



Loading rate effect on mixed mode I/II brittle fracture behavior of PMMA using inclined cracked SBB specimen

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ABSTRACT

This paper investigates the effect of loading rate on the mixed-mode I/II fracture behavior of Polymethylmethacrylate (PMMA). Experimental studies are performed using a novel fracture test specimen which has recently been developed, called short bend beam (SBB), in different loading rates of 1 mm/min, 20 mm/min, and 200 mm/min. Fracture test data are presented for pure mode I, pure mode II, and mixed-mode I/II conditions, thanks to the geometrical configuration offered by the specimen covering a full spectrum of mode mixities. Two fracture criteria including maximum tangential stress (MTS), and generalized maximum tangential stress (GMTS) are employed to predict the onset of fracture and the fracture initiation angle in different loading rates. The MTS criterion only considers the singular crack tip terms associated with the stress intensity factors (SIFs) in the solution while the GMTS includes the T -stress effect as well as the SIF-related terms. While enhanced predictions provided by the GMTS criterion, as compared to MTS results, reveals the significance of including the T -stress term in the theoretical calculations, the critical distance (r_c) is also found as another major parameter playing an important role in the fracture behavior. It is shown that the size of r_c decreases by increasing the loading rate. Therefore, for the first time, a variable and loading rate dependent r_c value is utilized for employment of the GMTS criterion and it is demonstrated that considering variable critical distance can provide acceptable theoretical predictions for different loading rates.

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1. Introduction

As a transparent and lightweight thermoplastic polymer exhibiting low density, high impact strength, good abrasion resistance, hardness, and stiffness, Polymethylmethacrylate (PMMA) has been widely used in aerospace and biomedical applications (Dar and Zhang, 2015; Bruyn Neto et al., 2016; Tedjojuwono et al., 2013; Bisht et al., 2019; Ali et al., 2015; Dhanopia and Bhargava, 2017; Mousa et al., 2000). Particular attention exists in aerospace industries where PMMA has been used for years to manufacture aircraft windshields requiring high impact resistant performing under extreme environments. The long-term mechanical performance, resistance to chemicals, heat, cracking, and shock, while being transparent has made PMMA a great choice in the design of aircraft windows. Exhibiting a high degree of compatibility with biomaterials, PMMA is also used in many biomedical applications including bone implantations (Dhanopia and

Bhargava, 2017; Mousa et al., 2000) and fabricating dentures (Karthick et al., 2014). Offering superior wear and mechanical properties while being compatible with the body environment, PMMA can be an excellent candidate in denture material applications where withstanding heavy chewing forces under different loading rates is a first-order design priority. Moreover, being weather and scratch resistant together with displaying favorable processing conditions (soluble in many general organic solvents) have made PMMA a competing candidate to be used not only in aerospace and biomedical applications, but in a wide range of engineering applications including optical, pneumatic actuation, sensor, and conductive devices (Henry et al., 2000; Lee and Chen, 2001; Shah et al., 2006; Adhikari and Majumdar, 2004).

A thorough understanding of the mechanical response, strength, and failure behavior of the material is required for designing and manufacturing of engineering components made of PMMA. To address the response of the material under extreme loading conditions, the fracture and mechanical behavior of PMMA under different loading rates and temperatures have been of particular interest to researchers (Acharya and Mukhopadhyay,

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2014; Almaraz et al., 2017; Bauwens-Crowet, 1973; Qiu et al., 2019; Wada et al., 1996; Wu et al., 2004; Zhou et al., 2013; Weerasooriya et al., 2006a; Fleck et al., 1990). Wada et al. (Wada et al., 1996) studied the effects of loading rate and plate thickness on the fracture toughness measurements of PMMA under impact loading conditions. They revealed that the rate-dependent mode I fracture toughness of PMMA becomes larger as the plate thickness decreases which agrees with the case of static loading as moving from plane strain to the plane stress conditions. An experimental study performed by Qiu et al. (Qiu et al., 2019) on the PMMA material using the caustics method also reveals the dependency of critical mode I stress intensity factor (or apparent fracture toughness) on the applied loading rate. While extensive studies can be found in the literature about the failure response (e.g., (Weerasooriya et al., 2006a; Almaraz et al., 2017), and tensile and compressive behavior (e.g., (Acharya and Mukhopadhyay, 2014; Wu et al., 2004) of PMMA under different temperatures and loading rates, a limited number of them are dedicated to combined normal/shear loading conditions. As one of the few attempts, Zhou et al. (Zhou et al., 2013) investigated the rate-dependent failure and post-failure of PMMA under combined shear/compression loading conditions where they developed an empirical criterion capable of predicting the loading rate-dependent failure behavior of PMMA under combined normal/shear stresses. However, they did not report any test data describing the mechanical response of PMMA under pure shearing conditions.

A literature survey reveals that most fracture studies of PMMA are devoted to pure mode I loading conditions (e.g., (Zhou et al., 2006; Rittel and Maigre, 1996) and relatively fewer published works exist addressing the mixed-mode I/II/III fracture behavior. However, the understanding of mixed-mode fracture behavior of the materials is critical when designing structural components subjected to combined loading situations. Under static loading conditions, Maccagno and Knott (Maccagno and Knott, 1989) reported a brittle behavior for mixed-mode I/II fracture of PMMA in room temperature tested by four-point bending specimens. A study by Mirsayar (Mirsayar, 2015), leading to development of extended maximum tangential strain (EMTSN) criterion, indicated that the mixed-mode I/II fracture behavior in PMMA can be predicted properly through a strain-based framework. Later, it was shown that the EMTSN criterion can successfully predict mixed-mode I/II fracture toughness and fracture angle in a wide range of specimen types made of brittle and quasi-brittle materials (Mirsayar et al., 2016; Mirsayar et al., 2018; Mirsayar and Park, 2016; Mirsayar, 2018; Mirsayar, 2017). Aliha and co-workers (Aliha et al., 2015; Ayatollahi et al., 2006; Aliha et al., 2016a; Mousavi et al., 2020a; Bahmani et al., 2020; Aliha et al., 2016b; He et al., 2021) have investigated mode III (Aliha et al., 2015), and mixed mode I/II (Ayatollahi et al., 2006; Aliha et al., 2016a; Mousavi et al., 2020a; Bahmani et al., 2020) and I/III (He et al., 2021; Sundaram and Tippur, 2017) fracture toughness and fracture trajectory in PMMA. Using different specimen types, they measured the fracture toughness and crack propagation angle in PMMA and examined several fracture criteria to predict the test data. Depending on the specimen type, they reported that the first non-singular crack tip stress term, called T -stress, may substantially influence the theoretical predictions. An article by Sundaram and Tippur (Sundaram and Tippur, 2017) can be considered as one of the very limited published works on the dynamic mixed-mode I/II fracture behavior of PMMA. Using digital gradient sensing together with high-speed photography, full-field measurements have been conducted to obtain mixed-mode I/II fracture toughness values. However, due to the geometrical restrictions of the specimen used in their research, they have not been able to conduct any measurements under pure mode II conditions.

In this paper, for the first time, the effect of loading rate on the mixed-mode I/II fracture toughness of PMMA is reported covering a full range of mode mixities from pure mode I to pure mode II conditions. Using a recently developed specimen by the authors (Mousavi et al., 2020a), called short bend beam (SBB), the mixed mode I/II fracture toughness values were obtained under different mode mixities and loading rates. The SBB specimen is described as a short beam having an aspect ratio (i.e. length to width ratio) of approximately less than four, offering more cost efficiency due to the reduced amount of material needed for construction compared to its traditional versions. Moreover, the specimen can cover a full spectrum of mode mixities (I, II, I/II), unlike the previously three-point bend beam samples (Suresh et al., 1990; Araki et al., 2005; Pereira et al., 2019; Zappalorto et al., 2013; Xeidakis et al., 1996). The influence of loading rate on the onset of mixed mode I/II fracture is predicted using generalized maximum tangential stress (GMTS) criterion (Aliha et al., 2012, 2013; Ayatollahi and Aliha, 2009; Xeidakis et al., 1996). It is shown that the load bearing capacity and fracture resistance of PMMA is reduced significantly by increasing the rate of applied loading.

2. Test specimen and experimental program

Fig. 1 shows the overall configuration of the SBB specimen employed for the experimental part of this research. A short beam with length and width of L and W , respectively is utilized for this specimen in which L/W ratio is less than 4. The specimen contains an inclined crack with length a that makes angle α relative to the vertical direction. By three-point bend loading of the SBB specimen and changing the affecting parameters (i.e., crack length, crack inclination angle and loading span ($2S$)), different combinations of crack-tip opening/sliding displacements (i.e. mixed mode I/II) is produced. While other conventional three-point bend beam samples (with vertical edge crack and large loading span ratio $2S/L > 0.8$) fail to produce complete mode I + II mixities, the SBB specimen can provide full mixities of mode I and mode II thanks to its inclined crack, shorter length of SBB sample, and smaller loading span ratio S/W . The ability of this specimen for introducing pure mode II and mixed mode I/II in addition to pure mode I loading has already been proved by extensive finite element analyses performed by the authors [34], (Aliha and Mousavi, 2020; Aliha et al., 2018a; Aliha and Shaker, 2020; Aliha et al., 2019a; Shaker et al., 2019). It is worth mentioning that the edge crack beam specimen subjected to four-point bend loading can also be utilized for conducting mixed mode I/II fracture tests on brittle materials including the PMMA (Maccagno and Knott, 1989; Aliha et al., 2009; Aliha and Ayatollahi, 2010; Bhattacharjee and Knott, 1995).

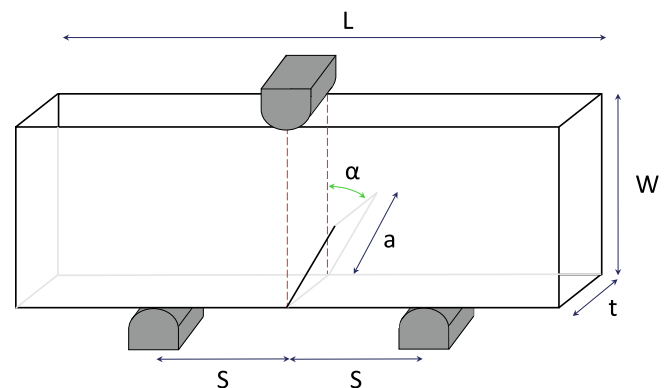


Fig. 1. Schematic representation of (short bend beam) SBB specimen utilized for mixed mode I/II fracture testing.

. In this specimen, by adequate selection of the distances of two upper and two bottom loading rollers relative to the crack, different combinations of mode I and mode II are obtained. However, this test method requires four-point bend fixture that may not be available in any laboratory. Furthermore, simultaneous application of four loading rollers on the upper and lower edges of the beam specimen is not easy and requires precise setting of the test fixture and needs accurate manufacturing process of beam to produce parallel lower and upper surfaces. Otherwise, this test beam shape sample will actually be loaded by the three of first contacting support rollers in an undesirable manner.

In the SBB specimen, mode I and mode II stress intensity factors are function of applied load P and geometry factors Y_I and Y_{II} as:

$$K_{I(SBB)} = \frac{P}{Wt} \sqrt{\pi a} Y_I(a/W, S/W, \alpha) \quad (1)$$

$$K_{II(SBB)} = \frac{P}{Wt} \sqrt{\pi a} Y_{II}(a/W, S/W, \alpha) \quad (2)$$

By defining a mode mixity parameter as: $M^e = \frac{2}{\pi} \tan^{-1} (K_I/K_{II}) = \frac{2}{\pi} \tan^{-1} (Y_I/Y_{II})$, the variations of Y_I and Y_{II} can be calculated from finite element analysis for any desired mode mixity or M^e value. To do that, first a finite element (FE) model (including the geometry, boundary conditions and material property) of the SBB specimen is created in a FE software such as ABAQUS. Then by applying a reference load and performing the static and elastic analysis via J-integral method developed in the ABAQUS code, the corresponding values of stress intensity factors (K_I and K_{II}) and consequently their normalized forms (i.e., Y_I and Y_{II}) are determined directly for any desired a/W , S/W and α values in the modeled SBB specimen. More details about the computation of these geometry factors can be found in previous works of the authors (e.g., (Mousavi et al., 2020a; Aliha and Mousavi, 2020). Fig. 2 shows the variations of Y_I and Y_{II} with M^e in the SBB specimen and for typical loading conditions of $a/W = 0.5$ and $S/W = 0.5$. According to this Figure, the SBB specimen is able to provide a full range of mode mixities from $M^e = 1$ (i.e. Pure mode I) to $M^e = 0$ (i.e. pure mode II). However, the conventional three-point bend specimen with vertical crack can only produce dominantly mode I loading conditions.

The SBB specimen is employed here for investigating the effect of loading rate on mixed mode I/II fracture and cracking behavior of PMMA. The dimensions of the SBB specimen for conducting the mixed mode fracture toughness tests were considered as: $L = 36$ mm, $W = 12$ mm and $a = 6$ mm. All the SBB test samples were cut from a PMMA sheet with thickness of $t = 4$ mm using a laser cut

machine. An artificial narrow notch was also introduced at the middle edge of samples using the laser cut machine. The length of manufactured notches was slightly less than 6 mm. The tip of each notch was sharpened further by pressing a razor blade to create a sharp crack with final length of 6 mm. This method is a common technique for creating a sharp crack in the PMMA material (Ayatollahi et al., 2016; Ayatollahi et al., 2019; Lach and Grellmann, 2017; Sundaram et al., 2018; Weerasooriya et al., 2006b) instead of using fatigue cracking process that is more common for metallic materials. The length of pre-notch was constant and equal to 6 mm in all samples, but the crack inclination angle was varied from the vertical direction (i.e. $\alpha = 0^\circ$) to $\alpha = 39^\circ$ to capture full mode mixities from pure mode I to pure mode II. Hence, five inclination angles (i.e. $\alpha = 0^\circ, 15^\circ, 26^\circ, 33^\circ, 39^\circ$) were introduced during the manufacturing of test samples. Using three-point bend fixture, the SBB specimens were loaded with three different loading rates of 1, 20 and 200 mm/min. A constant span length of $S = 6$ mm was also considered for testing the whole SBB samples of this research. For each mode mixity (or crack inclination angle) and for each same condition, three replicate samples were tested. Fig. 3 shows the SBB test setup made of manufactured PMMA material and some examples of load-displacement curves obtained for different mode mixities and loading rates. It is seen that except for very negligible non-linearity at the beginning of test (that can be related to the lack of complete contact between the specimen and loading rollers), the overall loading behavior of the tested PMMA material follows a linear and brittle fracture in which each SBB sample is failed suddenly at a critical peak load. However, the magnitude of this peak load depends significantly on both mode mixity and loading rate. The fracture load is increased by moving towards the pure mode II condition and increasing the crack inclination angle. In addition, increasing the loading rate can result in substantial reduction of the SBB fracture load.

Following the linear elastic fracture mechanics framework, the critical values of stress intensity factors (K_{Ic} and K_{IIc}) can be determined from the below equations:

$$K_{Ic(SBB)} = \frac{P_{cr}}{Wt} \sqrt{\pi a} Y_I \quad (3)$$

$$K_{IIc(SBB)} = \frac{P_{cr}}{Wt} \sqrt{\pi a} Y_{II} \quad (4)$$

where P_{cr} is the fracture load. For each tested mode mixity, the corresponding values of geometry factors (Y_I and Y_{II}) were determined from the curves given in Fig. 2. Table 1 illustrates the geometry factors for the testing conditions (i.e., $a/W = 0.5$, $S/W = 0.5$ and the tested crack inclination angles).

3. Results and discussion

Fig. 4a presents and compares the mixed mode fracture toughness data obtained for the SBB specimen and under different loading rates. This figure obviously shows the influence of loading mode and loading rate on the mixed mode fracture behavior of PMMA material such that by increasing the loading rate from 1 to 200 mm/min the mixed mode fracture toughness envelope becomes very smaller. The stress intensity factor rate ($\dot{K}_c$) defined as: $\dot{K}_c = K_c/t_c$ (in which K_c is the fracture toughness and t_c denotes the interval from the start of loading to the onset of fracture), was varied from 0.1 to 10 for the tested SBB specimen made of PMMA material of this research. Compared to the available strain rates considered in the literature for study of loading rate or strain rate effect on the fracture and cracking behavior of materials (which varies typically between 10^{-2} and 10^6 (Shannahan et al., 2015; Faye et al., 2014; Sahraoui and Lataillade, 1994; Faye et al.,

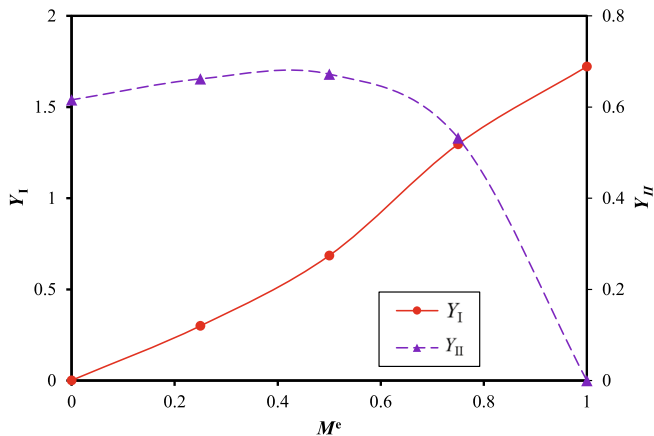


Fig. 2. Variations of mode I and mode II geometry factor (Y_I and Y_{II}) versus mode mixity parameter (M^e) for the SBB specimen.

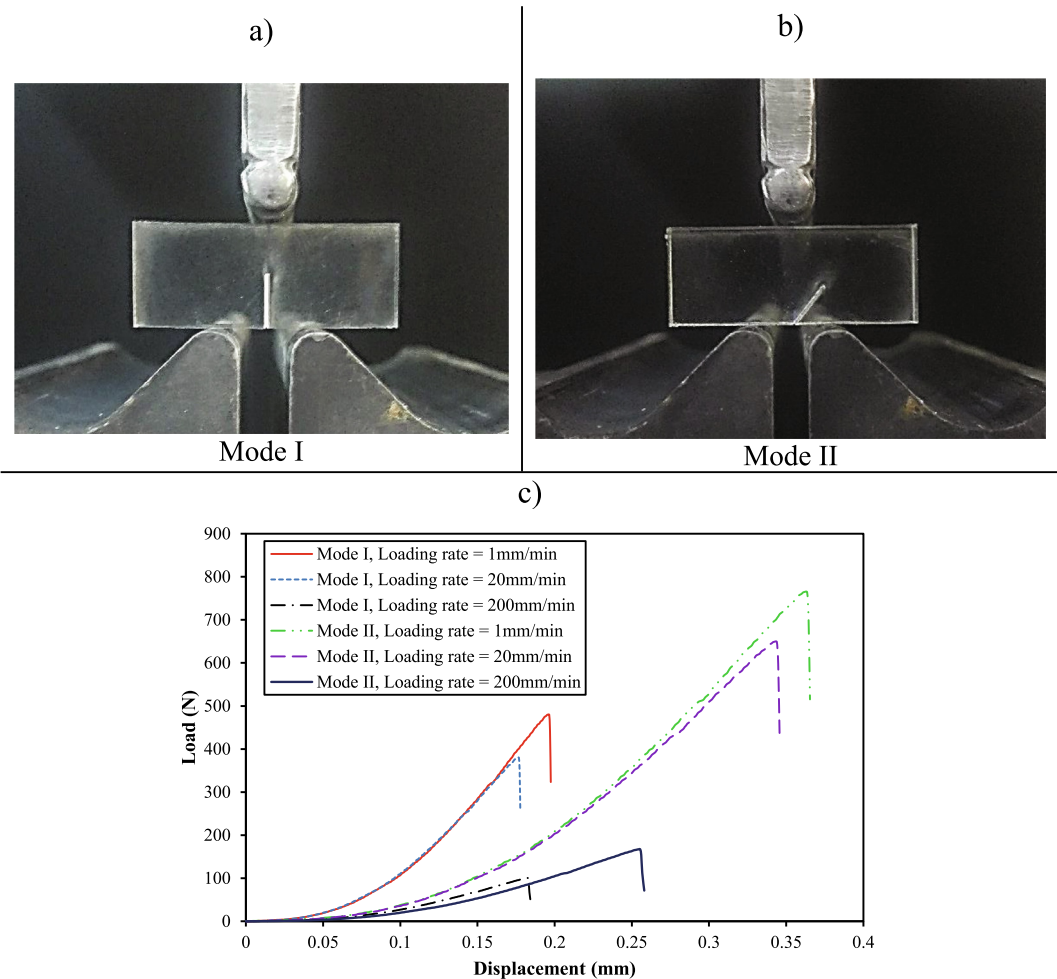


Fig. 3. a, b) Mode I and II loading setup for SBB, c) load-displacement curves obtained for different loading rates and mode mixities.

Table 1
Corresponding values of Y_I and Y_{II} for the tested SBB specimens under different mode mixities.

M^e	1		0.75		0.5		0.25		0	
α (degree)	0		15		26		33		39	
Geometry factor	Y_I	Y_{II}	Y_I	Y_{II}	Y_I	Y_{II}	Y_I	Y_{II}	Y_I	Y_{II}
	1.72	0.00	1.30	0.53	0.69	0.67	0.30	0.66	0.00	0.62

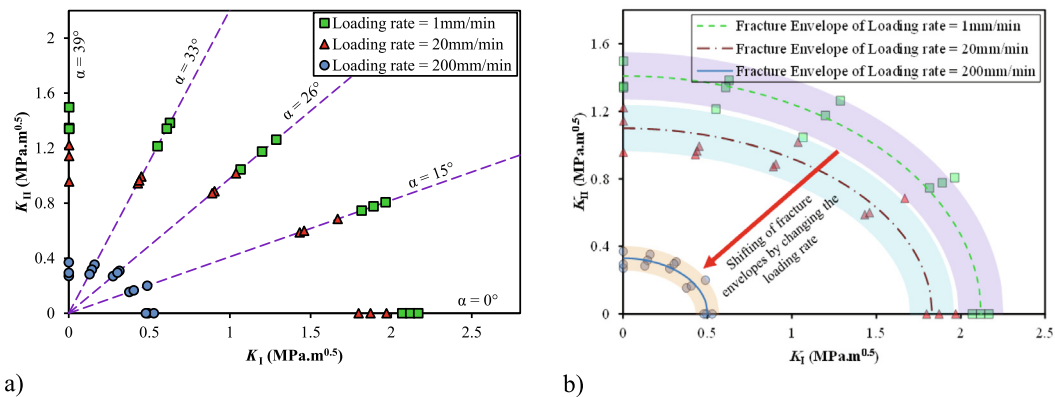


Fig. 4. (a) Mixed mode fracture results obtained for the SBB specimen made of PMMA and with different loading rates, (b) the effect of loading rate on shifting the fracture envelopes.

2016; Hitoshi, 1992; Erdogan and Sih, 1963; Chang, 1981)), the tested range in the current research for the PMMA material lies in a narrow range. This range (i.e., $\dot{K}_c$ less than 10) is usually categorized as predominantly static (i.e., quasi-static) loading regime. According to the available works published in this regard, the critical value of stress intensity factor (K_{Ic}) or the critical value of fracture energy parameter (like J-integral, J_c) decreases by increasing the SIF rate or J rate (i.e., $\dot{K}$ and $\dot{J}$) in this strain rate regime. For example, by pure mode I testing of PMMA material using the single edge notch beam specimen with different loading rates, Faye and coworkers (Chang, 1981) showed that J_c decreases linearly by a factor of about 3.5 when the J parameter decreases from 0.1 to 10. This observation is in good agreement with the findings of the current research which shows similar reduction in the value of K_{Ic} by increasing the SIF rate (i.e., $\dot{K}$) from 0.1 to 10. The quasi-static mode I fracture toughness value for the PMMA material obtained from the SBB testing method varies typically in the range of 0.5 to 2 MPa.m^{0.5} for different loading rates. Such range for the K_{Ic} of PMMA has also been reported in other works using different test specimens (Ayatollahi et al., 2019; Lach and Grellmann, 2017; Razavi et al., 2018; Wei et al., 2017a) showing the validity of SBB testing method utilized in this research. However, for higher strain rates (i.e., $\dot{K} > 10^4$) that is usually known as dynamic loading in the literature, the available test results reported in the papers indicate that the fracture toughness or fracture energy parameters increases significantly by increasing the strain rate (Sahraoui and Lataillade, 1994; Erdogan and Sih, 1963; Chang, 1981; Razavi et al., 2018; Wei et al., 2017a). Indeed, opposite trend and behavior is seen between the fracture behavior of PMMA under static and dynamic loading rate regimes.

As shown in Fig. 4b the fracture envelopes obtained from the average points of tested mode mixities (or α values) are nearly parallel and a shift in the fracture envelope is seen by changing the loading rate. This implies that the loading rate has nearly the same influence on the fracture behavior and fracture mechanism of tensile, shear and combined tensile-shear loading modes. It is also observed from the test data that the value of fracture toughness is decreased by moving from pure mode I towards pure mode II. For example it is seen that the pure mode I fracture toughness (K_{Ic}) is approximately 1.4 times the pure mode II fracture toughness (K_{IIc}) value. This relation is valid for different loading rates investigated here for the PMMA material.

It is interesting to examine the ability of theoretical fracture criteria for predicting or assessing the obtained mixed mode I/II results of SBB samples made of PMMA and tested with different loading rates. Several stress/strain and energy-based fracture models exist in the literature for investigating the mixed mode I/II fracture problems (Ayatollahi et al., 2011; Richard, 1984; Sih, 1974; Hussain et al., 1974; Aliha et al., 2017a; Mousavi et al., 2020b; Aliha et al., 2019b; Aliha et al., 2019c; Aliha et al., 2017b). These criteria often use the crack tip stress/strain field (that can be expressed in terms of the fracture parameters such as the stress intensity factors etc.) for predicting the onset of fracture. For example, a well-known generalized maximum tangential stress criterion (GMTS) assumes that the mixed fracture initiates from the crack tip and along a direction in which the tangential stress is maximum. At the onset of fracture, the corresponding value of tangential stress along this direction reaches a critical value that is related to a material characteristic strength. By considering the tangential stress component at the crack tip (Ayatollahi and Aliha, 2009):

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] + T \sin^2 \theta + O(r^{1/2}) \quad (5)$$

one can estimate both direction of fracture initiation angle (θ_0) and the onset of mixed mode fracture based on the hypothesis of the generalized maximum tangential stress (GMTS) criterion as equations (6) and (7) respectively (Ayatollahi and Aliha, 2009; Aliha et al., 2012), as:

$$\begin{aligned} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0, \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \\ \rightarrow \therefore [K_I \sin \theta + K_{II} (3 \cos \theta - 1)] - \frac{16T}{3} \sqrt{2\pi r_c} \cos \theta \sin \frac{\theta}{2} = 0 \\ K_{Ic} = \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] + \sqrt{2\pi r_c} T \sin^2 \theta \end{aligned} \quad (7)$$

In Equations (6) and (7), K_I , K_{II} are the modes I and II stress intensity factors and the T -stress is the non-singular stress term that is independent from the crack tip distance. The T -stress depends on the geometry of test specimen, loading type applied to the specimen and the state of mode mixity. This term for the SBB specimen can be written as following equation in terms of the geometry and loading condition of specimen:

$$T_{(SBB)} = \frac{P}{Wt} T^*(a/W, S/W, \alpha) \quad (8)$$

where the T^* is the non-dimensional form of T -stress. To utilize the GMTS criterion for predicting the mixed mode I/II fracture behavior of SBB specimen, it is necessary to know the value of T -stress. The corresponding values of T^* for the SBB specimen are presented in Table 2 for different crack inclination angles tested in the current study (i.e. for $a/W = 0.5$ and $S/W = 0.5$). These data were extracted from our previous research works performed using numerical analysis of the SBB specimen (Mousavi et al., 2020b). Similar to the geometry factors (Y_I and Y_{II}), the corresponding value of T^* for any given a/W , S/W and α parameters can be computed directly from the J-integral method.

Having calculated three fracture parameters (i.e. K_I , K_{II} and T -stress) or their normalized forms (Y_I , Y_{II} and T^*) for the SBB specimen and replacing them into Equations (6) and (7), both θ_0 and onset of mixed mode I/II fracture can be estimated for any desired mode mixity or crack inclination angle. However, the value of critical distance (r_c) should also be known. This parameter that is related to the size of damage or fracture process zone is often considered as the material constant in many previous works (Torabi et al., 2019; Xiongzhou et al., 2021; Dehghany et al., 2017; Hou et al., 2019; Aliha et al., 2018b; Seitzl et al., 2018; Aliha et al., 2017b; Sangsefidi et al., 2020; Yin et al., 2020; Xing et al., 2020; Wei et al., 2018; Wei et al., 2017b; Fuan et al., n.d.; Aliha et al., 2019d). But, the rate of applied load during the fracture testing can affect the process of initiation and development of the fracture process zone and consequently may change the size of this region. By increasing the loading rate, the material could fail more rapidly with lower loads because of lack of enough time for developing and creating the plastic zone. Therefore, it is rational if we assume that the critical distance or size of plastic zone ahead of the crack tip becomes smaller when the speed of loading applied to the test specimen is increased. The size of critical distance or plastic zone can be estimated using the crack tip stress field from the following well-known relation:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] \quad (9)$$

Under the onset of pure mode I fracture case for instance, in which $\theta = 0^\circ$ and $K_I = K_{Ic}$ and $\sigma_{\theta\theta} = \sigma_t$ (material's tensile strength), the critical value of r that is equal to r_c is determined from:

Table 2Corresponding values of T^* for different crack inclination angles of SBB specimen ($a/W = 0.5$ and $S/W = 0.5$).

M^*	0	0.25	0.5	0.75	1.0
α (degree)	39	33	26	15	0
T^*	1.29	1.12	0.79	0.01	-0.72

$$r_c = \frac{1}{2\pi} \left(\frac{K_{Ic}}{\sigma_t} \right)^2 \quad (10)$$

By replacing the mode I fracture toughness values obtained from the experiments under different loading rates, into Eq. (10) the corresponding values of r_c are determined. The tensile strength of PMMA material is considered equal to 65 MPa from previous works of the authors (Mousavi et al., 2020b). Table 3 presents the calculated r_c values for different loading rates. It is seen that the size of critical distance becomes smaller when the loading rate is increased from 1 to 200 mm/min. The variations of r_c versus loading rate in the logarithmic form (shown in Fig. 5b) demonstrates that a linear relation exists between the $\log(r_c)$ and the loading rate. Therefore, the below equation can be used for estimating the critical distance of PMMA material for different loading rates.

$$\log(r_c) = -0.0062LR - 0.8079 \quad (11)$$

where “LR” is the loading rate in (mm/min).

To predict the mixed mode I/II fracture results of the tested SBB specimens made of PMMA under different loading rates, the equations of GMTS criterion can be rewritten in terms of the normalized fracture parameters of the SBB specimen (i.e. $Y_{I(SBB)}$, $Y_{II(SBB)}$ and $T^*_{(SBB)}$) as:

$$[Y_{I(SBB)} \sin \theta_0 + Y_{II(SBB)} (3 \cos \theta_0 - 1)] - \frac{16T^*_{(SBB)}}{3} \sqrt{\frac{2r_c}{a}} \cos \theta_0 \sin \frac{\theta_0}{2} = 0 \quad (12)$$

$$\frac{K_{Ic}}{K_{eff}} = \cos \frac{\theta_0}{2} \left[\frac{Y_{I(SBB)}}{\sqrt{Y_{I(SBB)}^2 + Y_{II(SBB)}^2}} \cos^2 \frac{\theta_0}{2} - \frac{3}{2} \frac{Y_{I(SBB)}}{\sqrt{Y_{I(SBB)}^2 + Y_{II(SBB)}^2}} \sin \theta_0 \right] + \frac{T^*_{(SBB)}}{\sqrt{Y_{I(SBB)}^2 + Y_{II(SBB)}^2}} \sqrt{\frac{2r_c}{a}} \sin^2 \theta_0 \quad (13)$$

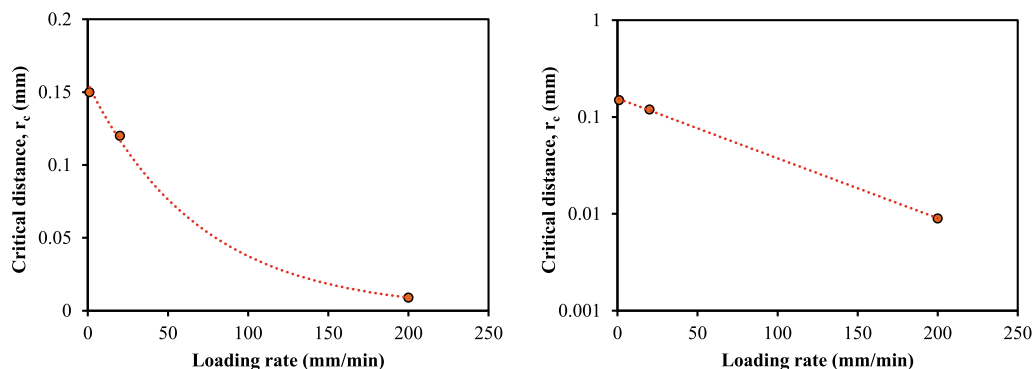
Table 3

Corresponding values of critical distance for the PMMA material tested under different loading rates.

Loading rate (mm/min)	1	20	200
r_c (mm)	0.15	0.12	0.009

The corresponding values of such fracture parameters were presented in previous sections for different mode mixities or crack inclination angles of the SBB sample. These values only depend on the specimen geometry and loading support distances and are not related to the material type or loading rate applied to the specimen. Accordingly, by replacing the corresponding values of $Y_{I(-SBB)}$, $Y_{II(SBB)}$ and $T^*_{(SBB)}$ into Equation (13) and using the r_c values presented in Table 2, the mixed mode I/II fracture envelopes predicted by the GMTS criterion are obtained for the tested PMMA material under different loading rates. Fig. 6 presents the theoretical prediction curves of the SBB sample obtained from the MTS and GMTS criteria. As seen, the theoretical curves presented by the GMTS criterion are in good agreement with the experimental mixed mode I/II fracture toughness data and the criterion can successfully address the effects of geometry, mode mixity, and the loading rate on fracture behavior of the tested PMMA material. Under pure mode I or mode I dominant conditions, both MTS and GMTS provide similar theoretical prediction. However, the discrepancy between the two criteria increases as one approaches mode II conditions. Fig. 7 also compares the mixed mode fracture envelopes (obtained from both experimental tests and theoretical predictions of the GMTS criterion) for the three investigated loading rates. The GMTS criterion that uses a variable critical distance dependent on the loading rate can properly estimate the fracture behavior and load carrying capacity of brittle PMMA loaded with different rates.

Recently, Aliha and coworkers [91] developed a model for estimating the fracture behavior of cracked materials subjected to mixed mode loading. Based on this model that is called “tangential stress contour (TSC)”, the fracture of a cracked body subjected to external applied load F , initiates from the boundary of a damage zone in the vicinity of crack tip along the direction in which the tangential stress ($\sigma_{\theta\theta}$) is maximum. The onset of fracture also takes place when the maximum value of $\sigma_{\theta\theta}$ at a critical distance r_c from the crack tip (which is representative of the damage zone) reaches the tensile strength (σ_t) of brittle material. If the boundary of damage or crack tip process zone is assumed to be a circular region of radius r_c with origin located at the crack tip, it is only required to perform a simple finite element analyses and obtain the tangential stress contour ahead of the crack tip for any arbitrary mixed mode tensile-shear loading. Fig. 8 shows the tangential stress value ($\sigma_{\theta\theta}$) determined from the finite element analyses of the tested SBB

**Fig. 5.** Relation between the size of the critical distance r_c and the loading rate in the investigated SBB samples made of PMMA.

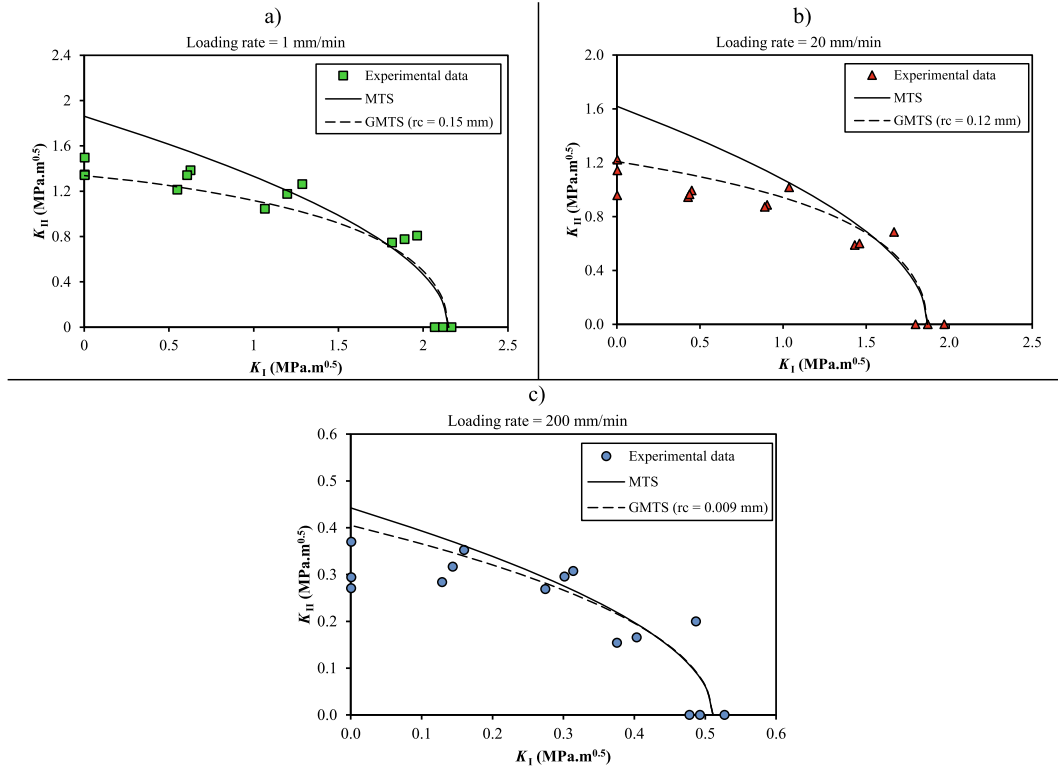


Fig. 6. Predictions of the MTS and GMTS criteria for mixed mode I/II fracture experiments of the PMMA material with different loading rates, (a) 1 mm/min, (b) 20 mm/min and (c) 200 mm/min.

samples under mode I loading conditions. In the finite element models the critical fracture loads obtained for the loading rates 1, 20 and 200 mm/min were applied as the input load and the corresponding crack tip stress field (i.e. $\sigma_{\theta\theta}$ versus distance from the

crack tip, r) was plotted. As seen from this Figure, the tangential stress value tends to infinity by moving towards the crack tip. However, since the brittle materials cannot withstand any stress above their ultimate tensile strength, it can be assumed that the volume of material located in front of the crack tip and subjected to stress values higher than the tensile strength of material is damaged due to excessive high strains. The main fracture starts from the boundary of this zone and the length or radius of this zone is known as the fracture or damage zone. The intersection of horizontal line shown in Fig. 8a corresponds to the tensile strength of tested PMMA sample (i.e. 65 MPa) and the crack tip tangential stress curves of the SBB samples with different loading rates can be utilized for determining and estimating the crack tip damage zone size. Accordingly, the critical distances (r_c) for the PMMA tested under different loading rates (i.e. 1, 20 and 200 mm/min) are determined from this model equal to 0.15, 0.12 and 0.009 mm, respectively that are in quite well agreement with the r_c values employed in Fig. 7 for predicting the PMMA mixed I/II fracture toughness tests. Similar TSC method was utilized for determining the critical distance and size of damage zone under mixed mode I/II and pure mode II loading conditions of tested SBB sample with different loading rates. Shown in Fig. 9 are the corresponding tangential stress curves plotted for the SBB specimen at crack inclination angles $\alpha = 26^\circ$ (mixed mode I/II) and 39° (pure mode II). Again, the intersection of these curves with the fixed tensile strength value of 65 MPa for the PMMA material, illustrates the corresponding value of r_c for any investigated loading rate. It is seen that for any desired loading rate, nearly constant and fixed value of r_c is obtained for different mode mixities, demonstrating the independency of critical distance from the state of mode mixity. Indeed, nearly the same critical distances obtained in Fig. 8 for loading rates of 1, 20 and 200 mm/min under pure mode I ($M^e = 1$) were also obtained for other mode mixities as shown in Fig. 9 for the cases $M^e = 0.5$ (mixed mode I/II) and $M^e = 0$ (pure mode II).

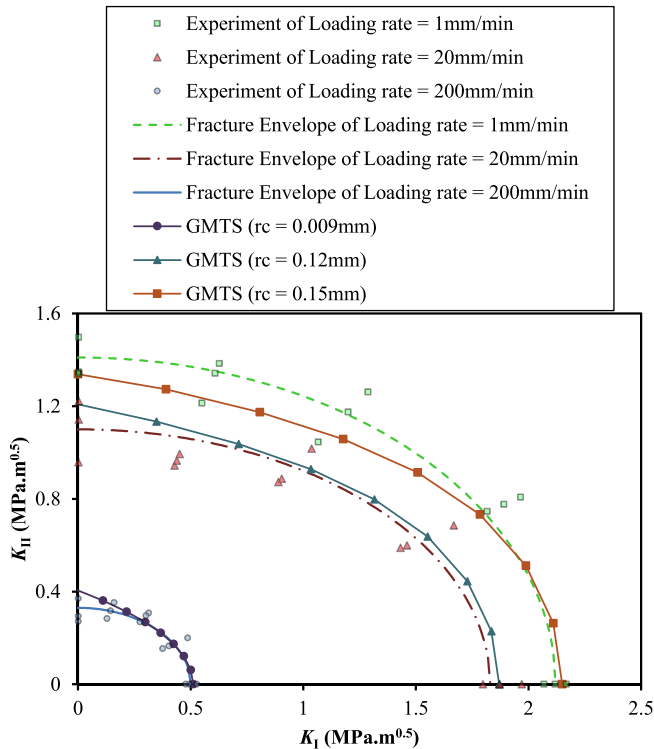


Fig. 7. Comparison of experimental mixed mode I/II fracture results and theoretical predictions using the GMTS criterion for the tested SBB samples at three loading rates of 1, 20 and 200 mm/min.

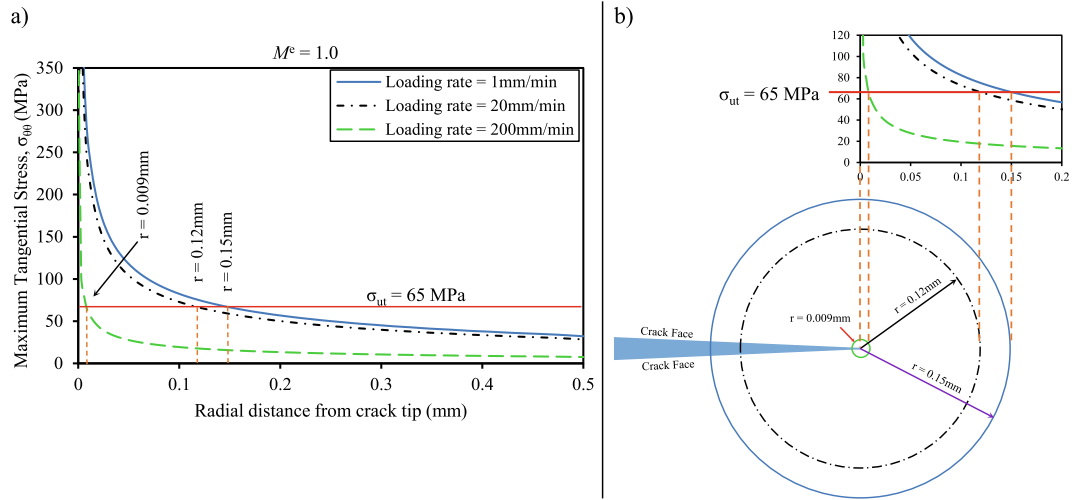


Fig. 8. (a) Maximum tangential stress versus crack tip distance curve and (b) zoomed view of the crack tip region stress field obtained using the TSC model via the finite element model for the tested PMMA samples at different loading rates.

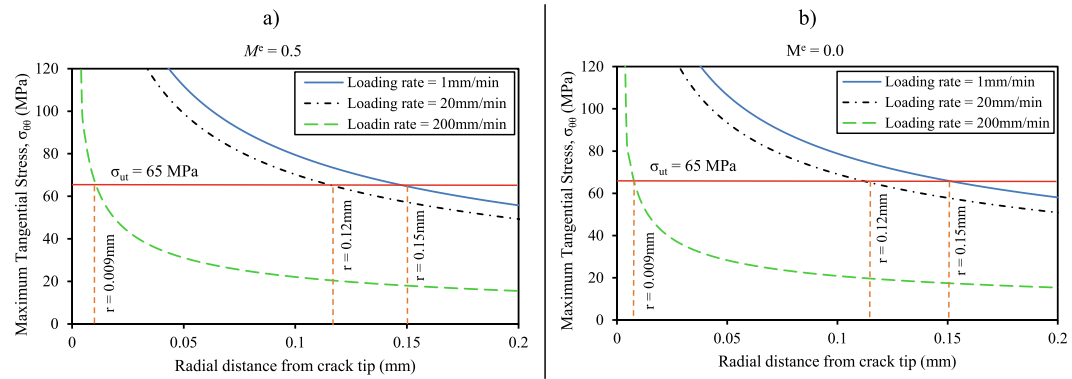


Fig. 9. prediction of critical distance (r_c) for the tested PMMA samples at three loading rates of 1, 20 and 200 mm/min using the TSC model for (a) $M^e = 0.5$ and (b) $M^e = 0$.

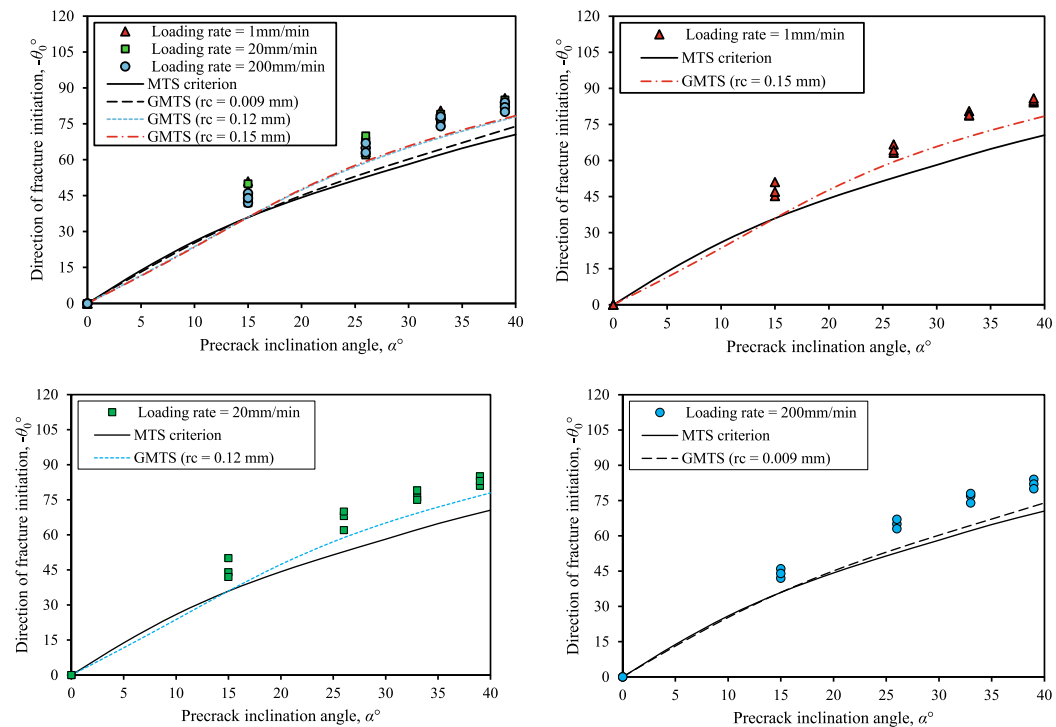


Fig. 10. Variations of fracture initiation direction for different mode mixities and loading rates in the tested SBB samples made of PMMA.

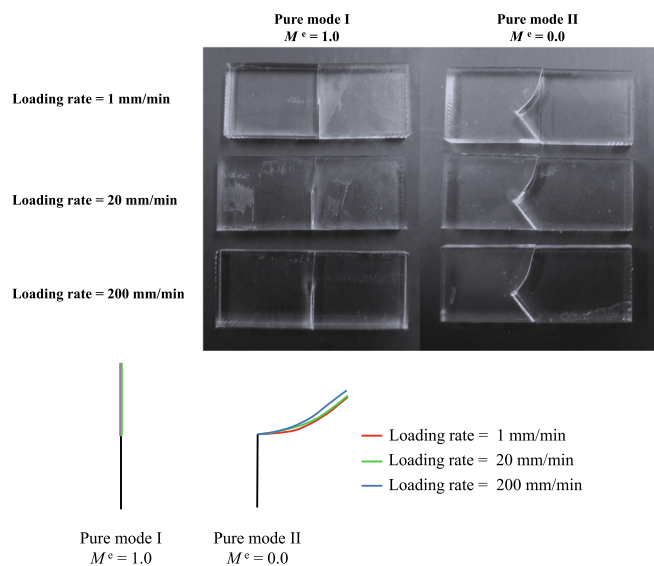


Fig. 11. Comparison of fracture trajectory in the tested PMMA samples at different loading rates.

It is finally noted that in the previous research works utilizing the generalized fracture criteria such as generalized maximum tangential stress (Ayatollahi et al., 2006; Aliha et al., 2016a; Ayatollahi and Aliha, 2009), generalized maximum tangential strain (Mirsayar, 2015; Mirsayar et al., 2016; Mirsayar et al., 2018), average strain energy density criterion (Sih, 1974; Aliha et al., 2017c; Hou et al., 2019) and etc., the mixed mode fracture prediction has been done by considering a fixed critical distance value. However, we introduced for the first time a variable and loading rate dependent r_c value for accurate and true estimation of the mixed mode fracture phenomenon in brittle materials like PMMA. Increasing the applied loading rate will result in noticeable reduction in the crack tip plasticity which can lead to decrease in the fracture load and resistance capacity of material against fast and brittle fracture. Indeed, the size of critical distance or plastic zone (in small scale yielding range) is reduced by increasing the quasi-static loading rate and this results in reducing the load carrying capacity of material because less deriving force or strain energy is dissipated for creating the process zone when the loading rate is increased in the tested PMMA material.

However, our experiments showed that the loading rate has no sound effect on the direction of fracture initiation and the path of mixed mode I/II fracture. For example, Fig. 10 presents the variations of fracture initiation angle (θ_0) measured from the tip of broken SBB samples made of PMMA at different loading rates and mode mixities. Similarly, the path of fracture trajectory for different loading rates and different mode I and II mixities are compared in Fig. 11. It is seen from Figs. 10 and 11 that fracture propagation stages of the tested PMMA samples are nearly the same and independent of the loading rate for any desired mode mixity. Since the shape of crack tip damage zone and its rotation about the crack tip controls mainly the direction of fracture initiation, it can be concluded that altering the loading rate cannot affect the shape and orientation of the crack tip damage zone and a simple scale down or scale up in the shape of fracture process zone is occurred by changing the loading rate for different mode mixities.

4. Conclusions

- A novel sub-sized short beam sample called the SBB specimen was utilized for conducting the mixed mode fracture toughness experiments on PMMA at different loading rates.

- It was shown that the SBB specimen can introduce the full range of mode mixities from pure mode I to pure mode II and for the investigated loading rates ranging from 1 to 200 mm/min.
- The fracture loads and corresponding fracture toughness values for any desired mode mixity were decreased drastically by increasing the loading rate from 1 to 200 mm/min. This was mainly because of the reduction in the crack tip plasticity level for higher loading rates.
- A theoretical model called tangential stress contour (TSC) was employed for determining the critical distance value (r_c) under different loading rates. It was shown that the size of critical distance increases from 0.15 mm to 0.009 mm when the loading rate increases from 1 to 200 mm/min.
- The obtained experimental mixed mode I/II fracture toughness data for different loading rates were predicted successfully using the generalized maximum tangential stress (GMTS) and considering especial value of r_c for each loading tested loading rate.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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