frac{\partial^2 u_3}{\partial x_2^2}+e_{33}\frac{\partial^2 u_3}{\partial x_3^2}-\epsilon_{11}\frac{\partial^2\phi}{\partial x_1^2}-\epsilon_{11}\frac{\partial^2\phi}{\partial x_2^2}-\epsilon_{33}\frac{\partial^2\phi}{\partial x_3^2}=0.
\end{aligned}
\tag{4}
$$

Now consider a harmonic wave propagating in x_1 direction, thus all physical quantities only depend on in-plane variables (x_1,x_2), and are independent of x_3. This case is termed as generalized plane strain problem. In this situation, Eq. (4) is simplified into

$$
\begin{aligned}
&\left(C_{11}\frac{\partial^2}{\partial x_1^2}+C_{66}\frac{\partial^2}{\partial x_2^2}-\rho_p\frac{\partial^2}{\partial t^2}\right)u_1+(C_{12}+C_{66})\frac{\partial^2 u_2}{\partial x_1\partial x_2}=0\\
&(C_{12}+C_{66})\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\left(C_{66}\frac{\partial^2}{\partial x_1^2}+C_{22}\frac{\partial^2}{\partial x_2^2}-\rho_p\frac{\partial^2}{\partial t^2}\right)u_2=0\\
&\left(C_{44}\frac{\partial^2}{\partial x_1^2}+C_{44}\frac{\partial^2}{\partial x_2^2}-\rho_p\frac{\partial^2}{\partial t^2}\right)u_3+e_{15}\frac{\partial^2\phi}{\partial x_1^2}+e_{15}\frac{\partial^2\phi}{\partial x_2^2}=0\\
&e_{15}\frac{\partial^2 u_3}{\partial x_1^2}+e_{15}\frac{\partial^2 u_3}{\partial x_2^2}-\epsilon_{11}\frac{\partial^2\phi}{\partial x_1^2}-\epsilon_{11}\frac{\partial^2\phi}{\partial x_2^2}=0.
\end{aligned}
\tag{5}
$$

Obviously, it can be seen that (u_1, u_2) is decoupled with (u_3, ϕ). The first two equations of Eq. (5) show that (u_1, u_2) may constitute purely elastic Rayleigh wave whereas the last two equations indicate that (u_3, ϕ) could comprise the Bleustein–Gulyaev wave.

3. Dispersion relations

3.1. Description of the problem

The problem in question is shown in Fig. 1. The piezoelectric material occupies the half-space $x_2 < 0$ and the liquid covers the half-space $x_2 > 0$. x_3 axis is parallel to the axis of symmetry.

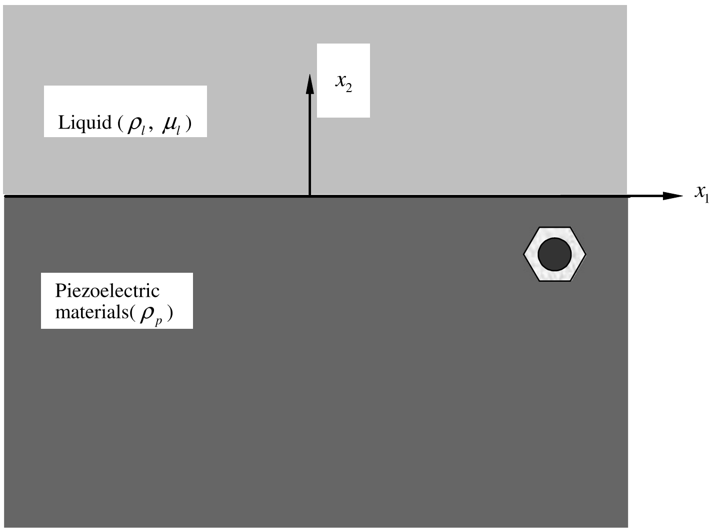

Fig. 1. Schematic illustration of the problem and the coordinate system.

The liquid is assumed to be viscous and nonconductive. Suppose the motion of liquid is induced only by wave propagation in the piezoelectric material and also propagates in the form of harmonic wave. For this problem, the embroil inertial term in the Navier–Stokes equation can be omitted. Moreover, the pressure gradient also can be ignored since only shear deformation occurs during wave propagation (McMullan et al., 2000). Therefore, the governing equation for liquid is simplified to

$$\frac{\partial v_3}{\partial t} - \frac{\mu_l}{\rho_l}\nabla^2 v_3 = 0, \tag{6}$$

where ρ_l is the mass density of liquid, μ_l the dynamic viscous coefficient of the liquid and v_3 is the liquid particle velocity in the x_3 direction.

Due to the effect of viscosity, the B–G wave propagating in the piezoelectric material in contact with viscous liquid is attenuated. Other effects such as polarization relaxation, acoustoelectric effect and viscoelasticity may also cause energy dissipation. However, the influences of these effects on wave propagation make no difference when the viscous liquid is present or absent. Thus, their influences can be readily taken into account and deducted. Therefore, in this paper, these effects are not considered.

Interfacial mechanical conditions are continuity of particle velocity and stress components at the interface. Assume that the liquid is electrically insulated and its permittivity is much less than that of the piezoelectric material. The electrical conditions at the interface can be classified into two categories, i.e., (1) open circuit: electric displacement $D_2|_{x_2=0} = 0$, (2) metalized surface: electric potential $\phi|_{x_2=0} = 0$.

3.2. Derivation of dispersion relations

For a harmonic plane progressive wave traveling in x_1 direction, the displacement component u_3 and electric potential ϕ can be assumed to be in the following form

$$u_3 = W(x_2)e^{ik(x_1-vt)} = W(x_2)e^{i(kx_1-\omega t)}, \quad \phi = \Phi(x_2)e^{ik(x_1-vt)} = \Phi(x_2)e^{i(kx_1-\omega t)}, \tag{7}$$

where k is wave number, v phase velocity of the wave and ω angular frequency, i is the imaginary unit, and $W(x_2)$ and $\Phi(x_2)$ are functions to be determined.

Substituting Eq. (7) into the last two equations of Eq. (5) yields

$$\begin{aligned} &C_{44}W'' + e_{15}\Phi'' + k^2(\rho_p v^2 - C_{44})W - k^2 e_{15}\Phi = 0 \\ &e_{15}W'' - \epsilon_{11}\Phi'' - k^2 e_{15}W + k^2\epsilon_{11}\Phi = 0, \end{aligned} \tag{8}$$

where the double prime indicates the second derivative with respect to x_2.

Eliminating Φ in Eq. (8) leads to

$$(C_{44}\epsilon_{11} + e_{15}^2)W'' + k^2[\epsilon_{11}(\rho_p v^2 - C_{44}) - e_{15}^2]W = 0. \tag{9}$$

Then, solving this equation gives $W(x_2) = C_2 e^{\lambda_2 k x_2}$, where $\lambda_2^2 = \frac{e_{15}^2 - \epsilon_{11}(\rho_p v^2 - C_{44})}{C_{44}\epsilon_{11} + e_{15}^2}$, $\mathrm{Re}(\lambda_2) > 0$ and C_2 is a constant.

Rewrite the second expression of Eq. (8) into

$$(\epsilon_{11}\Phi - e_{15}W)'' - k^2(\epsilon_{11}\Phi - e_{15}W) = 0. \tag{10}$$

Solving this equation, we get

$$\epsilon_{11}\Phi(x_2) - e_{15}W(x_2) = -kC_3 e^{kx_2}. \tag{11}$$

Finally, we obtain the expression of function $\Phi(x_2)$ as follows

$$\Phi(x_2) = \frac{C_3}{\epsilon_{11}}e^{kx_2} + \frac{e_{15}}{\epsilon_{11}}C_2 e^{\lambda_2 k x_2}, \tag{12}$$

where $\mathrm{Re}(k) > 0$ and C_3 is a constant.

Similarly, for the motion of liquid, we assume

$$v_3 = V_3(x_2)e^{ik(x_1-vt)}, \tag{13}$$

where v is the phase velocity and k the wave number.

Substitution of Eq. (13) into Eq. (6) gives

$$V_3''(x_2) + \left(\mathrm{i}kv\frac{\rho_l}{\mu_l} - k^2\right)V_3(x_2) = 0. \tag{14}$$

From this equation, we obtain $V_3(x_2) = C_1 e^{\lambda_1 x_2}$, where $\lambda_1^2 = k^2 - \mathrm{i}kv\frac{\rho_l}{\mu_l}$, $\mathrm{Re}(\lambda_1) < 0$ and C_1 is a constant.

In the piezoelectric material, the shear stress and electric displacement are represented by

$$\begin{aligned} \tau_{23} &= C_{44}\frac{\partial u_3}{\partial x_2} + e_{15}\frac{\partial \phi}{\partial x_2} = \left[C_2 C_{44}\lambda_2 k e^{\lambda_2 k x_2} + e_{15}\left(k\frac{C_3}{\epsilon_{11}}e^{kx_2} + \lambda_2 k C_2 \frac{e_{15}}{\epsilon_{11}}e^{\lambda_2 k x_2}\right)\right]e^{\mathrm{i}k(x_1 - vt)} \\ D_2 &= e_{15}\frac{\partial u_3}{\partial x_2} - \epsilon_{11}\frac{\partial \phi}{\partial x_2} = -C_3 k e^{kx_2} e^{\mathrm{i}k(x_1 - vt)}. \end{aligned} \tag{15}$$

Thus, the velocity of material particle in the piezoelectric material is given by $\dot{u}_3 = -vk\mathrm{i}C_2 e^{\lambda_2 k x_2} e^{\mathrm{i}k(x_1 - vt)}$.

In the liquid, the velocity of material particle and shear stress are equal to

$$\begin{aligned} v_3 &= C_1 e^{\lambda_1 x_2} e^{\mathrm{i}k(x_1 - vt)} \\ \tau_{23} &= \mu_l \frac{\partial v_3}{\partial x_2} = C_1 \mu_l \lambda_1 e^{\lambda_1 x_2} e^{\mathrm{i}k(x_1 - vt)} \end{aligned} \tag{16}$$

By imposing the continuity of stress and velocity at the interface between liquid and solid, we have

$$\begin{aligned} C_1\mu_l\lambda_1 &= C_2 k\lambda_2\left(C_{44} + \frac{e_{15}^2}{\epsilon_{11}}\right) + C_3 k\frac{e_{15}}{\epsilon_{11}} \\ -C_2 vk\mathrm{i} &= C_1. \end{aligned} \tag{17}$$

From the open circuit condition, we infer $C_3 = 0$. For the metalized surface condition, we deduce $C_3 + C_2 e_{15} = 0$.

From these relations, we obtain the dispersion relation for open circuit condition as

$$-v\mu_l\lambda_1\mathrm{i} = \lambda_2\left(C_{44} + \frac{e_{15}^2}{\epsilon_{11}}\right). \tag{18}$$

After some mathematical manipulations, this relation can be rewritten to

$$k^2 = \frac{\omega^2(\rho_p\xi + \mathrm{i}\mu_l\rho_l\omega)}{(\xi^2 + \mu_l^2\omega^2)}, \tag{19}$$

where μ_l is the viscosity of the liquid, ω is the angular frequency of wave and $\xi = (C_{44}\epsilon_{11} + e_{15}^2)/\epsilon_{11}$.

Similarly, the dispersion relation for metalized surface condition can be derived as

$$-\mathrm{i}\mu_l\lambda_1 v = \lambda_2\left(C_{44} + \frac{e_{15}^2}{\epsilon_{11}}\right) + \frac{e_{15}^2}{\epsilon_{11}}. \tag{20}$$

The above expression can be put into the following form

$$(\mathrm{i}\mu_l\rho_l\omega + \rho_p\xi)^2\omega^4 + [4\xi\eta^2\rho_p - 2(\xi^2 + \eta^2 + \omega^2\mu_l^2)(i\omega\rho_l\mu_l + \rho_p\xi)]\omega^2 k^2 + [(\xi^2 + \eta^2 + \omega^2\mu_l^2)^2 - 4\xi^2\eta^2]k^4 = 0, \tag{21}$$

where $\eta = e_{15}^2/\epsilon_{11}$.

Once the wave number is obtained, the phase velocity is calculated by $v = \omega/\mathrm{Re}(k)$. The imaginary part of wave number k represents the attenuation per unit length in the propagation direction. When the piezoelectric material is not in contact with liquid, from Eqs. (19) and (21) it is easily deduced that the phase velocity of B–G wave in 6mm piezoelectric materials for open circuit condition is $v_o \doteq \sqrt{\xi/\rho}$, and wave velocity for metalized surface condition is $v_m = \sqrt{\frac{C_{44}}{\rho}(1 + \frac{\eta}{\xi})}$.

4. Numerical results

We consider a PZT-5H piezoelectric ceramic half-space in contact with viscous liquid. Material properties of PZT-5H are taken from Fang et al. (2000) and listed below

$$C_{44} = 2.3 \times 10^{10}\ \mathrm{N/m^2}, \quad \rho = 7.5 \times 10^3\ \mathrm{kg/m^3}$$
$$e_{15} = 17.0\ \mathrm{C/m^2}, \quad \epsilon_{11} = 227.0 \times 10^{-10}\ \mathrm{F/m}.$$

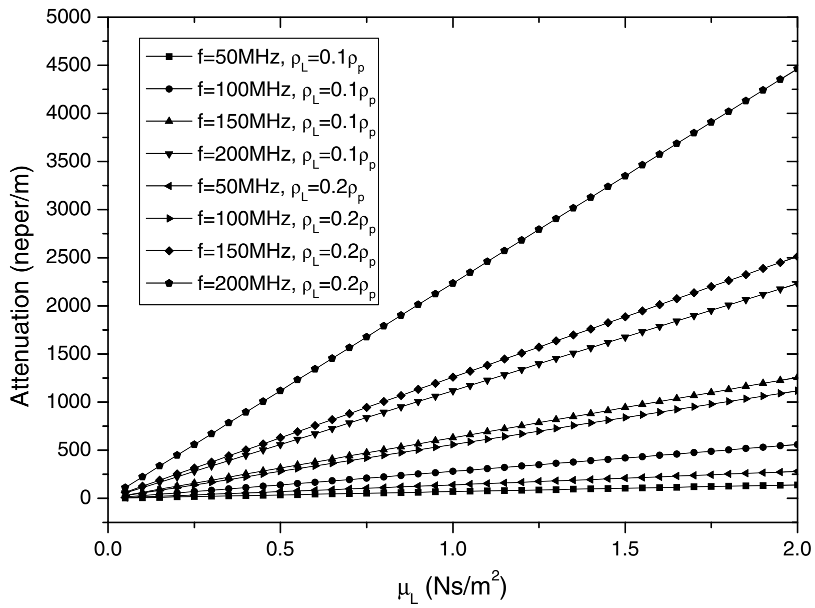

Fig. 2. The attenuation as a function of liquid viscosity for open circuit condition.

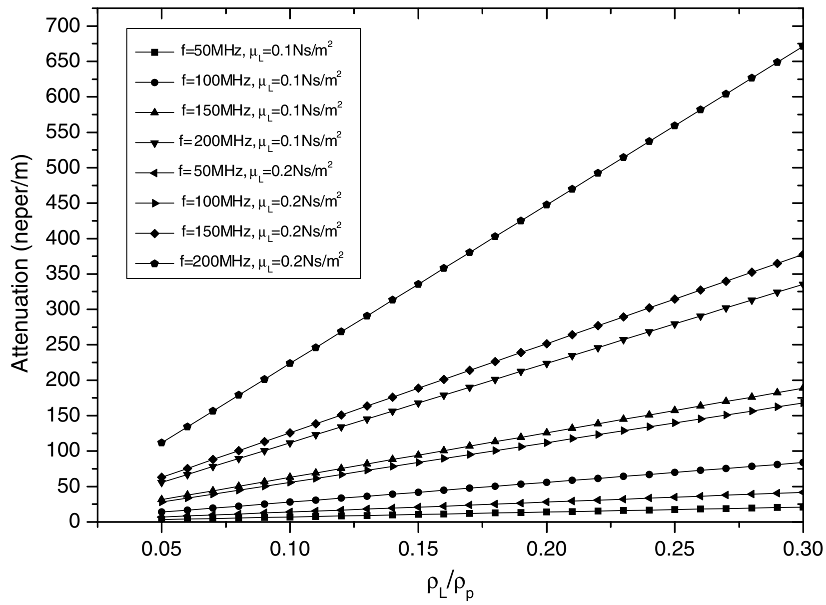

Fig. 3. The attenuation as a function of the density ratio ρ_l/ρ_p for open circuit condition.

From these data, it can be easily calculated that $v_o = 2111.34$ m/s and $v_m = 2005.90$ m/s. The electro-mechanical coupling factor is defined as $K^2 = 2(v_o - v_m)/v_o$, which expresses the ability of material to transform an electric signal into an elastic surface wave or vice versa. For PZT-5H piezoelectric ceramic, $K = 0.316$.

From the analysis in the last section, we know the velocity of liquid particle can be expressed as $v_3 = C_1 e^{\lambda_1 x_2} e^{i(kx_1 - \omega t)}$, in which $\lambda_1 = \sqrt{k^2 - i\omega \frac{\rho_l}{\mu_l}}$. In the piezoelectric material, we have $u_3 = C_2 e^{\lambda_2 k x_2} e^{i(kx_1 - \omega t)}$ for both open circuit and metalized surface conditions, where $\lambda_2 k = \sqrt{k^2 - \rho_p \frac{\omega^2}{\xi}}$. In the open circuit condition, we get $\phi = \frac{e_{15}}{\epsilon_{11}} C_2 e^{\lambda_2 k x_2} e^{i(kx_1 - \omega t)}$ while the electric potential in the metalized surface condition has this form

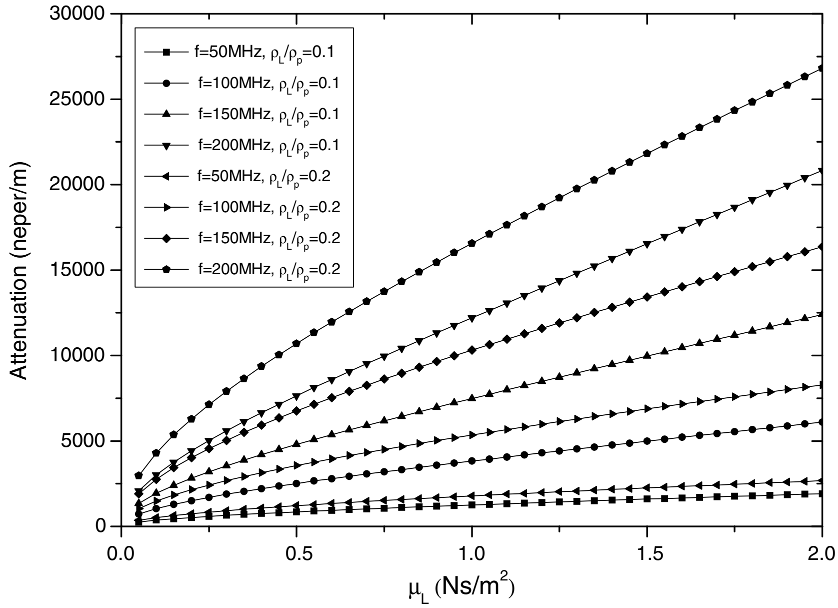

Fig. 4. The change of attenuation with liquid viscosity for metalized surface condition.

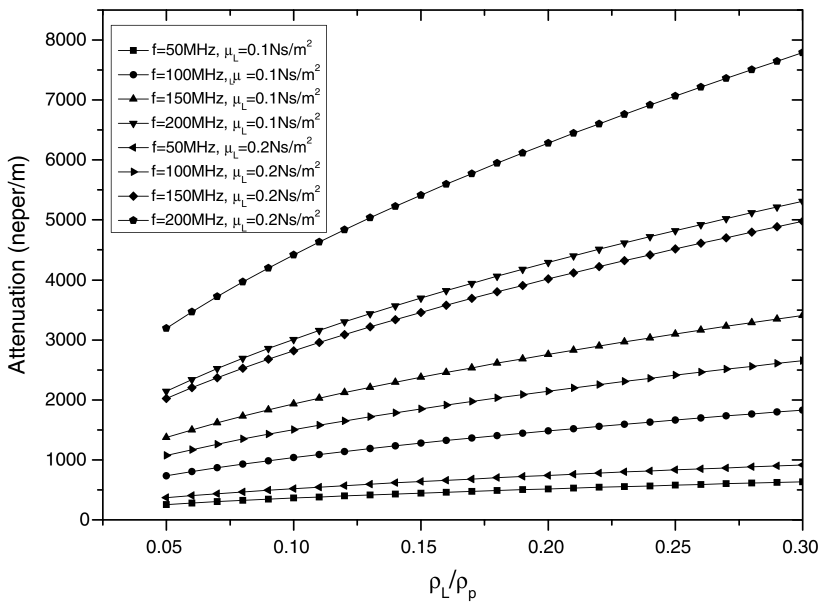

Fig. 5. The change of attenuation with the density ratio ρ_l/ρ_p for metalized surface condition.

$\phi = \frac{e_{15}}{\epsilon_{11}} C_2 (e^{\lambda_2 k x_2} - e^{k x_2}) e^{i(k x_1 - \omega t)}$. The wave length equals to $\lambda = \frac{v}{f} = \frac{2\pi v}{\omega} = \frac{2\pi}{\mathrm{Re}(k)}$, where f is circular frequency, ω angular frequency and v the phase velocity.

The numerical results of wave velocity and attenuation are shown in Figs. 2–10. Figs. 2 and 3 show change of attenuation with liquid viscosity at different values of liquid density under the open circuit condition. Figs. 4 and 5 illustrate the attenuation as a function of liquid density at different values of liquid viscosity under the

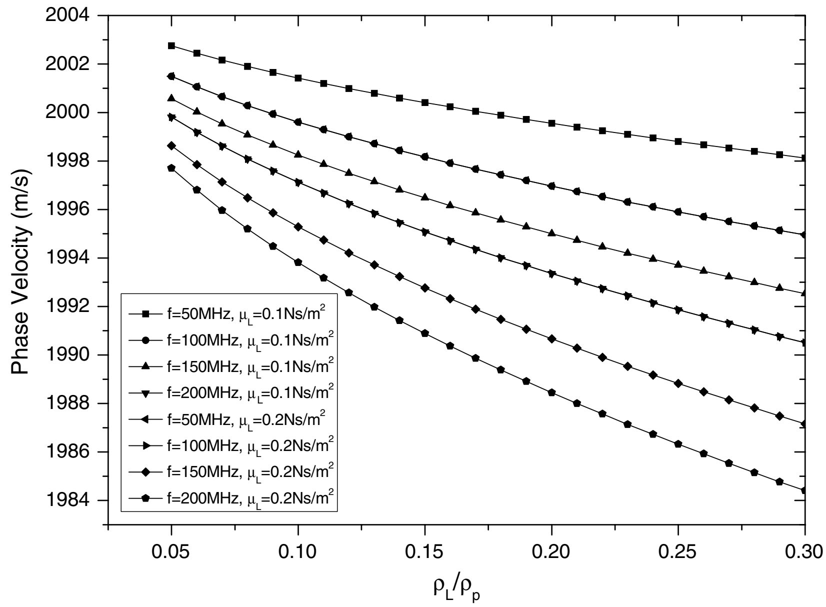

Fig. 6. The phase velocity as a function of density ratio ρ_l/ρ_p for metalized surface condition. (The curve for the case $f = 100$ MHz, $\mu_l = 0.1$ NS/m^2 coincides with that for $f = 50$ MHz, $\mu_l = 0.2$ NS/m^2 and the curve for $f = 200$ MHz, $\mu_l = 0.1$ NS/m^2 coincides with that for $f = 100$ MHz, $\mu_l = 0.2$ NS/m^2.)

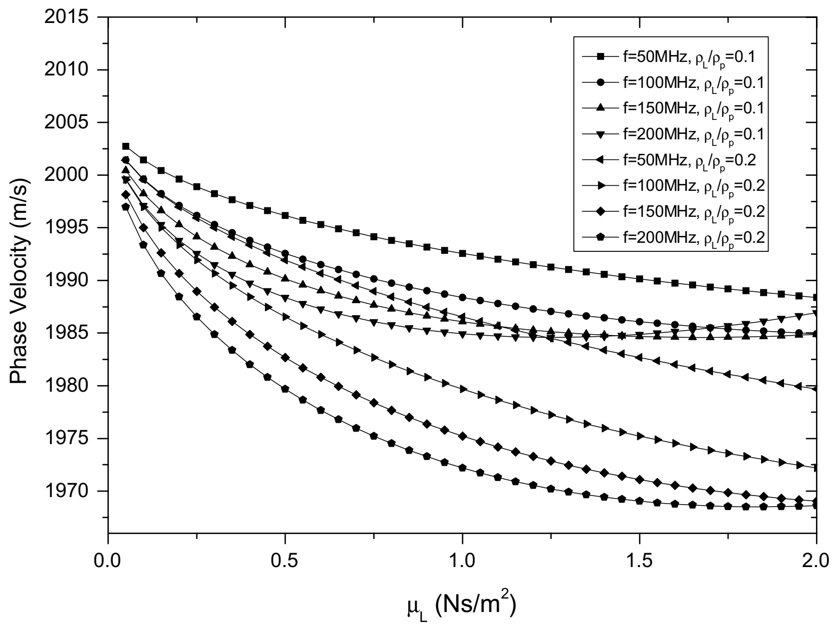

Fig. 7. The phase velocity as a function of the liquid viscosity for metalized surface condition.

metalized surface condition. It is seen from these figures that the relationship of attenuation with viscosity or density is approximately linear in the open circuit condition. The attenuation for metalized surface condition is obviously larger than that for the open circuit condition. Besides, the relationship between the attenuation and liquid viscosity or density ratio in the metalized surface condition is obviously nonlinear, which is different from the approximately linear relationship for the open circuit condition. Figs. 6 and 7 demonstrate the effect of viscosity and liquid density on the phase velocity in the metalized surface condition. We can see that in most cases, the wave velocity decreases with the increase of viscosity or liquid density. Specifically, with increasing viscosity, the wave velocity is found to decrease smoothly, reach a minimum, and then increase mildly with

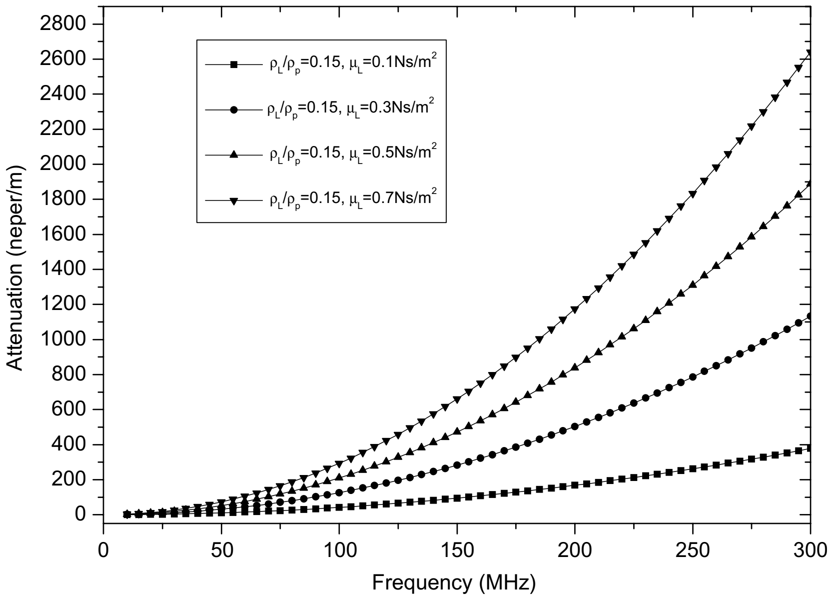

Fig. 8. Change of attenuation with the wave frequency for open circuit condition.

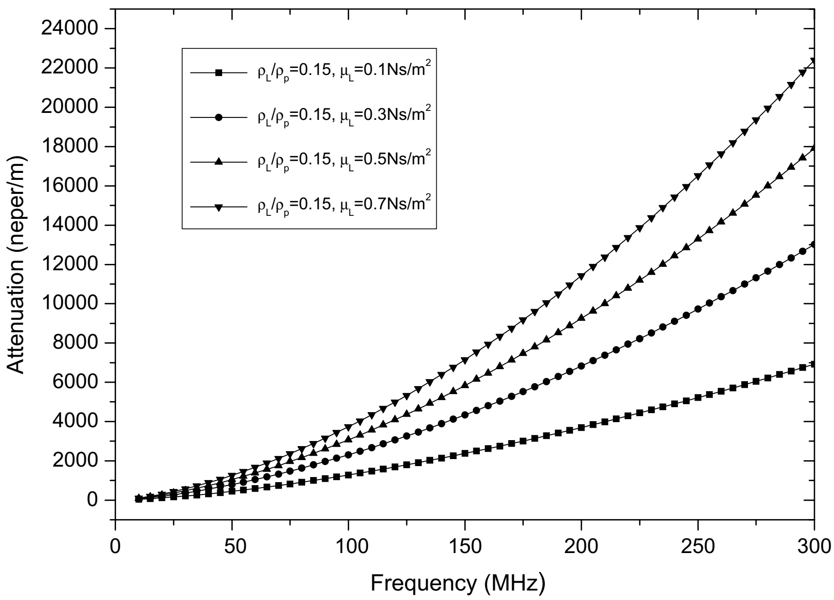

Fig. 9. Change of attenuation with the wave frequency for metalized surface condition.

increasing viscosity (see Fig. 7). Similar results were also reported by Zaitsev et al. (2001). On the contrary to the metalized surface condition, numerical results show that the change of wave velocity due to the presence of liquid is negligible for the open circuit condition. The influence of wave frequency on the attenuation and phase velocity are shown in Figs. 8–10. It can be seen that in general, the attenuation increases monotonically with the increase of wave frequency, and phase velocity falls monotonically with the increase of wave frequency.

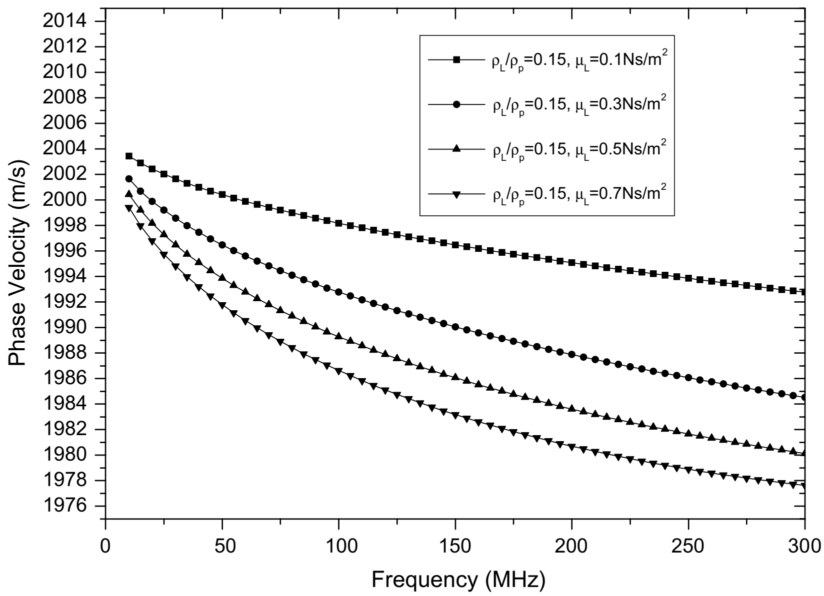

Fig. 10. Change of phase velocity with the wave frequency for metalized surface condition.

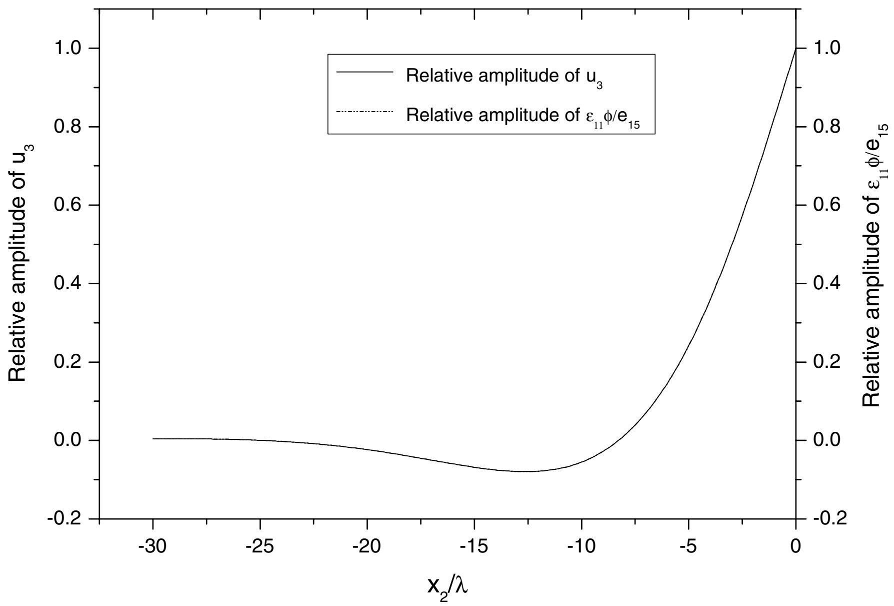

Fig. 11. Distribution of displacement and electric potential in PZT-5H under open circuit condition ($f = 200$ MHz, $\mu_l = 0.3$ NS/m^2, $\rho_l/\rho_p = 0.15$).

Figs. 11 and 13 show that the distribution of displacement component u_3 and electric potential ϕ along the depth direction in the piezoelectric material. On examining these figures, we found that the penetration depth for the open circuit condition is much larger than that for metalized surface condition. The penetration depth for open circuit condition is up to 30 wave lengths while the penetration depth for metalized surface condition is only about 3 wave lengths. These typical features of B–G wave account for the characteristics of attenuation and wave speed of B–G wave propagating in piezoelectric materials loaded with viscous liquid.

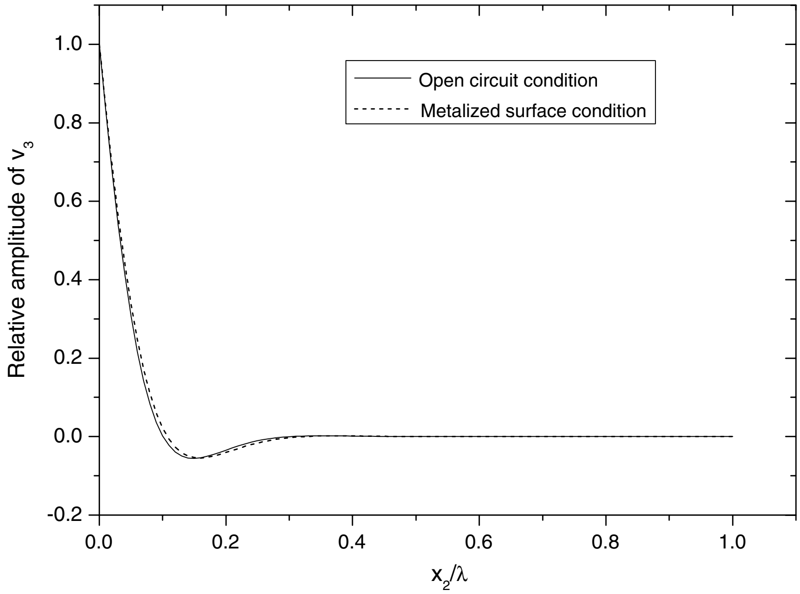

Fig. 12. Distribution of particle velocity in the liquid for both open circuit condition and metalized surface condition ($f = 200$ MHz, $\mu_l = 0.3$ NS/m^2, $\rho_l/\rho_p = 0.15$).

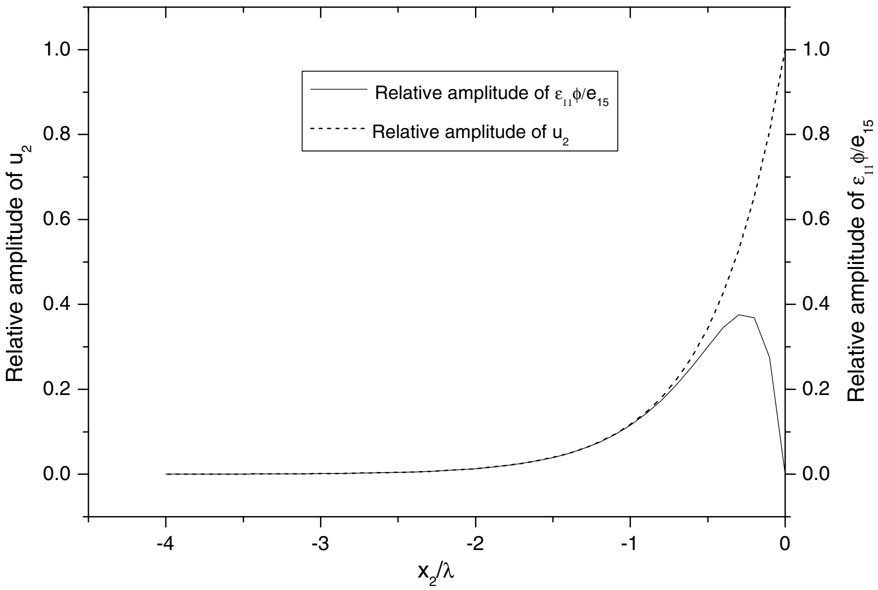

Fig. 13. Distribution of displacement and electric potential in PZT-5H under metalized surface condition ($f = 200$ MHz, $\mu_l =$ 0.3 NS/m^2, $\rho_l\,/\rho_p = 0.15$).

Fig. 12 illustrates that the distribution of particle velocity in the liquid. It is seen that the distribution of particle velocity in the liquid are almost the same for both open circuit and metalized surface conditions.

5. Conclusions

In this paper, we studied propagation of B–G wave in a piezoelectric half-space of 6 mm symmetry in contact with viscous liquid. The explicit dispersion relations for both open circuit and metalized surface boundary conditions are derived. Numerical example of PZT-5H piezoelectric ceramic loaded with viscous liquid is calculated. Numerical results show that the attenuation increases with the increase of liquid viscosity and density. Furthermore, the attenuation in the metalized surface condition is much larger than that in the open circuit condition. The variation of wave velocity for open circuit condition is negligible while in metalized surface condition we see a noticeable change of wave velocity. This is coherent with the fact that in the open circuit condition the penetration length is greater than that in the metalized surface condition. Therefore, in the metalized surface condition energy concentrates more on the region near the interface, thus the B–G wave in the metalized condition is more sensitive to environmental disturbances. Compared with the case of metalized surface condition, the B–G wave in the open circuit condition is less sensitive to surrounding changes.

One advantage of B–G wave for liquid sensing application is that B–G wave has no multiple modes. This makes that inverse determination of liquid properties by utilizing B–G wave is easier than that by utilizing other types of waves, such as Love wave, Lamb wave, etc. Moreover, it is worthwhile to note that the conclusions drawn in this paper are expected to also hold true for B–G wave propagating in other piezoelectric materials of crystal class 6 mm in contact with viscous liquid. This paper provides useful data for the design and development of liquid sensing devices.

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