



Effect of a shear modified Gurson model on damage development in a FSW tensile specimen

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ABSTRACT

For a friction stir welded aluminum plate the resistance to ductile failure is studied by analyzing tensile test specimens cut out across the weldline. As the stress triaxiality is rather low in these tests, the Gurson material model is not expected to give a very accurate description of the void growth to coalescence. A recently proposed modified version of the Gurson model is used, in which an extra term in the damage evolution law allows for the prediction of failure even at zero or negative values of the mean stress. This modification of the Gurson model is purely phenomenological, such that the damage parameter does not really represent the void volume fraction. Various amounts of the additional damage evolution are compared with predictions of the original Gurson model. The analyses are carried out for different yield stress profiles transverse to the weld and for different specimen widths. It is found that the modification does provide additional damage development in the friction stir weld, which may help to fit experimental data. But the suggested modification depends strongly on the overall stress state, and may have a too strong effect in some cases where the stress triaxiality is rather high.

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1. Introduction

The friction stir welding method (FSW) was developed at The Welding Institute (TWI, Cambridge, United Kingdom) in 1991 (Thomas et al., 1991). This has become a widely used solid-state welding technique for joining a variety of aluminum alloys. The FSW process utilizes a spinning tool, consisting of a pin and a shoulder plate, which is lowered into the weldline until the shoulders are pressed in contact with the metal sheets to be welded. By forcing the spinning tool forward along the weldline a joint is created due to friction heating and extensive deformation of the material in the stir zone. The combined rotating and translational motion of the tool results in an asymmetric material flow (Schmidt et al., 2006), which creates an asymmetry in the final weld. Furthermore, the heat and deformation created between the shoulder plate and the material to be welded, results in a difference between the top (crown) and bottom (root) of the weld.

Compared to conventional fusion welds the FSW technique has proven to produce welds of better mechanical properties for the aluminum alloy (AA2024) considered here. However, a non-homogeneous variation of the mechanical properties transverse to the weldline is still found. For the aluminum alloy considered, a number of experimental studies for the mechanical properties (Lockwood et al., 2002; Genevois et al., 2004; Liu and Chao, 2005) and for the evolution of the microstructure in the weld region (Yang et al., 2004; Sutton et al., 2004) have been published. It is well known that a FS-weld may be divided into four regions; the material to be joined is the base material (BM); a heat affected zone (HAZ) is the material experiencing a

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significant increase in temperature; a thermo-mechanically affected zone (TMAZ) is closer to the weldline, where the material is exposed to significant heating as well as mechanical deformation; and a fine grained region in the middle of the weld is called the nugget zone (NG).

Based on published experimental work the variation of the most essential mechanical properties, transverse to the weldline, may be estimated (Liu and Chao, 2005; Nielsen, 2008). This variation of mechanical properties is known to strongly affect the ductile damage development in the weld region. As discussed by Nielsen (2008) the yields stress profile may be shown to have a significant influence on the mode of the final failure in a uni-axial tensile specimen cut out of a welded plate, with the weld running transverse to the tensile direction. It has been shown that a shift in the failure mode occurs as the yield stress level in the TMAZ becomes comparable to that of the NG, such that a shear band-like failure in the softer TMAZ region, is replaced by a neck governing failure in the NG zone. In all cases it was found that failure occurred under relatively low stress triaxiality, due to the applied uni-axial loading.

In the case of low stress triaxiality the Gurson model is known to give a less accurate description of the void volume fraction evolution. This is due to the formation of non-spherical voids at low stress triaxiality (Tvergaard, 1988; Gologanu et al., 1997). In the extreme case of pure shear, where the stress triaxiality is zero, the Gurson model gives no failure at all. Recently, a material model has been proposed that extends the Gurson model to also describe failure in pure shear (Nahshon and Hutchinson, 2008). Here, the damage parameter f is no longer only a void volume fraction, but is a damage parameter that can also grow in pure shear, or even under negative stress triaxiality, dependent on the value of the third invariant, J_3 .

The purpose of the present paper is to analyze the effect of this shear modified Gurson model (Nahshon and Hutchinson, 2008) on the ductile damage development in a tensile specimen with a FS-weld running transverse to the tensile direction. The suggested modification of the Gurson model is formulated such that it vanishes under an axi-symmetric stress state, thus reducing to the original Gurson model. The effect of the modification on the damage development as well as on the total force vs. elongation curves is presented and compared to the original Gurson model. Furthermore, the relative contribution of the additional damage term is studied for various cases, including a FS-welded tensile test specimen in 2D and 3D, respectively.

2. Flow rules and damage model

The applied finite element model is based on a total Lagrangian formulation of the field equations (Budiansky, 1964; Hutchinson, 1973). Hence, the dynamic principle of virtual work can be written in the reference state, in terms of the Lagrangian strain η_{ij} and the work conjugate Kirchhoff stress τ^{ij} as (1), by integrating over the volume, V , and surface, S , in the reference configuration.

$$\int_V \tau^{ij} \delta \eta_{ij} dV = \int_S T^i \delta u_i dS - \int_V \rho \frac{\partial^2 u^i}{\partial t^2} \delta u_i dV \quad (1)$$

with,

$$\eta_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i} + u_i^k u_{k,j}) \quad (2)$$

The contravariant components of Kirchhoff stress tensor, τ^{ij} , are here given on the convected base vectors, while the surface tractions, T^i , and displacements, u_i , are related to the reference coordinates. Furthermore, $(\cdot)_{,i}$ here denotes covariant differentiation in the reference frame. The Lagrangian strain increment is here taken to be the sum of the elastic and plastic contributions, $\dot{\eta}_{ij} = \dot{\eta}_{ij}^E + \dot{\eta}_{ij}^P$.

To model the ductile damage development a modified version of the Gurson model (Gurson, 1977; Tvergaard and Needleman, 1984), suggested by Nahshon and Hutchinson (2008) is applied. The potential surface is here assumed to take the form (3), where σ_M is the microscopic effective stress in the matrix material surrounding the voids, while σ^{ij} are the contravariant components of the macroscopic Cauchy stresses, describing the average stress field over the material in the convected coordinate system (Tvergaard, 1990)

$$\Phi = \frac{\sigma_e^2}{\sigma_M^2} + 2q_1 f^* \cosh \left(\frac{q_2}{2} \frac{\sigma_k^k}{\sigma_M} \right) - [1 + (q_1 f^*)^2] = 0 \quad (3)$$

Here,

$$f^*(f) = \begin{cases} f & \text{for } f \leq f_c \\ f_c + \frac{f_f - f_c}{f_f - f_c} (f - f_c) & \text{for } f > f_c \end{cases} \quad (4)$$

The correction (4) introduced by Tvergaard and Needleman (1984) accounts for coalescence of microvoids at some critical value, f_c , while final fracture occurs at $f^*(f_f) = f_f^* = 1/q_1$. It is seen that (3) reduces to the original Gurson model for $q_1 = q_2 = 1$ and $f^* = f$, and further to the standard Mises surface for $f = 0$.

The potential surface (3) is based on an assumption of spherical voids in an axi-symmetric stress state (Gurson, 1977). However, it is known that non-spherical voids develop at low stress triaxiality (Tvergaard, 1988; Gologanu et al., 1997; Pardoen and Hutchinson, 2000). In such cases the Gurson model may therefore only be used to approximately describe the evolution of the void volume fraction, f .

For zero or negative stress triaxiality the Gurson model predicts no increase in damage, if void nucleation is neglected. However, a continued softening and eventually fracture at zero or negative triaxiality is known to occur (Barsoum and Falek, 2007). To compensate this Nahshon and Hutchinson (2008) have introduced a modification to the damage growth rate, $\dot{f}$, which may be written as

$$\dot{f} = (1-f)G^{ij}\dot{\eta}_{ij}^p + \mathcal{D}\dot{\epsilon}_M^p + k_\omega f \omega(\sigma) \frac{s^{ij}\dot{\eta}_{ij}^p}{\sigma_e} \quad (5)$$

Here the first term, representing growth of existing voids, follows from plastic incompressibility, the second term describes nucleation of new voids, while the last term, introduced by Nahshon and Hutchinson (2008), is formulated to be consistent with the mechanism of void softening in shear. The modification introduced by Nahshon and Hutchinson (2008) is, however, purely phenomenological, hence f may no longer be considered as a measure of the void volume fraction, but instead either as an effective void volume fraction or simply as a damage parameter. The nucleation of voids in (5) was not included by Nahshon and Hutchinson (2008), but is needed to represent real materials that have been welded (Nielsen, 2008).

The parameter k_ω in (5), is defined by Nahshon and Hutchinson (2008) as the magnitude of the damage growth rate in pure shear, while $\omega(\sigma)$ is assumed to be given as

$$\omega(\sigma) = 1 - \left(\frac{27J_3}{2\sigma_e^3} \right)^2, \quad J_3 = \frac{1}{3} G^{ij} s_{kj} s_{il} s^{lk} \quad (6)$$

Where σ_e is the macroscopic effective Mises stress, G^{ij} are the contravariant components of the metric tensor for the convected frame and s_{ij} are the covariant components of the Cauchy stress deviator on the deformed base vectors. Nucleation of new voids is taken to be governed by a normal distribution as suggested by Chu and Needleman (1980), so that the coefficient $\mathcal{D}$ in (5) takes the form

$$\mathcal{D} = \frac{f_N}{s_N \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\epsilon_M^p - \epsilon_N}{s_N} \right)^2 \right] \quad (7)$$

Here, f_N is the volume fraction of second phase particles, from which new voids can nucleate, s_N is the standard deviation and ϵ_N is the mean nucleation strain.

To couple the microscopic and macroscopic values it is assumed that the rates of plastic work on each level are equal

$$\sigma^{ij}\dot{\eta}_{ij}^p = (1-f)\sigma_M\dot{\epsilon}_M^p \quad (8)$$

where the macroscopic plastic strain rate $\dot{\eta}_{ij}^p$ can be derived from the current potential surface as

$$\dot{\eta}_{ij}^p = A \frac{\partial \Phi}{\partial \sigma^{ij}}, \quad A = (1-f)\sigma_M\dot{\epsilon}_M^p \left[\sigma^{ij} \frac{\partial \Phi}{\partial \sigma^{ij}} \right]^{-1} \quad (9)$$

The effective microscopic plastic strain rate is here assumed to be governed by a potential law, given as (10). Thus, the material is represented as elastic-viscoplastic

$$\dot{\epsilon}_M^p = \dot{\epsilon}_0 \left[\frac{\sigma_M}{g(\epsilon_M^p)} \right]^{1/m}, \quad g(\epsilon_M^p) = \sigma_y \left(1 + \frac{E\epsilon_M^p}{\sigma_y} \right)^N \quad (10)$$

Here, m and $\dot{\epsilon}_0$ are the strain rate hardening exponent and the reference strain rate, respectively. Furthermore, the strain hardening function $g(\epsilon_M^p)$ is taken to be given as a power law, where N and σ_y are the strain hardening exponent and initial yield stress, respectively.

By formulating $\omega(\sigma)$ as (6), the modification vanishes at an axi-symmetric stress state, so that the model coincides with the Gurson model. Furthermore, it may be shown that $\omega(\sigma)$ lies in the interval $\omega(\sigma) \in [0, 1]$, with $\omega(\sigma) = 0$ for an axi-symmetric stress state and $\omega(\sigma) = 1$ for all states combined by pure shear and hydrostatic pressure. However, this modification of the Gurson model will not only contribute to damage development in shear, as is easily seen by considering a bi-axial stress state. At plane strain uni-axial tension one finds that $J_3 = 0$ for the stationary solution in the plastic domain, hence $\omega(\sigma) = 1$. The softening term added to the damage growth rate in (5) is thereby non-zero if $f > 0$, as is illustrated in Fig. 1. It is seen that the suggested modification has a significant effect on the damage development, even though no shear is present. This plane strain tensile test illustrates that the additional damage development due to the last term in (5) cannot be interpreted as solely due to shear. The curves in Figs. 1 and 2 is given by

$$f_{\text{Total}} = f_{\text{Growth}} + f_{\text{Nucleation}} + f_{\text{Modification}} \quad (11)$$

Here,

$$f_{\text{Growth}} = \int (1-f) G^{ij} \dot{\eta}_{ij}^p dt \quad (12)$$

$$f_{\text{Nucleation}} = \int \mathcal{D} \dot{\epsilon}_M^p dt \quad (13)$$

$$f_{\text{Modification}} = \int k_\omega f \omega(\sigma) \frac{s^{ij} \dot{\eta}_{ij}^p}{\sigma_e} dt \quad (14)$$

As seen in Fig. 2 the modification to the damage growth rate in (5) makes it possible to model damage development in a shear dominated condition. For simple shear in plane strain condition Fig. 2 shows that only nucleation of voids takes place in the Gurson model ($k_\omega = 0$), while no growth of microvoids is obtained. However, for $k_\omega > 0$ void nucleation is followed by an increase in damage due to the modification, which eventually causes final failure.

It is noted that the damage tensor $\Omega(\sigma)$, sometimes applied in continuum damage mechanics (Lemaitre and Chaboche, 1990) is not related to the coefficient $\omega(\sigma)$ in (5). In the present model based on (5) the only damage parameter is f .

3. Method of analysis

The dynamic form of the principle of virtual work (1) is discretized by either 8 or 20 node isoparametric planar 2D elements or solid 3D elements in the FEM code developed. This program is similar to the programs developed by Tvergaard and Needleman (2004), and the correctness of the program has been carefully tested against previous results. The integrals are evaluated by reduced integration. Furthermore, a lumped mass matrix is introduced to decouple the system of equations, which are solved by a standard explicit Newmark β -method. The dynamic analysis accounting for inertia has the advantage over a standard quasi static analysis that no stiffness matrix is needed, and the computation time in each increment is thereby significantly reduced. Under slow loading this dynamic method gives a very good approximation to the static solution. However, by use of the explicit dynamic method the maximum increment size is limited by a time increment fulfilling the Courant condition, $\Delta t \leq \Delta t_c$. As a consequence of the strong non-linearity in the elastic-viscoplastic material model (10), smaller time increments Δt are normally required. However, as shown by Peirce et al. (1984) the size of the critical time increment may be increased by introducing a forward gradient method for estimating the microscopic plastic strain rate in the following time increment (Peirce et al., 1984; Tvergaard, 1984). With this procedure incorporated the time increments applied are typically $\Delta t \approx 0.1 \Delta t_c$. The possibility of artificially increasing the time step size by introducing a non-physical

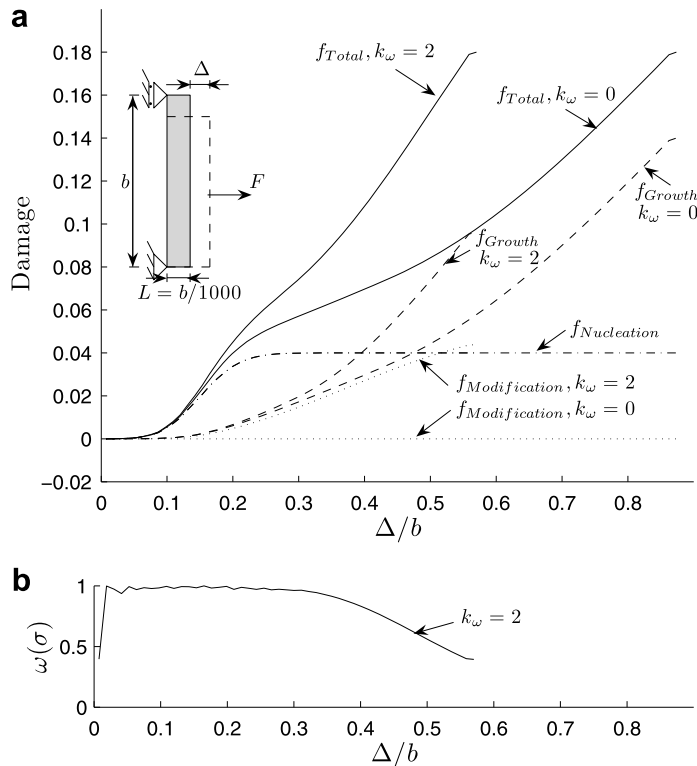


Fig. 1. Damage development vs. average strain in uni-axial plane strain tension. Distinction of the contributions to the total damage development, ($\epsilon_N = 0.15$, $s_N = 0.3\epsilon_N$, $f_c = 0.075$, $f_f = 0.2$, $q_1 = 2$ and $q_2 = 1$).

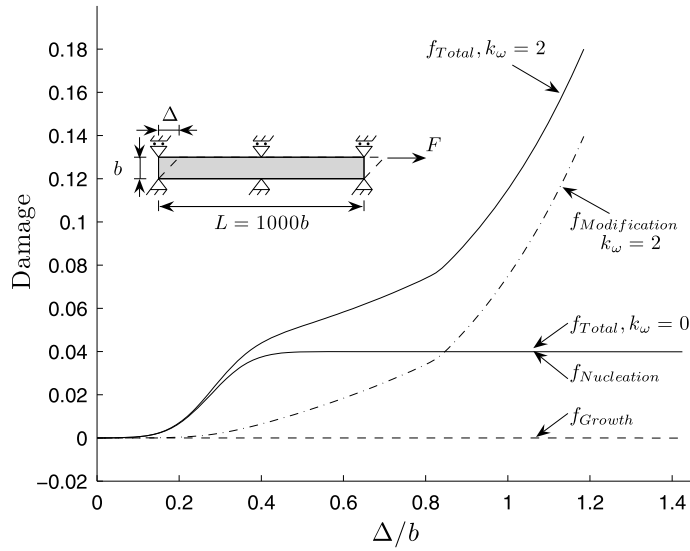


Fig. 2. Damage development vs. average strain at simple shear in plane strain condition. Distinction of the contributions to the total damage development, ($\varepsilon_N = 0.15$, $s_N = 0.3\varepsilon_N$, $f_c = 0.075$, $f_f = 0.2$, $q_1 = 2$ and $q_2 = 1$).

high specific mass, is not applied here. Thus, a small time step has been used for both the 2D and 3D calculations, such that a high number of time increments was needed.

As the potential surface shrinks due to an increase in the damage parameter, f , one may experience numerical difficulties when the stress carrying capacity of an element is reduced for $f \rightarrow f_f$. To deal with this the element vanishing technique is applied (Tvergaard, 1982). Damage development in a given Gauss point reaching $f = 0.9f_f$ is here turned off, while a given element is killed when 2 or 3 Gauss points in 2D or 3D, respectively, have been turned off (Tvergaard and Needleman, 2004). After killing the element the remaining forces from the element on neighboring elements are stepped down in the following 50 increments.

Both 2D and 3D tensile test specimens are modeled in order to study the effect of the modification (5) in the Gurson model. For the 2D case a plane strain condition is assumed, where the modification is known to have a significant influence on the damage development. However, as discussed in Section 2, the additional damage development cannot be interpreted as solely due to softening in shear. For a 3D specimen under uni-axial tension where $\omega(\sigma) = 0$ the modification in (5) would only contribute to the damage development if the stress state starts to deviate from uni-axial tension. It is analyzed how the variation of the specimen width w_0 affects such deviations from the uni-axial tension in the deformed region as localization occurs.

In all cases the tensile test specimens are assumed to be cut out transverse to the weldline, hence the FS-welded joint will be running transverse to the tensile direction. The emphasis of the following study is focused on the geometry of a FS-welded joint of 25 mm thick aluminum plates, described by Fonda and Bingert (2004). A similar weld was analyzed by Nielsen (2008), who estimated the weld dimensions and material parameters to be used.

To model the damage development in the tensile test specimen, a uniform mesh in the weld region is introduced, Fig. 3. By utilizing the symmetry of the problem and prescribing a uniform displacement in the x^1 -direction, the boundary conditions take the form

$$T^2 = 0, \quad T^3 = 0, \quad u^1 = \dot{U}\Delta t \quad \text{at} \quad x^1 = L_0 \quad (15)$$

$$T^2 = 0, \quad T^3 = 0, \quad u^1 = 0 \quad \text{at} \quad x^1 = 0 \quad (16)$$

$$T^1 = 0, \quad T^2 = 0, \quad u^3 = 0 \quad \text{at} \quad x^3 = 0 \quad (17)$$

$$T^i = 0, \quad \text{on remaining surfaces} \quad (18)$$

Furthermore, $u^3 \equiv 0$ for the 2D plane strain case, and here the smaller calculation time allows for a finer discretization. The same type of mesh, as shown in Fig. 3, is used for both 2D and 3D calculations.

Loading of the specimens is in all cases introduced as a uniform prescribed displacement in the x^1 -direction at $x^1 = L_0$. To avoid effects of the material inertia, due to a sudden prescribed displacement, a ramping of the velocity, $\dot{U}$, is applied (Needleman and Tvergaard, 1999)

$$\dot{U}(t) = \dot{U}_0 t / t_r \quad \text{for} \quad t \leq t_r \quad (19)$$

$$\dot{U}(t) = \dot{U}_0 \quad \text{for} \quad t > t_r \quad (20)$$

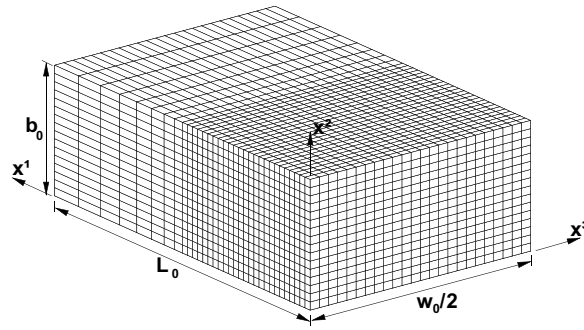


Fig. 3. 3D tensile test specimen with a typical mesh for $L_0/b_0 = 3$, $w_0/b_0 = 4$ and $L_0 = 75$ mm.

where t_r is the ramping time and $\dot{U}_0$ is the prescribed velocity. To obtain reasonable CPU times for the explicit analyses a prescribed velocity of $\dot{U}_0 = 1$ m/s has been used for all calculations. Comparison with solutions for lower velocity have indicated that inertia effects were limited.

4. Weld assumptions

As discussed by Nielsen (2008) the difference between the advancing (AS) and retreating (RS) sides of the weld is neglected. Thus, a symmetric hardness and initial yield stress profile transverse to the weldline is modeled as shown in Fig. 4. The geometry and reductions in the yield stress are estimated by Nielsen (2008) from experimentally measured hardness profiles presented by Junhui et al. (2006) and tensile test results of Genevois et al. (2004) and Liu and Chao (2005). The assumed yield stress profile is characteristic for FS-welded aluminum joints and may be found for various types of alloys.

The estimated yield stress profile is specified as the variation in the middle of the plate, while a linear variation between the crown^(C) and the root^(R) of the weld is assumed (Feng et al., 2007). According to Fig. 4 the dimension of the yield stress

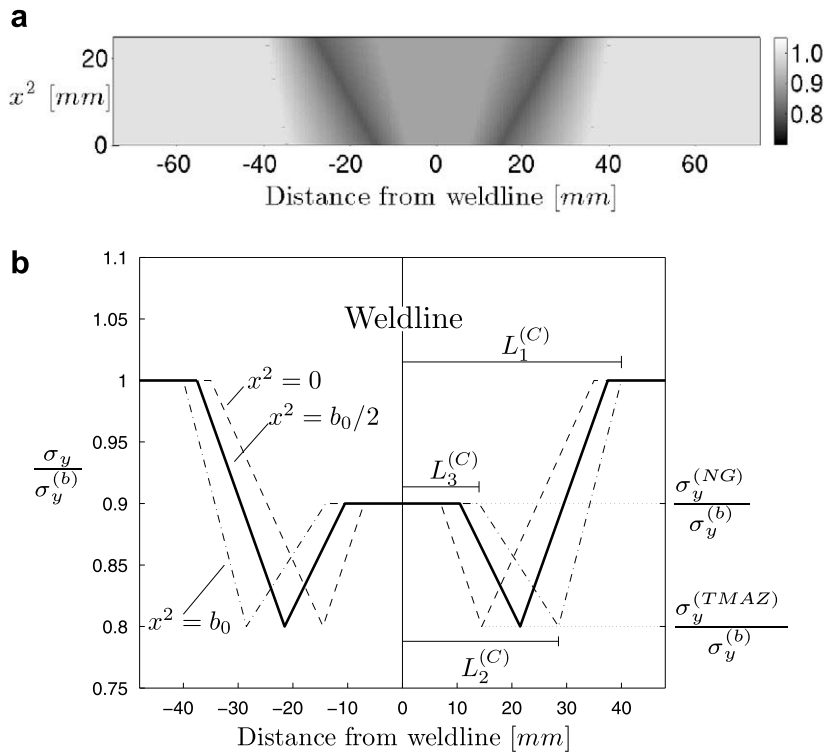


Fig. 4. Characteristic yield stress variation transverse to the weldline for FS-welded aluminum alloys. (a) Yield stress in weld cross-section, (b) modeled yields stress profile at the crown, root and middle of the weld cross-section, Fig. (a).

profile are $[L_1, L_2, L_3]^{(C)} = [40.0, 28.5, 14.0]$ mm and $[L_1, L_2, L_3]^{(R)} = [35.0, 14.5, 7.0]$ mm for the crown and root of the weld, respectively. Since both the process parameters and aging of the alloy is known to influence on the yield stress profile, two different cases are considered. First a yield stress profile with $\sigma_y^{(NG)}/\sigma_y^{(b)} = 1$ is studied, and subsequently a profile with $\sigma_y^{(NG)}/\sigma_y^{(b)} = 0.8$. Here, $\sigma_y^{(b)}$ denotes the yield stress of the base material. In both cases the yield stress in the TMAZ is assumed fixed at $\sigma_y^{(TMAZ)}/\sigma_y^{(b)} = 0.8$.

A change of the strain hardening exponent, N , has been observed experimentally in different regions of the welded joint for the aluminum alloy AA2024 (Lockwood et al., 2002; Genevois et al., 2004; Liu and Chao, 2005). To capture these experimental observations a yield stress dependent empirical correlation for the strain hardening, was introduced by Nielsen (2008) as specified by

$$N = \left[12.34 \frac{\sigma_y}{\sigma_y^{(b)}} \right]^{-7/10} \quad (21)$$

This expression for $N(\sigma_y)$ is also applied in the following study.

The motion of the welding tool gives very large plastic deformations as the materials on both sides of the weldline are forced to rotate and mix around the tool. This may introduce plastic an-isotropy with varying principal directions across the weld region. However, a systematic experimental determination of such an-isotropy has not been found. The material model used to describe plasticity and damage evolution assumes isotropic material behavior.

The remaining mechanical properties for aluminum AA2024 are here taken to be $E = 72$ GPa, $\nu = 0.33$ and $m = 0.01$, which are assumed constant everywhere in the specimen, inside and outside the weld. The yield stress of the base material is $\sigma_y^{(b)} = 380$ MPa. The damage parameters of the Gurson model for the current aluminum alloy, estimated by Nielsen (2008), are taken to be constant transverse to the weldline. This is to minimize the number of parameters to be varied. The applied damage parameters are then $\varepsilon_N = 0.15$, $s_N = 0.3\varepsilon_N$, $f_c = 0.075$, $f_f = 0.2$, $q_1 = 2$ and $q_2 = 1$. Furthermore, a constant level of second phase particles, $f_N = 0.04$, which are taken to initiate nucleation of new voids, is assumed. These damage parameters are chosen to represent the inclusion distributions typically observed experimentally (Yang et al., 2004; Barcellona et al., 2006) in FS-welded aluminum alloys. Also more serious initial damage such as channel defects has been observed in FS-welds (Chen et al., 2006), but such initial damage will not be present for appropriately chosen process parameters and welding tool.

For the present analyses all residual stresses are neglected and the tensile test specimen is considered stress free at $t = 0$. However, through the thermal effects and significant plastic deformation of the material due to the welding process, relatively large residual stresses are known to occur in the as-welded plate. The largest residual stresses are shown both experimentally and numerically by Feng et al. (2007) to occur along the weldline. Thus, in the analysis of a full welded structure the residual stress field will clearly affect the onset of plastic yielding and the subsequent stress and strain fields. However, for a 3D specimen a significant drop in residual stresses is shown to occur by Feng et al. (2007), since the stress component along the weldline is relaxed when the specimen is cut out from the welded plate. The approximation of neglecting residual stresses is therefore more reasonable for the thin 3D specimen than for the 2D plane strain case, or the thicker 3D specimen. It is planned to consider the effect of residual stresses more closely in future work.

5. Results

The response of the tensile test specimens, under plane strain conditions, are shown in Fig. 5 for both yield stress profiles and for four k_ω values. It is seen that k_ω has a large influence on the total damage development for both types of yield stress profiles. This is observed as a significant decrease in the fracture strain, as the value of k_ω is increased. This agrees with the interpretation of k_ω as the amplitude of the softening term added to the damage rate (5), as discussed by Nahshon and Hutchinson (2008).

The influence of k_ω on the damage development for the plane strain condition is partly due to the large values of $\omega(\sigma)$. As discussed in Section 2, one may show that $\omega(\sigma) = 1$ for the stationary plastic stress state under uni-axial plane strain conditions. For the present 2D calculations only a small deviation from $\omega(\sigma) = 1$ is observed in the specimen, even though a region of significant plastic localization occurs. Thus, $\omega(\sigma)$ can be considered as constant ($\omega(\sigma) \approx 1$) throughout the deformation so that the variation of $\omega(\sigma)$ has only a minor effect on the 2D results presented in the following.

Considerable damage due to the applied modification initiates relatively early in the deformation, since a change in the overall stress state should not be obtained for $\omega(\sigma) \neq 0$ to occur. This may also be seen from Fig. 6, where the contributions to the damage development from nucleation, growth and the modification, respectively, are traced for the region of the specimen obtaining the largest amount of damage. The results are here shown for the case of a shear band-like failure ($\sigma_y^{(NG)}/\sigma_y^{(b)} = 1$), where localization occurs in the TMAZ, Fig. 8. This corresponds to the Gauss point nearest $[x^1, x^2] = [(L^{(R)} + L^{(C)})/2, b_0/2]$, in the middle of the TMAZ. The different contributions to the total damage parameter, f , are shown for $k_\omega = 0$ and $k_\omega = 3$, corresponding to the Gurson model and the maximum value of k_ω , suggested by Nahshon and Hutchinson (2008), respectively. The damage development is shown for values up to $f = 0.05$. Beyond that a rapid increase in the total damage is observed, especially as f_c is exceeded, since void coalescence sets in. This may also be seen from the tendency of Fig. 6. The damage development for both values of k_ω in Fig. 6 is seen to initiate by nucleation of voids, after which growth sets in. For the case of $k_\omega = 3$ additional damage due to the modification is observed to slowly develop as

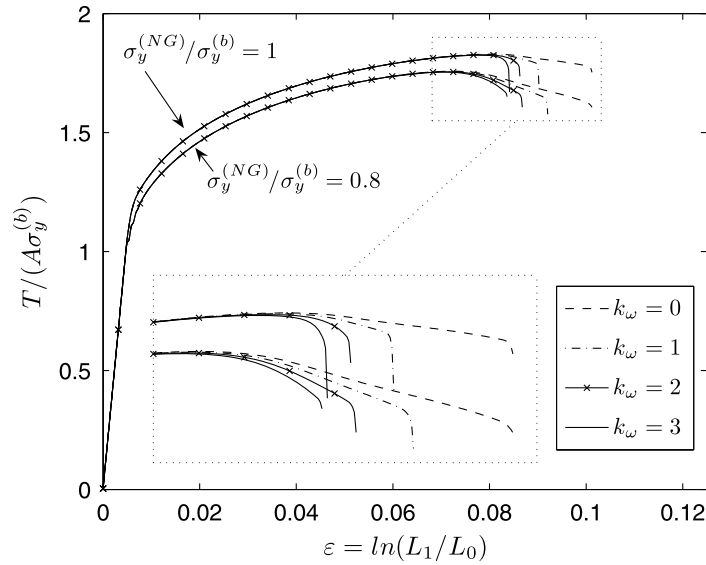


Fig. 5. Load vs. average axial strain curves for plane strain uni-axial tension. ($\sigma_y^{(NG)}/\sigma_y^{(b)} = 0.8$ or $\sigma_y^{(NG)}/\sigma_y^{(b)} = 1$).

the specimen is stretched. The relatively early initiation of the additional damage due to $\omega(\sigma) \neq 0$ is seen to result in a noticeable contribution to the total damage development, of the order of $f_{\text{Modification}} \approx 0.1f_c$. This additional damage of the specimen leads to earlier failure, Fig. 5. From Fig. 6 it is furthermore seen that for $k_\omega \neq 0$ the applied modification gives so much earlier failure that the growth contribution is less dominant.

When material softening occurs, somewhere after the load maxima in Fig. 5, it is well known that numerical predictions are mesh sensitive, as bands of localized plastic deformation will become as narrow as possible with the mesh applied. With a material length included in a non-local version of the Gurson model Tvergaard and Needleman (1995) have studied shear band localization. It was found that there is little mesh sensitivity prior to localization but after localization convergence depends on the material length. In the present study, as in many applications of damage mechanics, the element size represents the characteristic length of the analysis.

To illustrate the development of the additional damage contribution, compared to the development of the total damage parameter, f , contour plots of advanced states of the deformation are shown in Figs. 7 and 8. The contour plots are shown for both yield stress profiles, with $k_\omega = 3$. It is seen that in both cases a noticeable contribution from the additional softening term occurs in the regions of plastic localization, in the range of $f_{\text{Modification}} \approx 0.1f$. The reason for the nicely localized development of the additional softening term in the region of plastic localization is however partly due to the assumed strain controlled nucleation law. Since the applied modification aims to describe void softening and deformation, a dependence on

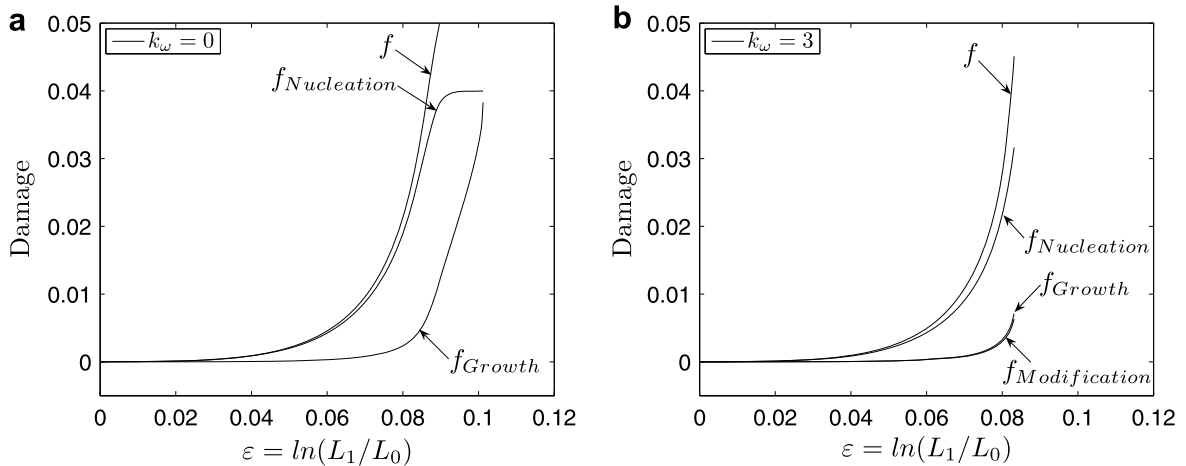


Fig. 6. Distinction of the contributions to the total damage development in Gauss points nearest $[x^1, x^2] = [(L_2^{(R)} + L_2^{(C)})/2, b_0/2]$ for $\sigma_y^{(NG)}/\sigma_y^{(b)} = 1$ in plane strain tension with $k_\omega = 0$ and $k_\omega = 3$, respectively.

already existing voids/damage is incorporated by Nahshon and Hutchinson (2008). Thus, for the case of no initial damage a dependence on the nucleation is present. Consequently, the regions with considerable contributions from the additional softening term correspond to the regions of significant nucleation. It is furthermore seen that for the two cases considered similar levels of additional damage are observed for both yield stress profiles, even though different failure modes occur.

As discussed, $\omega(\sigma)$ only has a minor influence on how the additional damage develops in the uni-axial plane strain state, since $\omega(\sigma) \approx 1$ throughout the specimen. This is not the case for a uni-axial tensile test specimen in 3D. For 3D uni-axial tension it may be shown that $J_3 = 2\sigma_e^3/27$, hence $\omega(\sigma) = 0$, both in the elastic and plastic domains. Therefore, additional damage development due to the applied modification requires a change from the overall uni-axial stress state, since $\dot{f}_{\text{Modification}} \neq 0$ requires that $\omega(\sigma) \neq 0$. Such a change may be found in the case of pronounced plastic localization, e.g., for the shear band-like failure, $\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 1$. As plastic localization occurs in a shear band-like region, a stress component along the weldline builds up, so that a change in both $\omega(\sigma)$ and in the stress triaxiality is observed. This change in the stress state and in $\omega(\sigma)$ may be enhanced by increasing the specimen width, since the constraint on the plastic flow is thereby enhanced in the band of localization. Thus, the additional damage development due to the applied modification is expected to increase for increasing specimen width.

The effect of the applied modification on the damage development in a 3D uni-axial specimen is illustrated in Figs. 9 and 12. The tensile response is shown for two different widths of the tensile specimen and for both yield stress profiles. Furthermore, the results for both 2D and 3D specimens are shown for the two yield stress profiles and for two k_ω values. It is seen, from Fig. 9, that k_ω only has a limited effect on the tensile response of the 3D specimen for the case where a necking type failure occurs in the NG, $\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 0.8$. However, an increasing influence of the modification is seen for increasing specimen width in both Figs. 9 and 12. This is due to the change in the overall stress state as localized deformation in the neck in the NG develops, since this leads to changes in $\omega(\sigma)$, Fig. 10. However, for the current yield stress profile only a limited changes in $\omega(\sigma)$ is observed in the neck region, while more significant changes are seen on the top surface and on the symmetry planes some distance from the neck, Fig. 10. The high level of $\omega(\sigma)$ in these areas do however not contribute to additional damage development, since only limited void nucleation takes place here. A larger influence of the modification is obtained in the case of a shear band-like failure in the TMAZ, Fig. 12. This is due to the large values of $\omega(\sigma)$ obtained in the regions of plastic localization, Fig. 13.

By comparing the tensile curves for the 2D and 3D specimens in Figs. 9 and 12, it is seen that for both yield stress profiles the largest effect of k_ω is found in the 2D plane strain case. This is however due to the significant difference in $\omega(\sigma)$ as discussed above. A higher strain at final failure is found for the 3D specimens than for the 2D specimens, mainly due to the lower stress triaxiality level in the 3D case. Thus, also at $k_\omega = 0$ damage development in the 2D case evolves more rapidly than in the 3D case due to the dependence of the stress triaxiality in the Gurson model.

Both in Figs. 10 and 11 and in Figs. 13 and 14 it is seen that the change in $\omega(\sigma)$ is directly reflected in the additional damage development in the weld region. Combined with the nucleation of voids in the regions of localized plastic deformation, a local increase in $\omega(\sigma)$ is seen to cause a considerable increase in the additional damage development. The largest contribution from the applied modification is obtained in the case of a wide tensile specimen, Figs. 11d and 14d. Especially in the case of a shear band-like failure were a high value of $\omega(\sigma)$ is seen to build up in the TMAZ, Fig. 13b. Thereby additional damage development occurs in the shear band-like region.

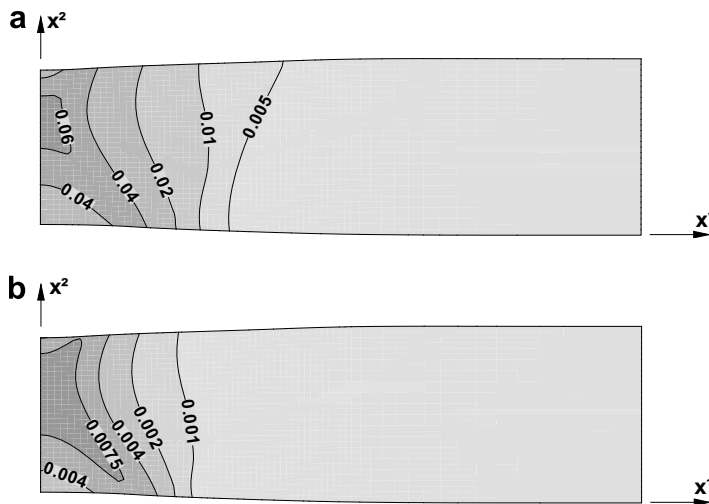


Fig. 7. Curves of constant (a) total damage f , (b) damage contribution from the modification term $f_{\text{Modification}}$, at plane strain tension. ($\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 0.8$, $k_\omega = 3$, $\varepsilon = 0.0831$).

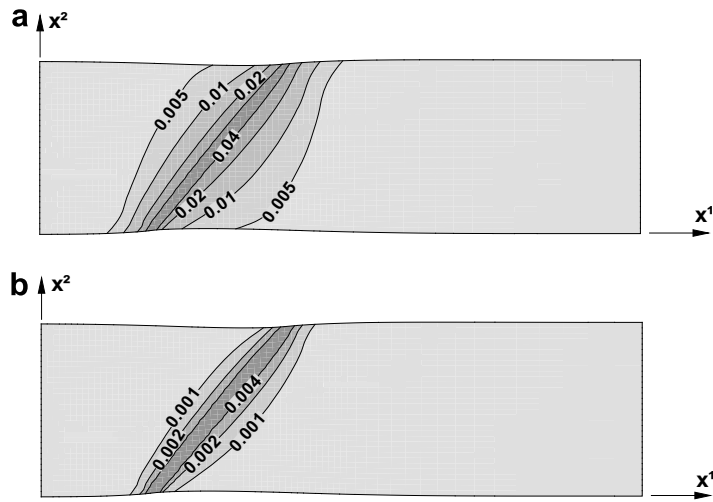


Fig. 8. Curves of constant (a) damage parameter f , (b) damage contribution from the modification term $f_{\text{Modification}}$, at plane strain tension. ($\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 1, k_\omega = 3, \varepsilon = 0.0831$).

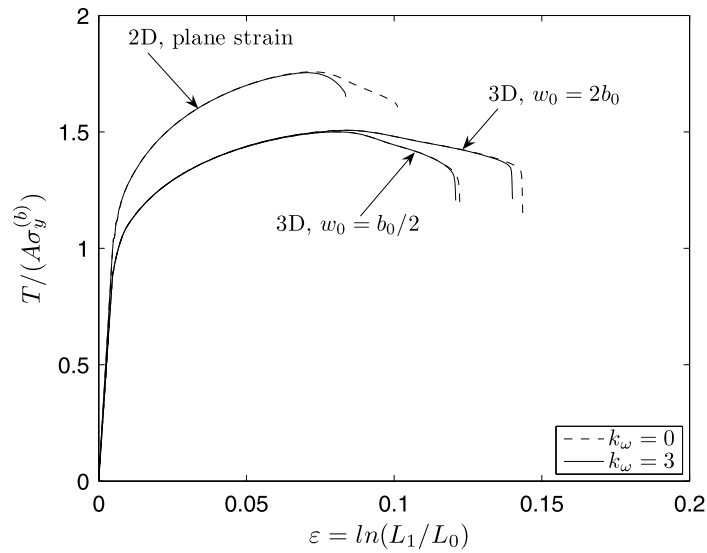


Fig. 9. Load vs. average axial strain curves. ($\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 0.8$).

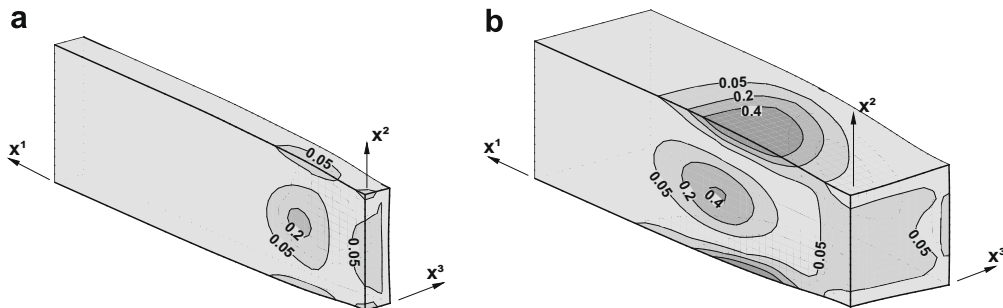


Fig. 10. Contours of $\omega(\sigma)$ in 3D test specimens, (a) $w_0 = b_0/2$ at $\varepsilon = 0.1173$, (b) $w_0 = 2b_0$ at $\varepsilon = 0.1358$. ($\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 0.8$ and $k_\omega = 3$).

5.1. Increasing specimen width

The effect of the constraint on localized plastic deformation in the 3D uni-axial stress state is expected to increase when the specimen width is increased, which will also increase the change in $\omega(\sigma)$. Therefore larger widths, up to $w_0/b_0 = 6$, have been analyzed. It was here found that, for the yield stress profile with $\sigma_y^{(NG)}/\sigma_y^{(b)} = 1$ and $\sigma_y^{(TMAZ)}/\sigma_y^{(b)} = 0.8$, a change in the damage development occurred. As shown in Fig. 15a damage development initiates in the TMAZ and is localized in a band inclined to the tensile direction. However, as the specimen is stretched damage also develops in a region of the NG, which eventually merges into the damaged zone in the TMAZ. Thereby, significant localization of plastic flow occurs in a band forming from the middle of the NG to the outer surface in the TMAZ, across different regions of the weld. This type of failure mode has not been observed in published experimental work for the range of tensile specimen dimensions studied here. For a tensile specimen with a FS-weld running transverse to the tensile direction, failure normally occurs in a single zone of the weld, corresponding to a critical region of the microstructure (neglecting welding defects). Typically this region is experimentally found to coincide with the region of minimum yield stress. However, localization in a band inclined to the tensile direction is known from uni-axial tensile tests in a thin strip of homogeneous material. As discussed by Tvergaard (1993) the Considère neck that occurs first in a tensile specimen with rectangular cross-section may subsequently localize in a narrow groove inclined to the tensile direction, depending on the width to height ratio, w_0/b_0 . Thus, it is the combined effect of the mechanical properties of the weld and the increased aspect ratio w_0/b_0 of the cross-section that leads to the localization seen in Fig. 15.

As discussed by Nielsen (2008) localization of the damage development in the TMAZ is significantly influenced by changes in both the yields stress profile and the distribution of second phase particles transverse to the weldline. To promote localized damage inside the weld a larger drop of the yield stress in the TMAZ is introduced in the following, $\sigma_y^{(TMAZ)}/\sigma_y^{(b)} = 0.5$. This larger drop of the yield stress in the TMAZ acts as a stronger imperfection in the tensile test specimen. The effect of changing the yield stress profiles is shown in Fig. 16, for two width of tensile specimens. The results are only shown for $k_w = 3$, but as in the previous study an increased localization were obtained for increasing k_w . By comparing Fig. 16a with Fig. 15 it is seen that a stronger localization of damage occurs in the region of low yield stress, while the NG of the weld is less damaged. Thus, localization of plastic flow and damage development mainly take place in a shear band-like mode in the TMAZ. However, still relatively large damage development is observed in the NG, which is partly due to the assumed uniform distribution of second phase particles.

To analyze the effect of the additional softening term both the contours of $\omega(\sigma)$ and the damage contribution from $f_{\text{Modification}}$ are shown in Fig. 17. It is seen that the additional damage development due to the applied modification does occur in the shear band-like region in the TMAZ. Thereby, the modification contributes to the overall damage development in the expected regions. Furthermore, a large portion of the TMAZ region experiences $\omega(\sigma) > 0.4$ near the center of the specimen,

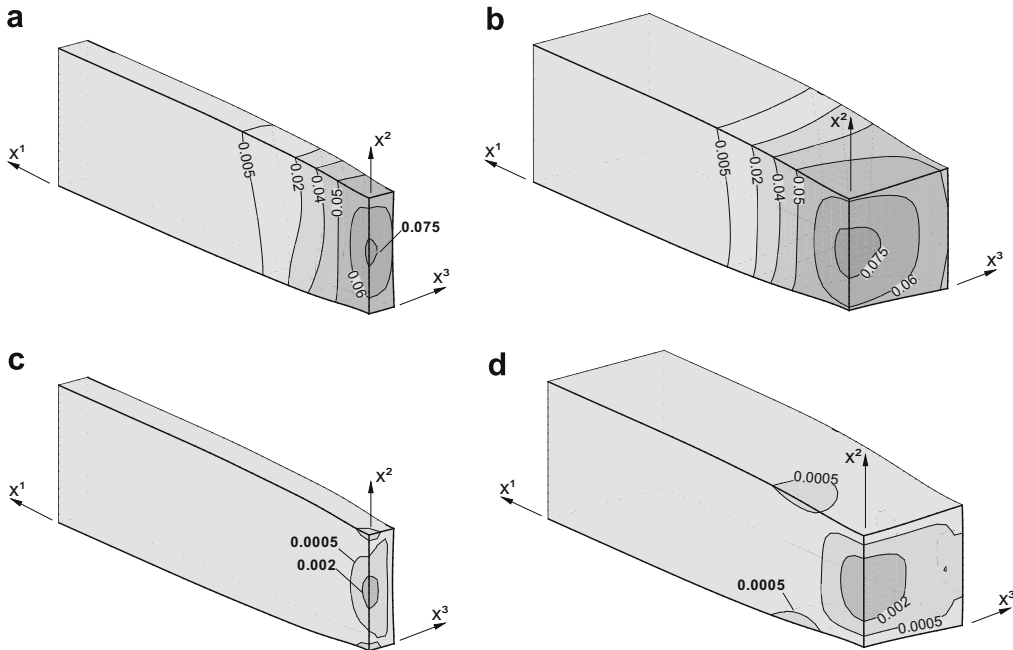


Fig. 11. Damage development in 3D test specimens, (a) contours of f for $w_0 = b_0/2$ at $\varepsilon = 0.1173$, (b) contours of f for $w_0 = 2b_0$ at $\varepsilon = 0.1358$, (c) contours of $f_{\text{Modification}}$ for $w_0 = b_0/2$ at $\varepsilon = 0.1173$ and (d) contours of $f_{\text{Modification}}$ for $w_0 = 2b_0$ at $\varepsilon = 0.1358$. ($\sigma_y^{(NG)}/\sigma_y^{(b)} = 0.8$, and $k_w = 3$).

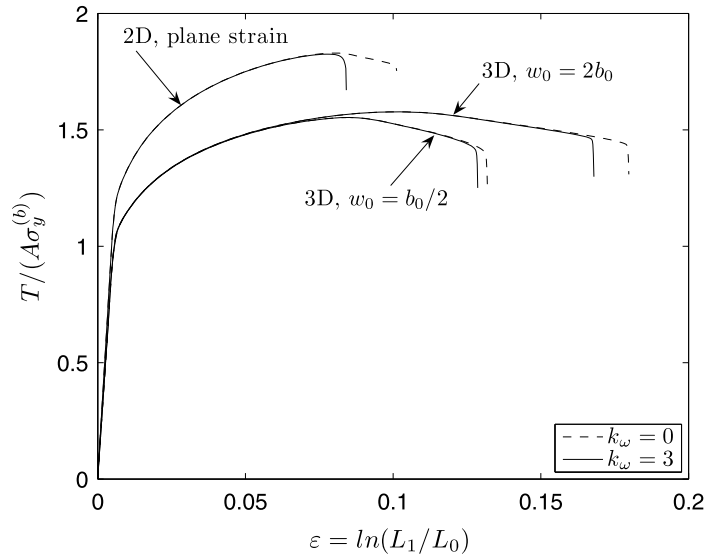


Fig. 12. Load vs. average axial strain curves. $(\sigma_y^{(NG)}/\sigma_y^{(b)} = 1)$.

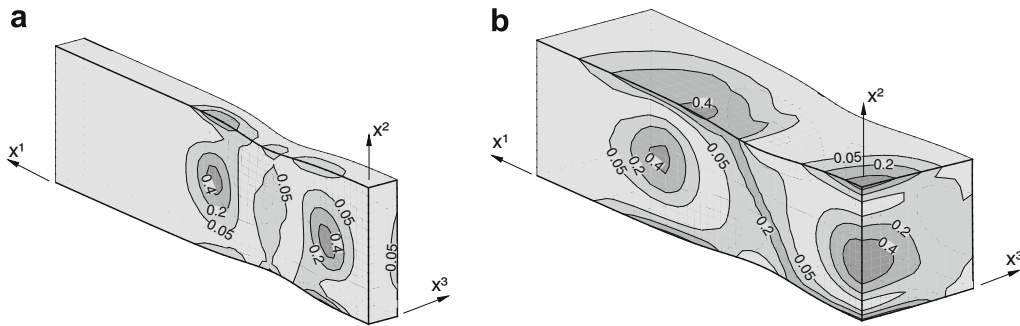


Fig. 13. Contours of $\omega(\sigma)$ in 3D test specimens, (a) $w_0 = b_0/2$ at $\varepsilon = 0.1253$, (b) $w_0 = 2b_0$ at $\varepsilon = 0.1648$. $(\sigma_y^{(NG)}/\sigma_y^{(b)} = 1$ and $k_\omega = 3$).

which indicates that the stress state has grown very different from uni-axial tension. Thus, the additional softening term does play a stronger role when the specimen width is increased.

6. Discussion

Application of the Gurson model to predict failure in friction stir welded AA2024 joints has been investigated by Nielsen (2008), making use of a number of published experimental results for this particular material joint. In particular the variation of the initial yield stress and of the rate of hardening across the weld were found to play an important role. Also the damage parameters, controlling the nucleation and growth of micro-voids in the Gurson model, were chosen based on the experimental results. The analyses of Nielsen (2008) were carried out for tensile test specimens cut out transverse to the weldline, using either planar analyses by assuming plane strain conditions or full 3D analyses. It was found that this material model gives a reasonable description of the observed ductile fracture of the test specimens, but failure occurred in a range of rather low stress triaxiality where the Gurson model does not perform optimally.

Recently, Nahshon and Hutchinson (2008) have proposed a modification of the Gurson model to be able to also describe fracture in shear. It is well known that the Gurson model does not predict any void growth when the mean stress is zero, as in pure shear, and that void nucleation does not change this, since nucleated voids do not grow any further. Therefore, Nahshon and Hutchinson (2008) suggested adding an extra term to the expression for the void growth rate. This extra term allows for the prediction of fracture in pure shear, but means that the parameter f in the model is not only a void volume fraction but more generally a damage parameter.

Even though the tensile tests for the friction stir welded plates do not have zero mean stress, the stress triaxialities are sufficiently low so that it is natural to apply the model of Nahshon and Hutchinson (2008), which is modified to predict frac-

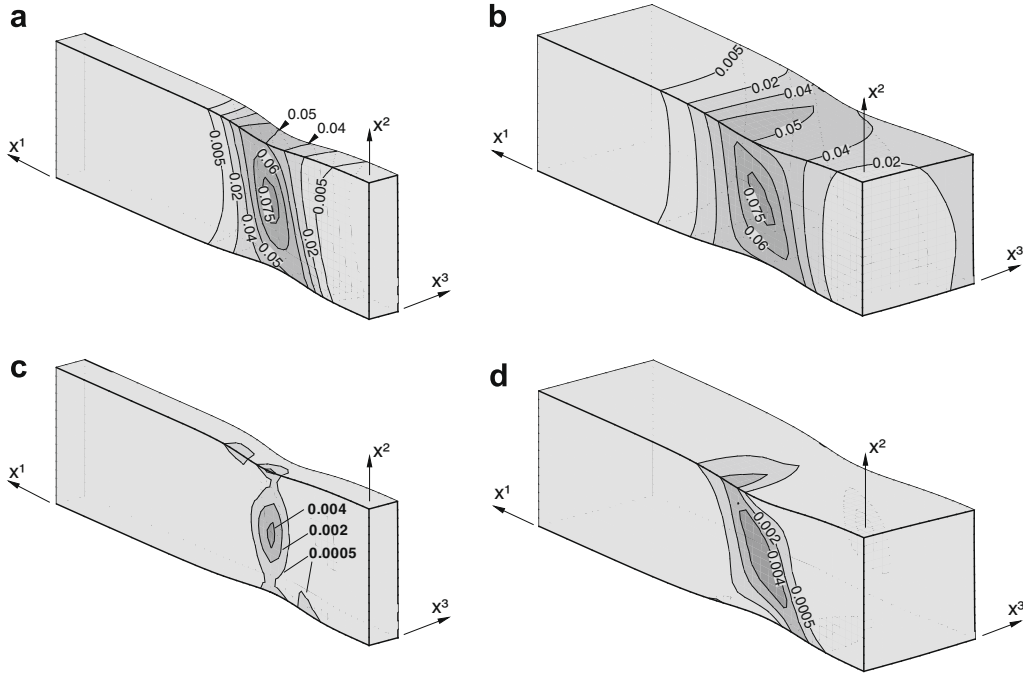


Fig. 14. Damage development in 3D test specimens, (a) contours of f for $w_0 = b_0/2$ at $\varepsilon = 0.1253$, (b) contours of f for $w_0 = 2b_0$ at $\varepsilon = 0.1648$, (c) contours of $f_{\text{Modification}}$ for $w_0 = b_0/2$ at $\varepsilon = 0.1253$ and (d) Contours of $f_{\text{Modification}}$ for $w_0 = 2b_0$ at $\varepsilon = 0.1648$. ($\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 1$ and $k_{\omega} = 3$).

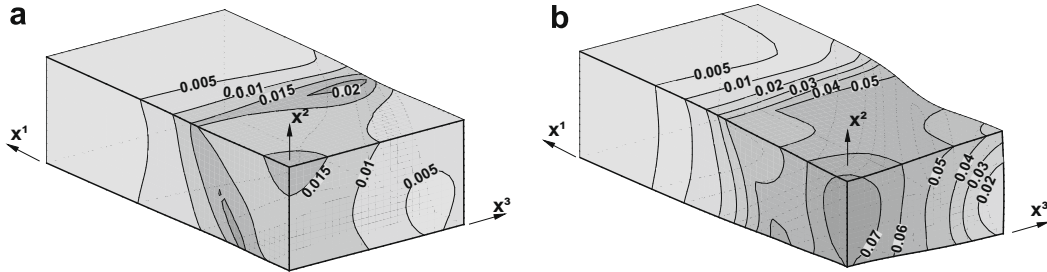


Fig. 15. Damage development, f , in a wide test specimen, $w_0 = 4b_0$, for yield stress profile $\sigma_y^{(\text{TMAZ})}/\sigma_y^{(b)} = 0.8$ and $\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 1$, (a) $\varepsilon = 0.1648$, (b) $\varepsilon = 0.2139$ ($k_{\omega} = 3$).

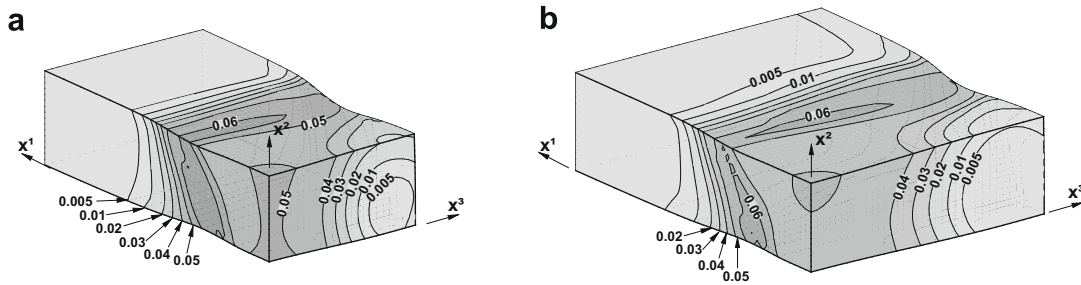


Fig. 16. Damage development, f , in wide 3D test specimens, (a) $w_0 = 4b_0$, (b) $w_0 = 6b_0$. ($\varepsilon = 0.1703$, $\sigma_y^{(\text{TMAZ})}/\sigma_y^{(b)} = 0.5$, $\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 1$ and $k_{\omega} = 3$).

ture at a mean stress that can be low, zero or even slightly negative. The analyses carried out here include results for the Gurson model ($k_{\omega} = 0$), corresponding to the results previously obtained by Nielsen (2008), and these results are compared with predictions obtained by incorporating the effect of the additional damage term ($k_{\omega} > 0$). In all the cases analyzed it is found that the modified model gives more rapid damage evolution and thus earlier fracture of the test specimens cut out

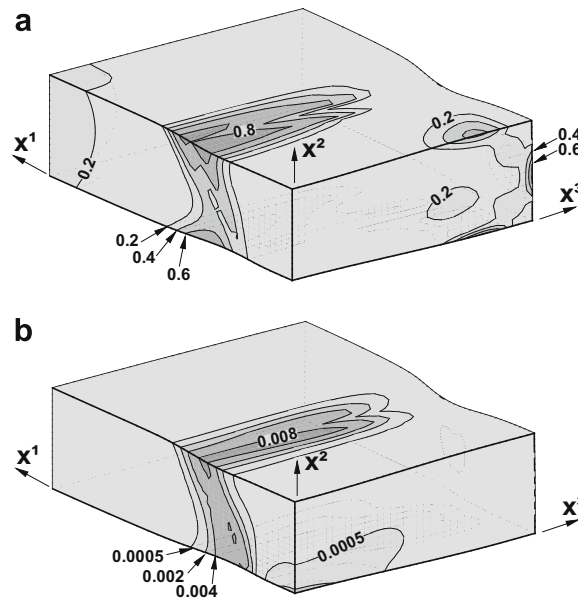


Fig. 17. (a) Contours of $\omega(\sigma)$, (b) contours of $f_{\text{modification}}$. ($w_0 = 6b_0$, $\varepsilon = 0.1703$, $\sigma_y^{(\text{TMAZ})}/\sigma_y^{(b)} = 0.5$, $\sigma_y^{(\text{NG})}/\sigma_y^{(b)} = 1$ and $k_\omega = 3$).

transverse to the weldline. This is found both for plane strain conditions and for the full 3D analyses, so it may be concluded that adding the extra damage term in the Gurson model has the expected effect of promoting fracture at the moderate levels of stress triaxiality found in these test specimens.

The analyses have shown significant differences in how the additional damage term performs in the 2D or 3D studies. In plane strain uni-axial tension the coefficient $\omega(\sigma)$ of the additional term has its maximum value ($\omega(\sigma) \approx 1$), while in the uni-axial tension modeled by the 3D analyses the value is at its minimum ($\omega(\sigma) = 0$). This means that in plane strain the additional damage term has maximum effect as soon as the value of f exceeds zero, and therefore in the present analyses the strong effect of this term is only delayed by the fact that the initial value assumed is $f = 0$ and that f only grows slowly due to the assumed strain controlled nucleation. On the other hand, in the 3D analyses for uni-axial tension the additional term has no influence at all whatever the value of f , as long as f is uniformly distributed, even though the stress triaxiality $\sigma^{kk}/(3\sigma_e) = 1/3$ is low. The 3D analyses show that here the influence of the additional damage term is due to the deviations from the uni-axial stress state that develop due to the strong imperfection represented by the different material properties in the weld. When localized plastic yielding occurs in the weld region the different stress state develops due to the constraints of the remaining part of the specimen on the localized plastic flow. This constraint increases with the specimen width and therefore the 3D analyses show an increasing effect of the additional damage term as the value of w_0/b_0 is increased, Figs. 9–17.

The additional damage term proposed by Nahshon and Hutchinson (2008) has the important effect of being able to predict the fracture that occurs in pure shear, and also failures at other states of low stress triaxiality, where the usual ductile fracture models do not perform well. However, at high stress triaxiality there is no reason to add to the void based ductile fracture models, which have been studied extensively by micromechanical models. A further improvement in the model of Nahshon and Hutchinson (2008) could involve an additional stress-dependent factor on the extra damage term, which should interpolate between the value unity at zero stress triaxiality and the value zero at a stress triaxiality well below one.

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