



Dynamic fracture of concrete – compact tension specimen

Joško Ožbolt^{a,*}, Akanshu Sharma^b, Hans-Wolf Reinhardt^a

^a Institute of Construction Materials, University of Stuttgart, 70560 Stuttgart, Germany

^b Reactor Safety Division, Bhabha Atomic Research Centre, Mumbai 400085, India

ARTICLE INFO

Article history:

Received 1 September 2010

Received in revised form 10 December 2010

Available online 4 February 2011

Keywords:

Concrete

Compact tension specimen

Dynamic fracture

Rate sensitivity

Crack branching

Finite element analysis

Microplane model

ABSTRACT

The behavior of concrete structures is strongly influenced by the loading rate. Compared to quasi-static loading concrete loaded by impact loading acts in a different way. First, there is a strain-rate influence on strength, stiffness, and ductility, and, second, there are inertia forces activated. Both influences are clearly demonstrated in experiments. Moreover, for concrete structures, which exhibit damage and fracture phenomena, the failure mode and cracking pattern depend on loading rate. In general, there is a tendency that with the increase of loading rate the failure mode changes from mode-I to mixed mode. Furthermore, theoretical and experimental investigations indicate that after the crack reaches critical speed of propagation there is crack branching. The present paper focuses on 3D finite-element study of the crack propagation of the concrete compact tension specimen. The rate sensitive microplane model is used as a constitutive law for concrete. The strain-rate influence is captured by the activation energy theory. Inertia forces are implicitly accounted for through dynamic finite element analysis. The results of the study show that the fracture of the specimen strongly depends on the loading rate. For relatively low loading rates there is a single crack due to the mode-I fracture. However, with the increase of loading rate crack branching is observed. Up to certain threshold (critical) loading rate the maximal crack velocity increases with increase of loading rate, however, for higher loading rates maximal velocity of the crack propagation becomes independent of the loading rate. The critical crack velocity at the onset of crack branching is found to be approximately 500 m/s.

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1. Introduction

There are various loading schemes, which are indicated as static, monotonic, dynamic, cyclic, impulsive, and maybe some more. Impact loading is a special case of dynamic loading and impulsive loading. Impact is associated with the collision of an object with a structure. The collision can be such that the object deforms considerably, which is indicated as soft impact and it can be such that the object does merely deform which causes a hard impact. The transition between the one and the other is smooth and depends on various parameters. The acting force on the structure depends strongly on the degree of deformation of the colliding object. When an impact is simulated numerically the collision of an object is often modelled by force acting on the surface of a structure or by prescribed displacement of the surface. The force or displacement is imposed at various rates, which means that a soft and a hard impact can be simulated. This method will be applied in the here discussed example (compact tension specimen).

The response of structures depends on time dependent loading through three different effects (Reinhardt, 1982; Bischoff and

Perry, 1991; Weerheijm, 1992; Ožbolt and Reinhardt, 2005, 2006; Pedersen et al., 2006; Larcher, 2009; Pedersen, 2009; fib, 2010): (1) through the rate dependency of the growing micro-cracks (influence of inertia at the micro-crack level), (2) through the viscous behaviour of the bulk material between the cracks (creep of concrete or viscosity due to the water content) and (3) through the influence of structural inertia forces, which can significantly change the state of stresses and strains of the material. The first two effects can be accounted for by the constitutive law and the third effect should be automatically accounted for through dynamic analysis where the constitutive law interacts with structural inertia forces. Depending on the material type and the loading rate, the first, second or third effect may dominate. For quasi-brittle materials, such as concrete, which exhibit cracking and damage phenomena, the first two effects are important for relatively low and medium strain rates. However, for higher strain rates (impact) the last effect dominates.

Regardless of the influence of strain rate, the complex transfer of energy between two or more colliding bodies or in case of explosion is an additional problem. For this reason, strain rate is not the only problem to be considered. According to Bentur et al. (1987) and Banthia et al. (1987), because of very high strain gradients it is experimentally difficult to meet the energy balance for very high loading rates. In contrary to this, numerically is relatively simple to

* Corresponding author.

E-mail addresses: ozbolt@iwb.uni-stuttgart.de (J. Ožbolt), akanshusharma@yahoo.co.in (A. Sharma), reinhardt@iwb.uni-stuttgart.de (H.-W. Reinhardt).

calculate the transfer of kinetic energy into other energy forms. Accordingly, it is useful to perform numerical simulations to better understand the processes that take place in a very short period of time.

There are different frameworks for modeling of rate dependent behavior of materials. The rate theory is employed to explain the first two above mentioned reasons for strain rate sensitivity (micro-cracking and creep) (Krausz and Krausz, 1988; Mihashi and Wittmann, 1980). The theory assumes that crack initiation on an atomic scale is governed by the activation energy, i.e. in a non-stressed material there are the same number of bond-breaking and bond-healing steps in time. When an external force acts on the material there is a surplus of energy, which causes more bond-breaking than bond-healing steps. Since the number of bond-breaking steps is assumed to be constant in time that means that a longer loading time causes more ruptures than a shorter loading time. Expressed in terms of strength, it means that sustained loading and creep causes a reduction of strength whereas very short loading causes an increase of strength. This means high loading rates increase the apparent strength of a material. According to Mihashi and Wittmann (1980) the increase of strength, compressive, tensile and flexural, can be expressed by:

$$f_d/f_s = (\dot{\sigma}/\dot{\sigma}^0)^\alpha \quad (1)$$

with f_d = dynamic strength, f_s = static strength under monotonic load, $\dot{\sigma}$ = stress rate under dynamic load, $\dot{\sigma}^0$ = stress rate under static load and α = parameter dependent on load and type of material and the way of loading. Eq. (1) does not consider the effect of inertial forces.

The model of Bažant et al. (2000a,b) is based on the same principle. Again, the influence of inertia forces is not considered. The influence of the loading rate is divided in two parts: (i) the effect of viscosity and (ii) the effect of strain-dependent growth of micro-cracks. The former effect is described by a relatively simple visco-elastic model. The influence of the strain-dependent growth of micro-cracks on the other hand is based on the theory of activation energy:

$$\sigma(\varepsilon) = \sigma^0(\varepsilon) \left[1 + C_2 \ln \left(\frac{2\dot{\varepsilon}}{C_1} \right) \right] \quad (2)$$

where σ = stress at dynamic load, σ^0 = stress at static load, $\dot{\varepsilon}$ = strain rate, C_1 and C_2 are constants determined from experiments. This model has been used in various forms in the microplane model for concrete (Bažant et al., 2000a,b; Ožbolt et al., 2006).

Inertia effects at the material level are taken into account in the model of Reinhardt and Weerheijm (1991) and by Lu and Xu (2004). The authors look at a cluster of penny shaped cracks in a plane, which is perpendicular to the tensile loading direction. When a crack propagates into the material with a certain velocity the crack faces move. For this movement, the energy balance is computed which is the sum of external work W , deformation energy V , kinetic energy T , fracture energy D , and initial energy E_0 at time t_0 , thus

$$-W + V + T + D - E_0 = 0 \quad (3)$$

The faster the crack propagates the larger is the kinetic energy and the deformation energy. The calculations show that the rate of energy supply becomes too high to be absorbed in the fracture process, resulting in an equilibrium in which a major part of the supplied energy is stored as kinetic and deformation energy around the crack tip. That means that the stress distribution around the crack tip changes and the stress intensity factor decreases with increasing loading rate. This is in agreement with Freund (1972a,b) who predicts that the stress intensity factor even vanishes when the crack propagates with the speed of the Rayleigh

wave. The consequence of this result is that the strength of a material increases with loading rate. The rate of strength increase is not constant for all loading rates, but is rather small for low and medium strain rates while there is a steep increase at high loading rates. At very high loading rates the strength increases with the cube root of strain rate (Kipp et al., 1980).

In the literature, there are various two- and three-dimensional phenomenological models, continuous or discrete, that are based on coupling of elastic, plastic, damage and viscous units in different combinations (Holmquist et al., 1993; Rabczuk and Belytschko, 2004; Larcher, 2009; Pedersen, 2010). Depending on the model, the strain rate dependent material behavior is in the entire strain rate range described either better or worse.

The result of several theories is summarized in Fig. 1. As can be seen, for strain rates larger than about 10 s^{-1} , the total resistance starts to increase progressively. It has been recently demonstrated that for low and medium strain rates up to 10 s^{-1} , the total resistance is controlled by viscosity and strain dependent crack growth (Ožbolt and Reinhardt, 2005; Ožbolt et al., 2006; Travaš, 2009). However, at very high strain rates (impact) the inertial forces dominate and cause progressive increase of structural resistance and have dominant influence on failure mode and crack propagation (Ožbolt and Reinhardt, 2005; Ožbolt et al., 2006; Travaš et al., 2009; Reinhardt et al., 2010).

Theoretical, experimental and numerical studies show that the failure mode, crack pattern and velocity of the crack propagation are influenced by the loading rate (Freund, 1972a,b; Ožbolt and Reinhardt, 2005; Ožbolt et al., 2006; Travaš et al., 2009). Principally, with increase of loading rate failure mode tends to change from mode-I to mixed mode. The responsible for this are inertia forces which homogenize material in the impact zone and force damage (crack) to move away from the zone of high inertia forces. Therefore, there is a change of mode I fracture to mixed mode. Moreover, when crack starts to propagate relatively fast, inertia forces at the crack tip tends to prevent crack propagation. Consequently, single crack split into two inclined cracks (branching), i.e. stress intensity factor at the crack tip decreases with increase of a crack speed.

The velocity of the crack propagation also depends on loading rate. The maximal theoretical crack velocity is equal to Rayleigh wave speed $v_R = C_R (G_c/\rho_c)^{0.5}$ where the constant C_R depends on Poisson's ratio, G_c is the shear modulus and ρ_c is specific weight of the material. For normal strength concrete Rayleigh wave speed is approximately equal to 2100 m/s. However, experimental (Curbach and Eibl, 1990), theoretical (Freund, 1972a,b) and numerical investigations (Rabczuk and Belytschko, 2004; Travaš et al.,

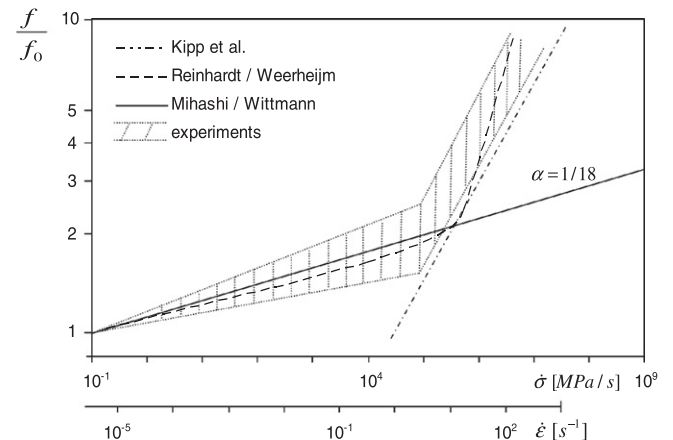


Fig. 1. The material resistance as a function of stress- and strain rate: (a) Mihashi and Wittmann (1980), (b) Reinhardt and Weerheijm (1991), (c) Kipp et al. (1980).

2009) indicate that a single mode-I type of crack cannot reach this theoretical speed of propagation. According to these investigations the maximal crack speed in concrete is between 500 and 600 m/s. Moreover, after crack speed reaches certain critical value there is crack branching (Freund, 1972a,b; Rabczuk and Belytschko, 2004; Kožar and Ožbolt, 2010; Travaš et al., 2009; Reinhardt et al., 2010).

Due to the complexity of the problem the above phenomena can be studied only numerically. In the numerical modeling, which can be of smeared or discrete type, different constitutive laws and modeling techniques can be used (Rabczuk and Belytschko, 2004; Ožbolt et al., 2006; Larcher, 2009; Pedersen, 2009). The influence of viscosity and strain rate dependent growth of micro-cracks is accounted for by the constitutive law. For this purpose some of the above discussed approaches can be used. The effect of structural inertia forces on the rate dependent crack growth should be automatically accounted for through dynamic analysis. However, it is important that constitutive law and (or) corresponding discretization method are employed, which are able to account for realistic interaction between structural inertial forces and material constitutive law.

To investigate the above mentioned phenomena for the case of rate dependent mode-I crack propagation, 3D FE analysis of compact tension specimen is carried out and discussed in the present paper. In the analysis the rate dependent microplane model for concrete is used. In the first part of the paper the model and computational background are described. Subsequently, results of static and dynamic 3D finite element analysis of compact tension specimen are shown and discussed. Finally, summary and conclusions are drawn out.

2. Constitutive law – rate sensitive microplane model for concrete

2.1. Rate insensitive microplane model for concrete

In contrary to classical macroscopic kind of constitutive laws, which are based on tensorial invariants of stresses and strains, in the microplane model the material response is calculated based on the monitoring of stresses and strains in different predefined directions. Integrating microplane stresses in a thermodynamically consistent way, from a known macroscopic strain tensor, it is possible to calculate macroscopic stress tensor. The constitutive framework is similar to discrete type of the models (e.g. random particle model) with the difference that the model is formulated in the framework of continuum. The physical concept behind the microplane model was already discussed at the beginning of last century by Mohr (1900) and Taylor (1938). The microplanes may be imagined to represent damage planes or weak planes in the microstructure, such as those that exist at the contact between aggregate and the cement matrix or slip planes in the theory of plasticity.

The here employed microplane model is based on the so-called relaxed kinematic constraint approach (Ožbolt et al., 2001). It is a modification of the M2 microplane model proposed by Bažant and Prat (1988). In a number of numerical applications it has been demonstrated that the model is able to realistically predict concrete response for rather complex stress–strain histories (Ožbolt et al., 2001; Ožbolt and Reinhardt, 2002; Pivonka et al., 2004). In the model each microplane is defined by its unit normal vector components n_i (see Fig. 2). Microplane strains are assumed to be the projections of macroscopic strain tensor ε_{ij} (kinematic constraint). On the microplane are considered normal (σ_N, ε_N) and two shear stress–strain components ($\sigma_K, \sigma_M; \varepsilon_K, \varepsilon_M$). To realistically model concrete, the normal microplane stress and strain components have to be decomposed into volumetric and deviatoric

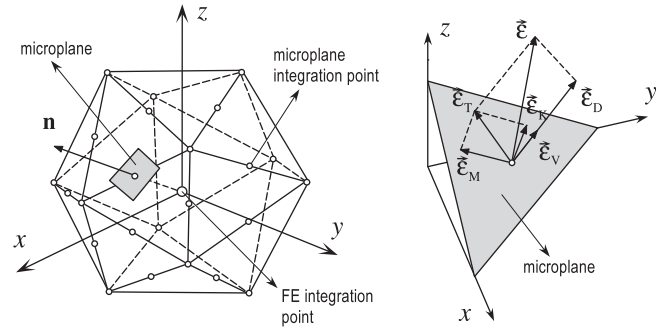


Fig. 2. Decomposition of the macroscopic strain vector into microplane strain components – normal (volumetric and deviatoric) and shear.

parts ($\sigma_N = \sigma_V + \sigma_D, \varepsilon_N = \varepsilon_V + \varepsilon_D$). Unlike to most microplane formulations for concrete, which are based on the kinematic constrain approach, to prevent unrealistic model response for dominant tensile load, in the present model kinematic constrain is relaxed (Ožbolt et al., 2001). Based on the micro–macro work conjugacy of volumetric–deviatoric split and using in advance defined microplane stress–strain constitutive laws, the macroscopic stress tensor is calculated as an integral over all possible, pre-defined, microplane orientations:

$$\sigma_{ij} = \sigma_V \delta_{ij} + \frac{3}{2\pi} \times \int_S \left[\sigma_D \left(n_i n_j - \frac{\delta_{ij}}{3} \right) + \frac{\sigma_K}{2} (k_i n_j + k_j n_i) + \frac{\sigma_M}{2} (m_i n_j + m_j n_i) \right] dS \quad (4)$$

where S denotes the surface of the unit radius sphere, δ_{ij} denotes Kronecker delta and k_i and m_i are directions of shear microplane components. The integration is performed by an optimal Gaussian integration formula for a spherical surface with 21 integration points (symmetric part of the sphere, see Fig. 2, left). It has been shown that this is an optimal number of integration points that still yields acceptable accuracy (Bažant and Oh, 1986). To account for large strains and large displacements, Green–Lagrange finite strain tensor is used. Furthermore, to account for the loading history of concrete, the co-rotational stress tensor is employed. Detailed discussion of the features and various aspects related to the finite strain formulation of the microplane model are beyond the scope of the present paper. For more details refer to Bažant et al. (2000a,b) and Ožbolt et al. (2001).

2.2. Rate sensitivity in the microplane model

The rate dependency in the here presented version of the microplane model for concrete accounts for two effects: (1) the rate dependency related to the formation (propagation) of the micro-cracks, which is the effect of inertia forces at the level of the micro-crack tip, and (2) the rate dependency due to the viscosity of concrete (bulk material) between the micro-cracks. In the rate dependent microplane model proposed by Bažant et al. (2000a,b), both effects are modelled by two separate contributions, i.e., viscosity is represented by a simple visco-elastic model and the rate dependent growth of micro-cracks is modelled by a rate process theory. In the present microplane model both effects are modelled in the integral sense based on the rate process theory, similar to the concept proposed by Mihashi and Wittmann (1980). It is assumed that the micro-cracks start to grow immediately after the application of load. The initial stiffness, which is in experiments always measured as secant stiffness, is assumed to be controlled by the rate of growing micro-cracks. Since in the model the out-com-

ing strength is proportional to the initial stiffness, an increase in initial stiffness, because of higher strain rate, also causes an increase in strength. The influence of structural inertia on the rate dependency is not a part of the constitutive law. As will be demonstrated later, this effect is automatically accounted for in dynamic analysis in which the constitutive law interacts with forces due to structural inertia.

The rate of strain $\dot{\varepsilon}$ in a continuum with a number of parallel cohesive cracks can be expressed as:

$$\dot{\varepsilon} = \frac{\dot{w}}{s_{cr}} + \frac{\dot{\sigma}}{E} \approx \frac{\dot{w}}{s_{cr}} \quad (5)$$

where ε = average macroscopic strain normal to the direction of parallel cracks, s_{cr} = spacing of the parallel cracks, E = effective Young's modulus of bulk material and $\dot{\sigma}/E$ is the elastic strain ratio which can be, compared to the crack opening ratio $\dot{w}$, neglected. After introducing a few reasonable simplifications into the concept that is based on the energy activation theory (Krausz and Krausz, 1988), the influence of the rate effect on the rate independent stress-strain relation $\sigma^0(\varepsilon)$ can be calculated according to (2) (Bažant et al., 2000a,b).

In the microplane model macroscopic response is obtained by integrating normal and shear microplane stresses over all microplanes. The rate independent microplane stress components $\sigma_{Mp}^0(\varepsilon_{Mp})$ (Mp = stands for microplane volumetric, deviatoric and shear components, respectively) are calculated from the known microplane strains ε_{Mp} using pre-defined microplane uniaxial stress-strain constitutive relations (Ožbolt et al., 2001). The strain rate independent model parameters are: Young's modulus, Poisson's ratio, uni-axial compressive and tensile strengths and fracture energy. These macroscopic properties are related to internal microplane parameters. For more details see Ožbolt et al. (2001). It seems reasonable to assume that the rate effect on each microplane component is of the same type as given by (2). Consequently, the rate dependency for each microplane component reads (Bažant et al. (2000a,b)):

$$\sigma_{Mp}(\varepsilon_{Mp}) = \sigma_{Mp}^0(\varepsilon_{Mp}) \left[1 + c_2 \ln \left(\frac{2\dot{\gamma}}{c_1} \right) \right] \text{ with } \dot{\gamma} = \sqrt{\frac{1}{2} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}} \quad c_1 = \frac{c_0}{s_{cr}} \quad (6)$$

where c_0 and c_2 are material rate constants, which have to be calibrated by fitting test data, $\dot{\varepsilon}_{ij}$ = components of the macroscopic strain rate tensor. From (6) is obvious that the rate magnitude is not measured on the individual microplanes, which would be not

objective, but on the macroscale. Furthermore, (6) applies to all microplane components except to volumetric compression, which is assumed to be rate insensitive. This is because for volumetric compression there is no crack development, i.e. the material is compacted.

The model parameters from (6) are calibrated (Ožbolt et al., 2006) based on the uni-axial compressive tests performed by Dilger et al. (1978). The tests have been carried out for three loading rates: 0.2 s^{-1} , $3.33 \times 10^{-3} \text{ s}^{-1}$ and $3.33 \times 10^{-5} \text{ s}^{-1}$.

Assuming average crack spacing of $s_{cr} = 100 \text{ mm}$, the following values are obtained from the calibration procedure: $c_0 = 0.0004$ and $c_2 = 0.032$ (Ožbolt et al., 2006). Using these parameters, the rate dependent uniaxial compressive stress-strain curves are plotted for three different loading rates in Fig. 3a. In the same figure, the test results are also shown. Fig. 3b shows the model response in the case of a sudden increase and decrease of the strain rate for the compression softening. Similar behaviour was observed in the experiments (Bažant et al., 2000a,b). The influence of strain rate on the uni-axial compressive strength and initial Young's modulus is shown in Fig. 4. In both figures, the static strength and the static Young's modulus correspond to the strain rate of 10^{-5} s^{-1} for which the average test value of the ratio between dynamic and static strengths is equal to one. As can be seen, for medium strain rates up to approximately 1.0 s^{-1} the microplane model prediction agrees well with the average trend observed in the experiments.

As mentioned above, the influence of structural inertia on the rate sensitive response is not a part of the constitutive model. By a number of numerical studies (Ožbolt and Reinhardt, 2005; Ožbolt et al., 2006; Travaš et al., 2009; Reinhardt et al., 2010) was demonstrated that inertia effect, which cause a kind of homogenization of the material, is accounted for automatically in dynamic analysis through the interaction between inertia forces (stresses) and constitutive law. The effect is here illustrated on the example of a concrete cylinder (Young's modulus $E_c = 35000 \text{ MPa}$, Poisson's ratio $\nu = 0.18$, tensile strength $f_t = 2.25 \text{ MPa}$, uniaxial compressive strength $f_c = 23 \text{ MPa}$ and concrete fracture energy $G_F = 0.08 \text{ N/mm}$; cylinder diameter $D = 50 \text{ mm}$ and height $H = 100 \text{ mm}$) loaded by different strain rates. To demonstrate the influence of inertia, rate sensitive static and dynamic analysis were carried out. The relation between the relative compressive strength (resistance) and strain rate is plotted in Fig. 5. The results of the rate dependent static analysis show almost linear increase of relative strength as a function of strain rate (log-log. scale). The outcome of the rate sensitive dynamic analysis shows for medium strain rates the same

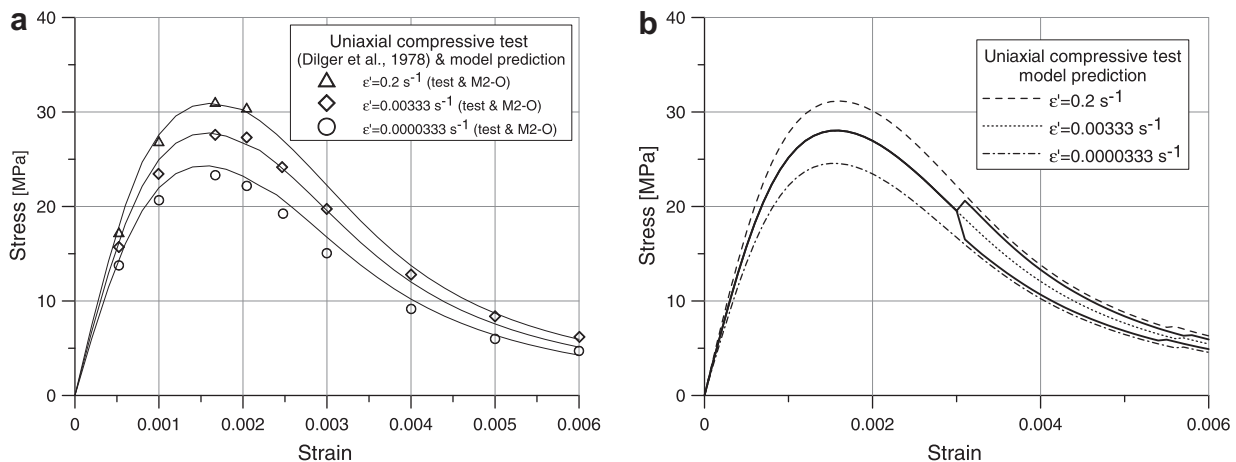


Fig. 3. Uniaxial compressive test (Ožbolt et al., 2006): (a) model prediction and test data (Dilger et al., 1978) and (b) model prediction – increase and decrease of the loading rate in the softening region.

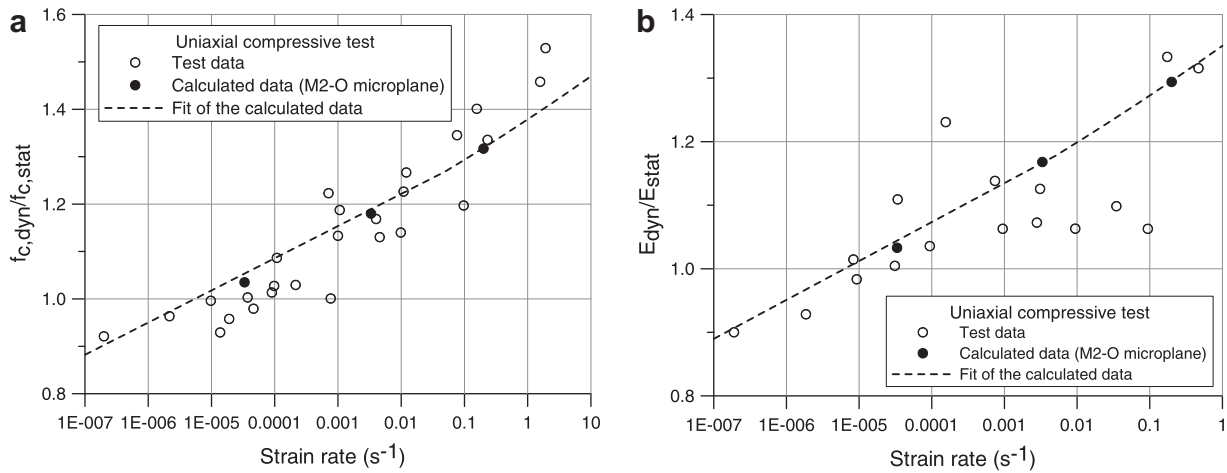


Fig. 4. Uniaxial compressive test (Özbolt et al., 2006): (a) rate dependent compressive strength – test data and model prediction and (b) rate dependent initial Young's modulus – test data and model prediction.

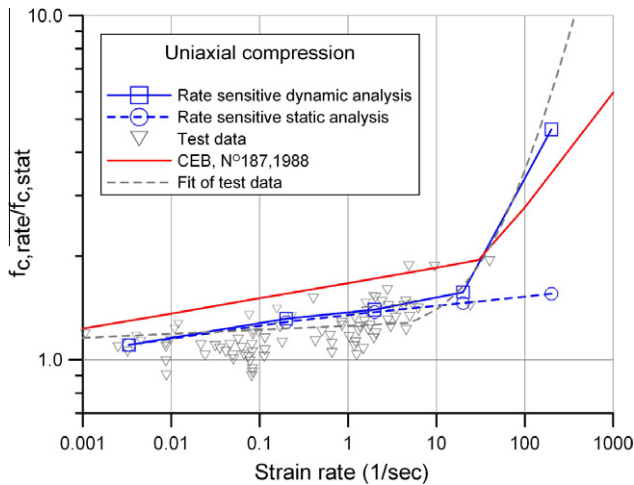


Fig. 5. Relative compressive strength as a function of strain rate.

tendency. However, similar to experiments (Bischoff and Perry, 1991), for strain rates higher than approximately 10 s^{-1} the increase of resistance becomes progressive. Obviously, structural inertia forces are responsible for the progressive increases obtained in the dynamic analysis. For comparison a curve proposed by CEN (1988) is also shown. The agreement between test data, CEB prediction formula and numerical results is very good. Note that the progressive increase of resistance for strain rates larger than 10 s^{-1} does not occur due to the increase of uni-axial compressive strength. This increase is mainly due to the contribution of inertia forces.

3. Three-dimensional FE analyses

In static and dynamic analysis of the compact tension specimen the rate sensitive microplane model discussed above is employed. Static analysis is performed using an implicit 3D FE code based on the incremental secant stiffness approach (Belytschko et al., 2001). In the 3D transient dynamic FE analysis the system of unknown displacements in each time step Δt is calculated by solving the following system of equations (Voigt notation):

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) - \mathbf{f}(t) = 0 \quad (7)$$

where $\mathbf{M}$ = mass matrix, $\mathbf{C}$ = damping matrix, $\ddot{\mathbf{u}}$ = nodal accelerations, $\dot{\mathbf{u}}$ = nodal velocities and $\mathbf{f}(t)$ = resulting nodal forces. The resulting nodal forces are calculated as:

$$\begin{aligned} \mathbf{f}(t) &= \mathbf{f}^{\text{ext}}(t) - \mathbf{f}^{\text{int}}(t) \\ \text{with: } \mathbf{f}^{\text{ext}}(t) &= \text{external nodal forces;} \\ \mathbf{f}^{\text{int}}(t) &= \text{internal nodal forces} \end{aligned} \quad (8)$$

The system of Eqs. (7) is solved using an explicit direct integration scheme (Belytschko et al., 2001). The external nodal forces are known nodal loads. The internal nodal forces are unknown and they are calculated by the integration of the stresses over the finite elements. In the FE code used in the numerical study the mass and damping matrices are assumed to be diagonal.

4. Numerical analysis of compact tension specimen

4.1. Geometry, FE discretization, material properties and loading

Because of relative compact size, compact tension specimen is frequently used to measure fracture properties of concrete and other quasi-brittle materials. As discussed above, the fracture properties and cracking response of concrete depend on the rate of loading. In general, higher loading rates lead to higher resistance. Moreover, with increase of loading rate failure mode also changes (Brandon, 1987; Banthia et al., 1983; Özbolt et al., 2006; Saatci and Vecchio, 2009; Travaš et al., 2009). To investigate the influence of the loading rate on the response of compact tension specimen, transient 3D finite element analysis using rate sensitive microplane model is carried out.

The geometry, boundary conditions and finite element mesh of the specimen are shown in Fig. 6. The dimensions of the specimen are: width-height-thickness = $200 \times 250 \times 50 \text{ mm}$ and the notch length is 100 mm. The analysis is carried out for normal strength concrete with the following macroscopic properties: Young's modulus $E_c = 30000 \text{ N/mm}^2$, Poisson's ratio $\nu = 0.18$, uni-axial compressive strength $f_c = 40.0 \text{ N/mm}^2$, tensile strength $f_t = 3.50 \text{ N/mm}^2$, fracture energy $G_F = 0.09 \text{ N/mm}$ and concrete mass density $\rho_c = 2400 \text{ kg/m}^3$. The parameters that control the rate dependent response (Eq. 6) are as specified in section 2 ($c_0 = 0.0004$ and $c_2 = 0.032$).

The spatial discretization is performed by standard four-node solid finite elements with constant strain field (see Fig. 6). To assure objectivity of the analysis with respect to the size of the finite

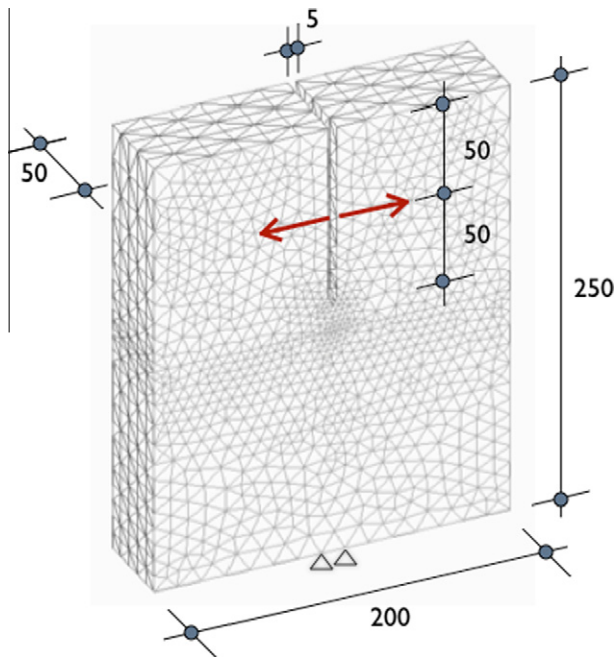


Fig. 6. Geometry of the compact tension specimen (all in mm), finite element mesh, loading (displacement control) and boundary conditions.

elements crack band method is used (Bažant and Oh, 1983). The load is applied by controlling horizontal displacement of the notch, applied 50 mm from the top of the specimen. To prevent local failure at the application of load, elements around the applied load are assumed to be linear elastic. The specimen is first analyzed assuming quasi-static loading condition. Subsequently, dynamic analysis is carried out with the following displacement rates $\Delta/dt = 10, 100, 500, 1000, 2500, 5000$ and 10000 mm/s. Moreover, in order to demonstrate the influence of inertia forces and rate sensitivity on the response of the specimen, additional static and dynamic analyses are carried out.

4.2. Results of the analysis and discussion

The calculated load displacement curves are shown in Fig. 7. It can be seen that with increase of displacement rate peak load increases and for high displacement rates it is much higher than the static resistance (Fig. 7a). The peak load measured relative to the quasi-static resistance is plotted as a function of displacement rate in Fig. 8a. It is assumed that quasi static load corresponds to displacement rate of 0.01 mm/s. As can be seen from Fig. 8a, up to displacement rate of 100 mm/s there is relatively low increase of resistance with increase of displacement rate. However, for higher displacement rates there is progressive increase of resistance. This progressive increase is due to structural inertia forces, which have in case of higher displacement rates a major influence

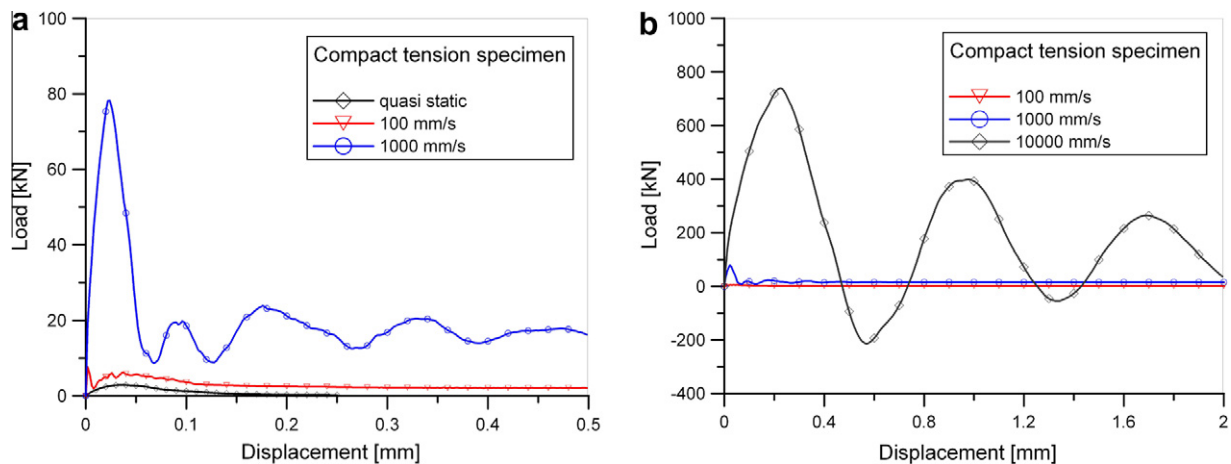


Fig. 7. Predicted load–displacement curves: (a) for displacement rates up to 1000 mm/s and (b) for displacement rates up to 10000 mm/s.

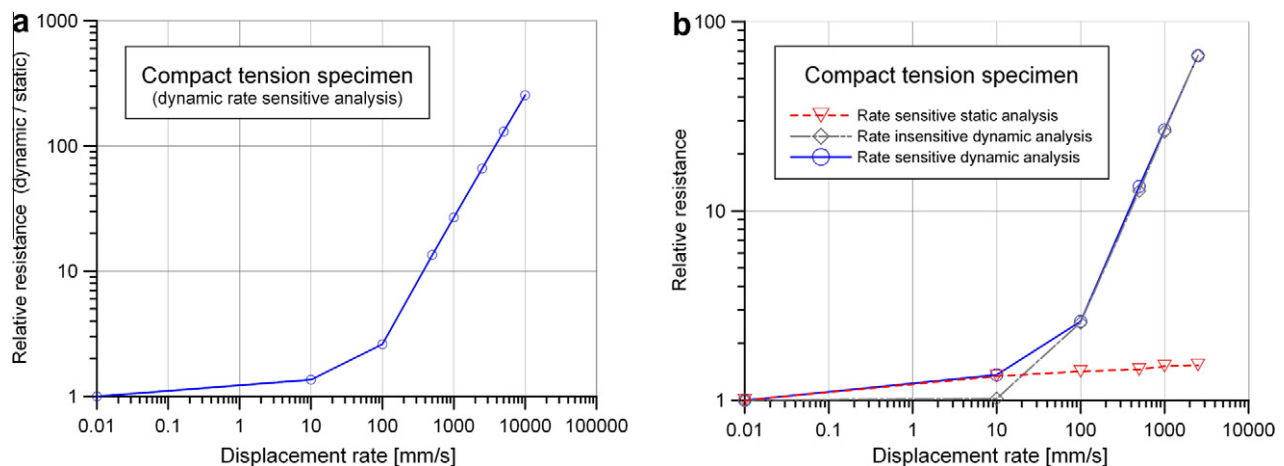


Fig. 8. Relative resistance (rate dependent resistance/ quasi static resistance) for: (a) dynamic analysis for range up to 10 m/s and (b) comparison between static analysis with rate sensitivity and dynamic analysis with and without rate sensitivity – loading range up to 2.5 m/s.

on structural resistance (Ožbolt et al., 2006). To confirm this, two additional series of analysis are carried out, i.e. static analysis with rate sensitivity and dynamic analysis without rate sensitivity. The results are summarised in Fig. 8b for displacement rate up to 2500 mm/s. The comparison of results shows that for displacement rate up to 10 mm/s strain rate controls the response, however, for higher displacement rates structural inertia dominates and strain rate has much less influence on the total resistance. The results of static rate sensitive analysis show that the resistance increases as a linear function of displacement rate (log–log scale).

In Fig. 9 are shown predicted crack patterns in terms of maximal principal strains, assuming critical crack width $w_{cr} = 0.2$ mm. It can be seen that for quasi-static load and for relatively low displacement rates (up to 100 mm/s) there is a single mode-I crack, which propagates starting from the notch tip. However, for higher displacement rates ($\Delta/dt > 500$ mm/s) there is a branching of the crack and multiple cracking. For instance, for displacement rate $\Delta/dt = 1000$ mm/s the crack starts as a single one from the notch tip and subsequently branches into two inclined cracks. For higher displacement rates ($\Delta/dt > 2500$ mm/s) the crack branching starts already at the crack tip at onset of cracking and subsequently additional branching (multiple branching) is observed. Obviously, for higher displacement rate there is a cluster of multiple cracks with rather complex mixed failure mode.

Responsible for the change of the failure mode are structural inertia forces. Namely, the analyses without inertia forces (static analysis with rate sensitivity) indicate no change of the failure mode, i.e. independent of the displacement rate there is mode-I fracture (single crack). In the contrary to this, dynamic analysis without rate sensitivity predicts the same failure modes and crack branching as shown in Fig. 9. When in dynamic analysis crack reaches the critical velocity of approximately 500–600 m/s crack branching is observed. This is illustrated in Fig. 10. The figure

shows crack velocities at different cracking stages ($\Delta/dt = 1000$ and 2500 mm/s). The data are obtained by the evaluation of numerical results. It is interesting to observe that crack branching takes place always when the crack velocity reaches approximately 500 m/s. Furthermore, Fig. 10 also shows that the crack velocity depends on the crack path. For instance in case of displacement rate of 1000 mm/s the maximal crack propagation of 480 m/s is observed before crack branching and the minimal crack velocity of 40 m/s is measured close before the termination of the analysis. Although the displacement rate is constant during entire loading history (1000 mm/s) there is a span of the crack velocity from 40 to 480 m/s.

The main reason for the crack branching are inertia forces at the crack tip which prevents mode-I propagation and force crack to split (branch) into two new cracks. The direction of propagation of new cracks depends on the propagation of loading waves in damaged structure, which depends on material properties, current fracture configuration, geometry and displacement rate. From theoretical investigations (Freund, 1972a,b) it is well known that with increase of the crack velocity the stress intensity factor at the crack tip decreases, i.e. if the crack speed approaches to Rayleigh wave speed the dynamic stress intensity factor approaches to zero. This blocks mode-I crack propagation and force the crack to branch in directions in which the stress intensity factor is higher. Although the inhomogeneity of concrete is not the major reason for crack branching, it also probably plays an important role. It can be expected that higher level of inhomogeneity tends to generate crack branching at lower crack velocities. Note that for the here used concrete Rayleigh wave speed $v_R = C_R (G_c/\rho_c)^{0.5} = 2140$ m/s with $C_R = 910$ (for crack velocity in m/s). The results are also in good agreement with theoretical and experimental measurements of the crack speed, which show that maximal crack speed cannot reach the theoretical limit (Rayleigh wave speed) (Freund,

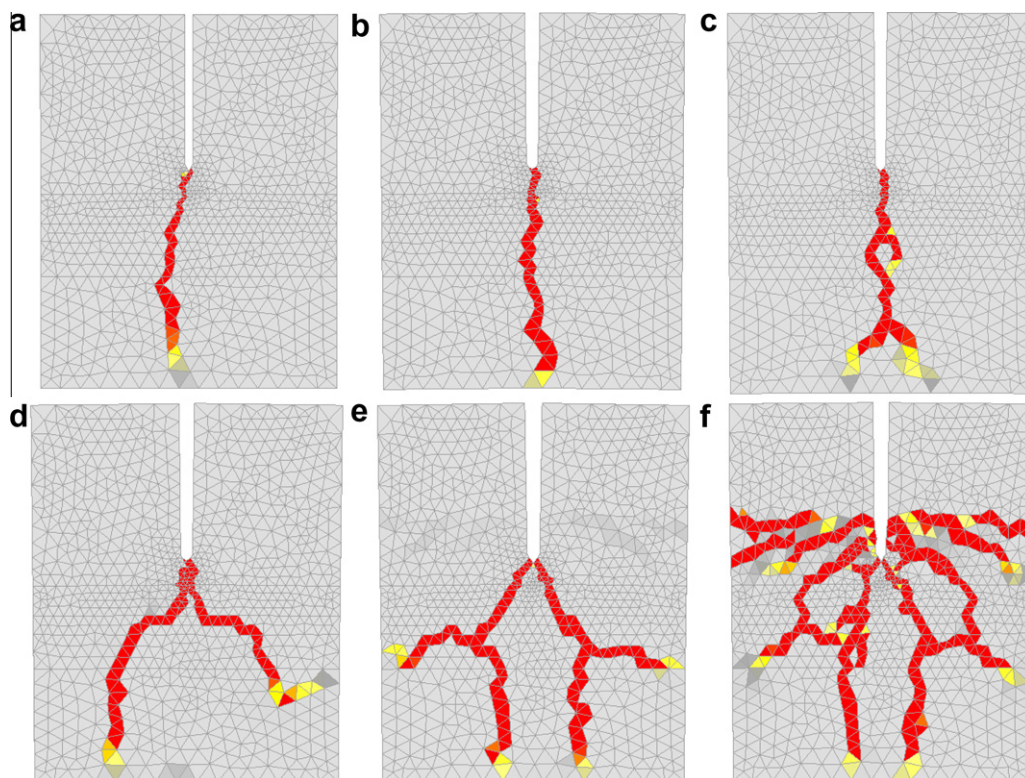


Fig. 9. Predicted failure modes: (a) quasi static analysis (b) $\Delta/dt = 100$ mm/s (c) $\Delta/dt = 500$ mm/s (d) $\Delta/dt = 1000$ mm/s (e) $\Delta/dt = 2500$ mm/s (f) $\Delta/dt = 5000$ mm/s.

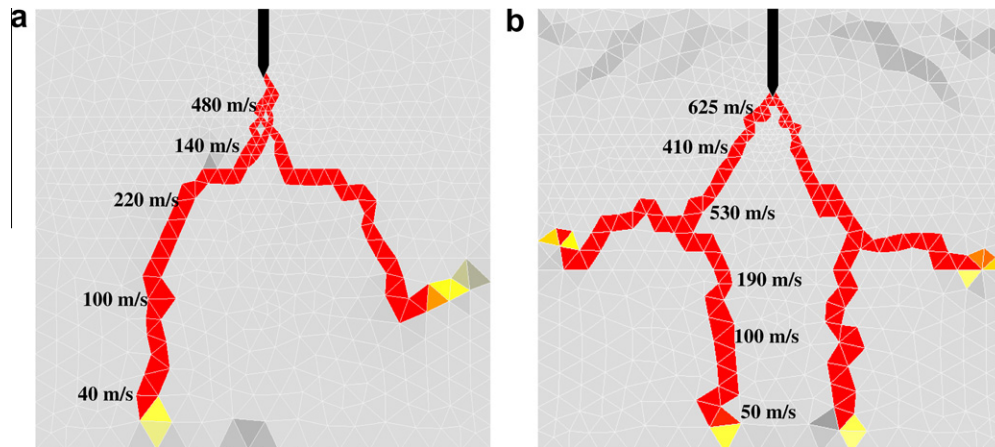


Fig. 10. Crack velocities for different displacement rates: (a) $\Delta/dt = 1000$ mm/s and (b) $\Delta/dt = 2500$ mm/s.

1972a,b; Curbach, 1987; Rabczuk and Belytschko, 2004; Larcher, 2009). The maximal measured crack velocity in concrete was approximately 500–1000 m/s (Curbach, 1987; Larcher, 2009).

As discussed before, the results of the analysis are obtained using spatial finite element discretization shown in Fig. 6, which is based on the randomly generated four node simple constant strain solid elements together with the localization limiter that is based on the energy criteria (crack band method). In order to confirm that the crack branching predicted by the finite element model from Fig. 6 is mesh independent, additional analysis is performed where relatively fine standard eight-node solid elements with linear strain field are employed. The comparison of the crack path and calculated crack velocities ($\Delta/dt = 1000$ mm/s) for two completely different spatial discretizations show that the results of the analysis are principally mesh independent (see Fig. 11) in the sense that the observed crack branching does not depend on the discretization. The crack path is slightly influenced by the mesh since in case of four-node finite elements the discretization is relatively coarse compared to the eight-node element mesh (hexa-mesh). Furthermore, for the finer hexa-mesh the evaluation of crack velocities is more precise due to the finer spatial discretization. Note that more serious mesh sensitivity study, which is out of the scope of the present paper, would require detailed and systematic variation of mesh size and shape.

The results of the analysis show that crack velocity depends on the crack path and on the displacement rate. Principally, the crack speed is the highest at the onset of crack branching where it reaches the velocity of approximately 500–600 m/s. Furthermore, the crack velocity at the onset of cracking (notch tip) depends on the displacement rate. It increases with the displacement rate and reaches maximal value of 625 m/s ($\Delta/dt = 10000$ mm/s). The crack velocity at the onset of cracking as a function of displacement rate is plotted in Fig. 12. It can be seen that the crack velocity increases with the increase of displacement rate up to $\Delta/dt = 2500$ mm/s. With further increase of displacement rate the crack speed at the onset of cracking does not increase, instead crack branching at onset of cracking is observed.

Finally, it should be pointed out that it is somewhat surprising that the used, relatively simple modelling technique, based on the standard finite elements, macroscopic constitutive law (micro-plane) and smeared crack approach, seems to be capable to correctly predict, not only the resistance, but also rate dependant failure mode, crack branching and crack velocity. Of course, experimental results are needed to confirm this. From the literature review, it was found that there is no continuous macroscopic constitutive law, which is able to predict all these phenomena. It has been shown that discrete models are able to predict these phenomena (Rabczuk and Belytschko, 2004), however, such models

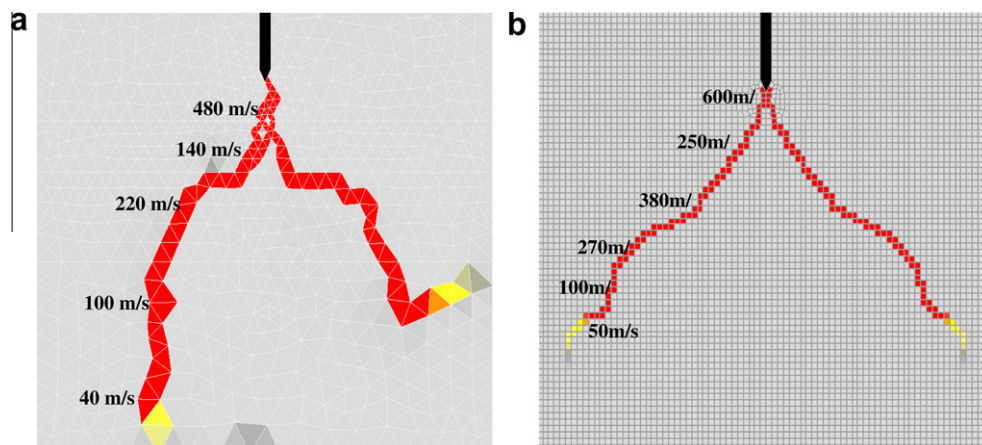


Fig. 11. Crack velocities and crack pattern for displacement rate $\Delta/dt = 1000$ mm/s (a) four-node solid elements (tetra-mesh), relatively coarse mesh and (b) eight-node solid elements (hexa-mesh), finer mesh.

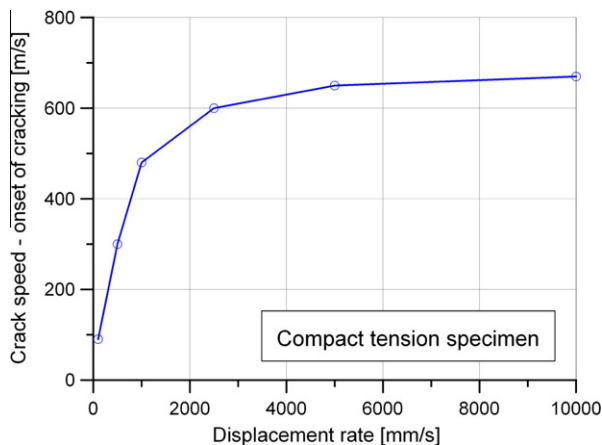


Fig. 12. Crack speed at onset of cracking as a function of displacement rate.

are more complex and cannot be used as a standard numerical tool in engineering practice.

5. Summary and conclusions

In the present paper the rate sensitive response of the compact tension specimen is numerically studied. The numerical analysis is performed using rate sensitive microplane model for concrete. Spatial discretization is performed by standard 3D solid finite elements. The formulation of the problem is performed in the framework of continuum mechanics, following basic principles of irreversible thermodynamics. Based on the numerical results obtained in the study of rate dependent failure of compact tension specimen, the following can be concluded. (1) Displacement rate has significant influence on the resistance and failure mode of the compact tension specimen. For lower displacement rates the resistance is influenced by strain rate dependent constitutive law. However, for higher displacement rates structural inertia forces control the response. (2) For quasi-static load and for relatively low displacement rates, lower than 500 mm/s, the failure mode is due to mode-I fracture. However, for higher displacement rates failure is due to mixed mode that takes in complexity with increase of displacement rate, i.e. multiple crack branching is observed. (3) The predicted rate dependent fracture and crack branching is in good agreement with experimental observations and theoretical predictions. Structural inertia forces and not the strain rates are responsible for the change of the failure mode and crack branching. (4) The crack velocity depends on the current crack pattern. The highest crack velocity is approximately 500–600 m/s and it is measured at the onset of crack branching. The maximal crack velocity in concrete cannot reach the theoretical limit (Rayleigh wave speed). (5) The maximal crack velocity at onset of cracking increases with increase of displacement rate, however, it reaches maximal value of approximately 600 m/s for displacement rate of 2500 mm/s. Further increase of displacement rate does not influence the crack propagation speed at onset of cracking, instead there is a crack branching. (6) Relative simple modeling approach based on the standard finite elements, macroscopic constitutive law (microplane) and smeared crack approach, seems to be capable to correctly predict, not only the resistance, but also rate dependant failure mode, crack branching and crack velocity. It is demonstrated that the observed crack branching and predicted crack velocities are principally not dependent on the finite element discretization. (7) Experimental tests must be performed to confirm the numerically obtained results.

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