ARTICLE IN PRESS

International Journal of Solids and Structures xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effect of mass diffusion on the damping ratio in micro-beam resonators

Ali Khanchehgardan, Ghader Rezazadeh *, Rasool Shabani

Mechanical Engineering Department, Urmia University, Urmia, Iran

ARTICLE INFO

Article history:
Received 26 November 2012
Received in revised form 7 December 2013
Available online xxxx

Keywords:
MEMS
Resonator
Internal damping
Thermoelastic
Mass diffusion

ABSTRACT

Present investigation is focused on studying the effect of mass diffusion on the quality factor of the micro-beam resonators. Equation of motion is obtained using Hamilton's principle and also the equations of thermo-diffusive elastic damping are established using two dimensional non-Fourier heat conduction and non-Fickian mass diffusion models. Free vibration of a clamped–clamped micro-beam with isothermal boundary conditions at both ends, and also a cantilever micro-beam with adiabatic boundary condition assumption at the free end, is studied using Galerkin reduced order model formulation for the first mode of vibration. Mass diffusion effects on the damping ratio are studied for the various micro-beam thicknesses and temperatures and the obtained results are compared with the results of a model in which the mass diffusion effect is ignored. In addition to the classic critical thickness of thermoelastic damping, a new critical thickness concerning mass diffusion is introduced.

1. Introduction

Micro-Electro-Mechanical-System (MEMS) is a combination of mechanical components, electronic sensors and electronic components. This mechanical structure is very small in size, and has grown tremendously in the last decade. This is due to the wide application of MEMS in different branches of medicine, medical engineering (analysis and synthesis of DNA and genetic code, drug delivery, diagnostic and imaging), transportation systems (converters, accelerometers, gyroscopes) and production (intelligent micro-robots) (Sadeghian and Rezazadeh, 2009).

Micro-mechanical resonator is a category of MEMS devices. To obtain high performance resonators, it is necessary to build a resonator that works with low power dissipation or in other words with high quality factor (Saeedivahdat et al., 2010). Quality factor of the resonator is a measure of the amount of energy loss. Air damping and clamping losses are the major extrinsic mechanisms of energy dissipation. Energy loss resulting from the thermoelastic damping (TED) is the main intrinsic mechanism of energy dissipation factors in micro-beam resonators (Nayfeh and Younis, 2004; Severine, 2006). TED is an important loss mechanism in high quality micro-structures, especially in those using flexural vibration modes (Lifshitz and Roukes, 2000; Duwel et al., 2003; Evoy et al., 2000; Rozshart, 1990). In precise measurements, TED acts as a source of mechanical thermal noise and contributes to reduce the quality factor and this causes an increase in energy consumption. The extrinsic losses such as air damping, can be minimized by a proper design and operating conditions. But intrinsic losses such as TED cannot be controlled as easily as extrinsic losses and they are almost impossible to eliminate (Nayfeh and Younis, 2004). Therefore it is important to find a way to reduce intrinsic losses, as much as possible and this is available by analyzing all factors, interfering in energy consumption.

Zener (1937) was the first one who explained the mechanism of TED. He also derived an analytical approximation to relate the energy dissipation and the material properties of a micro-beam structure. Copper and Pilkey (2002) demonstrated a thermoelastic solution method for beams with arbitrary quasi-static temperature distributions that create large transverse normal and shear stresses. They calculated the stress resultants and mid span displacements along a beam. Guo and Rogerson (2003) studied the thermoelastic coupling in a clamped–clamped elastic prism beam and examined its size-dependence. Lifshitz and Roukes (2000) studied TED of a beam with rectangular cross-sections, and found that after the Debye peaks, the thermoelastic attenuation will be weakened as the size increases. Sun et al. (2006) studied and analyzed the TED of micro-beam resonators by using both the finite sine Fourier transformation method combined with Laplace transformation and the normal mode analysis. Sun et al. (2008) investigated the vibration phenomenon during pulsed laser heating of a micro-beam and also they analyzed the size effect and the effect of different boundary conditions. Rezazadeh et al. (2009) discussed about the TED in capacitive micro-beam resonators using hyperbolic heat conduction model. They compared two dimensional parabolic and one dimensional hyperbolic heat conduction model to

* Corresponding author. Tel.: +98 914 145 1407; fax: +98 441 336 8033.
E-mail address: g.reazadeh@urmia.ac.ir (G. Rezazadeh).

http://dx.doi.org/10.1016/j.ijsolstr.2014.05.009
0020-7683/© 2014 Elsevier Ltd. All rights reserved.

one dimensional parabolic heat conduction model. Their comparison illustrated that considering these two cases has a negligible influence on the quality factor of the resonators. Therefore, neglecting longitudinal heat conduction and hyperbolic terms of equation would be proper assumptions. Vahdat and Rezazadeh (2011) studied the effects of axial and residual stresses on TED in capacitive micro-beam resonators. Their results showed that axial stress due to the stretching of the micro-beam decreases the TED ratio. Indeed, the effect of axial stress on the TED ratio gets importance when the applied bias DC voltage is near the pull-in voltage. Vahdat et al. (2012) investigated the TED in a micro-beam resonator tunable with a pair of piezoelectric layers bonded on its upper and lower surfaces. Their results showed that, increment of the thickness of piezoelectric layers increases the fundamental frequency and characteristic time of the resonator thus the TED critical thickness value of the resonator decreases.

Mass diffusion (MD) defined as the random movement, of an ensemble of particles, from regions of higher concentration to lower concentration. Recently there is an increasingly attention on the study of MD phenomenon because of its applications in electronic industries. In integrated circuit manufacture, diffusion is used to introduce dopants in controlled quantity into the semiconductor substance. Especially, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors and the source/drain regions in Metal Oxide Semiconductor (MOS) transistors, and dope poly-silicon gates in MOS transistors (Sherief et al., 2004).

In most of the previous investigations, the mass concentration is calculated using a simple law what is known as Fick's equation. Until recently, thermodiffusion in solids was considered as a quantity that is independent of the body deformation. Study of the phenomenon of MD shows that the process of thermodiffusion could have a very important effect upon the deformation of the body.

Nowacki (1974) developed the theory of thermoelastic diffusion using a coupled thermoelastic model. Dudziak and Kowalski (1989) and Olesiak and Pyryev (1995) respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion in an elastic layer. Sherief et al. (2004) developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speeds of propagation of waves. Sherief and Saleh (2005) investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time.

The above explanations declare that mass diffusion damping (MDD) is the other important intrinsic loss mechanism. Unfortunately researchers haven't attended on the effect of MDD in the micro-beam resonators. Therefore the aim of the present work is to examine MDD effect in micro-beam resonators taking into account the finite speed of heat and mass transfer. The question arises whether the previous works are still valid for design high quality factor resonators or the MDD effect has to be taken into account.

A Galerkin based reduced order model has been used to analysis the numerical results of coupled equations. Governing equation of coupled heat conduction has been extracted from the equation of non-Fourier heat conduction and the principle of energy conservation by neglecting heat conduction along the width direction and similar to the previous equation the equation of coupled mass diffusion has been resulted from the equation of non-Fickian mass diffusion and the principle of mass conservation by neglecting mass diffusion along the width direction.

2. Model description and assumptions

We consider small deflections of a thin elastic micro-beam with dimensions of length L $(0 \leqslant x \leqslant L)$, width b $(-b/2 \leqslant y \leqslant b/2)$ and thickness h $(-h/2 \leqslant z \leqslant h/2)$, as is shown in Fig. 1. We define the x axis along the axis of the beam and also y and z axes correspond to the width and thickness, respectively.

The usual Euler–Bernoulli assumption is made so that any plane cross-section, initially perpendicular to the axis of the beam, remains plane and perpendicular to the neutral surface during bending. Thus, the displacements can be given by:

$$u = -z\frac{\partial w}{\partial x} \tag{1}$$

$$T = T(x, z, t); \quad \frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial z} \tag{2}$$

$$C = C(x, z, t); \quad \frac{\partial C}{\partial x} \ll \frac{\partial C}{\partial z} \tag{3}$$

where u is the displacement of the beam in x direction, $T = T - T_0$, T is the absolute temperature and T_0 is the temperature of the beam in the natural state assumed to be equal to the ambient temperature, C is the mass concentration. Following Sherief et al. (2004) and Sadd (2009) the constitutive equation for an isotropic homogeneous elastic solid in terms of mass diffusion and heat conduction is:

$$\begin{aligned} &\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1 T - \beta_2 C) \\ &e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} + \alpha_t T\delta_{ij} + \alpha_c C\delta_{ij} \end{aligned} \tag{4}$$

where:

$$\beta_1 = \frac{E}{(1-2\nu)}\alpha_t, \quad \beta_2 = \frac{E}{(1-2\nu)}\alpha_c \tag{5}$$

where μ and λ are Lame's constants, α_t is the coefficient of the linear thermal expansion, α_c is the coefficient of the linear diffusion expansion σ_{ij} is the components of the stress tensor, u_i is the components of the displacement vector, e_{ij} is the components of the strain tensor, ν is Poisson ratio and δ_{ij} is the Kronecker delta.

Following Sadd (2009), Sherief et al. (2004) and Vahdat and Rezazadeh (2011) when the thickness (in z direction) and the width (in y direction) of a beam are small enough in comparison to the length (in x direction) of it, based on plane stress condition it can be concluded that the stress tensor components in z and y directions are zero ($\sigma_{yy} = \sigma_{zz} = \sigma_{xz} = \sigma_{yz} = \sigma_{xy} = 0$). Therefore the strain components can be simplified as following:

$$e_{xx} = -z\frac{\partial^2 w}{\partial x^2} = \frac{\sigma_{xx}}{E} + \alpha_t T + \alpha_c C \tag{6}$$

$$e_{yy} = \nu\left(z\frac{\partial^2 w}{\partial x^2}\right) + (1+\nu)\alpha_t T + (1+\nu)\alpha_c C \tag{7}$$

$$e_{zz} = \nu\left(z\frac{\partial^2 w}{\partial x^2}\right) + (1+\nu)\alpha_t T + (1+\nu)\alpha_c C \tag{8}$$

$$e_{xy} = e_{xz} = e_{yz} = 0 \tag{9}$$

For a narrow beam based on Euler–Bernoulli beam assumptions the trace of the strain tensor is as follows:

$$e = -(1-2\nu)\left(z\frac{\partial^2 w}{\partial x^2}\right) + 2(1+\nu)\alpha_t T + 2(1+\nu)\alpha_c C \tag{10}$$

and:

$$\sigma_{xx} = -\left(Ez\frac{\partial^2 w}{\partial x^2}\right) - E\alpha_t T - E\alpha_c C \tag{11}$$

Please cite this article in press as: Khanchehgardan, A., et al. Effect of mass diffusion on the damping ratio in micro-beam resonators. Int. J. Solids Struct. (2014), http://dx.doi.org/10.1016/j.ijsolstr.2014.05.009

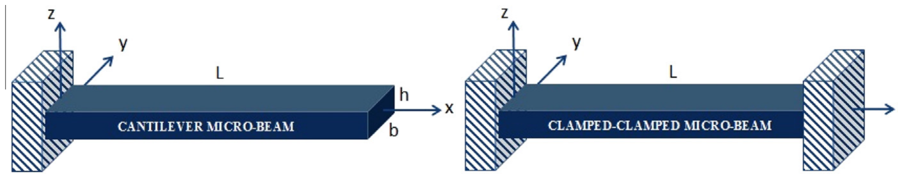

Fig. 1. Schematic illustration of the micro-beam set-up.

In contrast to previous case when the thickness of a beam is small enough in comparison to the length while the width of it is considerable, based on plane strain condition it can be concluded that the stress components in z direction and strain components in y direction vanish ($\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0$ and $e_{xy} = e_{xz} = e_{zy} = 0$). Therefore the components of the strain and stress tensors in terms of the displacement field considering Euler–Bernoulli assumptions can be expressed as following:

$$e_{xx} = -z\frac{\partial^2 w}{\partial x^2} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \alpha_t T + \alpha_c C \tag{12}$$

$$e_{yy} = 0 \tag{13}$$

$$e_{zz} = \frac{\nu}{(1-\nu)}\left(z\frac{\partial^2 w}{\partial x^2}\right) + \frac{(1+\nu)}{(1-\nu)}\alpha_t T + \frac{(1+\nu)}{(1-\nu)}\alpha_c C \tag{14}$$

$$\sigma_{yy} = \nu\sigma_{xx} - E\alpha_t T - E\alpha_c C \tag{15}$$

For a wide beam based on Euler–Bernoulli beam assumptions the trace of the strain tensor is as follows:

$$e = -\frac{(1-2\nu)}{(1-\nu)}\left(z\frac{\partial^2 w}{\partial x^2}\right) + \frac{(1+\nu)}{(1-\nu)}\alpha_t T + \frac{(1+\nu)}{(1-\nu)}\alpha_c C \tag{16}$$

and:

$$\sigma_{xx} = -\frac{E}{(1-\nu^2)}\left(z\frac{\partial^2 w}{\partial x^2}\right) - \frac{E\alpha_t}{(1-\nu)}T - \frac{E\alpha_c}{(1-\nu)}C \tag{17}$$

So for both plane stress and plane strain condition we have:

$$\sigma_{xx} = -\left(\tilde{E}z\frac{\partial^2 w}{\partial x^2}\right) - \beta_t T - \beta_c C \tag{18}$$

here β_t and β_c equal to $E\alpha_t$ and $E\alpha_c$ for a narrow beam (the plane stress condition) and also respectively equal to $E\alpha_t/(1-\nu)$ and $E\alpha_c/(1-\nu)$ for a wide beam (the plane strain condition). Note that for a wide beam, for which $b \geqslant 5h$, the effective modulus $\tilde{E}$ can be approximated by the plate modulus $E/(1-\nu^2)$, otherwise $\tilde{E}$ is Young's modulus E.

3. Motion equation of the micro-beam resonator

When a deflection in the micro-beam, take place the mechanical bending strain energy, U of the beam in terms of mass diffusion and heat conduction is given by:

$$U = \int_0^L \int_A \left(\frac{1}{2}\tilde{E}e_x^2 - \beta_t T e_x - \beta_c C e_x\right) dA\,dx \tag{19}$$

$$U = \frac{1}{2}\int_0^L \tilde{E}I\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \int_0^L \frac{\partial^2 w}{\partial x^2} M_T dx + \int_0^L \frac{\partial^2 w}{\partial x^2} M_C dx \tag{20}$$

where:

$$M_T = \int_{-h/2}^{h/2} b\beta_t T z dz; \quad M_C = \int_{-h/2}^{h/2} b\beta_c C z dz \tag{21}$$

where M_T and M_c are the thermal moment and the mass diffusive moment of the micro-beam. $I = bh^3/12$ is the moment of inertia of the cross-sectional area A. The kinetic energy of the micro-beam:

$$K = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial w}{\partial t}\right)^2 dx \tag{22}$$

By presenting Lagrangian $\mathcal{L}$ and extremizing it the equation of motion will be derived.

$$\mathcal{L} = K - U = \int_0^L F dx \tag{23}$$

$$\begin{aligned} &F = \frac{1}{2}\rho A\dot{w}^2 - \frac{1}{2}\tilde{E}Iw''^2 - (M_T + M_c)w'' \\ &w'' = \frac{\partial^2 w}{\partial x^2}; \quad \dot{w} = \frac{\partial w}{\partial t} \end{aligned} \tag{24}$$

According to the calculus of variation the following condition should be satisfied:

$$\delta\int_{t_1}^{t_2} \mathcal{L} dt = \int_{t_1}^{t_2}\int_0^L \delta F(w'', \dot{w}, M_T, M_C) dx\,dt = 0 \tag{25}$$

$$\begin{aligned} &\int_{t_1}^{t_2}\int_0^L \left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial F}{\partial w''}\right) - \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial \dot{w}}\right)\right]\delta w dx\,dt - \int_{t_1}^{t_2} \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w''}\right)\delta w\Big|_0^L dt \\ &\quad + \int_{t_1}^{t_2} \frac{\partial F}{\partial w''}\delta w'\Big|_0^L dt + \int_0^L \left(\frac{\partial F}{\partial \dot{w}}\right)\delta w\Big|_{t_1}^{t_2} dx = 0 \end{aligned} \tag{26}$$

The dynamic governing equation of the beam in terms of mass diffusion and heat conduction:

$$\begin{aligned} &\frac{\partial^2}{\partial x^2}\left(\frac{\partial F}{\partial w''}\right) - \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial \dot{w}}\right) = 0 \\ &\tilde{E}I\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 M_T}{\partial x^2} + \frac{\partial^2 M_C}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \end{aligned} \tag{27}$$

The initial conditions read:

$$\begin{aligned} &\left(\frac{\partial F}{\partial \dot{w}}\right)\delta w\Big|_{t_1}^{t_2} = 0 \\ &\dot{w}(x,t_2)\delta w(x,t_2) - \dot{w}(x,t_1)\delta w(x,t_1) = 0 \end{aligned} \tag{28}$$

The other equations resulted from the calculus of variation prescribed at $x = 0$ and $x = L$ as the boundary conditions:

$$\frac{\partial F}{\partial w''}\delta w'\Big|_0^L = (\tilde{E}Iw'' + M_T + M_C)\delta w'\Big|_0^L = 0 \tag{29}$$

$$\begin{aligned} &\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w''}\right)\delta w\Big|_0^L = 0 \\ &\left(\tilde{E}I\frac{\partial^3 w}{\partial x^3} + \frac{\partial M_T}{\partial x} + \frac{\partial M_C}{\partial x}\right)\delta w\Big|_0^L = 0 \end{aligned} \tag{30}$$

Please cite this article in press as: Khanchehgardan, A., et al. Effect of mass diffusion on the damping ratio in micro-beam resonators. Int. J. Solids Struct. (2014), http://dx.doi.org/10.1016/j.ijsolstr.2014.05.009

4. Equation of thermo-elasticity

4.1. Classical thermoelasticity

According to classical heat conduction, heat flux is directly proportional to the temperature gradient (Fourier's law) as:

$$q_j = -kT_{,j} \tag{31}$$

Following Sherief et al. (2004) the equation of energy conservation in terms of mass diffusion and trace of the strain tensor is as follow:

$$q_{j,j} = -\rho C_v \dot{T} - \beta_1 T_0 \dot{e} - aT_0 \dot{C} \tag{32}$$

where k and q are the thermal conductivity and heat flux vector, respectively. ρ is the density, C_v is the specific heat at constant strain, a is a measure of thermodiffusion effect.

By taking the divergence of both sides of Eq. (31) and using Eq. (32) and its time derivative, and assuming small deviations of temperature from equilibrium value, the coupled heat conduction equation for a classical thermoelastic body is given by:

$$kT_{,jj} = \rho C_v \frac{\partial T}{\partial t} + \frac{E\alpha_t}{(1-\nu^2)} T_0 \frac{\partial e}{\partial t} + aT_0 \frac{\partial C}{\partial t} \tag{33}$$

4.2. Generalized thermoelasticity

Thermoelasticity equation based on non-Fourier heat conduction equation has been proposed by Lord and Shulman (1967). Non-Fourier or hyperbolic heat conduction equation was introduced by Maxwell (1967) to eliminate the paradox of an infinite velocity peculiar to the classical theory by extension of Fourier law of heat conduction to the most general case involving heat flux and its first time derivative:

$$q_j + \tau_{0t} \dot{q}_j = -kT_{,j} \tag{34}$$

where τ_{0t} is thermal relaxation time. The constant τ_{0t} has a clear physical interpretation. It is the time required to establish the steady state of heat conduction in a volume element suddenly subjected to a temperature gradient. By taking the divergence of both sides of Eq. (34) and using Eq. (32) and its time derivative, we arrive at the equation of generalized heat conduction based on continuum theory frame:

$$\rho C_v(\dot{T} + \tau_{0t}\ddot{T}) + \beta_1 T_0(\dot{e} + \tau_{0t}\ddot{e}) + aT_0(\dot{C} + \tau_{0t}\ddot{C}) = KT_{,jj} \tag{35}$$

By substituting the trace of strain tensor the equation of coupled thermoelastic has been extracted:

$$\begin{aligned}&(\rho C_v + \gamma E\alpha_t^2 T_0)\frac{\partial T}{\partial t} + (\rho C_v \tau_{0t} + \gamma E\alpha_t^2 T_0 \tau_{0t})\frac{\partial^2 T}{\partial t^2} \\ &+ (aT_0 + \gamma E\alpha_t \alpha_c T_0)\frac{\partial C}{\partial t} + (aT_0\tau_{0t} + \gamma E\alpha_t\alpha_c T_0 \tau_{0t})\frac{\partial^2 C}{\partial t^2} \\ &- (\beta_t T_0) z \frac{\partial^3 w}{\partial x^2 \partial t} - (\beta_t T_0 \tau_{0t}) z \frac{\partial^4 w}{\partial x^2 \partial t^2} - k\frac{\partial^2 T}{\partial x^2} - k\frac{\partial^2 T}{\partial z^2} = 0\end{aligned} \tag{36}$$

where γ for the plane stress condition (wide beam) is $2(1+\nu)/(1-2\nu)$ and for the plane strain condition γ is $(1+\nu)/(1-2\nu)(1-\nu)$. As thermal boundary conditions assume both ends of the clamped–clamped micro-beam are isothermal and there is no heat flow through the free surfaces of the micro-beams.

5. Coupled mass diffusion equation with heat transfer

Following Sherief et al. (2004) analogous to Eq. (34) for the heat flux vector, we assume a similar equation for the mass flux vector of the form:

$$\chi_j + \tau_{0c}\dot{\chi}_j = -DP_{,j} \tag{37}$$

and similar to the equation of energy conservation we have the equation of mass conservation:

$$\chi_{j,j} = -\dot{C} \tag{38}$$

where:

$$P = -\beta_2 e + dC - aT \tag{39}$$

In the above equations, χ_j denotes the flow of the diffusing mass vector, P is the chemical potential per unit mass, D is the thermoelastic diffusion constant, d is a measure of diffusive effect. τ_{0c} is the diffusion relaxation time and this will ensure that the equation, satisfied by the concentration, will also predict finite speeds of propagation of matter. Taking the divergence of both sides of Eq. (37), and using Eq. (38) we arrive at:

$$C_j + \tau_{0c}\dot{C}_j = DP_{,jj} \tag{40}$$

substituting Eq. (39) into Eq. (40), we arrive at the equation of mass diffusion in our case, namely:

$$D\beta_2 e_{,ii} + DaT_{,ii} + (\dot{C} + \tau_{0c}\ddot{C}) - DdC_{,ii} = 0 \tag{41}$$

The governing equations of coupled mass diffusion based on non-Fickian two dimensional mass diffusion with one relaxation time in an elastic solid by neglecting MD along the y direction has been resulted from Eq. (41):

$$\begin{aligned}&\frac{\partial C}{\partial t} + \tau_{0c}\frac{\partial^2 C}{\partial t^2} + (Da + \gamma DE\alpha_t\alpha_c)\frac{\partial^2 T}{\partial z^2} + (-Dd + \gamma DE\alpha_c^2)\frac{\partial^2 C}{\partial z^2} \\ &+ (-Dd + \gamma DE\alpha_c^2)\frac{\partial^2 C}{\partial x^2} + (Da + \gamma DE\alpha_t\alpha_c)\frac{\partial^2 T}{\partial x^2} - (D\beta_c) z \frac{\partial^4 w}{\partial x^4} \\ &= 0\end{aligned} \tag{42}$$

Boundary conditions which accompanying for the equation of mass diffusion are in this form that both ends of the clamped–clamped micro-beam have the same concentration of the clamps and there is no mass flow through the free surfaces of the micro-beams.

Following dimensionless quantities are defined to transform Eqs. (27), (36), and (42) into nondimensional forms:

$$\begin{aligned}&\hat{w} = \frac{w}{h}; \quad \hat{x} = \frac{x}{L}; \quad \hat{z} = \frac{z}{h}; \quad \hat{T} = \frac{T}{T_o}; \quad \hat{C} = \alpha_c C \\ &\hat{t} = \frac{t}{t^*}; \quad t^* = L\sqrt{\frac{\rho}{\tilde{E}}}; \quad \hat{M}_T = \frac{M_T}{\tilde{E}bh^2}; \quad \hat{M}_c = \frac{M_c}{\tilde{E}bh^2}\end{aligned} \tag{43}$$

Applying these dimensionless quantities equations will take the following forms:

$$S_1\frac{\partial^4 \hat{w}}{\partial \hat{x}^4} + \frac{\partial^2 \hat{M}_T}{\partial \hat{x}^2} + \frac{\partial^2 \hat{M}_c}{\partial \hat{x}^2} + \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} = 0 \tag{44}$$

$$\begin{aligned}&\frac{\partial^2 \hat{T}}{\partial \hat{x}^2} + S_2\frac{\partial^2 \hat{T}}{\partial \hat{z}^2} - S_3\frac{\partial \hat{T}}{\partial \hat{t}} + S_4\hat{z}\frac{\partial^3 \hat{w}}{\partial \hat{x}^2 \partial \hat{t}} - S_5\frac{\partial^2 \hat{T}}{\partial \hat{t}^2} + S_6\hat{z}\frac{\partial^4 \hat{w}}{\partial \hat{x}^2 \partial \hat{t}^2} \\ &- S_7\frac{\partial \hat{C}}{\partial \hat{t}} - S_8\frac{\partial^2 \hat{C}}{\partial \hat{t}^2} = 0\end{aligned} \tag{45}$$

$$S_9\frac{\partial \hat{C}}{\partial \hat{t}} + S_{10}\frac{\partial^2 \hat{C}}{\partial \hat{t}^2} + S_{11}\frac{\partial^2 \hat{T}}{\partial \hat{z}^2} + S_{12}\frac{\partial^2 \hat{C}}{\partial \hat{z}^2} + S_{13}\frac{\partial^2 \hat{C}}{\partial \hat{x}^2} + S_{14}\frac{\partial^2 \hat{T}}{\partial \hat{x}^2} - S_{15}\hat{z}\frac{\partial^4 \hat{w}}{\partial \hat{x}^4} = 0 \tag{46}$$

in which:

$$S_1 = \frac{h^2}{12L^2};\quad S_2 = \frac{L^2}{h^2};$$
$$S_3 = (\rho C_v + \gamma E\alpha_t^2 T_0)\frac{L}{k}\sqrt{\frac{\tilde{E}}{\rho}};\quad S_4 = \frac{h^2\beta}{kL}\sqrt{\frac{\tilde{E}}{\rho}};$$
$$S_5 = (\rho C_v \tau_{0t} + \gamma E\alpha_t^2 T_0 \tau_{0t})\frac{\tilde{E}}{\rho};\quad S_6 = \frac{\tau_{0t}\beta\tilde{E}h^2}{\rho k L^2};$$
$$S_7 = (a + \gamma E\alpha_t\alpha_c)\frac{L}{\alpha_c k}\sqrt{\frac{\tilde{E}}{\rho}};\quad S_8 = (a\tau_{0t} + \gamma E\alpha_t\alpha_c\tau_{0t})\frac{\tilde{E}}{\alpha_c k\rho}; \tag{47}$$
$$S_9 = \frac{1}{\alpha_c L}\sqrt{\frac{\tilde{E}}{\rho}};\quad S_{10} = \frac{\tau_{0c}\tilde{E}}{\alpha_c\rho L^2};\quad S_{11} = (Da + \gamma DE\alpha_t\alpha_c)\frac{T_0}{h^2};$$
$$S_{12} = (-Dd + \gamma DE\alpha_c^2)\frac{1}{\alpha_c h^2};\quad S_{13} = (-Dd + \gamma DE\alpha_c^2)\frac{1}{\alpha_c L^2};$$
$$S_{14} = (Da + \gamma DE\alpha_c\alpha_t)\frac{T_0}{L^2};\quad S_{15} = D\beta_c\frac{h^2}{L^4}$$

6. Numerical solution

In order to analyze frequency of the micro-beam free vibration coupled with thermo-diffusive equations we used a Galerkin based reduced order model. Based on this model the dynamic deflection, temperature and concentration changes of the system can be approximated in terms of linear combinations of finite number of suitable shape functions with time dependent coefficients:

$$\hat{w}(\hat{x},\hat{t}) = \sum_{k=1}^{p}\varpi_k(\hat{t})\psi_k(\hat{x}) \tag{48}$$

$$\hat{T}(\hat{x},\hat{z},\hat{t}) = \sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j(\hat{z}) \tag{49}$$

$$\hat{C}(\hat{x},\hat{z},\hat{t}) = \sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}(\hat{t})\lambda_e(\hat{x})\Lambda_d(\hat{z}) \tag{50}$$

Substituting Eq. (49) into the M_T equation, it can be represented in nondimensional form as follows:

$$\hat{M}_T = \frac{M_T}{\tilde{E}bh^2} = \frac{T_0\beta_t}{\tilde{E}}\int_{-0.5}^{0.5}\hat{T}\hat{z}d\hat{z} = \frac{T_0\beta_t}{\tilde{E}}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i(\hat{x})\int_{-0.5}^{0.5}\hat{z}\phi_j(\hat{z})d\hat{z} \tag{51}$$

Substituting Eq. (50) into the M_C equation, it can be represented in nondimensional form as follows:

$$\hat{M}_c = \frac{M_c}{\tilde{E}bh^2} = \frac{\beta_c}{\alpha_c\tilde{E}}\int_{-0.5}^{0.5}\hat{C}\hat{z}d\hat{z} = \frac{\beta_c}{\alpha_c\tilde{E}}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}(\hat{t})\lambda_e(\hat{x})\int_{-0.5}^{0.5}\hat{z}\Lambda_d(\hat{z})d\hat{z} \tag{52}$$

Substituting Eqs. (48)–(52) into Eqs. (44)–(46) leads to following equations:

$$S_1\sum_{k=1}^{p}\varpi_k(\hat{t})\psi_k^{(IV)}(\hat{x}) + \frac{T_0\beta_t}{\tilde{E}}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i''(\hat{x})\int_{-0.5}^{0.5}\hat{z}\phi_j(\hat{z})d\hat{z} + \frac{\beta_c}{\alpha_c\tilde{E}}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}(\hat{t})\lambda_e''(\hat{x})\int_{-0.5}^{0.5}\hat{z}\Lambda_d(\hat{z})d\hat{z} + \sum_{k=1}^{p}\ddot{\varpi}_k(\hat{t})\psi_k(\hat{x}) = \epsilon_1 \tag{53}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i''(\hat{x})\phi_j(\hat{z}) + S_2\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j^{\circ\circ}(\hat{z})\\&- S_3\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u}_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j(\hat{z}) + S_4\hat{z}\sum_{k=1}^{p}\dot{\varpi}_k(\hat{t})\psi_k''(\hat{x})\\&- S_5\sum_{i=1}^{n}\sum_{j=1}^{m}\ddot{u}_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j(\hat{z}) + S_6\hat{z}\sum_{k=1}^{p}\ddot{\varpi}_k(\hat{t})\psi_k''(\hat{x})\\&- S_7\sum_{e=1}^{l}\sum_{d=1}^{h}\dot{\varsigma}_{ed}(\hat{t})\lambda_e(\hat{x})\Lambda_d(\hat{z}) - S_8\sum_{e=1}^{l}\sum_{d=1}^{h}\ddot{\varsigma}_{ed}(\hat{t})\lambda_e(\hat{x})\Lambda_d(\hat{z}) = \epsilon_2\end{aligned} \tag{54}$$

$$\begin{aligned}&S_9\sum_{e=1}^{l}\sum_{d=1}^{h}\dot{\varsigma}_{ed}(\hat{t})\lambda_e(\hat{x})\Lambda_d(\hat{z}) + S_{10}\sum_{e=1}^{l}\sum_{d=1}^{h}\ddot{\varsigma}_{ed}(\hat{t})\lambda_e(\hat{x})\Lambda_d(\hat{z})\\&+ S_{11}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i(\hat{x})\phi_j^{\circ\circ}(\hat{z}) + S_{12}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}(\hat{t})\lambda_e(\hat{x})\Lambda_d^{\circ\circ}(\hat{z})\\&+ S_{13}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}(\hat{t})\lambda_e''(\hat{x})\Lambda_d(\hat{z}) + S_{14}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(\hat{t})\varphi_i''(\hat{x})\phi_j(\hat{z})\\&- S_{15}\hat{z}\sum_{k=1}^{p}\varpi_k(\hat{t})\psi_k^{(IV)}(\hat{x}) = \epsilon_3\end{aligned} \tag{55}$$

where $\phi_j^{\circ\circ}(\hat{z}) = \partial^2\phi_j/\partial\hat{z}^2$ and $\Lambda_d^{\circ\circ}(\hat{z}) = \partial^2\Lambda_d/\partial\hat{z}^2$.

According to Galerkin method following conditions should be satisfied:

$$\int_0^1\psi_f(\hat{x})\epsilon_1 d\hat{x} = 0\quad f = 1,\ldots,p \tag{56}$$

$$\int_0^1\int_{-0.5}^{0.5}\varphi_q(\hat{x})\phi_g(\hat{z})\epsilon_2 d\hat{z}d\hat{x} = 0;\quad q = 1,\ldots,n,\ g = 1,\ldots,m \tag{57}$$

$$\int_0^1\int_{-0.5}^{0.5}\lambda_r(\hat{x})\Lambda_s(\hat{z})\epsilon_3 d\hat{z}d\hat{x} = 0;\quad r = 1,\ldots,l,\ s = 1,\ldots,h \tag{58}$$

Now applying Eqs. (56)–(58) to Eqs. (53)–(55) leads to following equations:

$$S_1\sum_{k=1}^{p}\varpi_k K_{fk}^{(1)} + \frac{T_0\beta_t}{\tilde{E}}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}K_{fi}^{(2)}K_j^{(3)} + \frac{\beta_c}{\alpha_c\tilde{E}}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}K_{ef}^{(4)}K_d^{(5)} + \sum_{k=1}^{p}\ddot{\varpi}_k K_{fk}^{(6)} = 0 \tag{59}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}G_{qi}^{(2)}G_{gj}^{(4)} + S_2\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}G_{qi}^{(1)}G_{gj}^{(5)} - S_3\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u}_{ij}G_{qi}^{(1)}G_{gj}^{(4)}\\&+ S_4\sum_{k=1}^{p}\dot{\varpi}_k G_{qk}^{(3)}G_g^{(6)} - S_5\sum_{i=1}^{n}\sum_{j=1}^{m}\ddot{u}_{ij}G_{qi}^{(1)}G_{gj}^{(4)} + S_6\sum_{k=1}^{p}\ddot{\varpi}_k G_{qk}^{(3)}G_g^{(6)}\\&- S_7\sum_{e=1}^{l}\sum_{d=1}^{h}\dot{\varsigma}_{ed}G_{eq}^{(7)}G_{gd}^{(8)} - S_8\sum_{e=1}^{l}\sum_{d=1}^{h}\ddot{\varsigma}_{ed}G_{eq}^{(7)}G_{gd}^{(8)} = 0\end{aligned} \tag{60}$$

$$\begin{aligned}&S_9\sum_{e=1}^{l}\sum_{d=1}^{h}\dot{\varsigma}_{ed}Q_{re}^{(1)}Q_{sd}^{(2)} + S_{10}\sum_{e=1}^{l}\sum_{d=1}^{h}\ddot{\varsigma}_{ed}Q_{re}^{(1)}Q_{sd}^{(2)} + S_{11}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}Q_{ri}^{(3)}Q_{js}^{(4)}\\&+ S_{12}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}Q_{re}^{(1)}Q_{sd}^{(5)} + S_{13}\sum_{e=1}^{l}\sum_{d=1}^{h}\varsigma_{ed}Q_{re}^{(6)}Q_{sd}^{(2)}\\&+ S_{14}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}Q_{ri}^{(7)}Q_{js}^{(8)} - S_{15}\sum_{k=1}^{p}\varpi_k Q_{rk}^{(9)}Q_s^{(10)} = 0\end{aligned} \tag{61}$$

in which:

$$\begin{aligned}&K_{fk}^{(1)} = \int_0^1\psi_f(\hat{x})\psi_k^{(IV)}(\hat{x})d\hat{x};\quad K_{fi}^{(2)} = \int_0^1\psi_f(\hat{x})\varphi_i''(\hat{x})d\hat{x};\\&K_j^{(3)} = \int_{-0.5}^{0.5}\hat{z}\phi_j(\hat{z})d\hat{z};\quad K_{ef}^{(4)} = \int_0^1\psi_f(\hat{x})\lambda_e''(\hat{x})d\hat{x};\\&K_d^{(5)} = \int_{-0.5}^{0.5}\hat{z}\Lambda_d(\hat{z})d\hat{z};\quad K_{fk}^{(6)} = \int_0^1\psi_f(\hat{x})\psi_k(\hat{x})d\hat{x}\end{aligned} \tag{62}$$

For the second coupled equation:

$$G_{qi}^{(1)} = \int_0^1 \varphi_q(\hat{x})\varphi_i(\hat{x})d\hat{x}; \quad G_{qi}^{(2)} = \int_0^1 \varphi_q(\hat{x})\varphi_i''(\hat{x})d\hat{x};$$
$$G_{qk}^{(3)} = \int_0^1 \varphi_q(\hat{x})\psi_k''(\hat{x})d\hat{x}; \quad G_{gj}^{(4)} = \int_{-0.5}^{0.5} \phi_g(\hat{z})\phi_j(\hat{z})d\hat{z};$$
$$G_{gj}^{(5)} = \int_{-0.5}^{0.5} \phi_g(\hat{z})\phi_j^{\circ\circ}(\hat{z})d\hat{z}; \quad G_g^{(6)} = \int_{-0.5}^{0.5} \hat{z}\phi_g(\hat{z})d\hat{z};$$
$$G_{eq}^{(7)} = \int_0^1 \lambda_e(\hat{x})\varphi_q(\hat{x})d\hat{x}; \quad G_{gd}^{(8)} = \int_{-0.5}^{0.5} \phi_g(\hat{z})\Lambda_d(\hat{z})d\hat{z} \tag{63}$$

And for the last coupled equation:

$$Q_{re}^{(1)} = \int_0^1 \lambda_r(\hat{x})\lambda_e(\hat{x})d\hat{x}; \quad Q_{sd}^{(2)} = \int_{-0.5}^{0.5} \Lambda_s(\hat{z})\Lambda_d(\hat{z})d\hat{z};$$
$$Q_{ri}^{(3)} = \int_0^1 \lambda_r(\hat{x})\varphi_i(\hat{x})d\hat{x}; \quad Q_{sj}^{(4)} = \int_{-0.5}^{0.5} \Lambda_s(\hat{z})\phi_j^{\circ\circ}(\hat{z})d\hat{z};$$
$$Q_{sd}^{(5)} = \int_{-0.5}^{0.5} \Lambda_s(\hat{z})\Lambda_d^{\circ\circ}(\hat{z})d\hat{z}; \quad Q_{re}^{(6)} = \int_0^1 \lambda_r(\hat{x})\lambda_e''(\hat{x})d\hat{x}$$
$$Q_{ri}^{(7)} = \int_0^1 \lambda_r(\hat{x})\varphi_i''(\hat{x})d\hat{x}; \quad Q_{sj}^{(8)} = \int_{-0.5}^{0.5} \Lambda_s(\hat{z})\phi_j(\hat{z})d\hat{z};$$
$$Q_{rk}^{(9)} = \int_0^1 \lambda_r(\hat{x})\psi_k^{(IV)}(\hat{x})d\hat{x}; \quad Q_s^{(10)} = \int_{-0.5}^{0.5} \hat{z}\Lambda_s(\hat{z})d\hat{z}; \tag{64}$$

Choosing suitable shape functions, which satisfy the boundary conditions, and solving Eqs. (59)–(61) concurrently in which:

$$\varpi_k = \bar{\varpi}_k e^{i\Omega_k\tau}; \quad u_{ij} = \bar{u}_{ij}e^{i\Omega_{ij}\tau}; \quad \varsigma_{ed} = \bar{\varsigma}_{ed}e^{i\Omega_{ed}\tau}; \tag{65}$$

complex frequencies are obtained.

Note that $\hat{T}$, $\hat{C}$ and $\hat{w}$ vibrate at the same frequency, therefore, $\Omega_k = \Omega_{ij} = \Omega_{ed} = \Omega$ (Sun et al., 2006). According to complex frequency approach the TED ratio (ζ) can be calculated as

$$\zeta = \left| \frac{\Im(\Omega)}{\sqrt{\Re^2(\Omega) + \Im^2(\Omega)}} \right| \tag{66}$$

where $\Re(\Omega)$ is the real part of the complex frequency and $\Im(\Omega)$ is its imaginary part.

7. Numerical results

The proposed micro-beam is a wide beam and has the following material and geometrical properties as shown in Table 1. Assume coefficient of linear thermal expansion, coefficient of linear diffusion expansion and thermal conductivity are constant. Following Sherief and Saleh (2005) we take the following values of related parameters for Copper:

The theoretical results obtained in the previous section are employed in this part to investigate the influence of the mass diffusion effect on the damping ratio. When an elastic solid is set in motion, it is taken out of equilibrium, having an excess of kinetic and potential energy. The coupling of the strain field to a temperature field provides an energy dissipation mechanism that allows the system to relax back to equilibrium. This process of energy dissipation called thermoelastic damping (Sun et al., 2006). Similarly, the coupling of the strain and temperature field to a concentration field provides another energy dissipation mechanism that this process of energy dissipation, called thermo-diffusive elastic damping (TDED) is what we will discuss in this paper.

The bending of the beam causes dilations of opposite signs to exist on the upper and lower halves. One side of the beam is compressed and heated, while the other side is stretched and cooled. Thus, in the presence of finite thermal expansion, a transverse temperature gradient and concentration gradient are produced. These phenomena are couple and a temperature gradient can also works as a driving force for mass diffusion, in a phenomenon which is called thermodiffusion and vice versa concentration gradient can works as a driving force for heat flux. The temperature gradient generates local heat currents, and also the concentration gradients generates local mass currents, which cause increase of the entropy of the beam and lead to energy dissipation and this is the thermo-diffusive elastic damping effect. TED critical thickness takes place when the thermal characteristic time (the time necessary for temperature gradients to relax) is equal to the inverse of the beam fundamental frequency (Vahdat et al., 2012).

Figs. 2 and 3 show that as the thickness increases the damping ratio first increases to attain its maximum value and the related thickness to this value can be known as TDED critical thickness and after this critical thickness damping ratio weakens as the beam

Table 1
Geometrical and material properties of micro-beam resonator.

Symbols	Parameters	Values
L	Length	2000 µm
b	Width	20 µm
E	Young's modulus	120 GPa
ν	Poisson's ratio	0.34
K	Thermal conductivity	383 W m^{-1} K^{-1}
D	Thermo-elastic diffusion constant	0.85×10^{-8} kg s m^{-3}
ρ	Density	8954 kg m^{-3}
C_v	Specific heat at constant volume	383.1 J kg^{-1} K^{-1}
α_t	Coefficient of linear thermal expansion	1.78×10^{-5} K^{-1}
α_c	Coefficient of linear diffusion expansion	1.98×10^{-4} kg^{-1} m^3
T_0	Ambient Temperature	293 K
a	A measure of thermodiffusion effect	1.2×10^4 m^2 s^{-2} K^{-1}
d	A measure of diffusive effect	9×10^5 kg^{-1} m^5 s^{-2}

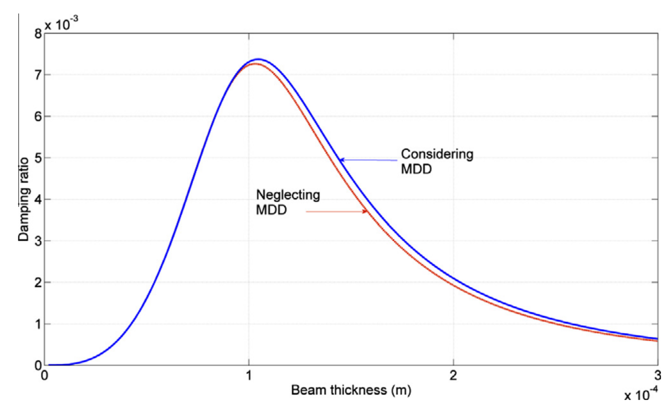

Fig. 2. Damping ratio versus beam thicknesses for the cantilever micro-beam.

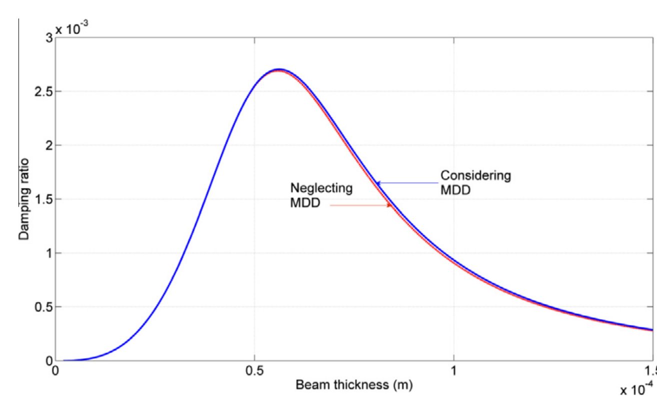

Fig. 3. Damping ratio versus beam thickness for the clamped–clamped micro-beam.

thickness increases. By considering the mass diffusion effect the value of the critical thickness does not change significantly and TDED and TED critical thicknesses are approximately equal.

Figs. 4 and 5 show that damping ratio of the beam changes considerably with changing in thickness or ambient temperature. Therefore in designing process of resonators with high quality factor the beam thickness and working temperature play important role.

From the above discussion we can conclude that damping ratio is sum of thermoelastic and mass diffusion damping. Figs. 2–13 show that influence of mass diffusion on damping ratio is much smaller than thermoelastic. Since the coefficients and constants related to the mass diffusion equation for materials (such silicon) which used in manufacturing of resonators are small therefore the thermoelastic is dominant damping and mass diffusion is negligible.

We have plotted damping ratio, in Figs. 6 and 7 respectively as a function of ambient temperatures for constant thickness ($h = 50\ \mu m$) and ($h = 150\ \mu m$) for the cantilever micro-beam and in Figs. 8 and 9 respectively we have plotted damping ratio as a function of ambient temperatures for constant thickness ($h = 30\ \mu m$) and ($h = 100\ \mu m$) for clamped–clamped micro-beam. The most important observation from the figures is that we can neglect the mass diffusion effect almost before the TED critical thickness but when the thickness of the micro-beam is larger than the critical thickness for high precision and low energy consumption applications it is important to consider the mass diffusion effect.

TR ratio can be defined as the quotient of the damping ratio with considering MDD (ζ_1) with respect to the damping ratio with neglecting MDD (ζ_2):

$$\mathrm{TR} = \frac{\zeta_1}{\zeta_2} \tag{67}$$

Figs. 10 and 11, are the graphical results of the TR ratio versus beam thicknesses respectively for the cantilever micro-beam and clamped–clamped micro-beam and according to these figures the thermo-diffusive elastic damping is equal to thermoelastic damping almost before the critical thickness but after critical thickness the TR ratio increases up to a specific thickness and after that the TR ratio has no considerable increase.

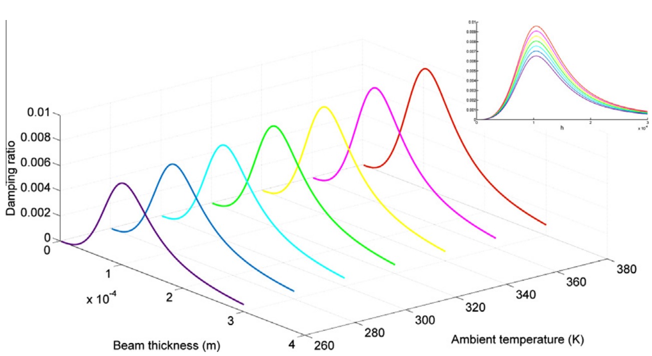

Fig. 4. Damping ratio versus beam thickness for different ambient temperatures for the cantilever micro-beam.

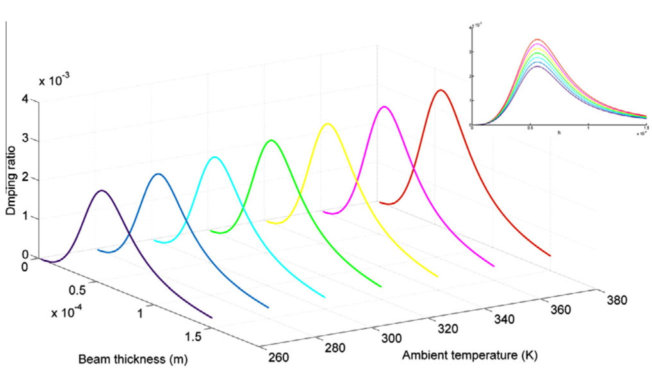

Fig. 5. Damping ratio versus beam thickness for different ambient temperatures for the clamped–clamped micro-beam.

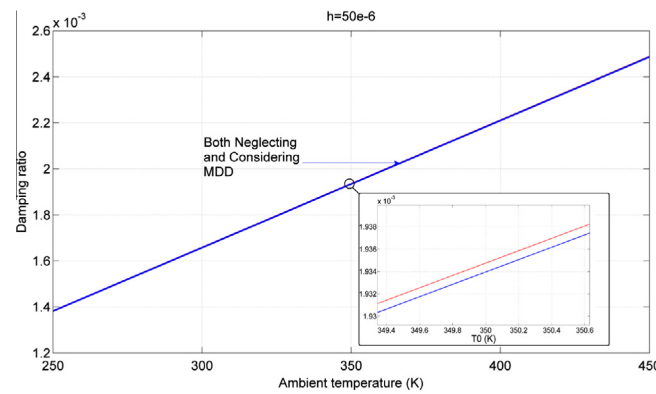

Fig. 6. Damping ratio versus ambient temperature for the cantilever micro-beam with thickness ($h = 50\ \mu m$).

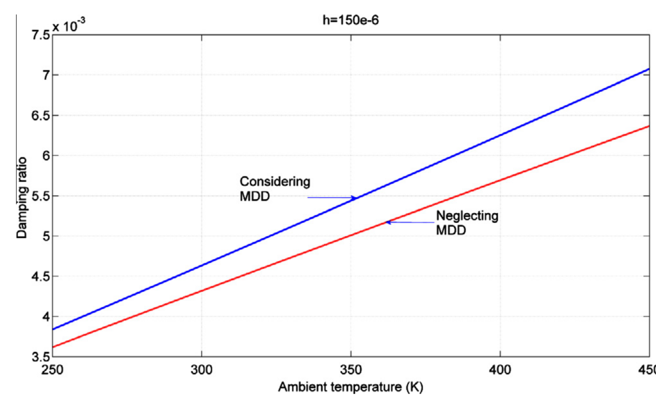

Fig. 7. Damping ratio versus ambient temperature for the cantilever micro-beam with thickness ($h = 150\ \mu m$).

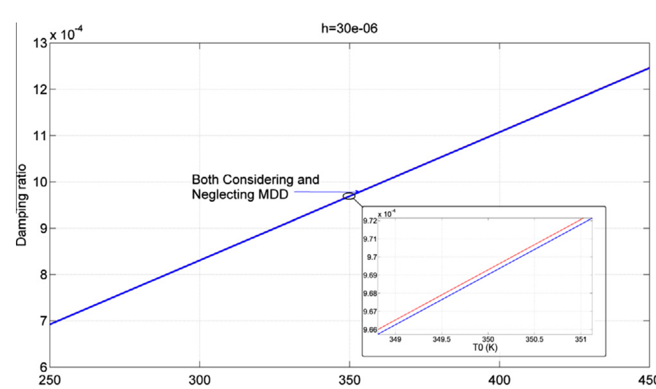

Fig. 8. Damping ratio versus ambient temperature for the clamped–clamped micro-beam with thickness ($h = 30\ \mu m$).

Please cite this article in press as: Khanchehgardan, A., et al. Effect of mass diffusion on the damping ratio in micro-beam resonators. Int. J. Solids Struct. (2014), http://dx.doi.org/10.1016/j.ijsolstr.2014.05.009

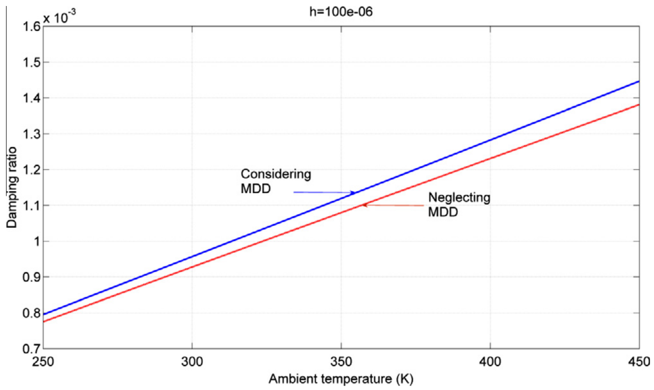

Fig. 9. Damping ratio versus ambient temperature for the clamped–clamped micro-beam with thickness ($h = 100$ μm).

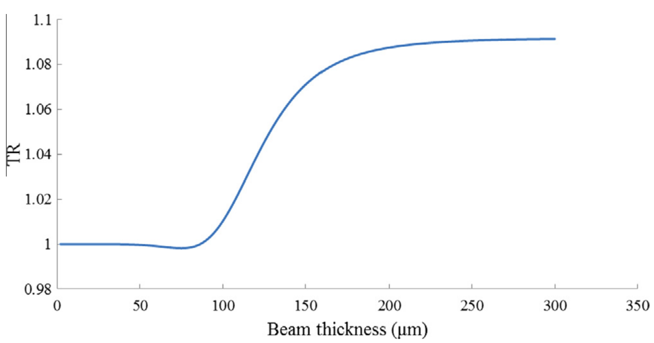

Fig. 10. TR ratio versus micro-beam thicknesses for the cantilever micro-beam.

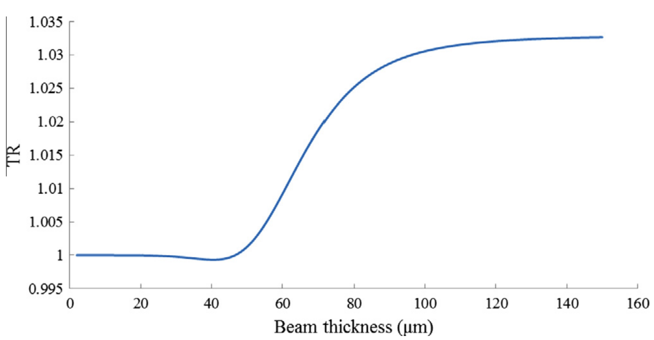

Fig. 11. TR ratio versus micro-beam thickness for the clamped–clamped micro-beam.

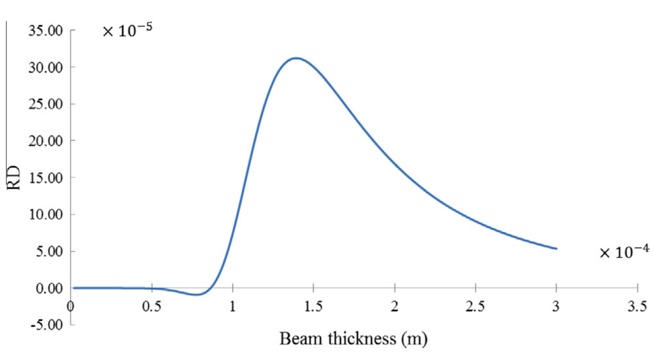

Fig. 12. Differences of damping ratios versus beam thickness for the cantilever micro-beam.

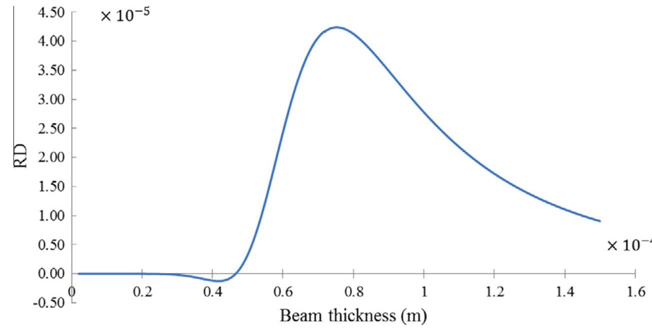

Fig. 13. Differences of damping ratios versus beam thickness for the clamped–clamped micro-beam.

Considering the coupled linear differential equations (44)–(46) and accepting the superposition principal, $RD = \zeta_1 - \zeta_2$ is introduced as the difference of TDED and TED which is capable to show successively the mass diffusion effects on the damping ratio. In Figs. 12 and 13, the values of RD are shown as a function of beam thickness for the cantilever and fixed–fixed micro-beams respectively. The maximum values of RD occur in a specific thickness other than critical thicknesses of TDED and TED. The critical thickness shown in Figs. 12 and 13 can be introduced as the MDD critical thickness in which the diffusion characteristic time (The time necessary for density gradients to relax) is equal to the inverse of the beam fundamental frequency and MDD effects is the maximum. It must be noted that as results show the effects of mass diffusion damping is small in comparison to the thermoelastic damping; therefore in the critical thickness of TDED, the effect of the TED is dominant.

8. Conclusion

In this paper the resonator was modeled as a thin isotropic homogeneous thermoelastic Euler–Bernoulli beam and the effect of mass diffusion on the damping ratios of the different types of micro-beams with different boundary conditions was investigated. The generalized theory of heat conduction and non-Fickian mass diffusion equations coupled with transversal motion of a micro-beam resonator were employed to evaluate TDED.

The results from numerical analysis showed that with increasing the thickness until to the TDED critical thickness the effect of mass diffusion on the damping ratio increases and this effect exposed itself on the quality factor of the micro-beam resonators. The mass diffusion effect almost after the TED critical thickness is considerable and also this effect for the cantilever micro-beam is more sensible than the clamped–clamped micro-beam. The critical thickness of the TDED was found and shown that approximately is equal to TED critical thickness and also the MDD critical thickness was introduced in which the effect of the mass diffusion is the greatest.

In addition was shown that by increasing the ambient temperature of the micro-beam the damping ratio increases and almost after the TED critical thickness damping ratio with considering MD increases faster than that with neglecting mass diffusion. As results showed for designing resonators with high quality factor and consequently minimizing energy consumption the effect of MDD in micro-beams with thickness greater than the TED critical thickness must be taken into account.

References

Copper, C.D., Pilkey, W.D., 2002. Thermoelasticity solutions for straight beams. Trans. ASME J. Appl. Mech. 69, 224–229.

Dudziak, W., Kowalski, S.J., 1989. Theory of thermodiffusion for solids. Int. J. Heat Mass Transfer 32, 2005–2013.

Please cite this article in press as: Khanchehgardan, A., et al. Effect of mass diffusion on the damping ratio in micro-beam resonators. Int. J. Solids Struct. (2014), http://dx.doi.org/10.1016/j.ijsolstr.2014.05.009

Duwel, A., Gorman, J., Weinstein, M., Borenstein, J., Warp, P., 2003. Experimental study of thermoelastic damping in MEMS gyros. Sens. Actuators A 103, 70–75.

Evoy, S., Oikhovets, A., Sekaric, L., Parpia, J.M., Craighead, H.G., Carr, D.W., 2000. Temperature-dependent internal friction in silicon nanoelectromechanical systems. Appl. Phys. Lett. 77, 2397–2399.

Guo, F.L., Rogerson, G.A., 2003. Thermoelastic coupling effect on a micro-machined beam machined beam resonator. Mech. Res. Commun. 30, 513–518.

Lifshitz, R., Roukes, M.L., 2000. Thermoelastic damping in micro- and nanomechanical systems. Phys. Rev. B 61, 5600–5609.

Lord, H., Shulman, Y., 1967. A generalized dynamical theory of thermoelasticity. J. Mech. Phys. Solid 15, 299–309.

Maxwell, J.C., 1967. On the dynamic theory of gases. Philos. Trans. R. Soc. London 157, 49–88.

Nayfeh, A.H., Younis, M.I., 2004. Modeling and simulations of thermoelastic damping in microplates. J. Micromech. Microeng. 14, 1711–1717.

Nowacki, W., 1974a. Dynamical problems of thermodiffusion in solids I. Bull. Acad. Pol. Sci. Ser. Sci. Technol. 22, 55–64.

Nowacki, W., 1974b. Dynamical problems of thermodiffusion in solids II. Bull. Acad. Pol. Sci. Ser. Sci. Technol. 22, 129–135.

Nowacki, W., 1974c. Dynamical problems of thermodiffusion in solids I. Bull. Acad. Pol. Sci. Ser. Sci. Technol. 22, 257–266.

Nowacki, W., 1974d. Dynamical problems of thermodiffusion in elastic solids. Proc. Vib. Prob. 15, 105–128.

Olesiak, Z.S., Pyryev, Y.A., 1995. A coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. Int. J. Eng. Sci. 33, 773–780.

Rezazadeh, G., Vahdat, A.S., Pesteii, S.M., Farzi, B., 2009. Study of thermoelastic damping in capacitive micro-beam resonators using hyperbolic heat conduction model. Sens. Trans. 108 (9), 54–72.

Rozshart, R.V., 1990. The effect of thermoelastic internal friction on the Q of the micromachined silicon resonators. In: IEEE Solid State Sensor and Actuator Workshop, Hilton Head Island, SC, USA, pp. 13–16.

Sadd, M.H., 2009. Elasticity Theory Application and Numerics. Elsevier Inc.

Sadeghian, H., Rezazadeh, G., 2009. Comparison of generalized differential quadrature and Galerkin methods for the analysis of micro-electro-mechanical coupled systems. Commun. Nonlinear Sci. 14 (6), 2807–2816.

Saeedivahdat, A., Abdolkarimzadeh, F., Feyzi, A., Rezazadeh, G., Tarverdilo, S., 2010. Effect of thermal stresses on stability and frequency response of a capacitive microphone. Microelectron. J. 41 (12), 865–873.

Severine, L., 2006. Stochastic finite element method for the modeling of thermoelastic damping in micro-resonators. Ph.D. Dissertation, University of Liege, Department of Aerospace and Mechanics. 1–38.

Sherief, H., Hamza, F., Saleh, H., 2004. The theory of generalized thermoelastic diffusion. Int. J. Eng. Sci. 42, 591–608.

Sherief, H., Saleh, H., 2005. A half-space problem in the theory of generalized thermoelastic diffusion. Int. J. Solids Struct. 42, 4484–4493.

Sun, Y., Fang, D., Soh, A.K., 2006. Thermoelastic damping in micro-beam resonators. Int. J. Solids Struct. 43, 3213–3229.

Sun, Y., Fang, D., Saka, M., Soh, A.K., 2008. Laser-induced vibrations of micro-beams under different boundary conditions. Int. J. Solids Struct. 45, 1993–2013.

Vahdat, A.S., Rezazadeh, G., 2011. Effects of axial and residual stresses on thermoelastic damping in capacitive micro-beam resonators. J. Franklin Inst. 348, 622–639.

Vahdat, A.S., Rezazadeh, G., Ahmadi, G., 2012. Thermoelastic damping in micro-beam resonator tunable with piezoelectric layers. Acta Mech. Solida Sin. 25 (1), 73–81.

Zener, C., 1937. Internal friction in solids. I. Theory of internal friction in reeds. Phys. Rev. 52, 230–235.

Please cite this article in press as: Khanchehgardan, A., et al. Effect of mass diffusion on the damping ratio in micro-beam resonators. Int. J. Solids Struct. (2014), http://dx.doi.org/10.1016/j.ijsolstr.2014.05.009