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Buckling behavior of a superconducting magnet coil

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Abstract

The buckling behavior of a superconducting magnet coil has been investigated for out of plane perturbations. It has been shown that such a coil becomes unstable due to its own magnetic field for out of plane bending at the operating values of electric currents. A technique devised by the author that computes the inductance change due to deformation, has been incorporated in this study. The evaluation shows that although the magnetic fields external to the coil are important and that there is additional structural stiffness provided by the surrounding structure, the effects of the self magnetic field should be evaluated and incorporated in the design of such coils. However, based on previous studies, both analytical and experimental, it has been established that the out-of-plane deformation is the most critical mode from a stability standpoint. Therefore, this mode has been exclusively addressed in this study.

Keywords: Buckling; Elastic bending; Inductance calculations; Magnetoelastic interactions; Structural stability; Superconducting magnets

1. Introduction

In a toroidal coil arrangement (tokamak) of a superconducting magnet structure used for fusion reactors, there are magnetic forces coming from the self magnetic fields of the individual coils as well as from adjacent coils and other external sources. For the case of perfect symmetry, the azimuthal components of the magnetic forces between the coils cancel each other. In the case of small deflection out of plane of a coil from the symmetrical configuration, the coil will experience a resultant out of plane force which will tend to increase its deflection. The magnetic force comes mainly from two sources, namely, (a) perturbed self magnetic field caused by the deflection of the coil and (b) imbalance of the magnetic fields caused in the overall system due to a perturbed motion of the individual coil under consideration. As a first step, the effects due

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to the perturbation of the self magnetic field in the coil will be considered. The effect of the force caused by the perturbed self magnetic field can be described by a negative magnetic stiffness proportional to the square of the current in the coil. The negative magnetic stiffness is superimposed on the positive elastic stiffness of the coil which tries to keep the coil in its original configuration. With increasing current, the total stiffness of the coil for out of plane perturbation is continually reduced and approaches zero when a critical current is reached at which point the coil "buckles". For the values of current exceeding the critical value the magnet arrangement becomes mechanically unstable within a linear framework and the amplitudes can only be predicted considering geometric nonlinearities. Such an analysis has been carried out in this paper.

In a study performed by the author (Chattopadhyay, 1979), the instability associated with a current-carrying elastic structure due to its own magnetic field was investigated. The instability of such a structure was shown to be dependent on the incremental inductance due to deformation. Numerical methods were employed in Chattopadhyay's (1979) work to compute the incremental inductance for simple elastic systems, such as beams and rings. The buckling effect in toroidal magnetic systems was experimentally investigated by Moon (1979) and Miya and Uesaka (1982). But there seems to be a gap between the small laboratory coils and the actual coils in a large torus of complex design. Geiger and Jungst (1991) have developed a generalized theoretical model for the calculation of buckling behavior of toroidal magnet systems. Experimental results obtained in their work seem to indicate that in a torus of complex design, the lateral stiffnesses of the coils may vary considerably with current. Therefore it seems that a generic analysis of instability in such structures is not feasible, and specific analyses must be performed for individual designs to reflect the contribution of various sources of magnetic forces.

Demachi et al. (1995) investigated the magnetoelastic buckling of a shallow arch and their numerical results agreed well with the experimental results. Zheng et al. (2001) investigated the dynamic behavior of current-carrying coils, and found that the dynamic stability is dependent on the magnitude of the steady current and the peak value and the duration of the pulse current. It is deemed appropriate that the contribution of the self magnetic field for an individual coil should be assessed for its own instability and will form an important design input for such coils. To this end, the out of plane bending motion of a superconducting coil has been investigated in this paper.

Based on the experimental results due to Moon (1979), the in-plane displacements of the coils produce stabilizing perturbation forces. Chattopadhyay (1979) has traced this stabilization to the hoop tension produced by the radial magnetic force distribution. Geiger and Jungst (1991) have surveyed the various modes of vibration and have concluded that the critical mode is the out-of-plane vibratory mode which they have termed as the "basic mode". This mode corresponds to the lowest value of the buckling current. We have therefore decided to pursue this case exclusively.

An approximate configuration of a specific design of a superconducting magnet coil (File et al., 1971) has been used in this study. The inductance of such a coil has been determined numerically as a function of the out of plane vibration amplitudes. The dynamics of the coil has been studied using a variational approach incorporating the kinetic, elastic and magnetic energies. The buckling and post-buckling responses have been determined. It is suggested that such an analysis will form a framework for detailed evaluation involving other coils forming the torus as well as other adjoining structural elements of the toroidal magnetic system.

2. Incremental inductance calculations

File et al. (1971) have suggested one particular coil configuration (Fig. 1) to be used in superconducting magnet for fusion power generation. A cylindrical structure supports a series of coils. These coils have to be designed in such a way that the conductors are in pure tension, and the supporting cylinder in compression. This will ensure no bending moments on the coil; also large forces will not have to be transmitted through

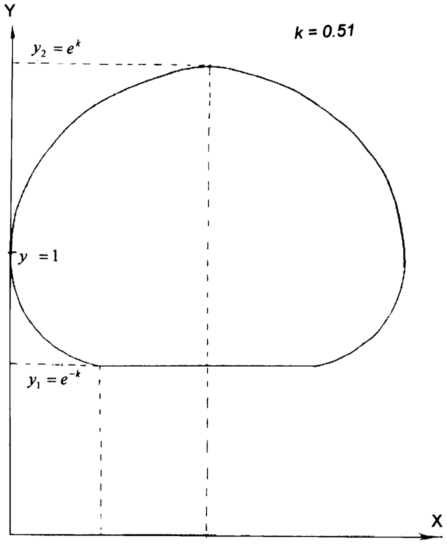

Fig. 1. Proposed shape of a typical coil.

the thermal insulation. If the conductors are to be in pure tension, then for a toroidal field, it can be shown that the axis of the conductor must lie along a curve whose x and y coordinates are related by

$$x = \int_1^y \frac{\pm \ln y}{(k^2 - \ln^2 y)^{1/2}}. \tag{1}$$

Each of the coils must carry 5 million ampere turns to produce the required field of 160,000 G. The details of the design can be found in the work of File et al. (1971).

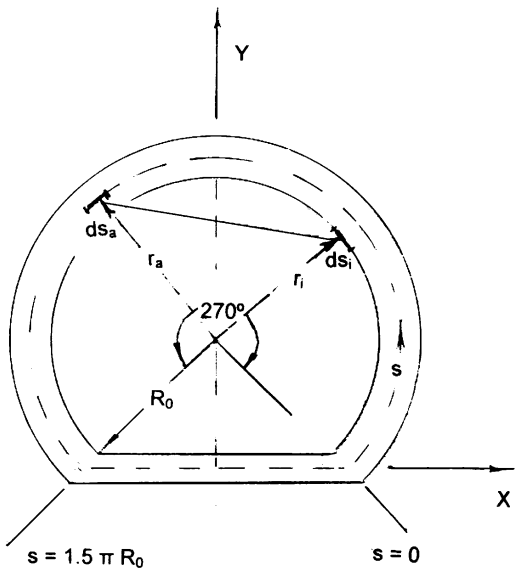

Fig. 2. Approximated shape of the coil.

The shape of the coil is approximated by an incomplete circular ring having an included angle of 270° as shown in Fig. 2. This approximation has also been made by File et al. (1971) for force reduction calculations. The coil is to be tethered at either end and therefore can be treated as a curved beam clamped at both ends. The corresponding out of plane mode shape is given by Felgar (1950) as

$$v = \alpha\left[\left(\cosh\frac{\lambda s}{L} - \cos\frac{\lambda s}{L}\right) - \eta\left(\sinh\frac{\lambda s}{L} - \sin\frac{\lambda s}{L}\right)\right], \tag{2}$$

where α is the amplitude for the out of plane deformation, s is the curvilinear distance measured along the axis of the curved beam, with $s = 0$, and $s = L$ denoting the ends of the beam. The parameters λ and η are equal to 4.73 and 0.9825, respectively.

The inductance of the coil has an external part and an internal part. The internal part results from the contribution of the interior of the conductor, and is small compared to the external part which arises from the field outside the coil due to the current in the coil itself. The internal part of the inductance, a function of the cross-section of the coil is assumed not to change with deformation. The external part of the inductance, which is of interest, is then approximated by the mutual inductance of two filaments, one running along the central axis, and the other along the inner edge of the coil. The mathematical expression of the inductance, $\overline{L}$, is then given by

$$\overline{L} = \frac{\mu_0}{4\pi}\iint\frac{\mathrm{d}s_{\mathrm{a}}\cdot\mathrm{d}s_{\mathrm{i}}}{|r_{\mathrm{a}} - r_{\mathrm{i}}|}, \tag{3}$$

where $\mathrm{d}s_{\mathrm{a}}$ is a differential distance vector along the central axis of the coil having a position vector r_{a}; $\mathrm{d}s_{\mathrm{i}}$ is a differential distance vector along the inner edge of the coil having a position vector r_{i}. The inductance of the

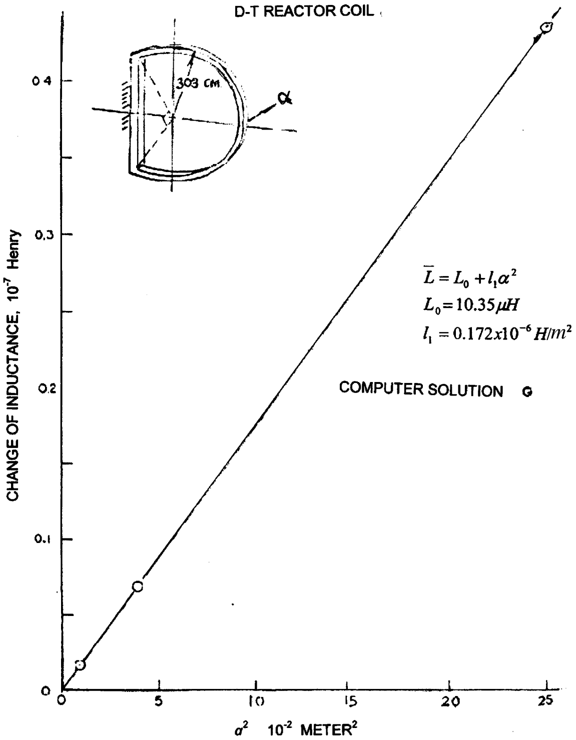

Fig. 3. Inductance calculations.

coil is evaluated by numerically integrating Eq. (3) for various out of plane deformations characterized by amplitude α in Eq. (1). The results are shown in Fig. 3.

From Fig. 3, it is seen that the incremental inductance due to small deformation is proportional to α^2. We define the incremental inductance l_1 by

$$\overline{L} = L_0 + l_1\alpha^2, \tag{4}$$

where L_0 is the inductance of the coil corresponding to the equilibrium position, $\alpha = 0$. l_1 is a constant for the vibratory mode.

3. Kinematics of deformation

For studying the kinematics of deformation, the approximated shape of the coil, namely an incomplete circular ring is used. The cross-sectional dimensions are taken the same as that for the actual design (see File et al., 1971). Let 'φ' and 'v' denote the angular displacement and the transverse (out of plane) displacement of the coil, respectively. Due to rotation 'φ', the surface of the ring becomes a conical surface with a curvature given by

$$\frac{\sin\varphi}{R} \approx \frac{\varphi}{R}. \tag{5}$$

The deflection 'v' produces a curvature 'ρ' in the principal plane x–y of the amount

$$\frac{1}{\rho} = \frac{\mathrm{d}^2 v}{\mathrm{d}s^2} = v''. \tag{6}$$

The net change in curvature about x-axis is therefore, subtracting Eq. (6) from Eq. (5),

$$\frac{1}{r_x} = \frac{\varphi}{R} - v''. \tag{7}$$

The angular displacement φ produces a twist of the amount $\mathrm{d}\varphi/\mathrm{d}s$ (written as φ'). The transverse displacement produces a twist of the amount $1/R\,\mathrm{d}v/\mathrm{d}s$ (written as $1/R\,v'$). The net twist is given by

$$\varphi_{\mathrm{t}} = \varphi' + \frac{v'}{R}. \tag{8}$$

4. Solution by Rayleigh–Ritz method

For a curved beam undergoing combined bending and torsion, the strain energy V, and the kinetic energy T are given by

$$V = \frac{1}{2}\int_0^L \mathrm{EI}_x\left(\frac{\varphi}{R} - v''\right)^2 \mathrm{d}s + \frac{1}{2}\int_0^L \mathrm{GK_T}\left(\varphi' + \frac{v'}{R}\right)^2 \mathrm{d}s, \tag{9}$$

$$T = \frac{1}{2}\int_0^L (\rho A\dot{v}^2 + \rho J_{\mathrm{P}}\dot{\varphi}^2)\,\mathrm{d}s, \tag{10}$$

where the superscripted dots indicate the time derivatives.

The magnetic energy W is given by

$$W = \frac{1}{2}\overline{L}I^2, \tag{11}$$

where, $\bar{L}$ is the inductance of the coil and I the current flowing through the coil. The following assumed mode shapes for 'v' and 'φ' are

$$v = \alpha Z_v(s)\mathrm{e}^{i\omega t}, \tag{12a}$$

$$\varphi = \delta Z_\varphi(s)\mathrm{e}^{i\omega t}, \tag{12b}$$

In Eqs. (12a) and (12b), 'α' and 'δ' are modal amplitudes. The Lagrangian of the system is formed as

$$\Im = T + W - V, \tag{13}$$

The maximum absolute value of the Lagrangian is given by

$$\begin{aligned}|\Im| = {} & \frac{1}{2}\rho A\omega^2\alpha^2\int_0^L Z_v^2\,\mathrm{d}s + \frac{1}{2}\rho J_{\mathrm{P}}\omega^2\delta^2\int_0^L Z_\varphi^2\,\mathrm{d}s + \frac{1}{2}(L_0 + l_1\alpha^2)I^2 - \frac{1}{2}\mathrm{EI}_x\alpha^2\int_0^L (Z_v'')^2\,\mathrm{d}s \\ & + \frac{\mathrm{EI}_x\alpha\delta}{R}\int_0^L Z_v'' Z_\varphi\,\mathrm{d}s - \frac{\delta^2}{2R^2}\mathrm{EI}_x\int_0^L Z_\varphi^2\,\mathrm{d}s - \frac{\mathrm{GK_T}}{2}\delta^2\int_0^L (Z_\varphi'')^2\,\mathrm{d}s - \frac{\mathrm{GK_T}\alpha\delta}{R}\int_0^L Z_\varphi' Z_v'\,\mathrm{d}s \\ & - \frac{\mathrm{GK_T}\alpha^2}{2R}\int_0^L (Z_v'')^2\,\mathrm{d}s.\end{aligned} \tag{14}$$

For $|\Im|$ to be stationary,

$$\frac{\partial|\Im|}{\partial\alpha} = 0, \quad \frac{\partial|\Im|}{\partial\delta} = 0. \tag{15}$$

For the linear problem Eq. (15) yield a set of homogeneous equations, linear and simultaneous in 'α' and 'δ'. For nontrivial solution, the determinants of the coefficients should be zero. This gives

$$\begin{vmatrix} \mathrm{EI}_x\left(\frac{\pi}{L}\right)^4 k_{11} + \frac{\mathrm{GK_T}}{R^2}\left(\frac{\pi}{L}\right)^2 k_{22} - \rho A\omega^2 - \overline{l_1}I^2 & \frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} + \frac{\mathrm{GK_T}}{R}\left(\frac{\pi}{L}\right)^2 k_{22} \\ \frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} + \frac{\mathrm{GK_T}}{R}\left(\frac{\pi}{L}\right)^2 k_{22} & \frac{\mathrm{EI}_x}{R^2} + \mathrm{GK_T}\left(\frac{\pi}{L}\right)^2 k_{22} - \rho J_{\mathrm{P}}\omega^2 \end{vmatrix} = 0. \tag{16}$$

In Eq. (16), we have used the following notations for $\overline{l_1}$, k_{11}, k_{22}, and k_{33} . These are given by Eq. (18)–(21). Furthermore the boundary conditions on the transverse displacement, v, and the twist φ, are the same; therefore the same mode shape could be assumed. Thus we have also used

$$Z_v(s) = Z_\varphi(s) = Z(s), \tag{17}$$

$$\overline{l_1} = \frac{l_1}{\int_0^L Z^2\,\mathrm{d}s}, \tag{18}$$

and

$$k_{11} = \left(\frac{L}{\pi}\right)^4 \frac{\int_0^L (Z''(s))^2\,\mathrm{d}s}{\int_0^L (Z(s))^2\,\mathrm{d}s}, \tag{19}$$

$$k_{22} = \left(\frac{L}{\pi}\right)^2 \frac{\int_0^L (Z'(s))^2\,\mathrm{d}s}{\int_0^L (Z(s))^2\,\mathrm{d}s}, \tag{20}$$

$$k_{33} = -\left(\frac{L}{\pi}\right)^2 \frac{\int_0^L Z''(s)Z(s)\,\mathrm{d}s}{\int_0^L (Z(s))^2\,\mathrm{d}s}. \tag{21}$$

With the notations

$$\beta = \frac{\mathrm{GK_T}}{\mathrm{EI}_x} \tag{22}$$

and

$$C = \frac{L}{\pi R}. \tag{23}$$

The critical current I_* from Eq. (16) is given by, setting $\omega = 0$, as

$$I_*^2 = \frac{\mathrm{EI}_x}{C^4 R^4 \overline{l_1}} \left[\frac{\beta k_{11} k_{22} + \beta C^4 k_{22} + C^2\{k_{11} - k_{33}^2 - 2\beta k_{22} k_{33}\}}{C^2 + \beta k_{22}} \right]. \tag{24}$$

The frequency current dispersion relation from Eq. (16) is then given by

$$I^2\left[\rho I_{\mathrm{P}}\omega^4 - \frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} - \frac{\mathrm{GK_T}}{R}\left(\frac{\pi}{L}\right)^2 k_{22}\right] + \left[\frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} + \frac{\mathrm{GK_T}}{R}\left(\frac{\pi}{L}\right)^2 k_{22}\right] I_*^2\left(1 - \frac{\omega^2}{\omega_0^2}\right) = 0, \tag{25}$$

where, ω_0 is the zero current natural frequency and is given by

$$\omega_0^2 = \frac{\left[\mathrm{EI}_x\left(\frac{\pi}{L}\right)^4 k_{11} + \frac{\mathrm{GK_T}}{R^2}\left(\frac{\pi}{L}\right)^2 k_{22}\right]\rho I_{\mathrm{P}} + \left[\frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} + \frac{\mathrm{GK_T}}{R}\left(\frac{\pi}{L}\right)^2 k_{22}\right]\rho A}{\left[\frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} + \frac{\mathrm{GK_T}}{R}\left(\frac{\pi}{L}\right)^2 k_{22}\right]\overline{l_1} I_*^2}. \tag{26}$$

Neglecting rotary inertia, the frequency current dispersion from Eq. (25) and the zero current natural frequency is simplified to

$$\frac{I^2}{I_*^2} = 1 - \frac{\omega^2}{\omega_0^2} \tag{27}$$

and

$$\omega_0^2 = \frac{\overline{l_1} I_*^2}{\rho A}. \tag{28}$$

5. Post-buckling amplitude

For this purpose, we need to include the higher order terms in the expression of strain. The expression of curvature in Eq. (7) is then modified to read as follows:

$$\frac{1}{r_x} = \frac{\varphi}{R} - v'' + \left(\frac{\varphi^3}{6R} + \frac{v' v''^2}{2}\right). \tag{29}$$

The higher order terms are included in the parentheses.

Within the next higher approximation, making use of Eq. (29), the strain energy expression in Eq. (9) would be increased by an amount, δV, where,

$$\delta V = -\frac{1}{2}\int_0^L \frac{\mathrm{EI}_x}{R}\varphi v'' v'^2 \,\mathrm{d}s + \frac{1}{2}\int_0^L \frac{\mathrm{EI}_x}{3R}\varphi^3 v'' \,\mathrm{d}s + \frac{1}{2}\int_0^L \mathrm{EI}_x v''^2 v'^2 \,\mathrm{d}s. \tag{30}$$

We have neglected some of the higher order terms that would have come from Eq. (29). Inclusion of δV from Eq. (30) into the expression of Lagrangian in Eq. (14) gives rise to an incremental Lagrangian $\delta|\Im|$ (to be added to the terms in Eq. (14)) given by

$$\delta|\Im| = \frac{\mathrm{EI}_x \alpha^3 \delta}{2R}\int_0^L Z_v''(Z_v')^2 Z_\varphi \,\mathrm{d}s - \frac{\mathrm{EI}_x \alpha \delta^3}{6R}\int_0^L Z_\varphi^3 Z_v'' \,\mathrm{d}s - \frac{\mathrm{EI}_x}{2}\alpha^4\int_0^L (Z_v'')^2 (Z_v')^2 \,\mathrm{d}s. \tag{31}$$

Next we define an additional set of constants, k_{44}, k_{55}, and k_{66}, similar to what was defined for k_{11}, k_{22}, and k_{33}, in Eqs. (19), (20) and (21) as

$$k_{44} = \left(\frac{L}{\pi}\right)^6 \frac{\int_0^L (Z'(s))^2 (Z''(s))^2 \,\mathrm{d}s}{\int_0^L (Z(s))^2 \,\mathrm{d}s}, \tag{32}$$

$$k_{55} = -\left(\frac{L}{\pi}\right)^4 \frac{\int_0^L Z''(s)(Z'(s))^2 \,\mathrm{d}s}{\int_0^L (Z(s))^2 \,\mathrm{d}s}, \tag{33}$$

$$k_{66} = -\left(\frac{L}{\pi}\right)^2 \frac{\int_0^L Z''(s)(Z(s))^3 \,\mathrm{d}s}{\int_0^L (Z(s))^2 \,\mathrm{d}s}. \tag{34}$$

From Eq. (16) for the linear problem at buckling we have the ratio of the amplitudes as

$$\frac{\delta}{\alpha} = -\left[\frac{\mathrm{EI}_x \left(\frac{\pi}{L}\right)^4 k_{11} + \frac{\mathrm{GK_T}}{R^2}\left(\frac{\pi}{L}\right)^2 k_{22} - \overline{l_1} I_*^2}{\frac{\mathrm{EI}_x}{R}\left(\frac{\pi}{L}\right)^2 k_{33} + \frac{\mathrm{GK_T}}{R^2}\left(\frac{\pi}{L}\right)^2 k_{22}}\right]. \tag{35}$$

Now including the higher order terms in the Lagrangian from Eq. (31) into (14) and invoking the first of Eq. (15), namely,

$$\frac{\partial|\Im|}{\partial\alpha} = 0,$$

we have

$$\alpha^3\left[2\mathrm{EI}_x\left(\frac{\pi}{L}\right)^6 k_{44} + \frac{3\mathrm{EI}_x}{2R}\left(\frac{\pi}{L}\right)^4 \frac{\delta}{\alpha} k_{55} + \frac{\mathrm{EI}_x}{6R}\left(\frac{\pi}{L}\right)^2 \left(\frac{\delta}{\alpha}\right)^3 k_{66}\right] - \alpha \overline{l_1} I^2 = 0. \tag{36}$$

Employing Eq. (35) and nondimensionalizing the current I by

$$\overline{I}^2 = \frac{\overline{l_1} R^4}{\mathrm{EI}_x} I^2. \tag{37}$$

We obtain the post-buckling amplitude as

$$\alpha^2 = \frac{R^2\left(\frac{L}{\pi R}\right)^6 \left(\overline{I}^2 - \overline{I}_*^2\right)}{k_{44} - \frac{3}{2} k_{55} C_\alpha + \frac{1}{6}\left(\frac{\pi R}{L}\right)^6 k_{66} C_\alpha^3}, \tag{38}$$

where

$$C_\alpha = \frac{1 + \beta\left(\frac{\pi R}{L}\right)^2 - I_*^2}{k_{22} + \beta k_{33}} \tag{39}$$

and

$$\beta = \frac{\mathrm{GK_T}}{\mathrm{EI}_x}. \tag{40}$$

6. Numerical evaluations of k_{11}, k_{22}, k_{33}, k_{44}, k_{55}, k_{66} and $\overline{l_1}$

Using the mode shape given by Eq. (1) the expressions for k_{11}, k_{22} and k_{33} as given in Eqs. (19), (20) and (21) are evaluated numerically. We also evaluate k_{44}, k_{55} and k_{66} numerically using (32), (33) and (34). The following values result:

$$k_{11} = 5.139,$$
$$k_{22} = 1.246,$$
$$k_{33} = 1.246,$$
$$k_{44} = 2.95,$$
$$k_{55} = 0.76,$$
$$k_{66} = 2.86.$$

The normalized incremental inductance coefficient $\overline{l_1}$ is obtained from Eq. (18) as

$$\overline{l_1} = l_1/1.25L.$$

7. Example problem

7.1. Geometric parameters

Radius of the coil = 3.03 m.
$C = L/\pi R = 1.5$ (based on an included angle of 270°).
Coil cross-section 200 mm × 330 mm; this leads to an $I_x = 2.2 \times 10^{-4}\ \mathrm{m}^2$.

The inductance calculations are indicated in Fig. 3. The computations give the incremental inductance coefficient as

$$l_1 = 0.172 \times 10^{-6}\ \mathrm{H/m}^2.$$

7.2. Material parameters

$$E = 8 \times 10^{10}\ \mathrm{N/m}^2.$$

The parameter $\beta = \frac{\mathrm{GK_T}}{\mathrm{EI}_x}$ is approximately unity for our case.

With the above values substituted in Eq. (24), we obtain

$$I_*^2 = 0.78 \frac{\mathrm{EI}_x}{R^4 \overline{l_1}}. \tag{41}$$

This gives the critical current as 3.68×10^6 A.

The numerical values when substituted in Eqs. (39) and (40) give the post-buckling amplitude, α, as,

$$\alpha^2 = 4.4R^2[\bar{I}^2 - \bar{I}_*^2]. \tag{42}$$

From the above equation, it is seen that if the current in the coil exceeds the critical value by 1%, the post-buckling amplitude becomes 26% of the radius of the coil.

8. Conclusions

We have shown that for a single coil using the design due to File et al. (1971), the critical current is 3.68 million amperes, which is well below the proposed value of 5 million amperes in the coil. At this value of the current in the coil, the coil should become unstable due to its own magnetic field under out-of-plane bending. However, there are certain points which need to be considered in the analysis. First, in the actual coil system for the fusion reactor, there are a series of such coils arranged over the periphery of a cylindrical structure, which provides additional stiffness in the coil support system. The effects of the magnetic fields of the neighboring coils on the stability of the coil under consideration remain to be seen. It is conceivable that there could be possible stiffening effect provided by the magnetic fields of the other coils comprising the toroidal magnet system. Also the effects of the initial tension on the stability of the coil need to be investigated.

Nevertheless, the preliminary investigation clearly demonstrates how important the effects of the perturbed magnetic fields due to deformation can be on the stability of the coil. This assessment provides an important first step input into the structural design of superconducting magnet coils.

In this work, the stability of an in-plane perturbation of the coil has not been pursued. Such an analysis can be performed in a similar manner. In the work of Chattopadhyay (1979) it was shown that a simply supported circular ring carrying current was unstable to an in-plane deformation, when the effects of hoop tension due to radial magnetic force distribution were ignored. However, that is certainly not the case. Moon (1979) has established based primarily on experimental observation, that the in-plane deformation of the coils produce stabilizing perturbation forces.

The propensities of the other out-of-plane vibratory modes are intricately related to the manner in which the coil is attached to the magnet structure, and can be addressed by appropriately modifying the boundary conditions. Such modifications to the boundary conditions of the coil are beyond the scope of the present study. Geiger and Jungst (1991) have treated the case outlined in the present study as the one they call the "basic mode," which occurs when the mid-point of the coil is not connected to the azimuthal inter-coil structure of the torus. If such a fixture were existent in the mid region of the coil, the so called basic mode will be excluded, and an "overturning mode" will become the controlling mode. However, this particular mode will correspond to a substantially larger buckling current as reported by Geiger and Jungst (1991).

It is therefore evident that the case considered in this study is the necessary and the most important first step in the design of a superconducting coil, and would be useful for preliminary sizing of components. Furthermore, this study is also relevant where the application deals with a single coil, where the influences of the neighboring coils are nonexistent.

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