International Journal of Solids and Structures 91 (2016) 102–114

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A fully coupled damage-plasticity model for unsaturated geomaterials accounting for the ductile–brittle transition in drying clayey soils

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ARTICLE INFO

Article history:
Received 1 April 2015
Revised 10 February 2016
Available online 13 April 2016

Keywords:
Constitutive laws
Damage
Plasticity
Unsaturated geomaterials
Effective stress

ABSTRACT

This paper presents a hydro-mechanical constitutive model for clayey soils accounting for damage-plasticity couplings. Specific features of unsaturated clays such as confining pressure and suction effects on elastic domain and plastic strains are accounted for. A double effective stress incorporating both the effect of suction and damage is defined based on thermodynamical considerations, which results in a unique stress variable being thermodynamically conjugated to elastic strain. Coupling between damage and plasticity phenomena is achieved by following the principle of strain equivalence and incorporating the double effective stress into plasticity equations. Two distinct criteria are defined for damage and plasticity, which can be activated either independently or simultaneously. Their formulation in terms of effective stress and suction allows them to evolve in the total stress space with suction and damage changes. This leads to a direct coupling between damage and plasticity and allows the model to capture the ductile/brittle behaviour transition occurring when clays are drying. Model predictions are compared with experimental data on Boom Clay, and the flexibility of the model is illustrated by presenting results of simulations in which either damage or plasticity dominates the coupled behaviour.

1. Introduction

Pressing needs for sustainable structures and safe geological repositories require the development of reliable models to predict the behaviour of natural geomaterials (e.g., soils, rocks) and engineered materials (e.g. compacted backfill materials, cement-based materials, ceramics etc.). One of the recurring modelling challenges is the prediction of deformation, stiffness and strength of porous media with a solid matrix containing clay minerals.

Experimental evidence show that clayey soils can exhibit either a brittle or a ductile behaviour (Dehandschutter et al., 2005). The transition between both behaviours depends on multiple factors including moisture content (Al-Shayea, 2001). Under deviatoric loading, clayey soils can undergo large permanent strains. Their properties, such as stiffness, strength, or permeability, are also known to be subject to changes after being submitted to hydric or mechanical solicitations. In clayey soils, these changes are related to several physical phenomena, such as the deterioration of cemented bonds, hence the destructuration of the material, or the change in water content, resulting in a decrease of suction-induced bonding.

Sophisticated plasticity models proposed for clayey soils allow capturing suction hardening and wetting collapse. However these models are not suited for stiffer or bonded materials, which can undergo both plastic deformation and stiffness degradation. A phenomenological variable d, called “damage”, can be defined at the continuum scale to quantify the energy dissipated by stiffness degradation. Note that d represents the effects of multiple microscopic dissipative processes that lead to the loss of adhesion between material surfaces, such as micro-crack propagation or debonding due to an increase of water saturation. Coupling damage and plasticity in a thermodynamically consistent framework raises many issues when one wants to ensure thermodynamical consistency while keeping the model simple enough to allow for easy calibration and incorporation into a numerical code. Models coupling damage and plasticity are often material and loading path specific, and difficult to generalise to a broader category of problems related to the coupled effects of mechanical stress and suction in unsaturated clay-bearing porous media. One of the fundamental issues that needs to be addressed is the choice of thermodynamic variables, in particular the stress variable involved in the yield and damage criteria.

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http://dx.doi.org/10.1016/j.ijsolstr.2016.04.007
0020-7683/

State-of-the art models are often designed to fit experimental data for specific materials subjected to specific stress-paths. By contrast, the modelling approach presented in this paper is aimed to predict the transition between brittle and ductile deformation regimes for different fabrics, clay contents and hydro-mechanical stress paths. Model calibration and numerical implementation are facilitated by the low number of constitutive parameters employed in the formulation (14 parameters in total, 8 for the mechanical part of the model and 6 for the hydraulic part). The proposed framework is flexible so that each component can be refined if one wants to adapt it to a specific material. The described model is designed in order to be adaptable to a wide range of geomaterials, ranging from stiff clayey soils to mixtures of clay and sand. To illustrate the versatility of the framework, the model was calibrated against experimental data obtained for a variety of geomaterials including Boom clay and mixtures of clay soil and sand.

The work presented in this paper provides a general method to couple damage and plasticity in porous materials that have a clay-bearing solid matrix. Clay minerals are expected to play a critical role in the deformation and retention properties of the damaged medium. Section 2 reviews the main modelling strategies available to date to model hydro-mechanical plasticity and damage in unsaturated porous media, and introduces the main concepts and states variables used to account for the effect of suction and damage. Then, the concept of double effective stress is introduced in Section 3, and its coupling with damage and plasticity is developed. In Section 4, the behaviour of the mechanical model is analysed, as well as its limits and its sensitivity to the main parameters. Section 5 presents the simulation of different laboratory tests performed on unsaturated clay-bearing geomaterials. The comparison between models predictions and experimental data reported in the literature is used as a basis to assess the performance of the model.

The sign convention used is the one of soil mechanics. Compressive stresses and strains take therefore positive values.

2. Damage in unsaturated clay-bearing porous media

2.1. Pore-scale hydro-mechanical couplings

In this study, we are interested in modelling multiphasic media made of a solid skeleton containing pores filled with a mixture of liquid and gas. The difference between gas and liquid pore pressures, $s = u_a - u_w$, is called suction. In the case in which air remains equal to the atmospheric pressure, water pressure is negative and suction takes a positive value. The air–water interface (called meniscus) starts to curve when suction increases. The radius of the meniscus decreases when suction increases, and once it becomes as small as the pore throats, air can invade the porous structure, which becomes unsaturated. The combination of the water surface tension and the negative pore water pressure results in a force that tends to pull the soil grains towards one another. The resulting force on the solid skeleton is similar to a compressive stress (Santamarina, 2003). An increase in suction will therefore lead to a decrease of the total volume (shrinkage), and wetting soils (i.e. decreasing suction) will usually make them swell. Suction also contributes to stiffen the soil against external loading thanks to grain bonding induced by water menisci in tension. The additional component of normal force at the contact will also prevent slippage between grains and thus increases the external force needed to cause plastic strains (Ridley et al., 2009). However, when wetting a soil under constant mechanical loading, the resaturation destroys the bonds formed by water menisci and may induce an irrecoverable volumetric compression (called collapse) (Muñoz Castelblanco et al., 2011). These main characteristics of unsaturated soils mechanical behaviour are represented in Fig. 1.

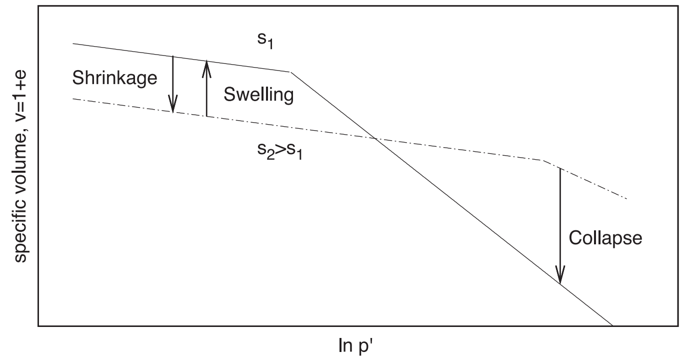

Fig. 1. Influence of suction on volumetric compression and volume changes due to wetting and drying, adapted from Alonso et al. (1990). p' is the net mean stress.

Changes in suction may also induce irreversible processes (plasticity or damage) during a drying process (Alonso et al., 2014; Wang et al., 2014).

2.2. State variables for unsaturated porous media

Unsaturated soil models are usually extensions of saturated soil ones. The most widely used of them is the Cam-clay model, first developed by Roscoe et al. (1958) and later modified by Roscoe and Burland (1968). Extension to unsaturated states requires the definition of specific state variables. A comprehensive review of the existing stress frameworks can be found in the paper of Nuth and Laloui (2008).

Houlsby (1997) demonstrated that, assuming the incompressibility of the solid matrix and the water phase, the work input to an unsaturated soil can be written as:

$$\dot{w} = [\boldsymbol{\sigma} - (S_r u_w + (1 - S_r)u_a)\mathbf{I}] : \dot{\boldsymbol{\varepsilon}} - (u_a - u_w)\phi\dot{S}_r \tag{1}$$

where $\boldsymbol{\sigma}$ is the total stress tensor, $\dot{\boldsymbol{\varepsilon}}$ the total strain rate tensor, u_a and u_w the air and water pore pressures, ϕ the porosity, S_r the degree of saturation, and $\mathbf{I}$ the identity matrix.

This formulation leads to the introduction of two state variables respectively conjugated to the strain rate, $\dot{\boldsymbol{\varepsilon}}$, and to the degree of saturation rate $\dot{S}_r$.

The stress quantity related to the strain increment is,

$$\begin{aligned}\boldsymbol{\sigma}^* &= \boldsymbol{\sigma} - (S_r u_w + (1 - S_r)u_a)\mathbf{I} \\ &= \boldsymbol{\sigma} - u_a\mathbf{I} + (u_a - u_w)S_r\mathbf{I} \\ &= \boldsymbol{\sigma}^{net} + sS_r\mathbf{I}\end{aligned} \tag{2}$$

which is a particular form of Bishop's effective stress (Bishop, 1959) in which the χ factor is taken equal to 1. This stress has been used by many authors and has been attributed different names, such as the average skeleton stress tensor (Jommi, 2000), the constitutive stress (Sheng et al., 2003) or the generalised effective stress (Laloui and Nuth, 2009). In the following, the term *constitutive stress* will be used.

Other expressions have been proposed for this constitutive stress, accounting for the energy of the air–water interface (Nikooee et al., 2012; Pereira et al., 2005), the different levels of porosity (Alonso et al., 2010), or the compressibility of the solid matrix through the Biot's coefficient (Chateau and Dormieux, 2002; Jia et al., 2007). However, for the sake of simplicity, we will keep the simple expression of Eq. (2), although the framework could easily accommodate a different expression for the constitutive stress.

According to Eq. (1), a second suction-related state variable, work-conjugated to the increment of degree of saturation is required. It will be called *modified suction* in the following and is written as:

$$s^* = \phi s \tag{3}$$

Constitutive equations are derived from an energy potential,

$$\boldsymbol{\sigma}^* = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e} \tag{4}$$

$$s^* = -\frac{\partial \psi}{\partial S_r} \tag{5}$$

in which ψ is the Helmholtz free energy, and $\boldsymbol{\varepsilon}^e$ is the elastic strain tensor.

Due to the presence of the term sS_r in the constitutive stress expression, the relationship between suction and degree of saturation has a great impact on the soil unsaturated mechanical behaviour.

For the sake of simplicity, hysteresis effects will be neglected in the present study, and the degree of saturation is expressed as a bijective function of the modified suction:

$$S_r = f(s^*) \tag{6}$$

The expression of van Genuchten (1980), in which the modified suction is used in place of suction, is used for the water retention properties:

$$S_r = \left(\frac{1}{1 + (\alpha_{vg} s^*)^{n_{vg}}} \right)^{m_{vg}} \tag{7}$$

where α_{vg}, n_{vg} and m_{vg} are parameters calibrated to fit experimental data.

It is worth noting that this stress framework choice provides many advantages in terms of numerical implementation. Indeed, for a degree of saturation $S_r = 1$, Eq. (2) becomes $\boldsymbol{\sigma}^* = \boldsymbol{\sigma} - u_w \mathbf{I}$, and Terzaghi's saturated effective stress is recovered. This smooth transition between unsaturated and saturated states makes it easy to simulate the behaviour of a soil submitted to negative as well as positive water pressures with a single model.

2.3. Dissipative mechanisms: hydro-mechanical plasticity and damage models

Under deviatoric loading, clayey soils can undergo large permanent strains. Their properties, such as stiffness, strength, or permeability, are also known to be subject to changes after being submitted to hydric or mechanical solicitations. In clayey soils, these changes can be related to the destructuration of the material or to the deterioration of water bonds induced by tension in the menisci. Several approaches have been used to model this degradation. Some models have been developed which assume elastic moduli to be functions of the amount of plastic strains (Gajo and Bigoni, 2008; Hueckel, 1976; Sulem et al., 1999). However, these models usually (except for Sulem et al., 1999) do not incorporate a strength reduction with plastic straining. Other models, developed for so-called structured, bonded or sensitive clays, focus on the increase in the size of the yield surface due to structure, which decreases at large strains to recover the yield surface of the reconstituted material (Baudet and Stallebrass, 2004; Karstunen et al., 2005; Kavvadas and Amorosi, 2000; Liu and Carter, 2002; Nova et al., 2003; Rouainia and Muir wood, 2000). A parameter is then introduced to account for the degradation of structure, which evolves with plastic strains in the aforementioned models. These models do not account for the concomitant degradation of elastic stiffness, although it has been shown to decrease during loading for stiff clays such as claystones (Chiarelli et al., 2003). Boom Clay samples extracted around a gallery have also shown a reduced small-strain shear modulus in the excavation damage zone (Dao et al., 2015) Another approach, the one that is used in this paper, is to use the framework of Continuum Damage Mechanics, first developed for metals and later extended to concrete and rocks. This approach assumes that the degradation of material properties is due to the initiation and propagation of microcracks in rocks. This approach has been used for concrete behaviour modelling (Grassl and Jirásek, 2006) as well as for stiff cemented clays (Einav et al., 2007). Several approaches were proposed to model the evolution of stiffness and the accumulation of irreversible deformation induced by anisotropic damage (Arson, 2014). Up to now, few attempts have been made to model damage in unsaturated geomaterials. Some models have been developed which consider damage in unsaturated geomaterials (Arson and Gatmiri, 2009), damage-plasticity couplings in saturated geomaterials (Chiarelli et al., 2003; Conil et al., 2004; Yu et al., 2013), damage and viscoplasticity in unsaturated geomaterials (Dufour et al., 2012), and even damage-plasticity couplings in unsaturated geomaterials (Hoxha et al., 2007; Jia et al., 2007). However, these models, initially formulated for rocks, ignore some specific important features of clayey soil behaviour, such as the dependence of elastic moduli to pressure. Moreover, damage-plasticity models proposed for rocks so far fail at predicting the transition between ductile and brittle behaviour associated with suction increase. Vaunat and Gens (2003) developed a model for bonded granular soils, based on microstructural considerations, which is able to reproduce both strength and stiffness degradation coupled with elastoplasticity. It has been later extended to other materials and loading scenarios (Cardoso et al., 2013; Pinyol et al., 2007; Yang et al., 2008). This model, however, has been developed for a very specific class of geomaterials. By contrast, our modelling framework can be applied to a broader range of materials, from quasi-brittle stiff clays for which damage occurs in the elastic strain domain, to soft clays for which plasticity is the predominant dissipative mechanism.

2.4. Principle of effective stress in Continuum Damage Mechanics

Introduced by Kachanov (1958), the effective stress in the sense of damage mechanics is based on the fact that the resisting section decreases when micro-cracks develop. This approach can be extended to a broad range of unsaturated clay-bearing geomaterials, in which damage also represents the loss of bonding due to water menisci. Note for instance that in some thermodynamic frameworks (Coussy et al., 2010), air–water interfaces are part of the apparent solid skeleton.

A scalar damage variable, d, is defined as an average of the proportion of damaged surfaces in the material. d ranges from $d = 0$ for an intact material to $d = 1$ for a totally damaged material with no residual resistance. Assuming that damage is isotropic and affects similarly all components of the stress tensor, the *effective stress* tensor then becomes

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{1 - d} \tag{8}$$

More complex expressions could be used in place of Eq. (8), in order to accommodate more sophisticated behaviours, such as anisotropic damage.

In this study, damage is a phenomenological variable that describes the combined effects of multiple mechanisms on elastic stiffness degradation.

3. Hydro-mechanical damage-plasticity model based on the concept of double effective stress

3.1. Introduction of a double effective stress accounting for suction and damage effects

The two previous sections allowed us to introduce two quantities describing the stress applied on the solid matrix. On the one hand, the constitutive stress, in unsaturated soils, takes into account the effect of water menisci in tension, acting like a compressive stress on the solid matrix. On the other hand, the effective stress, in the sense of damage mechanics, enables us to account for the decreasing material surface sustaining mechanical loads, resulting from the creation of micro-cracks and the modification of water bonds. There is a need to define a new quantity, representing the stress applied on the solid matrix when the material is affected by both suction and damage simultaneously. This quantity will be called the *double effective stress* and is assumed to control the porous material mechanical behaviour.

Two simple combinations of the previous effective stresses can be imagined to incorporate both damage and suction into this double effective stress, $\tilde{\boldsymbol{\sigma}}^*$:

$$\tilde{\boldsymbol{\sigma}}_1^* = \frac{\boldsymbol{\sigma} - u_a\mathbf{I} + sS_r\mathbf{I}}{1-d} = \frac{\boldsymbol{\sigma}^*}{1-d} \tag{9}$$

$$\tilde{\boldsymbol{\sigma}}_2^* = \frac{\boldsymbol{\sigma}}{1-d} - u_a\mathbf{I} + sS_r\mathbf{I} = \tilde{\boldsymbol{\sigma}} - u_a\mathbf{I} + sS_r\mathbf{I} \tag{10}$$

To choose between these two expressions, we assume that a damaged sample submitted to a change in suction should behave differently compared to the intact sample. This hypothesis has been considered by other authors, such as Carmeliet and Van Den Abeele (2000), who consider that damaged materials experience more swelling when wetted that intact ones.

Assuming that the total applied stress, the gas pressure, as well as damage are kept constant, the change in the double effective stress due to a suction increment would be:

$$\dot{\tilde{\boldsymbol{\sigma}}}_1^* = \frac{(s\dot{S}_r + S_r\dot{s})\mathbf{I}}{1-d} \tag{11}$$

$$\dot{\tilde{\boldsymbol{\sigma}}}_2^* = (s\dot{S}_r + S_r\dot{s})\mathbf{I} \tag{12}$$

Thanks to the use of a hyperelastic formulation (Section 3.3) elastic strains are directly related to the double effective stress. The strain change due to suction change would therefore be the same for an intact and a damaged sample for the second expression.

We will thus choose the first expression for the ***double effective stress***:

$$\tilde{\boldsymbol{\sigma}}^* = \frac{\boldsymbol{\sigma} - u_a\mathbf{I} + sS_r\mathbf{I}}{1-d} \tag{13}$$

We define the following quantities:

- Mean stress: $p = \frac{1}{3}tr(\boldsymbol{\sigma})$
- Deviatoric stress tensor: $\boldsymbol{\sigma}_d = \boldsymbol{\sigma} - p\mathbf{I}$
- Deviatoric stress: $q = \sqrt{\frac{3}{2}\boldsymbol{\sigma}_d : \boldsymbol{\sigma}_d}$

Then the double effective triaxial variables are:

$$\tilde{p}^* = \frac{p - u_a + sS_r}{1-d} \tag{14}$$

$$\tilde{q}^* = \tilde{q} = \frac{q}{1-d} \tag{15}$$

It can be noted that with this definition of the double effective stress, suction effects are isotropic, and thus don't have any impact on the deviatoric stress.

The existence of a double effective stress, in which suction and damage effects on mechanical behaviour are included, is a key assumption in the following modelling developments. In the following sections, we will study how this double effective stress allows for damage and suction effects on elastic and dissipative behaviours to be reproduced.

3.2. Expression of Helmholtz free energy

We assume that the material state is described by the values of the following state variables: The elastic strain, $\boldsymbol{\varepsilon}^e$, the degree of saturation, S_r, damage, d, and a hardening variable, $\boldsymbol{\chi}$. We assume that elastic, plastic and hydraulic potential energy functions are decoupled and that processes are isothermal. We propose the following form for Helmholtz free energy:

$$\psi = \psi(\boldsymbol{\varepsilon}^e, S_r, d, \boldsymbol{\chi}) = \psi^e(\boldsymbol{\varepsilon}^e, d) + \psi_l(S_r) + \psi^p(\boldsymbol{\varepsilon}^p, \boldsymbol{\chi}) \tag{16}$$

In order to build a damage constitutive model, an extra assumption has to be added to the concept of effective stress.

Concerning the damage-elastic part of Helmholtz free energy, we choose to use the form proposed by Ju (1989),

$$\psi^e(\boldsymbol{\varepsilon}^e, d) = \psi_0^e(\boldsymbol{\varepsilon}^e)(1-d) \tag{17}$$

which, after derivation gives the following expression of the constitutive stress:

$$\boldsymbol{\sigma}^* = \frac{\partial \psi^e}{\partial \boldsymbol{\varepsilon}^e} = (1-d)\frac{\partial \psi_0^e}{\partial \boldsymbol{\varepsilon}^e} \tag{18}$$

The double effective stress is therefore related to elastic strains through the following constitutive relationship:

$$\tilde{\boldsymbol{\sigma}}^* = \frac{\boldsymbol{\sigma}^*}{1-d} = \frac{\partial \psi_0^e}{\partial \boldsymbol{\varepsilon}^e} \tag{19}$$

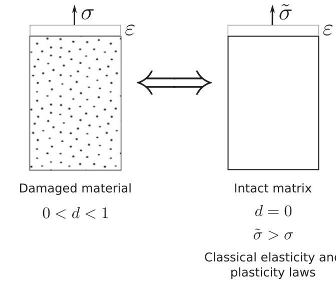

Fig. 2. Principle of strain equivalence.

The relationship given in Eq. (19) implies that, in a damaged material, the double effective stress will be linked to elastic strains with the same relationships that the constitutive stress in an intact material. This is the principle of strain equivalence defined by Lemaitre and Chaboche (1978) which states that the strain associated with a damaged state under the applied stress is equivalent to the strain associated with its undamaged state under the effective stress. The principle of strain equivalence is illustrated in Fig. 2.

This approach has the advantage of being easily extensible to plasticity, by replacing stresses by double effective stresses in classical equations.

3.3. Elasticity

For the sake of simplicity, the elasticity is assumed to be linear in the following developments. However, experimental evidence show that bulk and shear moduli of geomaterials increase with confining pressure, which may have an important effect on the material behaviour, especially if a large confining pressure range is considered. The present framework can be adapted to non-linear elasticity. In order to ensure the conservation of the elastic deformation energy, it is necessary to formulate the model within the framework of hyper-elasticity (Zytynski et al., 1978). Challenges related to the degradation of pressure dependent elastic moduli in porous material were discussed in Le Pense (2014).

Incorporating linear elasticity into Eq. (19) gives:

$$\begin{Bmatrix} \tilde{p}^* \\ \tilde{q} \end{Bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} \varepsilon_v^e \\ \varepsilon_s^e \end{Bmatrix} \tag{20}$$

in which K is the bulk modulus, G the shear modulus, ε_v^e the volumetric elastic strain, and ε_s^e the deviatoric elastic strain.

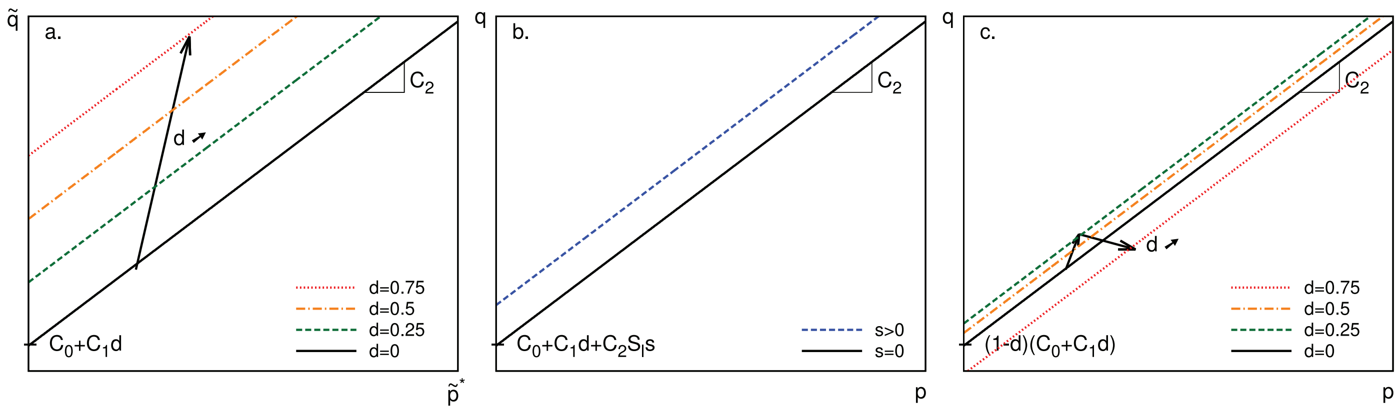

Fig. 3. Shape of damage criterion. (a) Double effective stress space, (b and c) total stress space (b. effect of suction, c. effect of damage).

This gives the following apparent stiffness matrix when expressed in terms of constitutive stresses:

$$\begin{Bmatrix} p^* \\ q \end{Bmatrix} = \begin{bmatrix} K(1-d) & 0 \\ 0 & 3G(1-d) \end{bmatrix} \begin{Bmatrix} \varepsilon_v^e \\ \varepsilon_s^e \end{Bmatrix} \tag{21}$$

Coupling of the principle of strain equivalence with a damaged effective stress therefore leads to a degradation of apparent elastic moduli with damage, without the need to explicitly express them as functions of the damage parameter.

3.4. Damage onset and evolution

Since suction has no effect on deviatoric stress, we will not consider the anisotropy induced by damage in this paper. We adopt Drucker–Prager damage criterion, which is expressed in terms of double effective stresses, so as to follow the principle of strain equivalence:

$$f_d = \tilde{q} - C_2\tilde{p}^* - C_0 - C_1 d = 0 \tag{22}$$

in which C_1 is a hardening parameter. The lower C_1, the faster d will increase with deviatoric stress. The C_2 coefficient allows the dependence on confining pressure to be accounted for. Indeed, geomaterials are known to be more brittle at low confining pressure and more plastic at high confining pressures. C_0 enables the modification of the damage threshold.

The shape of the damage criterion in the double effective stress space is given in Fig. 3a for different values of damage. It can be seen that, when damage increases, the material is hardening with respect to effective stresses.

Expressed in total stresses, Eq. (22) becomes:

$$\frac{q}{1-d} - C_2\frac{p+sS_r}{1-d} - C_0 - C_1 d = 0 \tag{23}$$

$$q - C_2(p+sS_r) - (1-d)(C_0 + C_1\ d) = 0 \tag{24}$$

The shape of the corresponding damage criterion in the total stress space is given in Fig. 3.

Fig. 3b shows the evolution of the damage criterion with suction. Although suction does not have an effect on the damage criteria in the double effective stress space, suction increases the stress value for which damage is initiated when considering total stresses.

Fig. 3c shows the evolution of the damage criterion with damage. It can be seen that, although the intact fraction of the material is hardening, an apparent softening behaviour appears after a certain value of damage is reached, when considering total stresses.

Deriving Eq. (22) gives the consistency condition,

$$0 = \dot{f}_d = \frac{\partial f_d}{\partial \tilde{p}^*}\dot{\tilde{p}}^* + \frac{\partial f_d}{\partial \tilde{q}}\dot{\tilde{q}} + \frac{\partial f_d}{\partial d}\dot{d} \tag{25}$$

from which the damage evolution law can be deduced:

$$\dot{d} = \mathbf{A}_d(\tilde{\boldsymbol{\sigma}}) : \dot{\tilde{\boldsymbol{\sigma}}}^* \tag{26}$$

where $\mathbf{A}_d = \frac{1}{C_1}[-\frac{C_2}{3}\mathbf{I} + \frac{\tilde{\sigma}_d}{\tilde{q}}]$.

This expression of the damage evolution rate as well as the damage criterion implies that damage initiation and evolution are solely related to elastic strains. Note that some authors assumed that damage is initiated by an accumulation of plastic strains. By contrast, we decoupled damage and plasticity, which allows modelling a wide range of materials, subject to damage propagation only, or plastic dissipation only, or both damage and plastic dissipation.

3.5. Coupled damage and plasticity model: suction hardening and damage softening

According to Jommi (2000), extending a poromechanical model from saturated to unsaturated materials requires the two following steps:

- the substitution of the average skeleton stress for effective stress
- introduction in the basic saturated elastoplastic model of the modifications necessary to take into account the effects of the interfaces on the overall mechanical behaviour

According to Ju (1989), plasticity occurs only in the undamaged counterpart of the bulk, and the expression of plastic flow for the damaged material can be obtained by using effective properties and effective stress in the expression of plastic flow for undamaged materials. Therefore, the characterisation of the plastic response should be formulated in the damaged effective stress space and the stress tensor should be replaced by the damaged stress tensor, $\tilde{\boldsymbol{\sigma}}$, into the equations of plasticity. This follows the principle of strain equivalence.

Similarly to the Barcelona Basic Model (BBM) (Alonso et al., 1990), the most widely used model for unsaturated soils, we use the modified Cam-Clay model (Burland, 1965) as a basis to predict plasticity in saturated geomaterials. Based on Jommi's and Ju's recommendations, we formulate the yield criterion in terms of double effective stress to extend the model to damaged and unsaturated geomaterials. In addition, we introduce a dependence of the yield criterion to suction:

The ***yield surface*** is therefore taken of the following form:

$$f_p = \tilde{q}^2 - M^2\tilde{p}^*(p_c^*(p_0, s) - \tilde{p}^*) \tag{27}$$

in which p_c^* is the preconsolidation pressure, which is a function of suction and the saturated preconsolidation pressure, p_0 (Eq. (34)).

Cam-clay models have been developed in the framework of Critical State Soil Mechanics (Roscoe et al., 1958). The ***critical state*** concept states that soils and other granular materials, if continuously distorted until they flow as a frictional fluid, will come into a well-defined critical state. At the onset of the critical state, shear distortions occur without any further changes in mean stress, deviatoric stress or void ratio. The critical state is described in the $(\tilde{p}^*, \tilde{q})$ plane by the line of equation:

$$\tilde{q} = M\tilde{p}^* \tag{28}$$

In some recent models (such as BBM), non-associate flow rules are adopted to predict plastic volumetric strains under a variety of stress paths. For the sake of simplicity, we considered an associate flow rule (like in the original Cam-Clay model). The ***plastic potential*** is defined as:

$$g_p = f_p = \tilde{q}^2 - M^2\tilde{p}^*(p_c^* - \tilde{p}^*) \tag{29}$$

The plastic flow rule is:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\Lambda}_p \frac{\partial g_p}{\partial \tilde{\boldsymbol{\sigma}}^*} = \dot{\Lambda}_p \left(\frac{\partial g_p}{\partial \tilde{p}^*}\frac{\mathbf{I}}{3} + \frac{\partial g_p}{\partial q}\frac{3\tilde{\boldsymbol{\sigma}}_d}{2\tilde{q}} \right) \tag{30}$$

in which

$$\frac{\partial g_p}{\partial \tilde{\boldsymbol{\sigma}}^*} = \frac{M^2(2\tilde{p}^* - \tilde{p}_c{}^*)}{3}\mathbf{I} + 3\tilde{\boldsymbol{\sigma}}_d \tag{31}$$

The ***hardening law*** is defined as:

$$\dot{p}_0 = \frac{p_0}{\lambda - \kappa}\dot{\varepsilon}_v^p \tag{32}$$

In order to reproduce the extension of the elastic domain with suction, the preconsolidation pressure is sought in the form of a function of suction and saturated preconsolidation pressure (p_0):

$$p_c^* = p_c^*(p_0, s) \tag{33}$$

When drawn in the (p, s) plane, this curve is called the Loading-Collapse (LC) curve. Many different expressions were proposed for the equation of the LC curve (Alonso et al., 1990; Buisson et al., 2003; Jommi, 2000; Sheng et al., 2004; Sun et al., 2008). Experimental characterisation of clay-bearing unsaturated materials is often driven by the determination of BBM parameters. In order to facilitate the calibration of our model, we chose the LC equation proposed by Sheng et al. (2004), because Sheng et al.'s approach is the closest we found to the BBM model. The equation of the LC curve is:

$$p_c^* = p_r\left(\frac{p_0}{p_r}\right)^{\frac{\lambda-\kappa}{\lambda_s-\kappa}} + sS_r \tag{34}$$

$$\lambda_s = \lambda[(1-r)\exp(-\beta s) + r] \tag{35}$$

The shape of the yield criterion in the double effective stress space is given in Fig. 4a. As expected, the yield surface in the double effective stress space does depend on suction, but not on damage.

Expressed in total stresses, the equation of the yield surface (Eq. (27)) becomes

$$q^2 - M^2(p + sS_r)[(1-d)p_c^* - (p + sS_r)] = 0 \tag{36}$$

and the equation of the critical state line becomes

$$q = M(p + sS_r) \tag{37}$$

Fig. 4b shows the evolution of the yield surface with suction. With respect to total stresses, the elastic domain increases with suction. Suction also induces an apparent cohesion.

Fig. 4c shows the evolution of the yield surface with damage. Damage has a softening effect on the plastic behaviour. Although plastic and damage dissipative potentials were assumed to be decoupled, the assumption of a double effective stress, associated with the principle of strain equivalence, allows for a direct damage-plasticity coupling. Indeed, although damage and plasticity criteria are expressed in terms of the double effective stress, and consequently do not depend explicitly on damage and suction, they evolve with damage and suction in the total stress space.

The following section will illustrate how the proposed model behaves for its mechanical part, based on specific sets of parameters.

4. Illustration of the mechanical damage-plastic behaviour

The model has been designed in a flexible way, which enables the independent refinement of its basic components (effective stresses, elasticity, damage and plasticity equations) to fit specific materials behaviours. Analysis of the model behaviour, as well as its validation, will focus on clayey geomaterials, such as Boom clay, since these materials exhibit simultaneously a strong plastic behaviour, as well as damage.

4.1. Summary of Boom Clay data from the literature

Boom Clay has been selected as a possible host rock for deep radioactive waste disposal in Belgium. It is considered as an overconsolidated plastic clay.

Most experimental studies published on Boom Clay focus on the characterisation of physical properties (such as retention and permeability), or on the hydro-mechanical response of the soil in unsaturated conditions. Very few measures were done to study

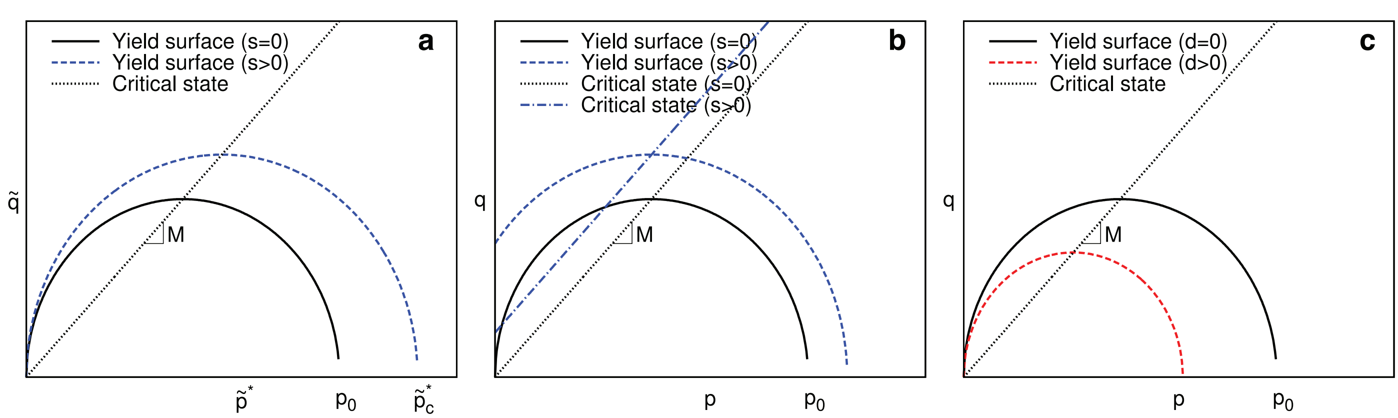

Fig. 4. Shape of yield surface. (a) Double effective stress space, (b and c) total stress space (b. effect of suction, c. effect of damage).

Table 1
List of the model parameters and their range of values as found in the literature.

	François et al. (2009)	Bésuelle et al. (2013)	Delahaye and Alonso (2002)	Della Vecchia et al. (2011)[a]	Wu et al. (2004)
Elasticity					
E (MPa)	200–400	150–500	70[b]		
ν	0.125–0.45		0.333		
Retention					
α_{vg} (MPa^{-1})				0.15(d)–0.5(w)	
m_{vg}				0.19(d)–0.22(w)	
n_{vg}				2.8(d)–2(w)	
Plasticity					
M			1	0.78	
$\lambda-\kappa$[c]			0.15	0.06	0.03–0.23
p_0 (MPa)	5.4–6			4	
r			0.564		0.015–0.3
β (MPa^{-1})			54.4		0.41–1.336
p_r (MPa)			0.06		0.595–1.2

[a] Results for natural Boom Clay, parameters for drying and wetting curves.
[b] Calculated from $K=\frac{k_i}{(1+e)p}$ (non-linear elasticity) for an initial state $e=0.59$ and $p=4.4\,$MPa.
[c] equivalent to $\frac{\lambda-\kappa}{1+e}$ in the cited references.

the degradation of stiffness with stress and suction, despite the proven existence of an Excavation Damaged Zone (EDZ) around underground openings. Excavation induced fractures were observed around galleries (Bastiaens et al., 2007; Bernier et al., 2007; Van Marcke and Bastiaens, 2010). Damage was also studied by means of seismic (Bastiaens et al., 2007) and acoustic (Lavrov et al., 2002) measurements. Boom clay can exhibit both ductile and brittle behaviours (Dehandschutter et al., 2005). More recent studies using advanced imaging technique provided evidence of cracks in Boom Clay samples (Bésuelle et al., 2013) The transition between the failure modes depends strongly on the confining pressure and is also influenced by the water content (Al-Shayea, 2001) and by the overconsolidation ratio.

Boom clay has been extensively studied either from experiments on undisturbed natural samples, or on samples prepared by compaction from Boom clay powder. Many experimental data on saturated natural Boom clay are available in the literature (Baldi et al., 1991; Coll, 2005; Sultan et al., 2010). However, concerning the unsaturated behaviour, most of the studies have been made on compacted (Bernier et al., 1997; Romero, 1999) or remoulded (Al-Mukhtar et al., 1996) samples, and only a few on undisturbed samples (Cui et al., 2007; Della Vecchia et al., 2011). Moreover, mechanical tests at different suctions are limited to oedometer and isotropic compression tests. By comparing experiments on natural and compacted samples, (Della Vecchia et al., 2011) concluded that the same constitutive framework seems to be applicable to natural Boom clay and to the material compacted from the clay powder. However, mechanical parameters have to be adapted for different microstructures.

Boom clay is a more complex material than other clay stones such as Callovo-Oxfordian argillites, which exhibit a less plastic behaviour. The following simulations will demonstrate that the modelling approach that we proposed above is suitable to predict the brittle/ductile transition in unsaturated geomaterials in which the behaviour is strongly influenced by plastic deformation, confining pressure, and water content.

Data available to calibrate our model involves tests performed on cores of different origins, taken at different depths. The mineral composition of the samples varied greatly from one experiment to the other, and therefore, a high variability was noted in the mechanical and physical properties. In the following numerical study, we used material parameters that fell in the range of values reported in the literature, and we adapted the set of parameters to the different soils tested, in order to match experimental test results.

Table 2
Parameters chosen as a basis for the parametric study.

Elasticity		Plasticity			Damage			Initial state
E MPa	ν	M	$\lambda-\kappa$	p_0 MPa	C_0 MPa	C_1 MPa	C_2	p MPa
300	0.4	1	0.05	5.5	0	10	0.5	3

Some values found in the literature for elasticity, plasticity, and retention parameters are given in Table 1. No similar damage model has been found which would allow to determine the range of values for our damage parameters.

The set of data chosen to study the sensitivity of the damage-plastic model to the different parameters is given in Table 2.

4.2. Damage model

Although the damage part of the model has been chosen to be formulated with the minimum number of parameters, and to be based on Drucker–Prager, no identical model, expressed in terms of the damaged effective stress has be found in the literature. The behaviour of this model, and the range of parameters for which it gives sensible results is studied in this section.

It can be seen in Fig. 5a that the model can exhibit a radial contraction under triaxial loading for certain sets of parameters.

This feature appears for sets of parameters which do not respect Eq. (38) (details of the calculation are given in Appendix A):

$$\frac{3(1-2\nu)}{1+\nu}\leqslant C_2\leqslant 3 \tag{38}$$

in which ν is the Poisson's ratio, and C_2 the slope of the damage criterion.

Fig. 5 shows the effect of the damage parameters C_2 (slope of the damage criterion) and C_1 (hardening parameter) on the stress–strain curves as well as the evolution of damage with axial strain. As expected from the theoretical developments presented earlier, the damage model can reproduce a hardening behaviour followed by a softening behaviour. It can be seen that variations of C_2 result mainly in a modification of the damage threshold, whereas changing C_1 modifies the damage evolution rate. High values of C_1 therefore result in higher peak stress values.

4.3. Damage-plasticity coupling

The plasticity part of the model is similar to BBM, expressed in terms of the unsaturated constitutive stress. It has been widely

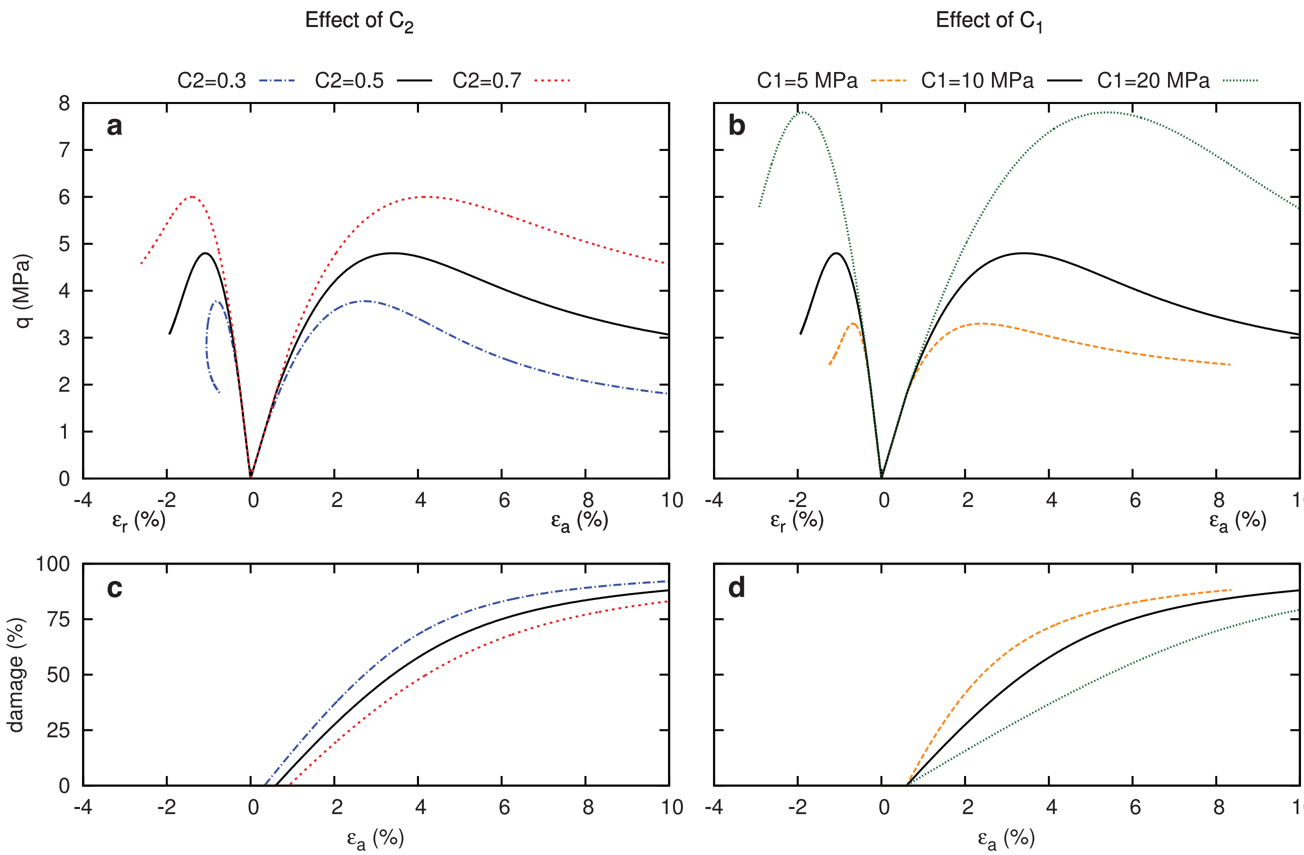

Fig. 5. Effect of C_2 (a,c) and C_1 (b,d) on the stress–strain curves (a,b) and damage evolution as a function of axial strain (c,d).

studied in the literature and sensitivity to its parameters will not be detailed here.

Fig. 6 compares the behaviour of the coupled damage plasticity model to the behaviour when only damage or plasticity is considered. Two cases are presented, one for which plasticity is the dominant dissipative phenomena, using the parameters of Table 2 (Fig. 6a,c,e), and one for which damage is dominant (Fig. 6b,d,f).

For the plasticity dominated case, it can be seen in Fig. 6a that the softening behaviour after deviatoric stress peak, characteristic of the damage model, is absent for the coupled model. The coupled model stress–strain behaviour also follows the same pattern as the plasticity model. This shows that, when using model parameters suitable for Boom clay, plasticity dominates damage effects. Fig. 6c shows, however, that damage is triggered, and develops up to 20%. When looking only at the stress–strain curve, one could mistake the non-linear behaviour as being the results of plasticity effects only. It should therefore be noted that the analysis of the strain–stress measurement only could hide the appearance of damage. The main effect of damage in that case, is to decrease the apparent yield stress of the material, as seen in Fig. 6a, which has a negligible effect on the final amount of plastic strain (Fig. 6e).

In order to demonstrate the capabilities of the model, Fig. 6b shows the behaviour of the model when damage dominates plasticity effects. The parameters chosen for this example are mostly the one of Table 2, but with $C_1 = 5\,\text{MPa}$ and $\lambda - \kappa = 0.01$, values chosen to increase the influence of damage, and decrease the influence of plasticity. These results show that the presented model can also reproduce a damage dominated behaviour, with a stress–strain behaviour of the coupled model similar to the one of the damage model. Plasticity effects result in greater strains, but have no effect on the peak deviatoric stress.

These illustrative simulations show that the current model is highly versatile and, depending on the set of parameters chosen, can reproduce damage-plasticity couplings, dominated either by plasticity or damage behaviours.

5. Simulation of hydro-mechanical experiments on Boom Clay

This section aims at comparing simulation results, using the model developed in the previous sections, with hydro-mechanical experiments results on clayey soils from the literature. As mentioned previously, parameters are adjusted to fit specific tests, but are chosen to lie within the range of values reported in the literature.

5.1. Elastic swelling

A significant advantage of the constitutive stress approach is the ability to capture suction induced strains without the need of extra parameters in addition to mechanical and retention parameters.

To illustrate this feature, an oedometer swelling test was simulated on a clay material (experimental data from Volckaert et al., 1996). The vertical stress was kept constant ($\sigma_v = 0.1\,\text{MPa}$) while suction was decreased from 230 to 0 MPa. The mechanical and retention properties chosen in the simulation are given in Table 3.

Swelling strains (volumetric strains) computed for an intact as well as a damaged material are represented in Fig. 7. Damage is assumed to remain constant during the test, and the suction state to be homogeneous within the sample.

Knowledge of the water retention properties in addition to mechanical stiffness parameters allows us to reproduce adequately the elastic swelling behaviour observed during wetting. Moreover, a different swelling behaviour is observed for intact and damaged samples, which is in accordance with other works

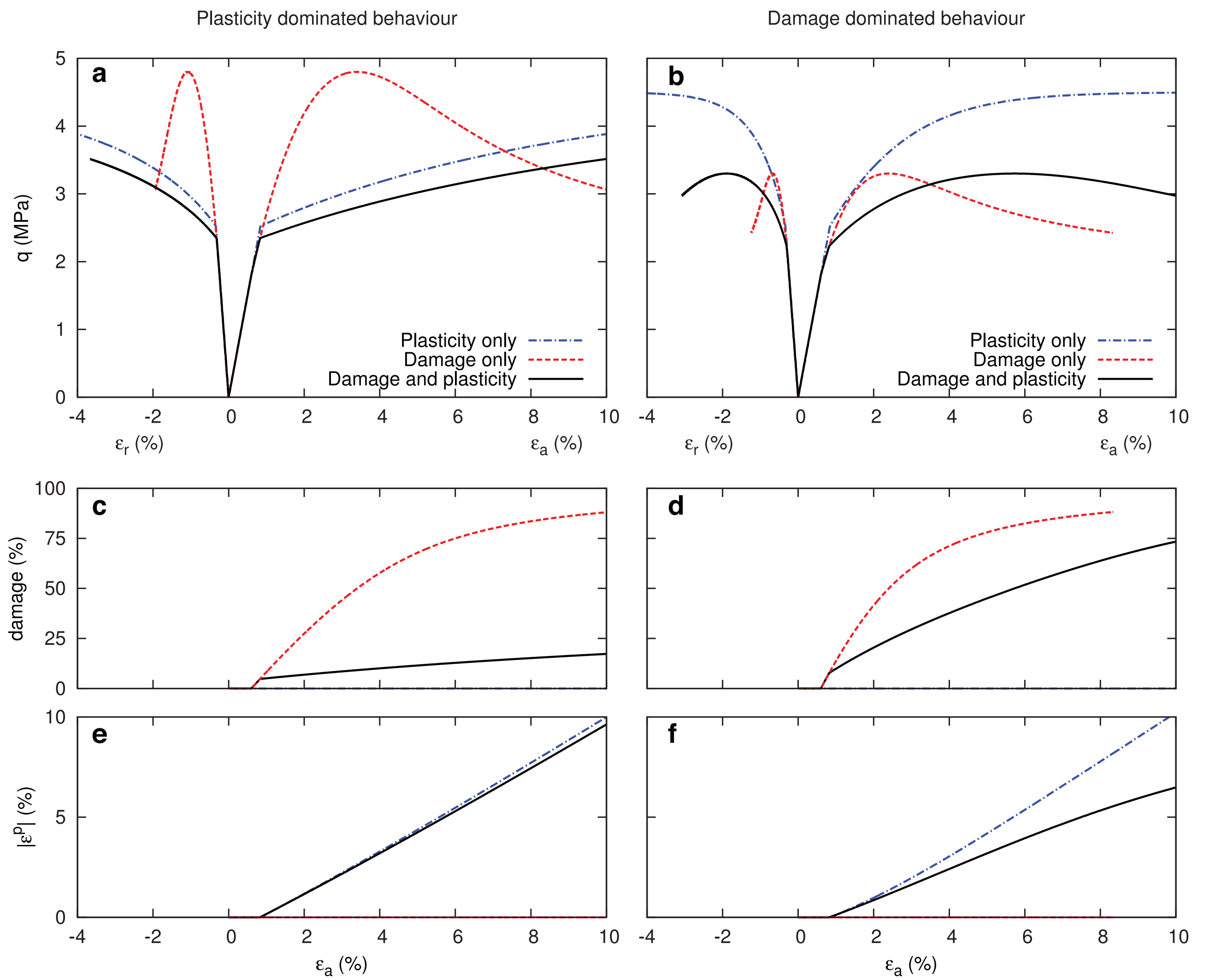

Fig. 6. Comparison of damage, plastic, and coupled model behaviour, for a plasticity dominated (a,c,e) and a damage dominated (b,d,f) coupling. (a,b) stress–strain curve, (c,d) damage evolution, (e,f) plastic strain evolution.

Table 3
Elasticity and retention parameters for the swelling test.

Elasticity		Retention			
K MPa	ν	S_r	α_{vg} kPa^{-1}	n_{vg}	m_{vg}
200	0.3	0	0.28.10^{-3}	2.3	0.21

(Carmeliet and Van Den Abeele, 2000). Indeed, the test is suction-controlled, therefore the volume of the sample can change as water tends to fill the pores during the wetting phase. Damaged samples are more compliant than undamaged samples: the resistance of the solid skeleton to pore filling and expansion is less in damaged materials, which tend to swell more than undamaged samples during wetting. This behaviour has been observed in oedometric swelling experiments on callovo-oxfordian argillite samples (Mohajerani et al., 2011), where it is seen that swelling capacity increases with damage.

5.2. Triaxial tests on saturated samples at different confining pressures

Triaxial drained compression tests with unloading–reloading cycles are simulated for two confining pressures (3 MPa and 4 MPa).

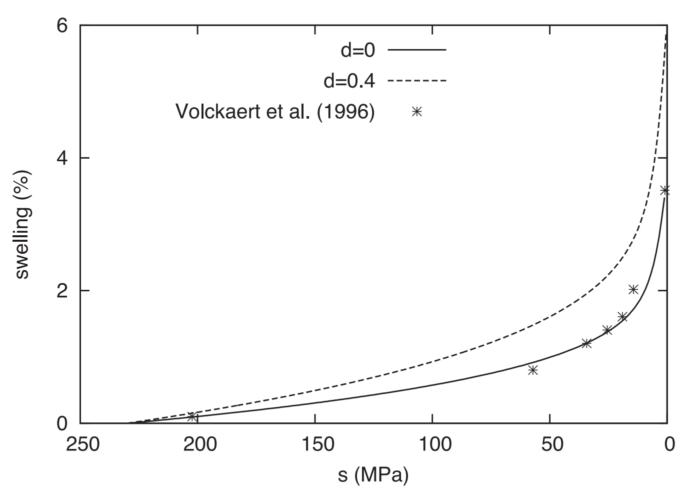

Fig. 7. Volumetric swelling strains in oedometric conditions under a vertical load $\sigma_v = 0.1$ MPa (compared with experimental data from Volckaert et al., 1996).

The experimental data (from Baldi et al., 1991) show the influence of the confining pressure on the deviatoric response. A degradation of the elastic modulus can also be seen from the unloading–reloading curves.

Table 4
Boom Clay mechanical parameters.

Elasticity		Plasticity			Damage		
K MPa	ν	M	$\lambda - \kappa$	p_0 MPa	C_0 MPa	C_1 MPa	C_2
300	0.4	1	0.05	5.5	0	4	0.5

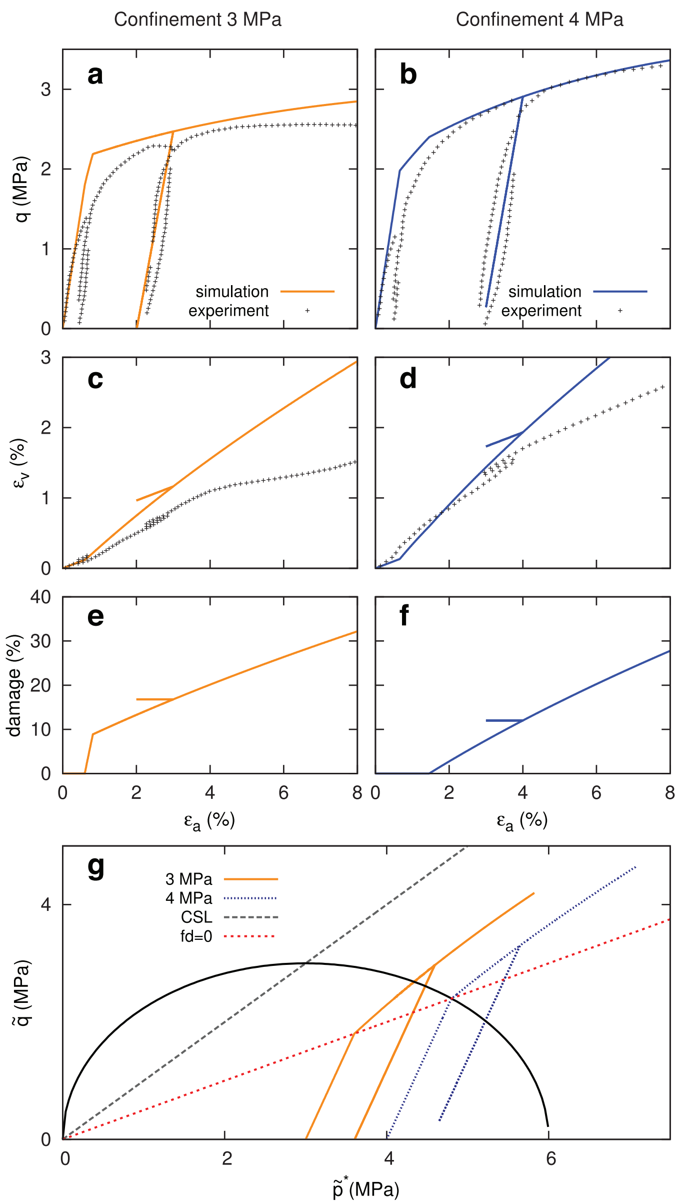

Fig. 8. (a–d) Triaxial test data from Baldi et al. for confinements (a,c) 3 MPa and (b,d) 4 MPa, compared with simulation results (a–f). (g) Double effective stress paths.

Elastic, plastic and damage parameters chosen in this study are summarised in Table 4. The preconsolidation pressure is taken equal to 6 MPa, which is in the range of values observed on samples from underground laboratories. The other mechanical parameters are chosen to fit the experimental results reported in Baldi et al. (1991).

Fig. 8 shows the comparison between experimental and numerical stress/strain curves. The main trends observed in the laboratory are captured by the model. For instance, it is noted that the stiffness measured during the unloading paths is less than the stiffness measured during the first loading paths. As expected, the loading stress supported by the sample before damage propagation is higher at higher confining pressure. However, the model does not capture well the smooth transition between elastic and plastic behaviour. This limitation of the model can be explained by the use of Cam-clay model, in which elasticity is assumed for all states of stress inside the yield surface. This behaviour could be improved by using more advanced versions of the Cam-Clay model, such as bounding surface plasticity (Dafalias, 1986) or continuous hyperplasticity (Puzrin and Houlsby, 2001). The volumetric behaviour could also be improved by using non-associated flow rules.

Fig. 8g shows the corresponding stress paths in the double effective stress space. It can be seen that for low confining pressure, the stress path attains the damage criterion earlier, which allows for more damage to be developed before the critical state is reached. The activation of the two competitive dissipation phenomena, damage and plasticity, depends on the confining pressure.

5.3. Simulation of the ductile/brittle transition with suction increase

Although no experimental data have been found in the literature about Boom clay, Al-Shayea (2001) showed that materials with high clay content exhibit a ductile/brittle behaviour transition when their water content decreases (see Fig. 9a). Ductile behaviour is characterised by the ability to sustain large plastic strains during plastic hardening. Brittle behaviour is characterised by abrupt failure at a well-defined peak strength with strong softening. Fig. 9a also shows higher shear strength for low water contents.

Although the experimental data from Al-Shayea (2001) are difficult to interpret and the exact experimental procedure cannot be reproduced in simulation due to the lack of data (retention properties, preconsolidation pressure, unloading–reloading curves), we will show that our model can reproduce a similar transition between a ductile and brittle behaviour when suction increases.

Triaxial compression tests under constant suction (0 MPa, 0.5 MPa, 1 MPa) are simulated. The confining pressure is taken equal to 200 kPa, and since samples are compacted in the experiments taken as reference, the preconsolidation pressure is taken equal to 500 kPa. The complete list of parameters chosen for this study are given in Table 5. The stress–strain curves obtained for different suctions are given in Fig. 9b. The corresponding effective stress paths can be seen in Fig. 9e. The evolution of damage and plastic strains with axial strain is given in Fig. 9c–d. Fig. 9b shows that our model can adequately reproduce the transition from a ductile behaviour for low suction, to a brittle behaviour for higher suctions.

At low suction, the plastic yield stress is low, and the plastic criterion is reached before the damage criterion. This leads to the development of large plastic strains, and damage remains low because of the lack of increase of the deviatoric stress . At higher suctions, the elastic domain is enlarged. The damage criterion is therefore reached before the plastic criterion. The deviatoric stress, and therefore damage, reaches higher values before the triggering of plasticity.

6. Discussion and conclusions

A constitutive modelling framework allowing for damage-plasticity couplings in unsaturated porous media has been proposed. This framework is based on the assumption of a double effective stress, accounting for damage and suction effects, which controls the material mechanical behaviour.

The principle of strain equivalence has been chosen for its ability to provide a straightforward way of coupling damage and plasticity. Damage and suction effects are taken into account by replacing the total stress by the double effective stress into elasticity

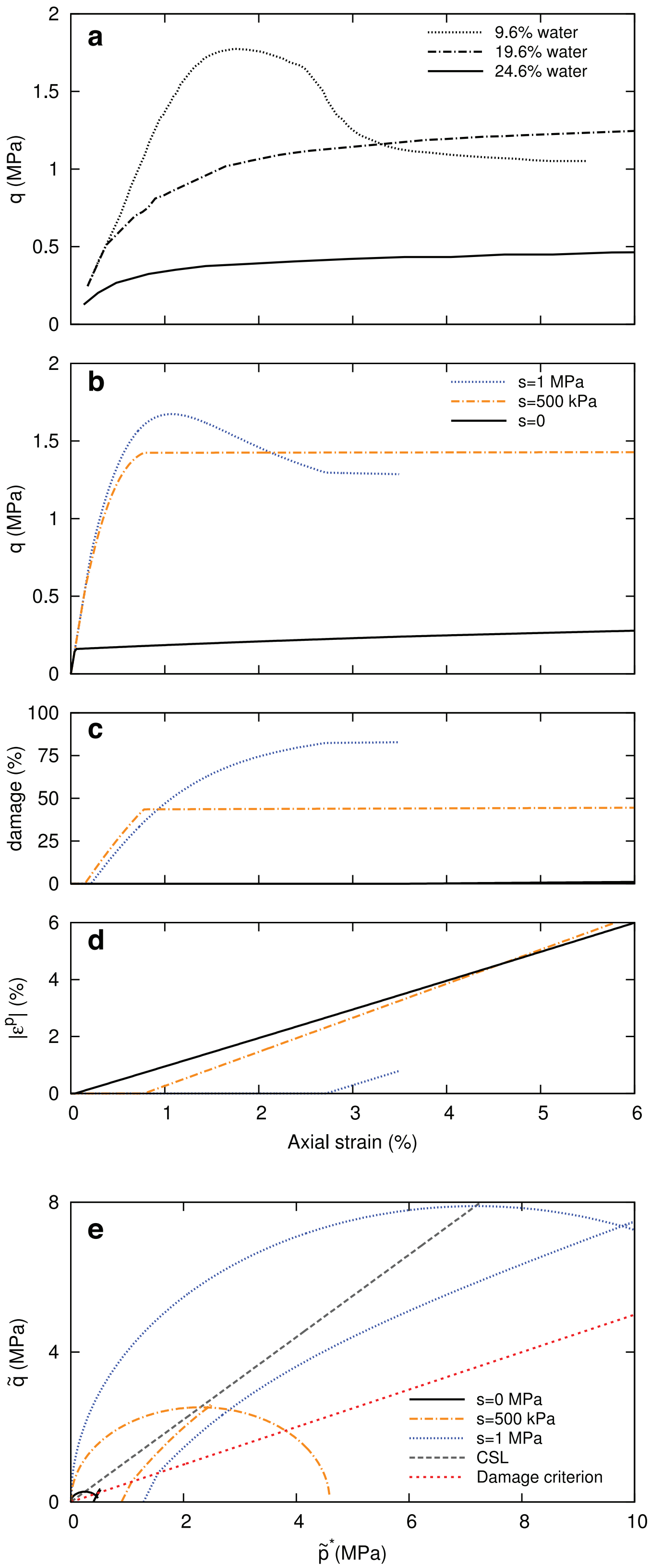

Fig. 9. Stress–strain behaviour of a clay-sand mixture (40% clay) at different moisture contents: (a) Experimental data from Al-Shayea (2001). (b) Present model. (c) Damage evolution. (d) Plastic strains. (e) Double effective stress path.

Table 5
Material mechanical parameters – ductile/brittle transition test.

Elasticity		Plasticity			LC curve			Damage			Retention		
K MPa	ν	M	$\lambda-\kappa$	p_0 kPa	p_r MPa	β MPa^{-1}	r	C_0 MPa	C_1 MPa	C_2	α_{vg} MPa^{-1}	n_{vg}	m_{vg}
300	0.3	1.1	0.2	500	0.001	2	0.6	0	3	0.5	0.28	2.3	0.21

and plasticity equations, which means that damage and plasticity criteria and evolution laws are expressed in terms of the double effective stress. This allows for a direct dependence of damage and plasticity criteria on suction and damage in the total stress space.

Illustrative examples have shown that the model is highly versatile, and can reproduce damage-plasticity coupled behaviour, dominated either by plasticity, or damage. This framework has therefore the potential to be adapted to various materials, from quasi-brittle stiff clays, in which damage and stiffness degradation are the dominating dissipative phenomena, to soft clays, in which plastic strains are predominant.

The developed model has then been used to reproduce experimental results on clayey soils, from the literature presented. Realistic parameters have been chosen so as to adequately represent a selected set of laboratory mechanical tests. Triaxial compression test at different suctions have then been simulated in order to highlight how the developed model capture the ductile/brittle transition due to suction increase.

The presented modelling framework, based on the combined assumptions of the existence of a double effective stress and the principle of strain equivalence, presents many advantages. The numerical implementation is straightforward and is able to accommodate different plasticity and damage models without the need of heavy code modifications.

Once implemented into a finite element code, this modelling framework will enable the modelling of fully coupled hydro-mechanical problems, such as desiccation-induced damage or the creation of the excavation damage zone around underground galleries.

Acknowledgements

The authors would like to thank ENS Cachan for funding the Ph.D. thesis of the first author.

Appendix A. Damage model, limits for which the sample experiences radial contraction during a triaxial test

For a triaxial stress state, the elasticity law reads:

$$\dot{\varepsilon}_r = -\frac{\nu}{E}\dot{\tilde{\sigma}}_a + \frac{1-\nu}{E}\dot{\tilde{\sigma}}_r \tag{A.1}$$

in which E is the Young's modulus, and ν the Poisson's ratio. Subscript a is related to the axial direction, and r to the radial direction.

Damage increment for a triaxial stress state:

$$\dot{d} = \frac{1}{C_1}(\dot{\tilde{q}} - C_2\dot{\tilde{p}}) = \dot{\tilde{\sigma}}_a\left(\frac{1}{C_1}\left(1-\frac{C_2}{3}\right)\right) - \dot{\tilde{\sigma}}_r\left(\frac{1}{C_1}\left(1+2\frac{C_2}{3}\right)\right) \tag{A.2}$$

$$\dot{\tilde{\sigma}}_a = \frac{3C_1}{3-C_2}\dot{d} + \frac{3+2C_2}{3-C_2}\dot{\tilde{\sigma}}_r \tag{A.3}$$

$$\dot{\varepsilon}_r = -\frac{\nu}{E}\frac{3C_1}{3-C_2}\dot{d} + \left[-\frac{\nu}{E}\frac{3+2C_2}{3-C_2} + \frac{1-\nu}{E}\right]\dot{\tilde{\sigma}}_r \tag{A.4}$$

Triaxial test $\Longrightarrow \dot{\sigma}_r = 0,\ \sigma_r = cste = \sigma_0$

$$\tilde{\sigma}_r = \frac{\sigma_r}{1-d} \Longrightarrow \dot{\tilde{\sigma}}_r = \frac{\dot{\sigma}_r}{1-d} + \frac{\dot{d}}{(1-d)^2}\sigma_r = \frac{\dot{d}}{(1-d)^2}\sigma_0 \tag{A.5}$$

$$\dot{\varepsilon}_r = -\frac{\nu}{E}\frac{3C_1}{3-C_2}\dot{d} + \left[-\frac{\nu}{E}\frac{3+2C_2}{3-C_2} + \frac{1-\nu}{E}\right]\frac{\dot{d}}{(1-d)^2}\sigma_0 \tag{A.6}$$

$$\dot{\varepsilon}_r = \left[-\nu\frac{3C_1}{3-C_2} - \nu\frac{3+2C_2}{3-C_2}\frac{\sigma_0}{(1-d)^2} + (1-\nu)\frac{\sigma_0}{(1-d)^2}\right]\frac{1}{E}\dot{d} \tag{A.7}$$

We want $\varepsilon_r \leq 0$. Since $\dot{d} \geqslant 0$:

$$-\nu\frac{3C_1}{3-C_2} - \nu\frac{3+2C_2}{3-C_2}\frac{\sigma_0}{(1-d)^2} + (1-\nu)\frac{\sigma_0}{(1-d)^2} \leqslant 0 \tag{A.8}$$

$$-\nu\frac{(1-d)^2}{3-C_2} - \nu\frac{3+2C_2}{3-C_2}\frac{\sigma_0}{3C_1} + (1-\nu)\frac{\sigma_0}{3C_1} \leqslant 0 \tag{A.9}$$

We can see that this expression is always true when there is no confinement, i.e. for $\sigma_0 = 0$ (true if $C_2 \leq 3$).

Otherwise:

$$(1-d)^2 + \left[(6+C_2) - \frac{3-C_2}{\nu}\right]\frac{\sigma_0}{3C_1} \geqslant 0 \tag{A.10}$$

If one wants it to be true for complete damage, i.e. $d = 1$, this gives the following relationship between the slope of the damage criterion, C_2, and the Poisson's ratio, ν:

$$C_2 \geqslant \frac{3(1-2\nu)}{1+\nu} \tag{A.11}$$

For a given set of parameters, it is also possible to determine from which value of damage radial contraction will start:

$$d = 1 - \sqrt{\left[\frac{3-C_2}{\nu} - (6+C_2)\right]\frac{\sigma_0}{3C_1}} \tag{A.12}$$

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