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Hygro-mechanically coupled modelling of creep in wooden structures, Part I: Mechanics

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ABSTRACT

In the paper at hand, a modelling approach for the simulation of the anisotropic long-term behaviour of wooden structures subjected to mechanical loading at constant moisture content for the use within the framework of the Finite Element Method is presented. Using a standard-solid body-model in serial combination with a BINGHAM-element allows for the differentiation between linear viscoelastic and non-linear viscoelastic-viscoplastic behaviour with respect to the applied stress level. Creep failure is considered by means of the concept of strain-energy density. Due to the cylindrical anisotropy of wood, the four material properties required for the description of the creep behaviour are determined depending on material direction and loading type (tension, compression, shear) by means of the re-calculation of experimental results. Finally the approach is applied to the re-calculation of two test-series in bending and to the analysis of a face staggered joint.

The paper at hand is the first part of a more complex model and deals with the mechanical long-term behaviour of wood. In the second part, the hygro-mechanical coupling is covered with respect to the consideration of the influence of moisture content on mechanically induced creep and mechano-sorptive effects caused by simultaneous mechanical loading and moisture changes.

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1. Introduction

The time-dependent behaviour of wooden structures subjected to mechanical and hygro-mechanical long-term loading is characterised by complex physical processes. Even at constant moisture content (MC), wood exhibits a non-linear stress-strain-relationship depending on the stress level (SL), the ratio of applied load and short-term strength. Furthermore, the stress-strain-relationship is influenced by the amount of (constant) MC. In terms of simultaneous mechanical loading and moisture changes, in addition to the hygro-expansion (swelling and shrinkage), mechano-sorptive effects are observed, which significantly scale up the creep deformations.

Although there exists a large variety of approaches for the simulation of particular features of the complex multi-physical behaviour, no model is known to the authors, which entirely captures the mentioned phenomena. Hence, a novel modelling approach under consideration of the

- cylindrically anisotropic material behaviour of wood,
- non-linear stress-strain-relationship, respectively strain-time-relationship depending on SL,
- occurrence of creep failure,
- influence of constant MC on the creep behaviour and
- mechano-sorptive effects

for the use within the framework of the Finite Element Method (FEM) is developed. The model may be characterised as phenomenological and refers to the macroscopic scale. A basic idea in the course of model development is the use of as few as possible physically motivated input properties (in contrast to pure model parameters) to describe the long-term behaviour of wood under consideration of failure processes. The fitting of the model to particular experimental creep curves is explicitly not aspired. Rather, the development of a general model with wide spectrum of application is aimed at.

The achievement of the final goal requires a structured proceeding with respect to model development. To capture the before mentioned features, several modelling steps are necessary, which build upon each other. Thus, step n is the precondition for step $n + 1$ with respect to modelling and algorithmic implementation as well as with respect to the material properties to be identified

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Table 1

Steps of development with respect to modelling of the long-term behaviour of hygro-mechanically loaded wooden structures.

	Additionally captured material characteristic	SL	MC
1.	Linear viscoelastic creep	$< LL$	Constant
2.	Non-linear viscoelastic-viscoplastic creep	$> LL$	Constant
3.	Creep failure	$> LL$	Constant
4.	Influence of constant MC	All	Constant
5.	Mechano-sorptive effect	All	Changing

by means of recalculating experimental results. In Table 1, the steps in development are shown. In the framework of the paper at hand, the numerical modelling of the purely mechanical behaviour is dealt with. The influence of MC as well as mechano-sorptive effects are described in the second part of this publication. For the sake of clarity, the different material features of wood as well as the state of the art with respect to modelling are presented in the particular sections.

2. Creep-model for wooden structures

2.1. Material characterisation

Viscoelastic creep of wood may be defined as the time-dependent increase of deformations at constant mechanical and hygical loading. At this, as mentioned above, a non-linear relationship with respect to SL is observed. Nevertheless, in literature, wood is often characterised as linear viscoelastic (e.g. Schniewind and Barrett, 1972) with constant ratio of stress and time-dependent strain. Thus, the principle of superposition is valid and the order of the application of particular loading steps is insignificant for the final result.

However, the assumption of linear viscoelasticity is only valid for wood, as long as SL stays below a specific value, the so-called limit of linearity (LL). When the applied SL exceeds LL, a disproportional increase of creep deformations is observed in comparison to the linear viscoelastic range. Furthermore, the principle of superposition is not longer valid. At high values of SL, creep failure occurs. Following the majority of references (e.g. Becker, 2002; Hunt, 1989), no permanent deformations develop for loadings below LL – creep deformations are entirely reversible. In contrast, when SL exceeds LL, irreversible deformations arise. For this reason, LL is sometimes denoted as yield limit and defined as the maximum SL with a creep velocity tending to zero and the deformations tending to a final state (Gressel, 1983). Thus, the existence of a creep limit is assumed in this case, which is also confirmed in other publications (e.g. Becker, 2002; Hunt, 1989).

Considering the creep velocity, three phases of wood creep may be distinguished (see Fig. 1)

- primary creep: linear viscoelastic with decreasing creep velocity,
- secondary creep: non-linear viscoelastic with constant creep velocity,
- tertiary creep: non-linear viscoelastic with increasing creep velocity followed by creep failure.

At this, as to be seen in Fig. 1, occurrence and temporal extent of the creep phases depend on SL.

So far, the qualitative characteristics of wood creep at constant MC are roughly described. With respect to quantification, creep deformations are influenced by several factors. Except SL, the type and direction of loading significantly influences the amount of creep deformations. In this respect, creep deformations due to

compressive loading at the same SL are considerably larger than those due to tensile loading. Creep resulting from shear and torsional loading, again, is more pronounced than compressive creep. Concerning the direction of loading, creep due to loading perpendicular to the grain of wood is much more distinctive than creep due to loading parallel to the grain, when the same SL is applied. More information on this topic can e.g. be found in Becker (2002), Gressel (1983) and Ranta-Maunus (1993).

With respect to modelling and description of the time-dependent behaviour of wood, in addition to the phenomenological approaches introduced subsequently, several structurally based models have been developed. In contrast to the phenomenological models, which merely characterise the reaction of the material to a particular loading, structural models are based on microscopic, sub-microscopic, respectively molecular features of the material's structure. Since those approaches are not of further interest for the following investigations, they will not be explained in the framework of the paper at hand, but rather, for the sake of completeness, mentioned giving related references:

- theory of hydrogen bond breaking and remaking, e.g. Bodig and Jayne (1993) and Hanhijärvi (1997),
- theory of polymer reaction, e.g. Gressel (1983), Hanhijärvi (1997) and van der Put (1989),
- theory of physical aging, e.g. Hunt and Gril (1996),
- lenticular trellis model, e.g. Hanhijärvi (1997), Boyd (1982) and
- theory of slip plane formation in compression, e.g. Hanhijärvi (1997) and Hoffmeyer and Davidson (1989).

2.2. Basic constitutive equations

The modelling approach introduced subsequently can be classified as continuum-macro-mechanical model and is formulated for small strains, since this assumption is sufficient with respect to the majority of applications in wood structural engineering. Resulting from the natural structure of wood, the cylindrically anisotropic material directions radial (r), tangential (t) and longitudinal (l) have to be distinguished (see Fig. 2). The transformation from global coordinates to local material directions is explained in detail in Resch and Kaliske (2005).

The material model is developed for use within the framework of the FEM. Since, in this first part of the entire work, the influence of MC is disregarded, the displacements $\mathbf{u}$ for the three directions are used as degrees of freedom for each node. On the basis of the static balance of momentum, the weak form of equilibrium is derived utilising the principle of virtual displacements with the test function $\delta \mathbf{u}$

$$G_u = \int_B \delta \nabla \mathbf{u} \cdot \underline{\sigma} \, dV - \int_B \delta \mathbf{u} \cdot \rho_0 \mathbf{b} \, dV - \int_{\partial B} \delta \mathbf{u} \cdot \mathbf{t} \, dA = 0, \quad (1)$$

with $\underline{\sigma}$: mechanical stresses, $\mathbf{b}$: body forces and $\mathbf{t}$: tractions at the boundaries ∂B of the investigated body B . Since Eq. (1) is a non-linear function of the field variable $\mathbf{u}$, it has to be consistently linearised for the use within the FEM

$$G_{u,lin} = G_u(\delta \mathbf{u}, \mathbf{u}) + \Delta G_u(\delta \mathbf{u}, \mathbf{u}; \Delta \mathbf{u}). \quad (2)$$

After subsequent spatial discretisation into particular elements, one arrives at

$$\mathbf{K}_{F,u} \cdot \mathbf{u} = \mathbf{F}^{apl} - \mathbf{F}^{nr}. \quad (3)$$

At this, $\mathbf{K}_{F,u}$ is the tangent matrix obtained from the second term of Eq. (2), $\mathbf{F}^{nr}$ are the internal nodal forces and $\mathbf{F}^{apl}$ the applied external mechanical loading

$$\mathbf{F}^{nr} = \int_V \mathbf{B}_u^T : \underline{\sigma} \, dV, \quad (4)$$

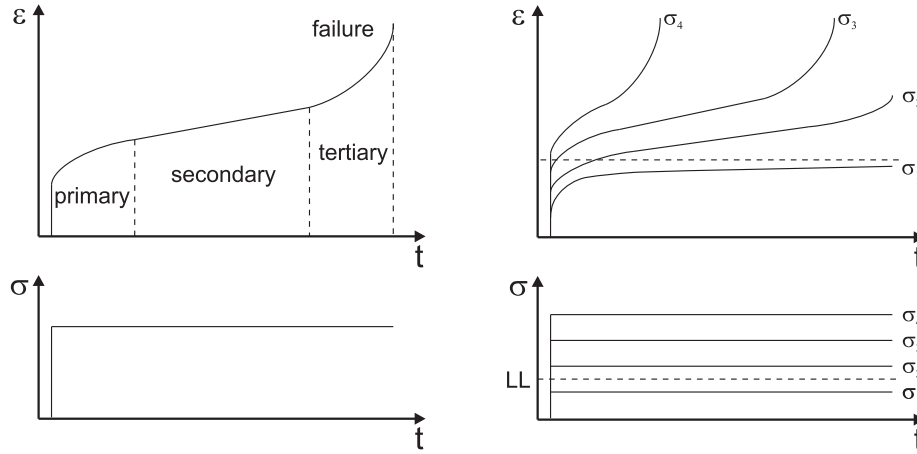


Fig. 1. Creep phases with dependency on stress level, respectively loading.

$$\mathbf{K}_{F,u} = \frac{\partial \mathbf{F}^{nr}}{\partial \mathbf{u}} = \int_V \mathbf{B}_u^T : \frac{\partial \boldsymbol{\sigma}}{\partial \mathbf{u}} dV = \int_V \mathbf{B}_u^T : \underline{\mathbf{C}} : \mathbf{B}_u dV, \quad (5)$$

with $\mathbf{B}_u$: operator matrix, $\underline{\mathbf{C}}$: mechanical material tangent. By means of the modelling approach presented subsequently, a physical content is given to the previous equations. At this, Voigt's notation is used for the representation of tensors.

2.3. Modelling of linear viscoelastic creep

2.3.1. State of the art

For the characterisation and modelling of the time-dependent behaviour of materials, numerous phenomenological approaches exist, which are frequently dealt with in literature and, thus, are only briefly described in the following. Basically, mechanical (rheological) models and purely mathematical models have to be differentiated. Mechanical models exhibit the advantage of simple physical interpretability, which is mostly difficult in case of purely mathematical approaches. In contrast, mechanical approaches often demand a large number of input quantities, which, in case of wood, have to be identified for the three material directions r , t and l as well as for the different loading types (compression, tension, shear).

In the framework of rheological approaches, all models composed of the basic elements “spring” (to capture the elastic behaviour) and “dashpot” (to capture the time-dependent features) can be characterised as linear viscoelastic. At this, the series of spring and dashpot is denoted as MAXWELL-element, whereas the parallel arrangement is called KELVIN-element. The generalised forms of both approaches are shown in Fig. 3 and are frequently used for the description of wooden structures (e.g. Bazant, 1985; Liu, 1994; Hering and Niemz, 2012).

A special case of the generalised KELVIN-model is the standard-solid body-model, which is characterised by the serial of a spring and one KELVIN-element. Connecting one KELVIN-element and one MAXWELL-element in series is called BURGERS-model. Both approaches are compared in Cai et al. (2002). While the standard-solid body-model sufficiently captures primary creep, the BURGERS-model, due to the serially added dashpot, shows a constant creep velocity and, thus, adequately describes the secondary creep phase (compare Section 2.1, Fig. 1).

Mathematical approaches, in contrast to the rheological models, may be described as pure approximation functions, that are calibrated by means of experimental results. Advantageous are the comparably low number of required input quantities and the good adaptability to the measured data. Amongst others, power law

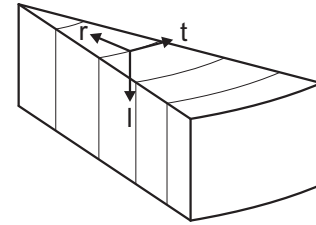


Fig. 2. Cylindrically anisotropic material directions of wood.

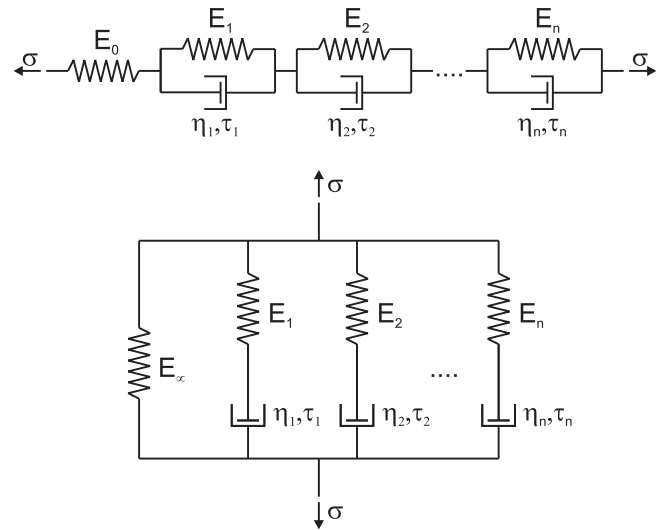


Fig. 3. Generalised KELVIN-model and generalised MAXWELL-model.

functions, exponential functions and logarithmic functions are used. Further information on this topic can e.g. be found in Schniewind and Barrett (1972), Gressel (1983), Bodig and Jayne (1993) and Cai et al. (2002).

2.3.2. Utilised modelling approach

With respect to the material characteristics of wood for loadings below LL, the following preconditions are assumed:

- Only primary creep exists.
- For $t \rightarrow \infty$, the time-dependent deformations tend to a creep limit.
- Linear viscoelastic deformations are fully reversible, no permanent deformations occur.

Due to the convenient physical interpretability of the model parameters, a rheological model is utilised for the simulation of the long-term behaviour. Since, furthermore, the work at hand aims at the application of as few as possible input properties (see above), the standard-solid body-model is used. This comparatively simple approach, which, in addition to the elastic properties, merely requires two material parameters, is preferred to the more complex generalised KELVIN- or MAXWELL-models, although the precise approximation of creep curves mostly demands a combination of several relaxation times τ (compare Fig. 3) of a wide spectrum of magnitudes (see e.g. Liu, 1994). However, since, as already mentioned, the work at hand aims at the development of a preferably general model with a large variety of applications, the advantage with respect to the input properties outweighs possible inaccuracies. In this context, the significant natural variability of the material properties of wood has to be kept in mind, which can for example be captured by means of the consideration of uncertain data (see e.g. Jenkel et al., 2011; Pannier et al., 2013).

For the subsequent investigations, all components of stress and strain are considered depending on time. The designation of the time-dependency is refrained from with regard to a clear notation.

2.3.2.1. Viscoelastic deformations and material tangent. Using the standard-solid body-model, time-dependency of material behaviour is captured by the KELVIN-element. Following Hanhijärvi and Mackenzie-Helnwein (2003) and Mackenzie-Helnwein and Hanhijärvi (2003), the respective stress-strain-relationship may be written as

$$\dot{\underline{\epsilon}}^{ve} + \frac{1}{\tau} \cdot \underline{\epsilon}^{ve} = \frac{1}{\tau} \cdot (\underline{\mathbf{C}}_1)^{-1} : \underline{\sigma}, \quad (6)$$

with $\underline{\mathbf{C}}_1$: material tensor of the three-dimensional, anisotropic KELVIN-spring. It can be seen that anisotropy in Hanhijärvi and Mackenzie-Helnwein (2003) and Mackenzie-Helnwein and Hanhijärvi (2003) is only considered with respect to the material properties of the spring. The relaxation time of the dashpot is taken as the same scalar value for all material directions. In the framework of the paper at hand, the generalisation of the approach with respect to the consideration of relaxation times depending on the material direction is obtained by means of the definition of tensor $\underline{\mathbf{T}}_f$

$$\underline{\mathbf{T}}_f = \begin{bmatrix} \frac{1}{\tau_{11}} & \frac{1}{\tau_{12}} & \frac{1}{\tau_{13}} & 0 & 0 & 0 \\ \frac{1}{\tau_{21}} & \frac{1}{\tau_{22}} & \frac{1}{\tau_{23}} & 0 & 0 & 0 \\ \frac{1}{\tau_{31}} & \frac{1}{\tau_{32}} & \frac{1}{\tau_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\tau_{44}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau_{55}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_{66}} \end{bmatrix}, \quad (7)$$

which leads to

$$\dot{\underline{\epsilon}}^{ve} + \underline{\mathbf{T}}_f : \underline{\epsilon}^{ve} = \underline{\mathbf{T}}_f : (\underline{\mathbf{C}}_1)^{-1} : \underline{\sigma}. \quad (8)$$

For utilisation in the framework of the FEM, a discretisation with respect to time is necessary (see also Hanhijärvi and Mackenzie-Helnwein, 2003; Mackenzie-Helnwein and Hanhijärvi, 2003). The time span to be investigated for this purpose is split into discrete time steps, whereas each investigated point of time t_{n+1} differs from the previous point of time t_n by the time increment Δt , $t_{n+1} = t_n + \Delta t$. Hence, the solution for time t_n is known, when the simulation at time t_{n+1} is carried out. Within time step Δt , a linear change of stresses is assumed.

Under definition of

$$\underline{\mathbf{E}}_t = \begin{bmatrix} e^{-T_{f,11} \cdot \Delta t} & e^{-T_{f,12} \cdot \Delta t} & e^{-T_{f,13} \cdot \Delta t} & 0 & 0 & 0 \\ e^{-T_{f,21} \cdot \Delta t} & e^{-T_{f,22} \cdot \Delta t} & e^{-T_{f,23} \cdot \Delta t} & 0 & 0 & 0 \\ e^{-T_{f,31} \cdot \Delta t} & e^{-T_{f,32} \cdot \Delta t} & e^{-T_{f,33} \cdot \Delta t} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-T_{f,44} \cdot \Delta t} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-T_{f,55} \cdot \Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-T_{f,66} \cdot \Delta t} \end{bmatrix}, \quad (9)$$

$$\underline{\mathbf{Q}} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

and

$$\tilde{\underline{\mathbf{E}}}_t = \underline{\mathbf{Q}} - \underline{\mathbf{E}}_t \quad (11)$$

the solution of the inhomogeneous, linear differential equation at time t_{n+1} leads to the stress-strain-relationship for the linear viscoelastic deformations of the KELVIN-element

$$\underline{\epsilon}_{n+1}^{ve} = \underline{\mathbf{E}}_t : \underline{\epsilon}_n^{ve} + \left(-\underline{\mathbf{E}}_t + \frac{1}{\Delta t} \cdot \tilde{\underline{\mathbf{E}}}_t : \underline{\mathbf{T}}_f^{-1} \right) : (\underline{\mathbf{C}}_1)^{-1} : \underline{\sigma}_n + (\underline{\mathbf{C}}_1)^{-1} : \underline{\sigma}_{n+1} - \frac{1}{\Delta t} \cdot \tilde{\underline{\mathbf{E}}}_t : \underline{\mathbf{T}}_f^{-1} : (\underline{\mathbf{C}}_1)^{-1} : \underline{\sigma}_{n+1}. \quad (12)$$

From the total differential of Eq. (12), the material tangent $\underline{\mathbf{C}}_{n+1}^{ve}$ of the KELVIN-element at time $n+1$ is derived

$$d\underline{\epsilon}_{n+1}^{ve} = (\underline{\mathbf{C}}_{n+1}^{ve})^{-1} : d\underline{\sigma}_{n+1}, \quad (13)$$

$$(\underline{\mathbf{C}}_{n+1}^{ve})^{-1} = (\underline{\mathbf{C}}_1)^{-1} - \frac{1}{\Delta t} \cdot \tilde{\underline{\mathbf{E}}}_t : \underline{\mathbf{T}}_f^{-1} : (\underline{\mathbf{C}}_1)^{-1}. \quad (14)$$

Finally, Eqs. (13) and (14) have to be supplemented with respect to the consideration of the serial spring necessary for the standard-solid body-model. Concerning the determination of stresses and strains, we know

$$\underline{\epsilon} = \underline{\epsilon}^{el} + \underline{\epsilon}^{ve} \quad (15)$$

and

$$\underline{\sigma} = \underline{\sigma}^{el} = \underline{\sigma}^{ve}. \quad (16)$$

Using Hooke's law of elasticity, the conditional equation for the stresses reads as

$$\underline{\sigma} = \underline{\mathbf{C}}^{el} : (\underline{\epsilon} - \underline{\epsilon}^{ve}), \quad (17)$$

with $\underline{\mathbf{C}}^{el}$: elasticity tensor. The combination of Eqs. (17) and (12) and rearrangement with respect to $\underline{\epsilon}_{n+1}^{ve}$ leads to the linear viscoelastic deformations of the standard-solid body-model at time $n+1$

$$\underline{\epsilon}_{n+1}^{ve} = \tilde{\underline{\mathbf{D}}} : \left(\underline{\mathbf{E}}_t : \underline{\epsilon}_n^{ve} + \left(-\underline{\mathbf{E}}_t + \underline{\mathbf{A}} \right) : \underline{\gamma} : (\underline{\epsilon}_n - \underline{\epsilon}_n^{ve}) + \underline{\mathbf{B}} : \underline{\epsilon}_{n+1} \right), \quad (18)$$

with

$$\underline{\gamma} = (\underline{\mathbf{C}}_1)^{-1} : \underline{\mathbf{C}}^{el}, \quad (19)$$

$$\underline{\mathbf{A}} = \frac{1}{\Delta t} \cdot \tilde{\underline{\mathbf{E}}}_t : \underline{\mathbf{T}}_f^{-1}, \quad (20)$$

$$\underline{\mathbf{B}} = (\underline{\mathbf{Q}} - \underline{\mathbf{A}}) : \underline{\gamma}, \quad (21)$$

$$\underline{\underline{\mathbf{D}}} = \begin{bmatrix} \frac{1}{1+B_{11}} & \frac{1}{1+B_{12}} & \frac{1}{1+B_{13}} & 0 & 0 & 0 \\ \frac{1}{1+B_{21}} & \frac{1}{1+B_{22}} & \frac{1}{1+B_{23}} & 0 & 0 & 0 \\ \frac{1}{1+B_{31}} & \frac{1}{1+B_{32}} & \frac{1}{1+B_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{1+B_{44}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{1+B_{55}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{1+B_{66}} \end{bmatrix}. \quad (22)$$

The linear elastic-viscoelastic material tangent $\underline{\underline{\mathbf{C}}}_{n+1}^{elve}$ of the standard-solid body-model is achieved by means of the inverse summation of the parts of spring and KELVIN-element

$$\left(\underline{\underline{\mathbf{C}}}_{n+1}^{elve}\right)^{-1} = \left(\underline{\underline{\mathbf{C}}}^{el}\right)^{-1} + \left(\underline{\underline{\mathbf{C}}}_{n+1}^{ve}\right)^{-1}. \quad (23)$$

2.3.2.2. Algorithmic implementation. The consideration of anisotropic material properties for the description of linear viscoelastic material behaviour is not possible offhand. Hence, in the framework of algorithmic implementation of the equations derived before, the additive split of tensors proposed in [Holzapfel and Gasser \(2001\)](#) and applied e.g. in [Schmidt \(2009\)](#) is utilised.

The tensors $\underline{\underline{\mathbf{A}}}, \underline{\underline{\mathbf{B}}}, \underline{\underline{\mathbf{E}}}, \underline{\underline{\mathbf{D}}}, \underline{\underline{\mathbf{g}}}, \underline{\underline{\mathbf{C}}}^{el}$ and $\left(\underline{\underline{\mathbf{C}}}_1\right)^{-1}$, which all exhibit the structure

$$\underline{\underline{\mathbf{X}}} = \begin{bmatrix} a & g & i & 0 & 0 & 0 \\ g & b & h & 0 & 0 & 0 \\ i & h & c & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & f \end{bmatrix}, \quad (24)$$

for this purpose are additively split into the sparsely populated tensors $\underline{\underline{\mathbf{X}}}_a, \underline{\underline{\mathbf{X}}}_b, \underline{\underline{\mathbf{X}}}_c, \underline{\underline{\mathbf{X}}}_d, \underline{\underline{\mathbf{X}}}_e, \underline{\underline{\mathbf{X}}}_f, \underline{\underline{\mathbf{X}}}_g, \underline{\underline{\mathbf{X}}}_h$ and $\underline{\underline{\mathbf{X}}}_i$. In the following, these tensors are denoted as $\underline{\underline{\mathbf{X}}}_m$. The components $m = a, b, c, \dots, i$ of the original tensors, thus, are considered separately to be able to allocate independent viscoelastic properties.

For each part m , the viscoelastic deformations $\underline{\underline{\mathbf{e}}}_{n+1,m}^{ve}$ and the material tangent $\left(\underline{\underline{\mathbf{C}}}_{n+1,m}^{elve}\right)^{-1}$ are determined. Finally, the particular solutions are summarised

$$\underline{\underline{\mathbf{e}}}_{n+1}^{ve} = \sum_{m=1}^9 \underline{\underline{\mathbf{e}}}_{n+1,m}^{ve} \quad (25)$$

$$\left(\underline{\underline{\mathbf{C}}}_{n+1}^{elve}\right)^{-1} = \sum_{m=1}^9 \left(\underline{\underline{\mathbf{C}}}_{n+1,m}^{elve}\right)^{-1} \quad (26)$$

and the stresses at time $n+1$ are calculated

$$\underline{\underline{\sigma}}_{n+1} = \underline{\underline{\mathbf{C}}}^{el} : \left(\underline{\underline{\mathbf{e}}}_{n+1} - \underline{\underline{\mathbf{e}}}_{n+1}^{ve}\right). \quad (27)$$

2.3.3. Identification of material parameters

Following [Niemz \(1993\)](#), a decrease of creep deformations is observed, when the density of the investigated piece of wood increases. Additionally, a nearly linear correlation between the reciprocal value of the short-term YOUNG's modulus E^{el} and occurring creep deformations and creep velocities is reported for mechanical loadings below LL (see e.g. [Becker, 2002](#); [Hanhijärvi, 1997](#); [Mohager and Toratti, 1993](#)). Accordingly, linear viscoelastic material properties of wood may be expressed subject to the short-term properties. With respect to the work at hand, the moisture-dependent short-term characteristics, namely the elastic properties and strength values depending on loading type and material direction, have been determined in the framework of

previous investigations for several wood species (see e.g. [Saft and Kaliske, 2011, 2012, 2013](#)) and may be taken as known. Hence, merely the additional time-dependent features have to be identified.

For the description of the time-dependent deformation behaviour of materials, the definition of a time-dependent creep-number $\varphi(t)$ as relation between time-dependent and instantaneous elastic deformations is possible

$$\varphi(t) = \frac{\varepsilon(t) - \varepsilon^{el}}{\varepsilon^{el}}. \quad (28)$$

Presuming the existence of a creep limit, a time-independent creep-number φ_{∞} may be defined, specifying the ratio of maximum linear viscoelastic and elastic deformations

$$\varphi_{\infty} = \frac{\varepsilon_{\infty}^{ve}}{\varepsilon^{el}}, \quad E_{\infty} = \frac{E^{el}}{1 + \varphi_{\infty}}. \quad (29)$$

Due to the convenient physical interpretability of this value, φ_{∞} in addition to the relaxation time τ is used in the following for the description of the creep curve.

Under consideration of the above mentioned linear relationship between elastic and linear viscoelastic deformations, φ_{∞} according to Eq. (29) acts as constant of proportionality and is defined as constant value depending on loading type and material direction. Hence, the influence of wood species on the linear viscoelastic deformations is covered by the specific elastic properties.

2.3.3.1. Determination of $\underline{\underline{\mathbf{C}}}_1$. Eqs. (18) and (23) derived above do not require the creep-numbers φ_{∞} , but rather the material tensor $\underline{\underline{\mathbf{C}}}_1$ of the KELVIN-spring as input quantity. The conversion succeeds with the help of the solution of the KELVIN-element's differential equation in the one-dimensional case for a constant loading σ (E^{ve} : YOUNG's modulus of the KELVIN-spring)

$$\varepsilon^{ve} = \frac{\sigma}{E^{ve}} \cdot \left(1 - e^{-\frac{t}{\tau}}\right). \quad (30)$$

While the bracketed term of Eq. (30) captures the time-dependent features of the creep function, the first term represents the asymptote and, thus, the creep limit with $\varepsilon_{\infty}^{ve} = \frac{\sigma}{E^{ve}}$. Since $\lim_{t \rightarrow \infty} (1 - e^{-\frac{t}{\tau}}) = 1$, Eq. (15) at time $t = \infty$ reads as

$$\varepsilon_{\infty} = \frac{\sigma}{E^{el}} + \frac{\sigma}{E^{ve}}. \quad (31)$$

The combination with Eq. (29) leads to $E^{ve} = \frac{E^{el}}{\varphi_{\infty}}$. Consequently, the material tensor $\underline{\underline{\mathbf{C}}}_1$ is defined by

$$\underline{\underline{\mathbf{C}}}_1 = \begin{bmatrix} \frac{C_{11}^{el}}{\varphi_{\infty,11}} & \frac{C_{12}^{el}}{\varphi_{\infty,12}} & \frac{C_{13}^{el}}{\varphi_{\infty,13}} & 0 & 0 & 0 \\ \frac{C_{21}^{el}}{\varphi_{\infty,21}} & \frac{C_{22}^{el}}{\varphi_{\infty,22}} & \frac{C_{23}^{el}}{\varphi_{\infty,23}} & 0 & 0 & 0 \\ \frac{C_{31}^{el}}{\varphi_{\infty,31}} & \frac{C_{32}^{el}}{\varphi_{\infty,32}} & \frac{C_{33}^{el}}{\varphi_{\infty,33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{44}^{el}}{\varphi_{\infty,44}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{C_{55}^{el}}{\varphi_{\infty,55}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{66}^{el}}{\varphi_{\infty,66}} \end{bmatrix}. \quad (32)$$

2.3.3.2. Quantification of the material properties. The quantification of $\varphi_{\infty,ij}$ and τ_{ij} , as far as possible, is carried out by means of the re-calculation of experimental results available in literature. However, the by far largest part of experimental data bases on bending experiments, which, as a combination of compression and tension, is not sufficient for the identification of the required material

Table 2
Creep-numbers and relaxation times for linear viscoelasticity.

		References	$\varphi_{\infty,ij}$ [–]	τ_{ij} [s]
Tension	rad	Hayashi et al. (1993), Ozyhar et al. (2013) and Ranz (2008)	0.45	10,000
	tan	Schniewind and Barrett (1972), Ozyhar et al. (2013) and Ranz (2008)	0.45	10,000
	long	Schniewind and Barrett (1972), Hayashi et al. (1993), Ozyhar et al. (2013) and Ranz (2008)	0.12	12,000
Compression	rad	Ozyhar et al. (2013) and Ranz (2008)	0.6	8500
	tan	Ozyhar et al. (2013) and Ranz (2008)	0.6	8500
	long	Ozyhar et al. (2013)	0.15	20,000
Shear	rt	– ^a	0.3	15,000
	tl	Schniewind and Barrett (1972)	0.3	15,000
	rl	Hayashi et al. (1993)	0.3	15,000

^a no literature source known for rolling shear, assumption of equal values as for longitudinal shear

properties. In Table 2, the identified quantities and the respective references are indicated. It has to be noted, that mean values are always used, when material properties are indicated in the framework of the paper at hand. The same holds for the second part.

2.3.3.3. Anisotropy. Considering the anisotropic material tensor, merely the entries of the primary diagonal have been discussed until now. However, since mechanical loading in a particular material direction also yields creep deformation perpendicular to the loading direction, the secondary diagonal components of the tensor have to be given thought to.

In the framework of experimental investigations, e.g. Hayashi et al. (1993) and Ozyhar et al. (2013), POISSON'S ratios, which originally were defined as constant values of the ratio of deformations perpendicular to the loading direction and deformations in loading direction, are found to depend on time. At this, the type of time-dependency differs with respect to the loading type. The absolute value of creep compliance (ratio of creep deformation and loading) decreases in tension and increases in compression (see Ozyhar et al., 2013), which leads to negative creep-numbers $\varphi_{\infty,ij}$. Tentative simulations using the negative values for compressive loading generated unphysical, dynamic responses of the investigated system. Thus, negative values of $\varphi_{\infty,ij}$ are not applicable in the framework of numerical simulations. It becomes obvious, that the appropriate description of the time-dependent transverse deformation behaviour of wooden structures, in experimental as well as in theoretical respect, requires fundamental and profound research. Further information on this topic can e.g. be found in Hilton (2001).

In the scope of the work at hand, secondary diagonal entries are ascribed the time-dependent properties of the respective quantities of the primary diagonal

$$\varphi_{\infty,12} = \varphi_{\infty,21} = \varphi_{\infty,22}, \quad \tau_{12} = \tau_{21} = \tau_{22}, \quad (33)$$

$$\varphi_{\infty,13} = \varphi_{\infty,31} = \varphi_{\infty,23} = \varphi_{\infty,32} = \varphi_{\infty,33}, \quad \tau_{13} = \tau_{31} = \tau_{23} = \tau_{32} = \tau_{33}. \quad (34)$$

2.4. Modelling of non-linear viscoelastic–viscoplastic creep

Wooden structures subjected to mechanical loadings larger than LL may not longer be described by means of the theory of linear viscoelasticity. Rather, due to the dependency of the creep curve on SL (see Fig. 1) non-linear viscoelastic behaviour and,

due to the occurrence of irreversible deformations, viscoplastic behaviour is observed. Following Gressel (1983), deformations resulting from loadings beyond LL are fully irreversible, whereas, following e.g. Becker (2002), those deformations are still partly reversible (increasing amount of plastic deformations with increasing SL).

2.4.1. State of the art

Using the creep functions $Y_i(t)$, non-linear time-dependent materials may be described by means of the GREEN–RIVLIN–approach (e.g. cited in Navi and Stanzl-Tschegg, 2009)

$$\begin{aligned} \varepsilon(t) = & \int_0^t Y_1(t-s_1) \cdot \frac{\partial \sigma(s_1)}{\partial s_1} ds_1 \\ & + \int_0^t \int_0^t Y_2(t-s_1, t-s_2) \cdot \frac{\partial \sigma(s_1)}{\partial s_1} \frac{\partial \sigma(s_2)}{\partial s_2} ds_1 ds_2 \\ & + \int_0^t \int_0^t \int_0^t Y_3(t-s_1, t-s_2, t-s_3) \\ & \cdot \frac{\partial \sigma(s_1)}{\partial s_1} \frac{\partial \sigma(s_2)}{\partial s_2} \frac{\partial \sigma(s_3)}{\partial s_3} ds_1 ds_2 ds_3 \\ & + \dots \end{aligned} \quad (35)$$

The experimental identification of the creep functions, however, is extremely difficult. Thus, in Liu (1994) (first proposed in Schapery, 1966) a non-linear modelling approach of the form

$$\varepsilon(t) = \frac{g_0 \cdot \sigma}{E} + \int_0^t Y'(t-s) \cdot \frac{\partial(g \cdot \sigma)}{\partial s} ds \quad (36)$$

is utilised for the simulation of the long-term behaviour of wood depending on SL and MC. At this, Y' is denoted as transient part of the creep function, $g = g(\sigma)$ and $g_0 = g_0(\sigma)$ are stress-dependent functions to capture the non-linear viscoelastic, respectively the non-linear elastic behaviour. An approach of this type is also introduced in Toratti (1992), where the g -function is directly related to the damage function $\alpha(t)$, basing on the damage accumulation concept (see Section 2.5.1).

In contrast to the before mentioned publications, which use non-linearity factors, that, following Becker (2002) do not adequately capture the second creep phase, a different strategy is pursued in Becker (2002). The linear model acting as starting point is additively extended by a non-linear stress term

$$\sigma_{nl}(t) = \int_0^t \vartheta_{nl} ds, \quad (37)$$

with

$$\begin{aligned} \vartheta_{nl} = & \pm A_{nl} \cdot (\sigma - \sigma_{VP})^{B_{nl}}, & \sigma > \sigma_{VP} \\ \vartheta_{nl} = & 0, & \sigma \leq \sigma_{VP}. \end{aligned} \quad (38)$$

The parameter σ_{VP} , at this, describes the moisture dependent limit of linearity. At constant stress, the additive term may be interpreted as a non-linear dashpot, which is activated, when SL exceeds LL. In Chassagne et al. (2006), non-linear viscoelasticity of wood is captured by means of a generalised MAXWELL-model with non-linear dashpots, depending on SL.

Non-linearity in terms of cracking is dealt with in Chazal and Dubois (2001). For the prediction of crack growth, viscoelastic stress intensity factors are related to viscoelastic opening displacement intensity factors by means of incremental fracture equations.

With respect to rheological models, approaches containing SAINT–VENANT-elements can a priori be characterised as non-linear approaches, since this element does not exhibit deformations until a certain yield strength σ_y is reached, respectively exceeded. Hence, the model behaviour depends on SL. The simplest model

of this type is the series of spring and SAINT-VENANT-element, which is used for the description of ideal elastic–plastic material behaviour. The parallel arrangement of dashpot and SAINT-VENANT-element represents a plastic-viscous liquid and is denoted as BINGHAM-model. For loadings below σ_y , the BINGHAM-model shows no deformation. When loading exceeds the yield strength, due to the dashpot, deformations increase with time.

2.4.2. Utilised modelling approach

To capture the material behaviour described above in the framework of numerical simulations, the improvement of the standard-solid body-model is required. At this, the following possibilities are conceivable:

- Additive extension by a dashpot (superposition principle applicable)
 - linear dashpot: irreversible deformations for each SL (BURGERS-model, e.g. used in Cai et al. (2002), Welling (1987) and Wang et al. (2012)),
 - non-linear dashpot: irreversible deformations depending on SL (e.g. used in Becker, 2002, see Section 2.4.1),
- Additive extension by a BINGHAM-element (e.g. used in Zienkiewicz and Corneau, 1974, superposition principle not applicable)
 - no irreversible deformation parts below the yield strength σ_y of the SAINT-VENANT-element,
 - equates to a non-linear dashpot with $\eta^{vp} = 0$ when $\sigma \geq \sigma_y$,
 - for $\eta^{vp} = \text{const.}$, a linearly increasing deformation part is added which captures secondary creep,
- extension of the formulation by a non-linear stress-function (Schapery, 1966, e.g. used in Liu, 1994; Toratti, 1992; Chazal and Dubois, 2001)
 - despite consideration of non-linear behaviour, entirely reversible deformations.

Since, in the framework of the paper at hand, a well-structured formulation with as few as possible input properties is aimed at, the extension of the standard-solid body-model by a further rheological element is obvious. Due to the distinct physical interpretability and the conformity of the model properties to the real material behaviour of wood, the standard-solid body-model is additively extended by a BINGHAM-element. For each material direction, consequently, the yield strength σ_y of the SAINT-VENANT-element and the viscoplastic stiffness η^{vp} of the dashpot are required to characterise the material behaviour. The extended standard-solid body-model is shown in Fig. 4 for the one-dimensional case.

For the entire anisotropic model, with respect to stresses and strains it is known

$$\underline{\epsilon} = \underline{\epsilon}^{el} + \underline{\epsilon}^{ve} + \underline{\epsilon}^{vp}, \quad (39)$$

$$\underline{\sigma} = \underline{\sigma}^{el} = \underline{\sigma}^{ve} = \underline{\sigma}^{vp}. \quad (40)$$

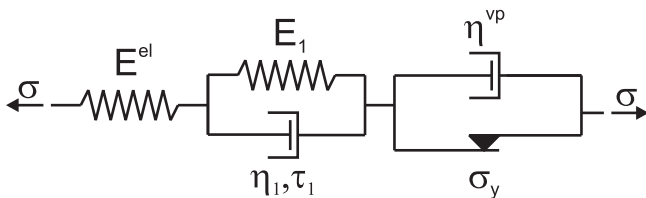


Fig. 4. Extended standard-solid body-model (one-dimensional).

Within the BINGHAM-element, the total stress $\underline{\sigma}^{vp}$ is composed of a SAINT-VENANT and a dashpot part

$$\underline{\sigma}^{vp} = \underline{\sigma} = \underline{\sigma}_d + \underline{\sigma}_{sv}, \quad (41)$$

the deformations of both parts are identical

$$\underline{\epsilon}^{vp} = \underline{\epsilon}_d^{vp} = \underline{\epsilon}_{sv}^{vp}. \quad (42)$$

Basing on the constitutive equation for the one-dimensional dashpot, the stresses $\underline{\sigma}_d$ are obtained from the viscoplastic stiffness tensor $\underline{\eta}^{vp}$ and the rate $\underline{\dot{\epsilon}}^{vp}$ of viscoplastic deformations

$$\underline{\sigma}_d = \underline{\eta}^{vp} : \underline{\dot{\epsilon}}^{vp}, \quad (43)$$

$$\underline{\eta}^{vp} = \begin{bmatrix} \eta_{11}^{vp} & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_{22}^{vp} & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{33}^{vp} & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_{44}^{vp} & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_{55}^{vp} & 0 \\ 0 & 0 & 0 & 0 & 0 & \eta_{66}^{vp} \end{bmatrix}. \quad (44)$$

Due to the application of a SAINT-VENANT-element, as mentioned above, the extended standard-solid body-model has to be characterised as non-linear approach. Analogous to short-term plasticity formulations (see e.g. Schmidt, 2009; Saft and Kaliske, 2011; Resch, 2001), the viscoplastic deformation rate has to be determined by means of an evolution equation for plasticity. This is solved numerically according to the “General Return-Mapping”-algorithm introduced in Simo and Hughes (1998) and e.g. used in Resch (2001). For the sake of a concise description, the algorithm for the model at hand is described in Section 2.5.2. Corresponding to approaches in the framework of short-term plasticity, the formulation bases on the KUHN-TUCKER-conditions using the consistency parameter $\Delta\gamma$ and the yield criterion $f(\underline{\sigma}_{sv})$

$$f(\underline{\sigma}_{sv}) \leq 0, \quad \Delta\gamma \geq 0, \quad f(\underline{\sigma}_{sv}) \cdot \Delta\gamma = 0 \quad (45)$$

combined with an associated flow rule

$$\underline{\dot{\epsilon}}^{vp} = \Delta\gamma \cdot \frac{\partial f(\underline{\sigma}_{sv})}{\partial \underline{\sigma}_{sv}}. \quad (46)$$

Additionally, the consistency condition

$$\Delta\gamma \cdot \dot{f} = 0 \quad (47)$$

is utilised.

2.4.2.1. Yield criterion. Next step is the definition of the yield criterion $f(\underline{\sigma}_{sv})$. This, in the scope of the work at hand, has got the function to model the limit of linearity and, thus, the limit between linear viscoelastic and non-linear viscoelastic-viscoplastic material behaviour. Limits of linearity exist for all types and directions of loading, whereas the values for tension and compression significantly differ (see e.g. Gressel, 1983). Hence, the yield criterion to be defined has to consider all types of loading and to capture the difference between tension and compression.

Although valid for all loading directions, the well known TSAI-WU single-surface yield criterion only partly meets the requirements, since, in case of the normal directions (as a distinction to the shear directions), the yield limits for tensile and compressive loading are considered in combination and, thus, tensile and compressive loadings are not distinguished. The multi-surface yield criterion according to Resch (2001) (see also Saft and Kaliske, 2011), due to the restriction to compressive loadings, differs between the loading types but is not defined for tensile and shear loading.

In case of the model at hand, as a combination of both approaches, the extended multi-surface yield criterion according to Resch (2001) is introduced. The function

$$f = \underline{\sigma}_{sv} : \underline{b} : \underline{\sigma}_{sv} - 1 \quad (48)$$

matches the approach of Resch (2001), when post-collapse hardening is disregarded. Deviating from Resch (2001), the strength tensor $\underline{b}$ is defined as

$$\underline{b} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{f_{v,rt}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{f_{v,tl}^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{f_{v,rl}^2} \end{bmatrix}, \quad (49)$$

with

$$b_{11} = \begin{cases} \frac{1}{f_{t,r}^2}, & \sigma_{sv,11}^{vp} \geq 0, \\ \frac{1}{f_{c,r}^2}, & \sigma_{sv,11}^{vp} < 0, \end{cases} \quad (50)$$

$$b_{22} = \begin{cases} \frac{1}{f_{t,t}^2}, & \sigma_{sv,22}^{vp} \geq 0, \\ \frac{1}{f_{c,t}^2}, & \sigma_{sv,22}^{vp} < 0, \end{cases} \quad (51)$$

$$b_{33} = \begin{cases} \frac{1}{f_{t,l}^2}, & \sigma_{sv,33}^{vp} \geq 0, \\ \frac{1}{f_{c,l}^2}, & \sigma_{sv,33}^{vp} < 0. \end{cases} \quad (52)$$

The values $\bar{f}$ of limit strength result from the product of the short-term strength f and the limit of linearity LL

$$\bar{f} = f \cdot LL, \quad (53)$$

$f_{c,i}$: compressive strength, $f_{t,i}$: tensile strength, $f_{v,ij}$: shear strength.

At this point, an anticipation to Part II of the publication is necessary. In spite of the type of loading, LL is also dependent on the magnitude m of MC (see e.g. Gressel, 1983). As extensively dealt with in Saft and Kaliske (2011, 2012, 2013), the same holds for the values of short-term strength. Since moisture-dependency shall be captured in the framework of the overall model, the values of limit strength are defined depending on MC already at this point.

Basing on bibliographical references, an exponential function is derived in Becker (2002) for the description of $\bar{f}(m)$. Since, for $m \rightarrow 0$, the exponential approach tends to ∞ , it is inconvenient for numerical simulations. In the framework of the model at hand, the approach of Becker (2002) is approximated by means of a quadratic function, which is also utilised for the description of the moisture-dependent strength values (see Saft and Kaliske, 2011, 2012, 2013, Table 8)

Table 4

Input parameters for viscoplastic stiffness $\eta_{kk,SL}^{vp}$ according to Eq. (56).

References			$\eta_{kk,start}^{vp} \left[\frac{N \cdot s}{mm^2} \right]$	$a_1 [-]$	$b_1 [-]$
Tension	rad/tan	Aicher et al. (1998)	$3.2 \cdot 10^7$	0.2 ^a	6 ^a
	long	Bach and Pentoney (1968), Bhatnagar (1964) and Soltis et al. (1989)	$1 \cdot 10^{10}$	0.2	6
Compression	rad/tan	– ^b	$1.024 \cdot 10^8$	0.2	6
	long	Keith (1972), Kingston and Budgen (1972) and Soltis et al. (1989)	$3.2 \cdot 10^{10}$	0.2	6
Shear	rt/tl/rl	– ^c	$3.2 \cdot 10^7$	0.2	6

^a experimental results only for one SL, assumption of equal dependency on SL as longitudinal tension

^b no usable data known, determination of $\eta_{kk,start}^{vp}$ assuming $\frac{\eta_{t,1}^{vp}}{\eta_{t,1}^{comp}} \approx \frac{\eta_{c,1}^{vp}}{\eta_{c,1}^{comp}}$

^c no usable data known, assumption of similar behaviour as tension perpendicular to grain

Table 3

Coefficients for moisture-dependent limits of linearity according to Eq. (54).

	Compression	Tension
$a[-]$	25	16
$b[-]$	–10	–7
$c[-]$	1.32	1.25

$$LL(m) = a \cdot m^2 + b \cdot m + c. \quad (54)$$

Consequently, the values of limit strength depending on type and direction of loading read as

$$\bar{f}(m) = f(m) \cdot LL(m). \quad (55)$$

The parameters a, b and c for tensile and compressive loading, determined basing on Becker (2002), are specified in Table 3. The original function of Becker (2002) has been derived for loading in longitudinal material direction. Since only few references are available for normal loading perpendicular to the grain and no reference is known to the authors considering shear loading, equal limits of linearity are assumed for all material directions in the following. For shear loading, as the less favourable case, the values for compressive loading are applied.

2.4.2.2. Quantification of the viscoplastic stiffness. In addition to LL , the tensor of viscoplastic stiffnesses $\underline{\eta}^{vp}$ is required as input property for the extended standard-solid body-model. When loading exceeds LL , the portion of secondary creep increases with increasing SL and the slope of the creep curve raises (compare Fig. 1). Transferred to the extended standard-solid body-model, the viscoplastic stiffness $\eta_{kk,SL}^{vp}$ for each material direction has to decrease with increasing SL . This is also proofed by means of the re-calculation of respective test series, e.g. Keith, 1972.

Unfortunately, only few useful references are available for the characterisation and quantification of $\eta_{kk,SL}^{vp}$. For this reason, a linear approximation-function would be sufficient. Nevertheless, this type of function exhibits the disadvantage, that negative values of $\eta_{kk,SL}^{vp}$ are calculated for high values of SL , which is physically unreasonable. To avoid this, the arctangent-function is utilised for the description of $\eta_{kk,SL}^{vp}$

$$\eta_{kk,SL}^{vp} = \eta_{kk,start}^{vp} \cdot \left[a_1 + \frac{1-a_1}{2} - \frac{1-a_1}{\pi} \cdot \arctan((SL-0.5) \cdot b_1) \right]. \quad (56)$$

At this, $\eta_{kk,start}^{vp}$ is the upper bound of the function, the lower bound is determined by a_1 as $a_1 \cdot \eta_{kk,start}^{vp}$. The inflection point of the curve is located at $SL = 0.5$, the parameter b_1 specifies the dilation in x-direction. In Table 4, the parameters determined by means of the re-calculation of experimental results are presented depending on type and direction of loading.

2.5. Modelling of creep failure

In case of loadings above LL, wooden structures are subjected to creep failure after a sufficiently long time, respectively at a sufficiently high SL (compare Fig. 1). Furthermore, wooden specimens tested after unloading from an endurance stress exhibit a lower strength compared to the initial short-term value. This phenomenon is denoted as duration-of-load-effect (DOL-effect). The time, a structure tolerates an endurance stress until failure occurs is called time to failure t_f .

In spite of the obvious relationship between creep failure and DOL-effect, both phenomena have been treated separately for a long time. While the deformation history at a given SL has priority with respect to creep, the focus considering the DOL-effect mainly lies at the determination of t_f . Nowadays, there is agreement among experts, that creep and DOL-effect have the same origin, are closely related and should be treated in combination (see e.g. Becker, 2002; Wang et al., 2012; Smith et al., 2003).

2.5.1. State of the art

Most approaches for the description of the DOL-effect base on the damage accumulation concept. At this, damage accumulating with time of loading is captured by means of the state variable α , which exhibits values between zero (no damage) and one (failure). The relationship between rate of damage $\frac{\partial \alpha}{\partial t}$ and applied loading is described by means of an empirical approximation function, that does not necessarily capture the physical reasons of damage accumulation. A summary of different damage accumulation models can e.g. be found in Smith et al. (2003).

For the use of most damage accumulation models, numerous empirical model parameters are required. To consider moisture-dependency, the original formulation has to be extended by additional terms (see e.g. Toratti, 1992). Following Becker (2002), using damage accumulation models in combination with a creep simulation leads to increasing deformation rates before creep failure occurs. Thus, tertiary creep is captured. Suchlike approaches are e.g. published in Wang et al., 2012 (combination with BURGERS-model) and Toratti (1992) (combination with g-function, see Section 2.4.1).

A fracture-mechanically motivated approach for the description of the DOL-effect is the theory of “Damaged Viscoelastic Material” (DVM-theory), according to Fuglsang Nielsen (1996). The DVM-theory is a method for the prediction of residual strength and time to failure of viscoelastic materials. Prerequisite is a creep function matching with a power law approach, which, in case of wood, is only true for primary creep. As input properties, creep characteristics at the microscopic crack tip and the strength of the uncracked material are required. Following Smith et al. (2003), the DVM-theory, in spite of its complexity, does not entirely capture the fatigue behaviour of wood. However, in contrast to damage accumulation models, the theory exhibits a profound theoretical basis.

A promising approach for the modelling of creep failure is the concept of strain-energy density, which was initially studied in Fridley et al. (1992) and e.g. in Becker (2002) and Chassagne et al. (2006) successfully utilised for the simulation of creep failure. Basis is the strain-energy density $e(t)$ for the one-dimensional case

$$e(t) = \int_0^t \sigma \cdot \dot{\epsilon} dt. \quad (57)$$

In the course of a numerical simulation, $e(t)$ is updated after each calculation step and compared to a critical value. Creep failure occurs as soon as the critical value is exceeded. For the identification of the critical strain-energy density e_{crit} , one has to distinguish between the particular types of loading. According to Becker, 2002,

assuming ideal elastic-brittle material behaviour, e_{crit} due to tensile loading may be determined by means of the short-term properties

$$e_{crit,t} = \frac{1}{2} \cdot \frac{f_t^2}{E}. \quad (58)$$

Since failure behaviour of wood is equal for tensile and shear loading (compare Saft and Kaliske, 2013), the same holds for $e_{crit,v}$

$$e_{crit,v} = \frac{1}{2} \cdot \frac{f_v^2}{G}. \quad (59)$$

The determination of $e_{crit,c}$ using the short-term properties, following Becker, 2002, is difficult due to the pronounced plastifiability of wood in compression. Basing on the re-calculation of different test series, the empirical value of

$$e_{crit,c} = 4 \cdot \frac{f_c^2}{E} \quad (60)$$

is identified. The advantage of the concept of strain-energy density lies in the modelling of failure in direct relation to the occurring deformations. Moreover, the consideration of the influence of MC is possible without changing the approach. However, according to Becker (2002), tertiary creep cannot be simulated using the concept of strain-energy density.

2.5.2. Utilised modelling approach

Using the extended standard-solid body-model, until now, allows the simulation of linear viscoelastic and non-linear viscoelastic-viscoplastic material behaviour. At this, the creep curve exhibits primary and secondary parts, depending on SL. The final step in the course of the development of a creep model for wood at constant climate is the enhancement of the approach with respect to the consideration of tertiary creep and creep failure. For that purpose, the concept of strain-energy density is applied, since no additional input properties are required in that case (see above). The disadvantage referred to in Becker (2002), that tertiary creep is not captured, is eliminated in the scope of the work at hand by means of the modification of the viscoplastic stiffness $\eta_{kk,SL}^{vp}$ using again an arctangent-function

$$\eta_{kk}^{vp}(SL, e) = \eta_{kk,SL}^{vp} \cdot \left[a_2 + \frac{1 - a_2}{2} - \frac{1 - a_2}{\pi} \cdot \arctan \left((e - 0.9 \cdot e_{crit}) \cdot \frac{b_2}{e_{crit}} \right) \right]. \quad (61)$$

When $e(t)$ exceeds 90% of the critical strain-energy density (inflection point at $(e - 0.9 \cdot e_{crit})$), the viscoplastic stiffness significantly decreases. As a result, the deformations markedly increase and the tertiary creep phase occurs. Analogous to Eq. (56), $\eta_{kk,SL}^{vp}$ specifies the upper and $a_2 \cdot \eta_{kk,SL}^{vp}$ the lower bound of the function.

The dilation in x-direction is described by the term $\frac{b_2}{e_{crit}}$. Regarding the creep curve, the parameter a_2 defines the slope of the region of tertiary creep and parameter b_2 the curvature of the transition between secondary and tertiary range.

Using the values given in Table 5 provides satisfying simulation results. As example, in Fig. 5 the time-dependent deformation and strain-energy density of spruce wood in tangential compression at $m = 0.12$ is presented for different values of SL. It is clearly shown, that the creep phases as well as the increase of the creep rate with increasing SL is captured by the modelling approach at hand.

2.5.2.1. Determination of deformation components and material tangent. As already mentioned in Section 2.4.2, due to the non-linearity of the modelling approach, the viscoelastic and viscoplastic deformation components have to be determined iteratively. Analogously to the “General Return-Mapping”-algorithm according to Simo and Hughes (1998), for the first “trial”-step of the

iterative procedure, the occurrence of irreversible deformations is assumed (see Section 2.4.2)

$$f > 0, \quad \Delta\gamma > 0. \quad (62)$$

These viscoplastic deformation components also have to be considered in the course of the derivation of viscoelastic deformations. Thus, Eq. (18) now reads

$$\begin{aligned} \underline{\epsilon}_{n+1}^{ve} &= \underline{\epsilon}_{n+1}^{ve,lin} + \underline{\epsilon}_{n+1}^{ve,nlin} \\ &= \underline{\tilde{D}} : \left(\underline{E}_t : \underline{\epsilon}_n^{ve} + \left(-\underline{E}_t + \underline{A} \right) : \underline{\gamma} : \left(\underline{\epsilon}_n - \underline{\epsilon}_n^{ve} - \underline{\epsilon}_n^{vp} \right) + \underline{B} : \underline{\epsilon}_{n+1} \right) \\ &\quad + \underline{\Omega} : \underline{\epsilon}_{n+1}^{vp}. \end{aligned} \quad (63)$$

The operator $\underline{\Omega}$ is defined using once again the additive split of tensors (see Section 2.3.2)

$$\underline{\omega} = \sum_{m=1}^9 -\underline{\tilde{D}}_m : \underline{B}_m : \underline{1}, \quad (64)$$

$$\underline{\Omega} = \text{diag}(\underline{\omega}), \quad (65)$$

with $\underline{1}$: second order unity tensor.

For the identification of the viscoplastic deformations at time $n+1$, the yield criterion as well as a number of residues have to be formulated as the next step. The yield criterion at time $n+1$ reads

$$f_{n+1} = f(\underline{\sigma}_{sv,n+1}). \quad (66)$$

As first residuum, the equilibrium of viscoplastic deformations is utilised

$$\underline{R}_{1,n+1} = -\underline{\epsilon}_{n+1}^{vp} + \underline{\epsilon}_n^{vp} + \Delta\underline{\epsilon}^{vp}, \quad (67)$$

with

$$\Delta\underline{\epsilon}^{vp} = \Delta\gamma \cdot \frac{\partial f}{\partial \underline{\sigma}_{sv,n+1}}, \quad (68)$$

$$\frac{\partial f}{\partial \underline{\sigma}_{sv,n+1}} = 2 \cdot \underline{b} : \underline{\sigma}_{sv,n+1}. \quad (69)$$

The second residuum is obtained from the equilibrium of stresses within the BINGHAM-element (Eq. (41)) and Eq. (43) discretised over time

$$\underline{R}_{2,n+1} = -\underline{\sigma}_{sv,n+1} + \underline{\sigma}_{n+1} - \frac{1}{\Delta t} \cdot \underline{\eta}^{vp} : (\underline{\epsilon}_{n+1}^{vp} - \underline{\epsilon}_n^{vp}). \quad (70)$$

Due to the dependence of the viscoplastic stiffness $\underline{\eta}^{vp}$ on the strain-energy density (Eq. (61)), which, again, is influenced by the current stress $\underline{\sigma}_{n+1}$, the equilibrium of strain-energy densities at time $n+1$ is introduced as third residuum

$$\underline{R}_{3,n+1} = -\underline{e}_{n+1} + \underline{e}_n + \underline{\dot{E}} : \underline{\sigma}_{n+1}, \quad (71)$$

with

$$\underline{\dot{E}} = \text{diag}(\underline{\epsilon}_{n+1} - \underline{\epsilon}_n). \quad (72)$$

Table 5

Input properties for strain-energy-density-dependent viscoplastic stiffness according to Eq. (61).

		a_2 [–]	b_2 [–]
Tension	rad/tan/long	0.01	100
Compression	rad	0.3	10
	tan	0.1	25
	long	0.1	25
Shear	rt/tl/rl	0.01	100

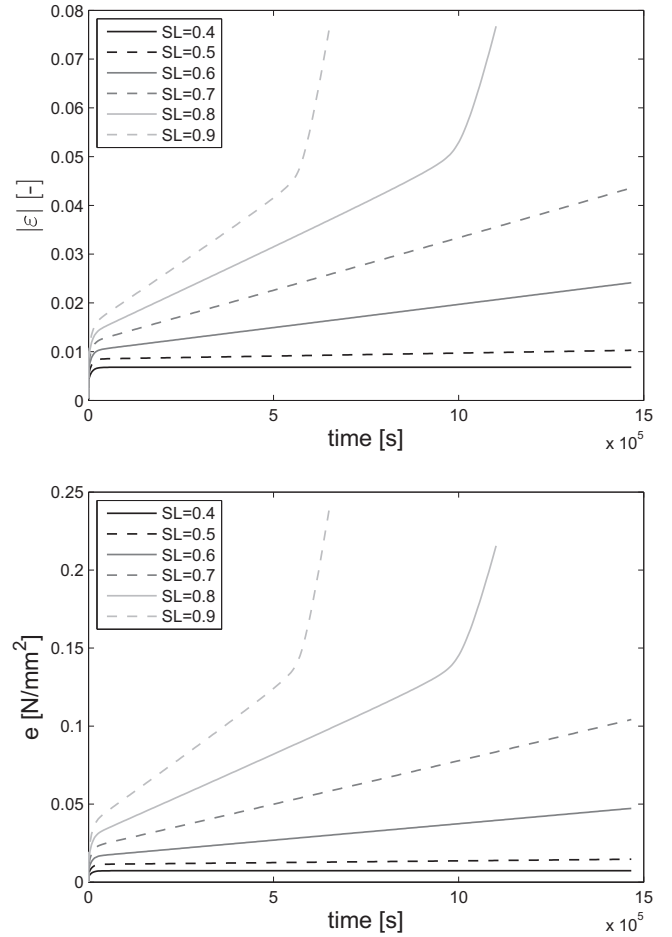


Fig. 5. Time-dependent deformation and strain-energy density of spruce wood in tangential compression depending on SL.

The fourth and last residuum is obtained from the non-linear part of the viscoelastic deformations according to Eq. (63) and reads

$$\underline{R}_{4,n+1} = -\underline{\epsilon}_{n+1}^{ve} + \underline{\epsilon}_{n+1}^{ve,lin} + \underline{\Omega} : \underline{\epsilon}_{n+1}^{vp}. \quad (73)$$

The iterative determination of the deformation components is carried out by means of the NEWTON-algorithm aiming at

$$f_{n+1} = 0 \quad (74)$$

and

$$\underline{R}_{1,n+1} = \underline{R}_{2,n+1} = \underline{R}_{3,n+1} = \underline{R}_{4,n+1} = \underline{0}. \quad (75)$$

For this purpose, the linearisation of the residues with respect to the iteration steps k is required

$$\begin{aligned} f_{n+1}^{k+1} &= f_{n+1}^k + \Delta f_{n+1}^k, \\ \underline{R}_{1,n+1}^{k+1} &= \underline{R}_{1,n+1}^k + \Delta \underline{R}_{1,n+1}^k, \\ \underline{R}_{2,n+1}^{k+1} &= \underline{R}_{2,n+1}^k + \Delta \underline{R}_{2,n+1}^k, \\ \underline{R}_{3,n+1}^{k+1} &= \underline{R}_{3,n+1}^k + \Delta \underline{R}_{3,n+1}^k, \\ \underline{R}_{4,n+1}^{k+1} &= \underline{R}_{4,n+1}^k + \Delta \underline{R}_{4,n+1}^k. \end{aligned} \quad (76)$$

The linearised terms are obtained from the derivations of the residues with respect to the unknown items $\Delta\gamma$, $\underline{\sigma}_{n+1}$, $\underline{\sigma}_{sv,n+1}$, $\underline{e}_{n+1}$ and $\underline{\epsilon}_{n+1}^{ve}$

$$\begin{bmatrix} \Delta \gamma_{n+1}^k \\ \Delta \mathbf{R}_{1,n+1}^k \\ \Delta \mathbf{R}_{2,n+1}^k \\ \Delta \mathbf{R}_{3,n+1}^k \\ \Delta \mathbf{R}_{4,n+1}^k \end{bmatrix} = \Delta : \begin{bmatrix} \Delta^2 \gamma^k \\ \Delta \boldsymbol{\sigma}_{n+1}^k \\ \Delta \boldsymbol{\sigma}_{sv,n+1}^k \\ \Delta \mathbf{e}_{n+1}^k \\ \Delta \mathbf{e}_{n+1}^{ve,k} \end{bmatrix}, \quad (77)$$

with Δ : operator, $\mathbf{1}$: fourth order unity tensor

$$\Delta = \begin{bmatrix} 0 & \mathbf{0} & \frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k} & \mathbf{0} & \mathbf{0} \\ \frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k} & (\mathbf{C}^{el})^{-1} & \Delta \gamma \cdot \frac{\partial^2 f}{(\partial \boldsymbol{\sigma}_{sv,n+1}^k)^2} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} + \frac{1}{\Delta t} \cdot \boldsymbol{\eta}^{vp} : (\mathbf{C}^{el})^{-1} & -\mathbf{1} & \underline{\Gamma} & \frac{1}{\Delta t} \cdot \boldsymbol{\eta}^{vp} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\underline{\Omega} : (\mathbf{C}^{el})^{-1} & \mathbf{0} & \mathbf{0} & -diag(\mathbf{1}) - \underline{\Omega} \end{bmatrix}, \quad (78)$$

$$\underline{\Gamma} = diag\left(-\frac{1}{\Delta t} \cdot \boldsymbol{\eta}^{k,vp} : \left(\mathbf{e}_{n+1}^k - \mathbf{e}_{n+1}^{ve,k} - (\mathbf{C}^{el})^{-1} : \boldsymbol{\sigma}_{n+1}^k\right)\right), \quad (79)$$

$$\boldsymbol{\eta}_{n+1}^{k,vp} = \begin{bmatrix} \frac{\partial \eta_{11}^{vp}}{\partial \mathbf{e}_{11,n+1}^k} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial \eta_{22}^{vp}}{\partial \mathbf{e}_{22,n+1}^k} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial \eta_{33}^{vp}}{\partial \mathbf{e}_{33,n+1}^k} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial \eta_{44}^{vp}}{\partial \mathbf{e}_{44,n+1}^k} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial \eta_{55}^{vp}}{\partial \mathbf{e}_{55,n+1}^k} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial \eta_{66}^{vp}}{\partial \mathbf{e}_{66,n+1}^k} \end{bmatrix} \quad (80)$$

and

$$\frac{\partial^2 f}{(\partial \boldsymbol{\sigma}_{sv,n+1}^k)^2} = 2 \cdot \underline{\mathbf{b}} \quad (81)$$

All input quantities related to time n are known from the previous calculation step. Furthermore, it is assumed, that the deformations $\mathbf{e}_{n+1}$ do not change in the course of the local iteration. The term $\mathbf{e}_{n+1}^{ve,lin}$ is specified previously and also constant.

With respect to the dependence of η_{kk}^{vp} on SL according to Eq. (56), it is assumed that $SL = \frac{\sigma}{f}$. This assumption leads to significant simplifications with respect to the local iteration, because $\frac{\partial \eta_{kk}^{vp}}{\partial \sigma_{n+1}} = 0$. Since the approach at hand is conceived to model the long-term behaviour of wood subjected to rather unchanging mechanical loadings, the change of stresses between two calculation steps is expected to be small and the assumption is reasonable.

As next step, the system of Eqs. (77) has to be solved to be able to determine the increments of the unknown items. Using the auxiliary variables

$$\underline{\Omega} = (diag(\mathbf{1}) + \underline{\Omega})^{-1}, \quad (82)$$

$$\underline{\mathbf{K}} = \left(\mathbf{1} + \frac{1}{\Delta t} \cdot \boldsymbol{\eta}^{vp} : (\mathbf{C}^{el})^{-1} + \underline{\Gamma} : \underline{\dot{\mathbf{E}}} - \frac{1}{\Delta t} \cdot \boldsymbol{\eta}^{vp} : \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1}\right)^{-1}, \quad (83)$$

$$\underline{\Xi} = \left((\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} - \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} + \Delta \gamma^k \cdot \frac{\partial^2 f}{(\partial \boldsymbol{\sigma}_{sv,n+1}^k)^2}\right)^{-1}, \quad (84)$$

$$\underline{\mathbf{L}} = -(\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} + \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}}, \quad (85)$$

$$\underline{\mathbf{M}} = -(\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} : \underline{\Gamma} + \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} : \underline{\Gamma}, \quad (86)$$

and

$$\underline{\mathbf{N}} = -\frac{1}{\Delta t} \cdot (\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} : \boldsymbol{\eta}^{vp} : \underline{\Omega} + \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1} : \underline{\mathbf{K}} : \boldsymbol{\eta}^{vp} : \underline{\Omega}, \quad (87)$$

the conditional equation for the increment $\Delta^2 \gamma^k$ of the consistency parameter is obtained

$$\Delta^2 \gamma^k = \frac{f_{n+1}^k - \frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k} : \underline{\Xi} : (\mathbf{R}_{1,n+1}^k + \underline{\mathbf{L}} : \mathbf{R}_{2,n+1}^k + \underline{\mathbf{M}} : \mathbf{R}_{3,n+1}^k + \underline{\mathbf{N}} : \mathbf{R}_{4,n+1}^k)}{\frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k} : \underline{\Xi} : \frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k}}. \quad (88)$$

Eq. (88) now allows for the determination of the increments of the remaining unknowns

$$\begin{aligned} \Delta \boldsymbol{\sigma}_{n+1}^k &= \underline{\mathbf{P}} : \left(-\mathbf{R}_{1,n+1}^k + \Delta^2 \gamma^k \cdot \frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k} - \underline{\Omega} : \mathbf{R}_{4,n+1}^k\right) \\ &\quad - \Delta \gamma^k \cdot \underline{\mathbf{P}} : \frac{\partial^2 f}{(\partial \boldsymbol{\sigma}_{sv,n+1}^k)^2} \\ &\quad : \left(\mathbf{R}_{2,n+1}^k + \underline{\Gamma} : \mathbf{R}_{3,n+1}^k + \frac{1}{\Delta t} \cdot \boldsymbol{\eta}^{vp} : \underline{\Omega} : \mathbf{R}_{4,n+1}^k\right), \end{aligned} \quad (89)$$

with

$$\underline{\mathbf{P}} = \left((\mathbf{C}^{el})^{-1} - \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1} + \Delta \gamma^k \cdot \frac{\partial^2 f}{(\partial \boldsymbol{\sigma}_{sv,n+1}^k)^2} : \underline{\mathbf{K}}^{-1}\right)^{-1}, \quad (90)$$

$$\begin{aligned} \Delta \boldsymbol{\sigma}_{sv,n+1}^k &= \underline{\Xi} : \left(-\mathbf{R}_{1,n+1}^k - \Delta^2 \gamma^k \cdot \frac{\partial f}{\partial \boldsymbol{\sigma}_{sv,n+1}^k}\right) \\ &\quad + \underline{\Xi} : \left(-\underline{\mathbf{L}} : \mathbf{R}_{2,n+1}^k - \underline{\mathbf{M}} : \mathbf{R}_{3,n+1}^k - \underline{\mathbf{N}} : \mathbf{R}_{4,n+1}^k\right), \end{aligned} \quad (91)$$

$$\Delta \mathbf{e}_{n+1}^k = \mathbf{R}_{3,n+1}^k + \underline{\dot{\mathbf{E}}} : \Delta \boldsymbol{\sigma}_{n+1}^k, \quad (92)$$

$$\Delta \mathbf{e}_{n+1}^{ve,k} = \underline{\Omega} : \mathbf{R}_{4,n+1}^k - \underline{\Omega} : \underline{\Omega} : (\mathbf{C}^{el})^{-1} : \Delta \boldsymbol{\sigma}_{n+1}^k \quad (93)$$

and

$$\Delta \mathbf{e}_{n+1}^{vp,k} = -\Delta \mathbf{e}_{n+1}^{ve,k} - (\mathbf{C}^{el})^{-1} : \Delta \boldsymbol{\sigma}_{n+1}^k. \quad (94)$$

At the end of each iteration step, the update of the unknown items takes place

$$\begin{bmatrix} \Delta \gamma^{k+1} \\ \boldsymbol{\sigma}_{n+1}^{k+1} \\ \boldsymbol{\sigma}_{sv,n+1}^{k+1} \\ \mathbf{e}_{n+1}^{k+1} \\ \mathbf{e}_{n+1}^{ve,k+1} \\ \mathbf{e}_{n+1}^{vp,k+1} \end{bmatrix} = \begin{bmatrix} \Delta \gamma^k \\ \boldsymbol{\sigma}_{n+1}^k \\ \boldsymbol{\sigma}_{sv,n+1}^k \\ \mathbf{e}_{n+1}^k \\ \mathbf{e}_{n+1}^{ve,k} \\ \mathbf{e}_{n+1}^{vp,k} \end{bmatrix} + \begin{bmatrix} \Delta^2 \gamma^k \\ \Delta \boldsymbol{\sigma}_{n+1}^k \\ \Delta \boldsymbol{\sigma}_{sv,n+1}^k \\ \Delta \mathbf{e}_{n+1}^k \\ \Delta \mathbf{e}_{n+1}^{ve,k} \\ \Delta \mathbf{e}_{n+1}^{vp,k} \end{bmatrix}. \quad (95)$$

The iteration is continued until the absolute values of the yield criterion and the residues fall below a tolerance limit to be defined.

To ensure a quadratic convergence of the incremental-iterative solution procedure, the derivation of the algorithmic material tangent is required as the last step

$$\underline{\underline{\mathbf{C}}}_{n+1}^{evp} = \frac{d\sigma_{n+1}}{d\epsilon_{n+1}}. \quad (96)$$

Starting point are the total differentials of the yield criterion

$$0 = \frac{\partial f}{\partial \sigma_{sv,n+1}} : d\sigma_{sv,n+1}, \quad (97)$$

the stress component of the SAINT-VENANT-element

$$d\sigma_{sv,n+1} = d\sigma_{n+1} - \frac{1}{\Delta t} \cdot \Delta \gamma \cdot \underline{\underline{\eta}}^{vp} : \frac{\partial^2 f}{(\partial \sigma_{sv,n+1})^2} : d\sigma_{sv,n+1} - \frac{1}{\Delta t} \cdot \underline{\underline{\eta}}^{vp} : \frac{\partial f}{\partial \sigma_{sv,n+1}} \cdot d\Delta \gamma - \frac{1}{\Delta t} \cdot \Delta \gamma \cdot \underline{\underline{\mathbf{a}}} : d\epsilon_{n+1}, \quad (98)$$

with

$$\underline{\underline{\mathbf{a}}} = \text{diag} \left(\underline{\underline{\eta}}_{n+1}^{vp} : \frac{\partial f}{\partial \sigma_{sv,n+1}} \right) \quad (99)$$

and the tensor of strain-energy densities

$$d\epsilon_{n+1} = \underline{\underline{\mathbf{E}}} : d\sigma_{n+1} + \underline{\underline{\mathbf{S}}} : d\epsilon_{n+1}, \quad (100)$$

with

$$\underline{\underline{\mathbf{S}}} = \text{diag}(\sigma_{n+1}). \quad (101)$$

Combining Eqs. (97), (98) and (100) leads to

$$0 = \frac{\partial f}{\partial \sigma_{sv,n+1}} : \underline{\underline{\mathbf{Q}}} : \left(\underline{\underline{\mathbf{c}}} : d\sigma_{n+1} - \frac{1}{\Delta t} \cdot \underline{\underline{\eta}}^{vp} : \frac{\partial f}{\partial \sigma_{sv,n+1}} \cdot d\Delta \gamma - \underline{\underline{\mathbf{d}}} : d\epsilon_{n+1} \right), \quad (102)$$

with the auxiliary variables

$$\underline{\underline{\mathbf{Q}}} = \left(\underline{\underline{\mathbf{1}}} + \frac{1}{\Delta t} \cdot \Delta \gamma \cdot \underline{\underline{\eta}}^{vp} : \frac{\partial^2 f}{(\partial \sigma_{sv,n+1})^2} \right)^{-1}, \quad (103)$$

$$\underline{\underline{\mathbf{c}}} = \underline{\underline{\mathbf{1}}} - \frac{1}{\Delta t} \cdot \Delta \gamma \cdot \underline{\underline{\mathbf{a}}} : \underline{\underline{\mathbf{E}}} \quad (104)$$

and

$$\underline{\underline{\mathbf{d}}} = \frac{1}{\Delta t} \cdot \Delta \gamma \cdot \underline{\underline{\mathbf{a}}} : \underline{\underline{\mathbf{S}}}. \quad (105)$$

Since Eq. (102) contains $d\Delta \gamma$ as additional unknown item, further conditional equations have to be defined, namely the total differential of the overall stresses $d\sigma_{n+1}$

$$d\sigma_{n+1} = \underline{\underline{\mathbf{C}}}^{el} : (d\epsilon_{n+1} - d\epsilon_{n+1}^{ve} - d\epsilon_{n+1}^{vp}) \quad (106)$$

and the total differential of residuum R_1

$$d\epsilon_{n+1}^{vp} = \frac{\partial f}{\partial \sigma_{sv,n+1}} \cdot d\Delta \gamma + \Delta \gamma \cdot \frac{\partial^2 f}{(\partial \sigma_{sv,n+1})^2} : d\sigma_{sv,n+1}. \quad (107)$$

Inserting Eqs. (13), (98) and (107) into (106) leads to

$$d\sigma_{n+1} = \underline{\underline{\mathbf{U}}} : (\underline{\underline{\mathbf{W}}} : d\epsilon_{n+1} + \underline{\underline{\mathbf{V}}} : d\Delta \gamma), \quad (108)$$

with

$$\underline{\underline{\mathbf{U}}} = \left((\underline{\underline{\mathbf{C}}}^{el})^{-1} + (\underline{\underline{\mathbf{C}}}^{ve})^{-1} + \Delta \gamma \cdot \frac{\partial^2 f}{(\partial \sigma_{sv,n+1})^2} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\mathbf{c}}} \right)^{-1}, \quad (109)$$

$$\underline{\underline{\mathbf{V}}} = \frac{\partial f}{\partial \sigma_{sv,n+1}} - \frac{1}{\Delta t} \cdot \Delta \gamma \cdot \frac{\partial^2 f}{(\partial \sigma_{sv,n+1})^2} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\eta}}^{vp} : \frac{\partial f}{\partial \sigma_{sv,n+1}} \quad (110)$$

and

$$\underline{\underline{\mathbf{W}}} = \underline{\underline{\mathbf{1}}} + \Delta \gamma \cdot \frac{\partial^2 f}{(\partial \sigma_{sv,n+1})^2} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\mathbf{d}}}. \quad (111)$$

The conditional equation for $d\Delta \gamma$ is obtained by means of the combination of Eqs. (102) and (108)

$$d\Delta \gamma = \frac{\underline{\underline{\beta}} : d\epsilon_{n+1}}{\alpha}, \quad (112)$$

with

$$\alpha = \frac{\partial f}{\partial \sigma_{sv,n+1}} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\mathbf{c}}} : \underline{\underline{\mathbf{U}}} : \underline{\underline{\mathbf{V}}} + \frac{1}{\Delta t} \cdot \frac{\partial f}{\partial \sigma_{sv,n+1}} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\eta}}^{vp} : \frac{\partial f}{\partial \sigma_{sv,n+1}} \quad (113)$$

and

$$\underline{\underline{\beta}} = \frac{\partial f}{\partial \sigma_{sv,n+1}} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\mathbf{c}}} : \underline{\underline{\mathbf{U}}} : \underline{\underline{\mathbf{W}}} - \frac{\partial f}{\partial \sigma_{sv,n+1}} : \underline{\underline{\mathbf{Q}}} : \underline{\underline{\mathbf{d}}}. \quad (114)$$

Finally, insertion of Eq. (112) into (108) and rearrangement leads to the conditional equation for the material tangent $\underline{\underline{\mathbf{C}}}_{n+1}^{evp}$ by analogy to Simo and Hughes (1998)

$$\underline{\underline{\mathbf{C}}}_{n+1}^{evp} = \underline{\underline{\mathbf{U}}} : \underline{\underline{\mathbf{W}}} - \underline{\underline{\mathbf{N}}}_1 \otimes \underline{\underline{\mathbf{N}}}_2, \quad (115)$$

with

$$\underline{\underline{\mathbf{N}}}_1 = \alpha^{-\frac{1}{2}} \cdot \underline{\underline{\mathbf{U}}} : \underline{\underline{\mathbf{V}}}, \quad (116)$$

$$\underline{\underline{\mathbf{N}}}_2 = \alpha^{-\frac{1}{2}} \cdot \underline{\underline{\beta}}. \quad (117)$$

3. Examples

The material formulation introduced above for the simulation of the long-term behaviour of wooden structures under constant climatic conditions is now applied for the re-calculation of several test series. A fitting of the respective creep curves is explicitly not aspired, rather, the input properties identified in the framework of the previous sections are used. Since those properties have been determined by means of the re-calculation of experimental investigations of mainly small specimens subjected to one particular type and direction of loading, the following examples aim at the verification and validation of the approach and the respective material properties on the basis of bending tests. Furthermore, the approaches are applied to the analysis of the time-dependent structural behaviour of a face staggered joint.

3.1. Linear viscoelasticity

First, the material approach for the simulation of linear viscoelastic behaviour for loadings below LL is used for the re-calculation of the experimental results of Welling (1987). By means of small specimens of the dimensions $L \times B \times H = 140 \times 9 \times 15$ [mm], the bending creep behaviour of spruce wood perpendicular to the grain is investigated for several constant values of MC and temperature. The bending stress of 30% of the short-time bending strength, at this, was applied using two point loads as a four-point bending test and kept constant for 6 h. The analysis of the results was carried out in Welling (1987) by means of the determination of the input properties for the BURGERS-model under consideration of the influence of temperature and MC.

In the framework of the re-calculation, a temperature of $T = 20^\circ\text{C}$ and a moisture content of $m = 0.12$ are assumed. The specimens are discretised by means of 20-node continuum-elements with displacement degrees of freedom. Depending on the investigated material direction (radial or tangential), the local

Table 6

Density ρ_0 in absolute dry conditions and POISSON's ratios ν_{ij} of spruce and generalised softwood according to Saft and Kaliske (2012, 2013).

	Spruce		Generalised softwood			Spruce		Generalised softwood	
$\rho_0 \left[\frac{\text{g}}{\text{cm}^3} \right]$	0.38	0.4			ν_{rt}	0.24	0.3		
					ν_{tl}	0.45	0.47		
					ν_{rl}	0.45	0.44		

material directions of the wooden structure are assigned to the specimens' dimensions

- radial: $L \times B \times H \Rightarrow r \times t \times l$,
- tangential: $L \times B \times H \Rightarrow t \times r \times l$.

As explained in detail in Section 2.3.2, the material properties for the description of the linear viscoelastic behaviour refer to the moisture-dependent short-term properties. These are based on an extensive literature study and are e.g. published in Saft and Kaliske (2012, 2013). The essential values are given in Tables 6–8. Following the theory of orthotropic elasticity (see e.g. Bodig and Jayne, 1993; Schmidt, 2009), the inverse of the elasticity tensor reads as

$$\left(\underline{\underline{C}}^{el} \right)^{-1} = \begin{bmatrix} \frac{1}{E_r} & -\frac{\nu_{rt}}{E_t} & -\frac{\nu_{rl}}{E_l} & 0 & 0 & 0 \\ -\frac{\nu_{rt}}{E_t} & \frac{1}{E_t} & -\frac{\nu_{tl}}{E_l} & 0 & 0 & 0 \\ -\frac{\nu_{rl}}{E_l} & -\frac{\nu_{tl}}{E_l} & \frac{1}{E_l} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{rt}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{tl}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{rl}} \end{bmatrix}, \quad (118)$$

with E_i : YOUNG's moduli, G_{ij} : shear moduli, ν_{ij} : POISSON's ratios. According to Sell (1997), the bending strength amounts to about 85–95% of the short-term tensile strength in longitudinal direction and, for the examples at hand, it is assumed as $f_b = 0.9 \cdot f_{t,l}$.

The experimental results approximated by means of the BURGERS-model as well as the simulation results are shown in Figs. 6 and 7 in the form of the midspan creep deformations related to the elastic deformations $\bar{\epsilon}^{ve} = \frac{\epsilon^{ve}}{\epsilon^{el}}$ at the tensile and compression face.

Table 7

Moisture-dependent YOUNG's and shear moduli of spruce and generalised softwood according to Saft and Kaliske (2012, 2013).

$X = X_{ref} \cdot (a_1 \cdot \rho_m^2 + a_2 \cdot \rho_m + a_3) \cdot (b_1 \cdot m^2 + b_2 \cdot m + b_3)$ for $\rho_m < 0.85 \frac{\text{g}}{\text{cm}^3}, 0.05 < m < 0.25, T = 20^\circ \text{C} (293.15 \text{ K})$		X_{ref} $\left[\frac{\text{N}}{\text{mm}^2} \right]$	a_1 $\left[\left(\frac{\text{cm}^3}{\text{g}} \right)^2 \right]$	a_2 $\left[\frac{\text{cm}^3}{\text{g}} \right]$	a_3 [–]	b_1 [–]	b_2 [–]	b_3 [–]
Spruce	E_r	682	7.7972	–1.6360	0.1535	0.7717	–4.0913	1.4794
	E_t	461	8.1430	–1.5446	0.0462	0.8422	–4.1608	1.4867
	E_l	10,236	–0.0067	2.2265	–0.0005	0.3149	–1.3457	1.1570
	G_{rt}	50	21.6327	–11.8463	1.9382	0.9558	–4.2683	1.4979
	G_{tl}	682	0.6881	1.8386	0.0330	0.2851	–2.0298	1.2391
	G_{rl}	695	0.6881	1.8386	0.0330	0.2851	–2.0298	1.2391
Generalised softwood	E_r	695	7.7230	–1.5798	0.1433	0.7819	–3.9049	1.4570
	E_t	471	8.0845	–1.5033	0.0388	0.8319	–3.9638	1.4632
	E_l	9955	–0.0086	2.2276	–0.0007	0.3764	–1.3612	1.1580
	G_{rt}	51	21.5955	–11.8167	1.9325	0.9024	–4.0491	1.4725
	G_{tl}	671	0.6872	1.8393	0.0329	0.3420	–2.0372	1.2392
	G_{rl}	684	0.6872	1.8393	0.0329	0.3420	–2.0372	1.2392

Bearing in mind, that no adjustment of elastic and viscoelastic material properties was carried out with respect to the recalculation, experimental and simulation results satisfactorily coincide, especially in case of tangential bending. Variations in curve shape are caused by the different modelling approaches. While the standard-solid body-model is used in the framework of the paper at hand, which is characterised by a creep limit and with that by a decreasing creep velocity, the BURGERS-model is used in Welling (1987). As mentioned above, due to the serially added dashpot, irreversible deformations are captured. Compared to the standard-solid body-model, the creep curves exhibit a larger creep velocity and with that a significantly larger slope.

3.2. Non-linear viscoelasticity–viscoplasticity

As second example, the experimental results of Cai et al. (2002) are taken into account. By means of four-point bending tests, the midspan creep deflection of small specimens with the dimensions $L \times B \times H = 660 \times 38 \times 38$ [mm] made of Southern Pine was investigated for stress levels higher than LL. At this, the local material directions of the wooden structure are assigned to the specimens' dimensions as $L \times B \times H \Rightarrow l \times t \times r$. The experiments were carried out at a constant climate leading to an equilibrium moisture content of about $m = 0.067$ with a loading time of one day. The results are analysed by means of the model parameters a, b, c and d for the approximation function

$$u = c - a \cdot t^{d+b \cdot t}, \quad (119)$$

with t : time [min], u : creep deflection [inch]. For each parameter, a minimum, a maximum and an average value as well as the corresponding SL are given (see Table 9). Analogous to the first example, the short-term bending strength is assumed to amount to 90% of the tensile strength. In the absence of the required short-term material properties of Southern Pine, generalised values for softwood are utilised (see Tables 6–8).

In Fig. 8, the approximation functions according to Cai et al. (2002) are shown together with the results determined by means of the numerical simulations for the stress levels given in Table 9. It becomes obvious, that the variations of the experimental results represented by the approximation functions are much larger than the influence of SL on the simulated creep curves. For the lowest stress level $SL = 0.6$, the experimental results seem to lie in the range of linear viscoelasticity and, thus, the creep deflections are much lower than the simulated ones. In contrast, the experimental results at $SL = 0.91$ exceed by far the simulated curve. However,

Table 8

Moisture-dependent tensile ($f_{t,i}$), compressive ($f_{c,i}$) and shear ($f_{v,ij}$) strength values of spruce and generalised softwood according to Saft and Kaliske (2012, 2013).

$X = a \cdot m^2 + b \cdot m + c, \quad m[-]$									
	a	b	c	a	b	c			
	$\left[\frac{\text{N}}{\text{mm}^2}\right]$	$\left[\frac{\text{N}}{\text{mm}^2}\right]$	$\left[\frac{\text{N}}{\text{mm}^2}\right]$	$\left[\frac{\text{N}}{\text{mm}^2}\right]$	$\left[\frac{\text{N}}{\text{mm}^2}\right]$	$\left[\frac{\text{N}}{\text{mm}^2}\right]$			
spruce	$f_{t,r}$	-36.7	3.7	3.9	$f_{v,rl}$	-69	5.5	7.9	
	$f_{t,t}$	-30	6	2.4	$f_{v,tl}$	-69	5.5	7.9	
	$f_{t,l}$	-879	140.6	84	$f_{v,rt}$	-16.5	1.7	1.8	
	$f_{c,r}$	27.2	2.7	-5.4					
	$f_{c,t}$	26.8	2.7	-4.4					
	$f_{c,l}$	113.9	136.7	-64					
Generalised softwood	$f_{t,r}$	-41.3	4.2	4.4	$f_{v,rl}$	-73.5	5.9	7.7	
	$f_{t,t}$	-27.8	5.1	2.3	$f_{v,tl}$	-73.5	5.9	7.7	
	$f_{t,l}$	-569.6	91.1	91.2	$f_{v,rt}$	-19.8	2	2.1	
	$f_{c,r}$	31.2	3.1	-5.9					
	$f_{c,t}$	35.3	3.6	-5.7					
	$f_{c,l}$	123	160.7	-69.7					

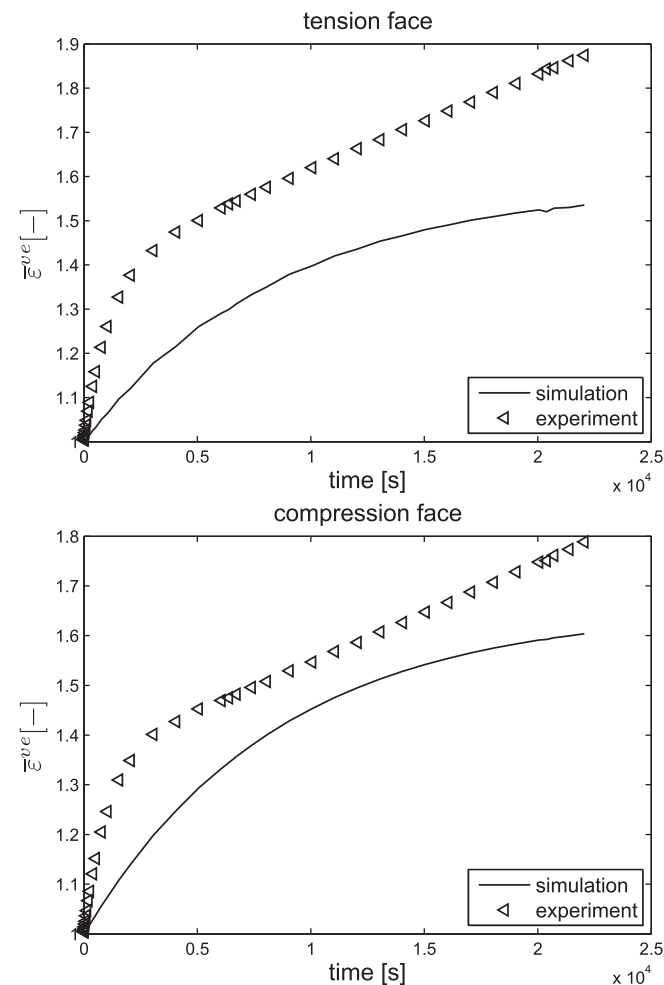


Fig. 6. Re-calculation of the experimental results of Welling (1987) for spruce wood in radial bending.

regarding the average value $SL = 0.8$, the experimentally determined and the simulated curve fit very well. This implies, that the average long-term behaviour of wood is adequately captured, when the modelling approach and the related material properties introduced above are applied.

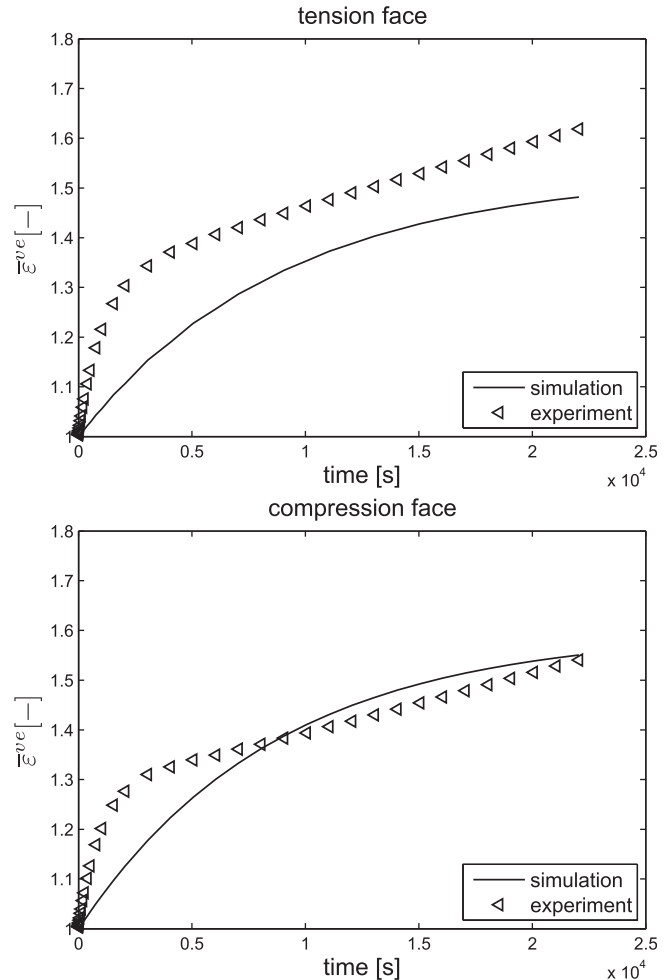


Fig. 7. Re-calculation of the experimental results of Welling (1987) for spruce wood in tangential bending.

3.3. Face staggered joint

Face staggered joints are connection types traditionally used for wooden constructions. Since no additional non-wooden connection device is required, this type of joint is implemented for hundreds of years. In Görlacher and Kromer (1991), face staggered joints of historical wooden constructions are analysed with regard to engineering design aspects. Here, the developed modelling approaches are applied for the investigation of the long-term structural behaviour. The dimensions of the analysed joint base on the data given in Görlacher and Kromer (1991) and are shown in Fig. 9. The structure is discretised by means of 20-node continuum-elements. Symmetry is used with respect to the numerical analysis, thus, only half of the structure is simulated.

The components of the analysed joint consist of spruce wood, the longitudinal material direction is marked by means of arrows in Fig. 9 and the material properties indicated in the previous sections and in Tables 6–8 are used. Loading is applied in terms of the surface load $p = 5 \frac{\text{N}}{\text{mm}^2}$ at the end face of the compression member and the long-term structural behaviour of the joint is simulated for a loading duration of 48 h.

In Fig. 10, the simulation results are shown. For the points P_1, P_2 and P_3 indicated in Fig. 9, the absolute values of time-dependent strains are depicted. Furthermore, the plastified parts of the structure are presented for three points of time (the darker the colour of the element, the more pronounced is the plastification). It can clearly be seen, that the applied loading causes the exceedance of

Table 9

Parameters of the approximation function of Cai et al. (2002) (Eq. (119)).

SL	a	b	c	d
0.69	0.0112	$-1.23 \cdot 10^{-3}$	0.0150	-0.4786
0.8	0.0718	$-6.32 \cdot 10^{-5}$	0.0629	-0.1167
0.91	0.1979	$-1.40 \cdot 10^{-6}$	0.1500	-0.2150

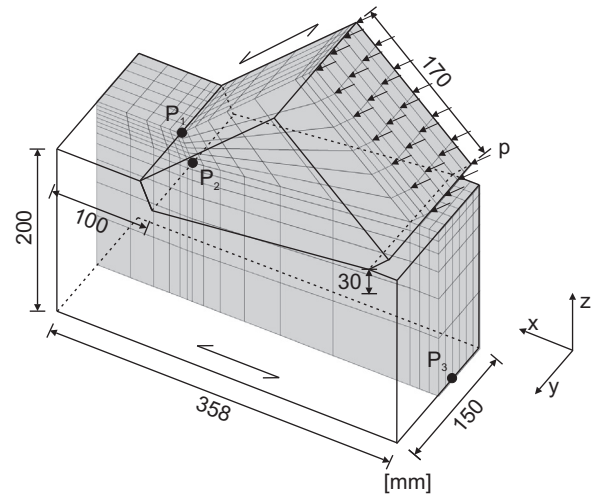
LL in large parts of the structure. Only slight changes of the plastified regions are observed with time, mainly in the area of point P_3 .

Regarding the strain–time–relationships, the cylindrically anisotropic nature of wood is obvious. While for point P_1 the shear strains τ_{tl} and the normal strains ε_t show non-linear viscoelastic-viscoplastic behaviour, a considerable amount of secondary creep is observed with respect to ε_t and τ_{tl} at point P_2 and ε_t , τ_{rt} and ε_r at point P_3 , respectively. Although a substantial part of the overall loading goes along with the longitudinal direction of the lower part of the joint, the longitudinal strain ε_l does not exhibit notable secondary creep. This is caused by the considerably larger YOUNG's modulus in grain direction compared to the values perpendicular to the grain (compare Table 7).

Having a closer look at the strain–time–relationship for τ_{tl} and ε_t at point P_1 , a discontinuity occurs at approximately $t = 38,00$ s. This exceptional curve shape provides evidence for the exceedance of the critical strain-energy density at that time. Due to the non-plastified bulk material around P_1 , a kind of support is generated. Thus, the overproportional increase of strains, which theoretically goes along with the tertiary creep phase (see Figs. 1 and 5) is prevented. Due to the consequential stress redistributions, the plastified parts of the structure slightly expand.

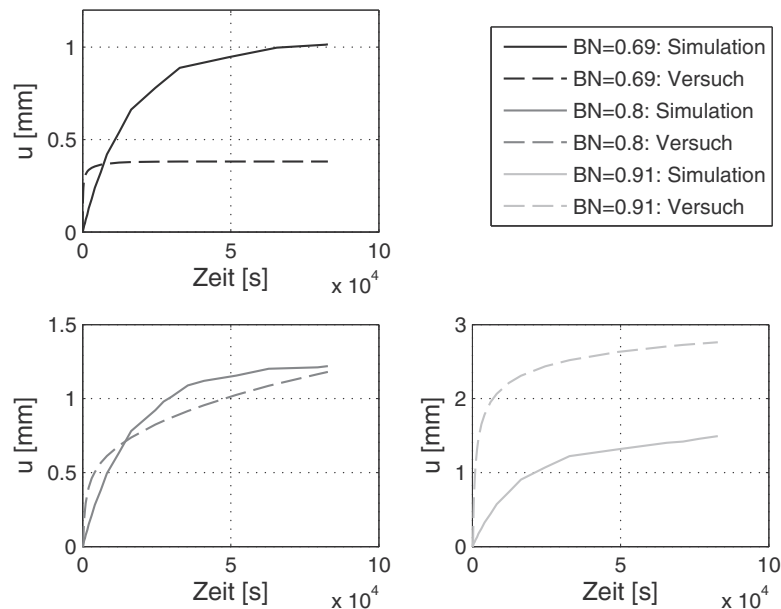
4. Summary and outlook

In the framework of the paper at hand, a material formulation for the simulation of the anisotropic long-term behaviour of wood at constant climate is introduced. Depending on the applied stress level, linear viscoelasticity and non-linear viscoelasticity–viscoplasticity are distinguished. This way, primary, secondary and tertiary creep and the influence of SL on the slope of the creep curve are adequately captured. Creep failure is considered by means of the concept of strain-energy density. To keep the approach usable to

**Fig. 9.** Geometry and discretisation of the face staggered joint.

a wide spectrum of applications, the utilisation of as few as possible, physically reasonable input properties was aimed at. In addition to the short-term properties taken as known, the number of required input quantities amounts to four ($\varphi_{\infty,ij}$, τ_{ij} , LL and η_{kk}^{vp}) for each loading case (combination of type and direction), which is few in relation to the variety of captured material characteristics and compared to other creep models for wood (e.g. Hanhijärvi, 1997; Chassagne et al., 2006).

The material properties are identified by means of the re-calculation of experimental data considering mainly small specimens subjected to one particular type and direction of loading. For the sake of verification and validation, the approach is applied to two test series in bending without adjusting the material parameters. The comparison of experimentally determined and simulated results is sufficient with respect to the average long-term behaviour of wood. Furthermore, the material formulation is utilised for the investigation of the long-term structural behaviour of a face staggered joint.

**Fig. 8.** Re-calculation of the experimental results of Cai et al. (2002).

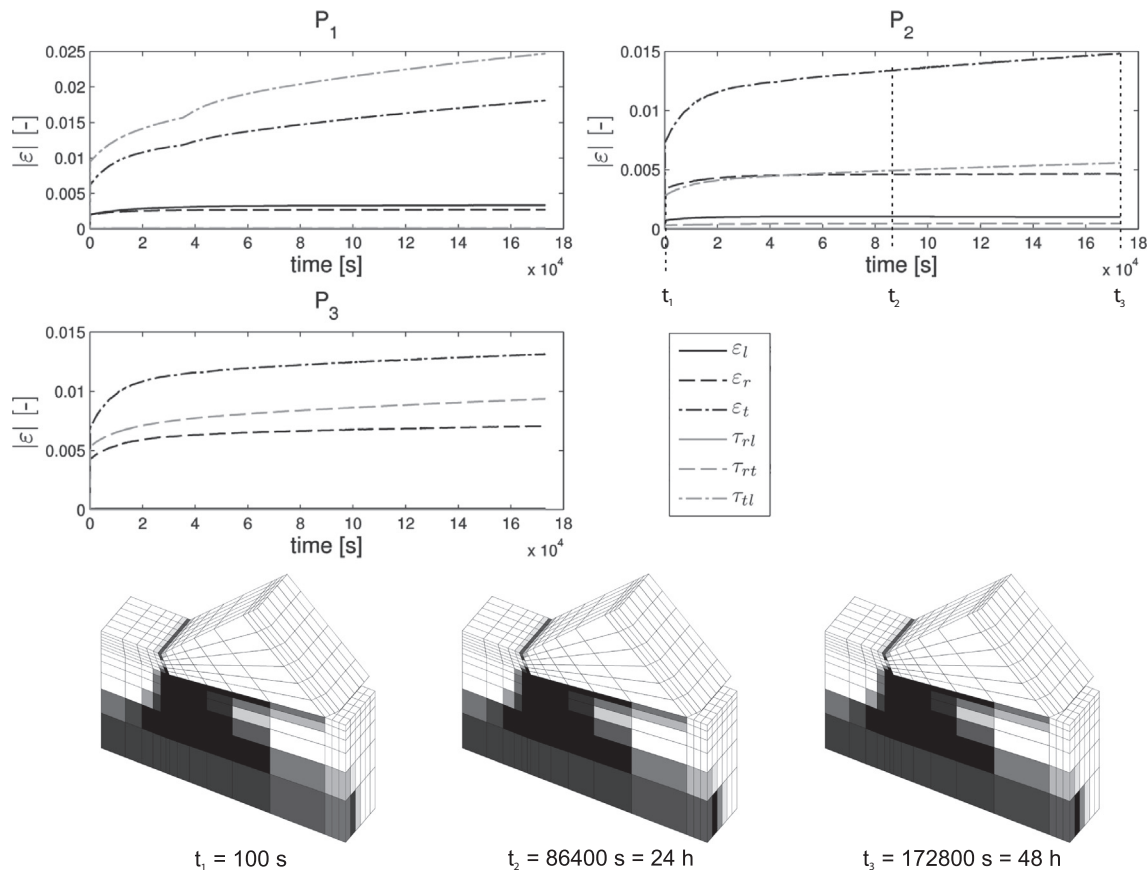


Fig. 10. Simulation results for the face staggered joint.

In the course of further working in the field of the long-term behaviour of wood, the use of an energetic criterion rather than a stress criterion to define the boundaries between linear viscoelastic and nonlinear viscoelastic–viscoplastic behaviour is conceivable. Since creep is characterised by dissipation processes due to viscosities, this could be a good alternative. In this conjunction, a further advancement of the approach may be possible with respect to the strain energy density. As e.g. done in Moutou Pitti et al. (2007, 2008), the utilisation of a double time integral could be applied for the determination of this value, since this allows for the consideration of viscous dissipation.

The paper at hand merely describes the first part of the entire work on the long-term behaviour of wood subjected to mechanical and hygrical loading. In the second part of this publication, the approach at hand is generalised and extended with respect to hygro-mechanical coupling to capture multiphysical phenomena. At this, both, the influence of MC on mechanically induced creep and the so-called mechano-sorptive effect resulting from simultaneous mechanical loading and moisture changes are considered.

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