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A new CDM-based approach to structural deterioration

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Abstract

Structural deterioration often occurs without visible manifestation. Continuum damage mechanics (CDM) enables one to predict the state of damage in such situations and to estimate residual strength/service life of an existing structure. The accumulation of damage is modeled as a dissipative process that is governed by the laws of thermodynamics. The rate of dissipation in a deformable system, $\mathscr{R}$, depends on the work done on the system and the evolution of the Helmholtz free energy, Ψ. Under certain thermodynamical conditions, the first variation of Ψ vanishes, and partial differential equations for damage growth in $\mathscr{R}$ prior to damage localization are obtained. This approach obviates the need of introducing arbitrary dissipation potential functions with undetermined constants in the damage growth equations. All solutions use only readily available material parameters. Assuming that damage occurs isotropically under uniaxial loading, closed-form solutions are obtained for ductile damage as a function of plastic strain, for creep damage as a function of time and for fatigue damage as function of number of cycles. The models are validated with published laboratory data.

Keywords: Continuum damage mechanics; Creep; Cyclic loads; Deformation; Engineering mechanics; Fatigue; Plasticity; Steel; Structural engineering; Thermodynamics

1. Introduction

Deterioration of a structure may occur in a variety of ways depending on its operating environment and service loading conditions. Examples of such deterioration include gross inelastic deformation, high-temperature creep and fatigue. The deleterious effects of these processes may, depending on the initial condition and service history, accumulate without any discernible manifestation (like the formation of a macroscopic crack) for a major portion of the service life. Existing models for assessing structural deterioration generally require either a measurable flaw (e.g., Paris–Erdogan law in fatigue crack growth) or a prior estimate of failure time (e.g., Miner's rule in fatigue) to be useful, and are generally unable to provide an estimate of residual strength of a degrading structure. Several state variable approaches to measuring damage have been proposed

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in the literature. These methods variously define damage as, (i) changes in the dynamic response of the structure (DiPasquale and Cakmak, 1989; Agbabian et al., 1991; Hearn and Testa, 1991), (ii) fraction of elastic springs fractured in an idealized material (Krajcinovic and Silva, 1982; Breysse, 1990; Kandarpa et al., 1996), (iii) the time-dependent cumulative failure probability (Diao, 1995), and (iv) changes in the acoustic emission intensity (Fang and Berkovits, 1995). These methods usually lack a strong micromechanical and thermodynamical framework in their formulations. Moreover, such models also are largely empirical in nature. The irreversible and dissipative nature of damage accumulation can be more suitably modeled in the context of continuum damage mechanics using thermodynamic principles (Hansen and Schreyer, 1994).

1.1. Basic concepts of CDM

Continuum damage mechanics (CDM) defines damage in terms of the material microstructure (Krajcinovic, 1984; Simo and Ju, 1987; Hult, 1987), and relates the state of damage in a structural component to globally measurable quantities—for example, its elastic modulus and Poisson's ratio. CDM makes it feasible to estimate the state of damage in the seemingly 'defect-free' stage. It also enables one to predict the effects of different damage-causing processes using one unified method, and to determine the residual strength or remaining service-life of an existing structure.

In CDM, the damage, $D(\underset{\wedge}{n})$, on an elemental cross-sectional plane (identified by the normal vector $\underset{\wedge}{n}$) is quantified by the surface density of cracks and cavities at that section,weighted by their average stress-raising effects (Lemaitre, 1985). If, however, damage can be considered the same regardless of the orientation of the cross-section on which it is measured, then damage is isotropic, and is quantified by one single scalar variable, D, a dimensionless number between zero and one. Damage will be considered isotropic in this paper.

The constitutive law for damaged materials is commonly derived from the principle of strain equivalence, which states (Chaboche, 1988; Kachanov, 1986; Lemaitre, 1985): a damaged volume of material under the nominal stress $\underset{\sim}{\sigma}$ shows the same strain response as a comparable undamaged volume under the effective stress $\underset{\sim}{\tilde{\sigma}}$, defined as

$$\underset{\sim}{\tilde{\sigma}} = \frac{\underset{\sim}{\sigma}}{1-D} \tag{1}$$

$\underset{\sim}{\sigma}$ being the nominal stress. Applying this principle to the elastic strain tensor, the damaged modulus of elasticity, E, can be related to the isotropic damage variable as,

$$E = E(1-D) \tag{2}$$

where E is the modulus of elasticity for undamaged material and it is implicitly assumed that the Poisson's ratio remains unaffected by damage (de Vree et al., 1995). Equation (2) provides a means to estimate the state of damage in a deformable material by experimentally determining its reduced modulus of elasticity by one of several non-destructive methods, e.g., a direct tension test, or a measurement of ultrasonic pulse velocity or change in electrical resistivity (Lemaitre, 1992).

1.2. Failure criteria

In CDM, failure occurs when the damage variable equals the critical damage$D_c \leqslant 1$; experimental data typically indicate values of D_c ranging between 0.15 and 0.85 (Lemaitre, 1992). In the

context of CDM, 'failure' is not necessarily fracture, but is the point at which the essential assumption of damage mechanics—namely, damage arising out of a volume-wide degradation of the material microstructure—ceases to be applicable. In other words, 'failure', is that point when the damage-causing process becomes localized and leads to the growth of a dominant defect (Chaboche, 1988). In slow ductile deformation, D_c is interpreted as the value of damage corresponding to the onset of rupture. In fatigue or creep, it is associated with initiation of a macroscopic crack (Lemaitre, 1984; Pasic, 1992; Dhar et al., 1996, Chow and Wei, 1991). The postulate that D_c is an intrinsic material property (see, for example, Chow and Wei, 1991) provides a tool to predict the time to failure in a complex loading situation by using the value of D_c obtained from a simple static tension test for the same material and temperature.

2. Existing CDM models of damage growth

Existing CDM-based approaches to modeling of damage growth can be broadly grouped into one of two categories:

(i) Postulating phenomenological or 'kinetic equations' of damage growth:

$$\dot{D} = f(D, \mathbf{x}; \omega) \tag{3}$$

where $\mathbf{x}$ is the set of internal variables (e.g., strain) and ω denotes material parameters. The pioneering work of Kachanov (1958) proposed the kinetic equation of damage growth under uniaxial tension (applicable to brittle fracture and creep) as:

$$\frac{dD}{dt} = A\left(\frac{\sigma}{1-D}\right)^n; \quad D = 0, t = 0 \tag{4}$$

where σ is the nominal applied stress, $A > 0$, $n \geqslant 1$ are material constants. This approach subsequently was used, among others, by Jun and Xing (1995), Carmeliet and Hens (1994) and Paas et al. (1993).

(ii) Postulating thermodynamic potential functions and differentiating them in order to obtain the damage growth rate:

$$\phi = \phi(Y, \dot{x}; \omega); \quad \dot{D} = -\frac{\partial \phi}{\partial Y} \tag{5}$$

where Y is the damage energy release rate. For example, Lemaitre's (1985) dissipation potential function and the resultant damage growth rate in uniaxial ductile deformation are

$$\phi = \frac{S}{a+1}\left(\frac{-Y}{S}\right)^{a+1}\dot{\varepsilon}_p \tag{6}$$

$$\dot{D} = \left(\frac{K^2}{2ES}\varepsilon_p^{2/M}\right)^a \dot{\varepsilon}_p \tag{7}$$

where S and a are material and temperature dependent constants, ε_p is the plastic strain, E is the elasticity modulus, K, M are Ramberg–Osgood hardening parameters and the initial condition is

$D = 0$, $\varepsilon_p \leqslant \varepsilon_0$. Similar thermodynamic potential functions have been adopted by Chow and Wei (1991), Chaboche (1988), Hansen and Schreyer (1994), Tie-Jun (1992) and Woo and Li 1992).

Both approaches described above introduce unknown material constants in the damage growth equations which may be difficult, if not impossible, to estimate numerically. The proliferation of undetermined material parameters has also been criticized by Krajcinovic and Mastilovic (1995). The unknown constants and the arbitrariness associated with the choice of the kinetic equations or the thermodynamic potential functions also pose an impediment to the use of damage mechanics in modeling structural deterioration.

3. Thermodynamic framework for damage accumulation

Consider a system $\mathscr{R}$ (defined by the closed boundary $\partial\mathscr{R}$) in contact with a heat reservoir whose (constant) absolute temperature is θ. Let W be the work done on $\mathscr{R}$, and let U and K_E be the system's internal and kinetic energies, respectively. The rate of heat flow into the system from the surrounding reservoir is Q and the increase in the entropy of $\mathscr{R}$ occurs at the rate of S, the dots indicating differentiation with respect to time t. The second law of thermodynamics requires that the rate of increase of entropy of the universe (the system and the reservoir) must be positive:

$$\dot{S} - \frac{Q}{\theta} \geqslant 0 \tag{8}$$

The Helmholtz free energy, $\Psi = U - \theta S$, is a function of the absolute temperature, the damage variable, and a set of variables, $\mathbf{x}$ (to be defined subsequently),

$$\Psi = \Psi(\theta, \mathbf{x}, D) \tag{9}$$

The nature of $\mathbf{x}$ depends on the degree of detail necessary to define the free energy. The second law inequality (8) can be written with the help of the first law (which states $K_E + U = Q + W$) and the free energy as:

$$-\dot{K}_E - \dot{\Psi} - \dot{\theta} S + \dot{W} \geqslant 0 \tag{10}$$

which can be expressed by differentiating eqn (9) and by noting that $\dot{\theta} = 0$:

$$-\dot{K}_E + \dot{W} - \frac{\partial \Psi}{\partial \mathbf{x}} \cdot \dot{\mathbf{x}} - \frac{\partial \Psi}{\partial D} \dot{D} \equiv \Gamma \geqslant 0 \tag{11}$$

The process is reversible (Prigogine, 1967) if the dissipation rate, Γ, is zero. It can be verified that for a body which is not undergoing deformation and not accumulating damage (i.e., $\dot{\mathbf{x}} = \mathbf{0}$, $\dot{D} = 0$), a reversible process implies $\Gamma = \dot{W} - \dot{K}_E = 0$; in words, reversible (non-frictional) work done on this body is fully converted to kinetic energy, a well-known fact in mechanics.

Let us now specify $\mathscr{R}$ as a deformable body in contact with a heat reservoir at constant temperature. The set of variables $\mathbf{x}$ in the free energy [eqn (9)] then is the symmetric strain tensor ε_{ij}, defined by $\varepsilon_{ij} = (1/2)(u_{i,j} + u_{j,i})$, where u_i is the deformation at a point ($i, j = 1, 2, 3$), and $u_{i,j}$ refers to its partial derivative with respect to the j displacement. The velocity and acceleration at the point are denoted by $\dot{u}_i$ and $\ddot{u}_i$, respectively. The strain rate tensor is, $\dot{\varepsilon}_{ij} = (1/2)(\dot{u}_{i,j} + \dot{u}_{j,i})$. The

stress tensor, $\sigma_{ij} = \partial\psi/\partial\varepsilon_{ij}$, is the partial derivative of the free energy per unit volume, $\psi = \partial\Psi/\partial V$, with regard to the strain tensor (Maugin, 1992).

A system in diathermal contact with a heat reservoir is in a state of equilibrium if the first variation of its Helmholtz Free Energy is zero (McLellan, 1980):

$$\delta\Psi = 0 \tag{12}$$

The work done on a system in a dissipative process at constant temperature, minus the increase in kinetic energy, is greater than the increase in the Helmholtz free energy (Sears and Salinger, 1975). Therefore,

$$\mathrm{d}\Psi = \mathrm{d}W_{\mathrm{nk}} - \mathrm{d}W_{\mathrm{d}}; \quad \mathrm{d}W_{\mathrm{d}} \geqslant 0 \tag{13}$$

where W_{nk} and W_{d} denote the non-kinetic work and the dissipation in the above process, respectively. The variation in the free energy at an arbitrary instant t_2 can be written as [integrating eqn (13)]

$$\delta\Psi(t_2) = \delta\Psi(t_1) + \delta\int_{t_1}^{t_2} (\dot{W} - \dot{K}_{\mathrm{E}})\,\mathrm{d}t - \delta\int_{t_1}^{t_2} \Gamma\,\mathrm{d}t \tag{14}$$

where the energy dissipation is assumed to be given by eqn (11) and the initial state is assumed to be one of thermodynamic equilibrium [i.e., $\delta\Psi(t_1) = 0$]. The variation $\delta\Psi(t_2)$ is in general a function

$$\delta\Psi(t_2) = g(\theta, \varepsilon_{ij}, D, \delta\theta, \delta\dot{\varepsilon}_{ij}, \delta\dot{D}, \ldots; t), \quad t \in [t_1, t_2] \tag{15}$$

which depends on the state of the system as well as on the choice of the variations in temperature, strain rate, damage and other terms, and is generally non-zero for an irreversible process or for a system yet to achieve equilibrium. However, damage growth prior to localization of defects is assumed to occur slowly and close to equilibrium, and the function $g(\cdot)$ is assumed to vanish for a suitable set of variations. Under this assumption (to be validated later by comparing the accuracy of the resulting models with experimental data), we can write [using eqn (11)]:

$$\delta\Psi(t_2) = \delta\int_{t_1}^{t_2} (\dot{W} - \dot{K}_{\mathrm{E}})\,\mathrm{d}t - \delta\int_{t_1}^{t_2} \left(\dot{W} - \dot{K}_{\mathrm{E}} - \frac{\partial\Psi}{\partial\varepsilon_{ij}}\dot{\varepsilon}_{ij} - \frac{\partial\Psi}{\partial D}\dot{D}\right)\mathrm{d}t \simeq 0 \tag{16}$$

Equation (16) can be rearranged as:

$$\delta\Psi(t_2) = \delta\int_{t_1}^{t_2} \left(\dot{W} - \dot{K}_{\mathrm{E}} + \frac{\partial\Psi}{\partial D}\dot{D}\right)\mathrm{d}t - \delta\int_{t_1}^{t_2} \left(\dot{W} - \dot{K}_{\mathrm{E}} - \frac{\partial\Psi}{\partial\varepsilon_{ij}}\dot{\varepsilon}_{ij}\right)\mathrm{d}t \tag{17}$$

$$= \int_{t_1}^{t_2} \delta I_1(t)\,\mathrm{d}t - \int_{t_1}^{t_2} \delta I_2(t)\,\mathrm{d}t \simeq 0 \tag{18}$$

where I_1 and I_2 refer to the two integrands in eqn (17) and the commutability of integration and variation has been used.

Let us consider δI_2 first. Body forces $F_i(t)$ and boundary traction $T_i(t)$ perform work on the system acting through the displacement field $u_i(t)$ on $\mathscr{R}$ and $\partial\mathscr{R}_1$, respectively, where $\partial\mathscr{R} = \partial\mathscr{R}_1 \bigcup \partial\mathscr{R}_2$ such

that displacements on only $\partial\mathscr{R}_2$ are specified at all time in $[t_1, t_2]$. The instantaneous variation in $I_2(t)$ caused by a sufficiently small variation $\delta u_i(t)$ in the velocity field (consistent with the above boundary condition) which does not alter the instantaneous force, acceleration and stress distributions, is

$$\delta I_2 = \int_{\mathscr{R}} F_i \delta \dot{u}_i \, \mathrm{d}V + \int_{\partial\mathscr{R}_1} T_i \delta \dot{u}_i \, \mathrm{d}\eta - \int_{\mathscr{R}} \rho a_i \delta \dot{u}_i \, \mathrm{d}V - \int_{\mathscr{R}} \sigma_{ij} \delta \dot{u}_{i,j} \, \mathrm{d}V \tag{19}$$

where use has been made of the symmetry of the stress tensor (Bhattacharya, 1997). Upon integration by parts, applying Green's theorem and noting that δu_i is identically zero on $\partial\mathscr{R}_2$, eqn (19) becomes,

$$\delta I_2 = \int_{\mathscr{R}} (F_i + \sigma_{ij,j} - \rho a_i) \delta \dot{u}_i \, \mathrm{d}V + \int_{\partial\mathscr{R}_1} (T_i - \sigma_{ij} n_j) \delta \dot{u}_i \, \mathrm{d}\eta \tag{20}$$

The expressions in parentheses in eqn (20) are each zero as they constitute the equilibrium equations of a deformable body (damaged or otherwise) on $\mathscr{R}$ and $\partial\mathscr{R}_1$, respectively (Krajcinovic and Sumarac, 1987). Thus, the second integral in eqn (18) is zero.

Consequently, the first integral in eqn (18) must also vanish. We assume that its integrand is stationary at every time instant:

$$\delta I_1(t) = \delta \left(\dot{W}(t) - \dot{K}_{\mathrm{E}}(t) + \frac{\partial \Psi(t)}{\partial D} \dot{D}(t) \right) = 0 \tag{21}$$

Let us suppose that the damage variable can be expressed as a function of the strain tensor. We apply a set of variations to the velocity field with the same boundary conditions as before, which are small enough not to alter the instantaneous force, acceleration and strain distributions of the body, and do not affect the rate of change in the free energy, $\psi_{\mathrm{D}} = \partial\psi/\partial D$, at that instant. Noting that $\delta D = \mathrm{d}(\delta D)/\mathrm{d}t$ and $\delta D = (\partial D/\partial \varepsilon_{ij}) \delta \varepsilon_{ij}$, eqn (21) becomes

$$\delta I_1(t) = \int_{\mathscr{R}} F_i \delta \dot{u}_i \, \mathrm{d}V + \int_{\partial\mathscr{R}_1} T_i \delta \dot{u}_i \, \mathrm{d}\eta - \int_{\mathscr{R}} \rho a_i \delta \dot{u}_i \, \mathrm{d}V + \int_{\mathscr{R}} \frac{\partial \psi}{\partial D} \frac{\partial D}{\partial \varepsilon_{ij}} \delta \dot{\varepsilon}_{ij} \, \mathrm{d}V = 0 \tag{22}$$

Proceeding similarly as in eqns (19)–(20), we arrive at the set of coupled partial differential equations,

$$F_i - \rho a_i - \left(\psi_{\mathrm{D}} \frac{\partial D}{\partial \varepsilon_{ij}} \right)_{,j} = 0 \quad \text{on } \mathscr{R} \tag{23}$$

$$T_i + \psi_{\mathrm{D}} \frac{\partial D}{\partial \varepsilon_{ij}} n_j = 0 \quad \text{on } \partial\mathscr{R}_1 \tag{24}$$

The solutions to eqns (23) and (24) may be computationally difficult for a body subjected to multiaxial straining. However, mechanical properties of engineering materials and experimental damage growth data usually are available for uniaxial loading. Thus, the uniaxial counterpart of eqn (24) (developed in the following), which can accommodate different damage-causing processes

and is amenable to closed-form solutions, is useful for validating theoretical models and for developing insights into structural damage growth.

4. Isotropic damage growth under uniaxial loading

For damage growth under uniaxial loading due to a far-field stress, σ_∞, acting normal to the surface, eqn (24) simplifies to the ordinary differential equation:

$$\frac{\mathrm{d}D}{\mathrm{d}\varepsilon} = -\frac{\sigma_\infty}{\psi_{\mathrm{D}}} \tag{25}$$

If the strain rate, $\dot{\varepsilon}$, is known, damage growth can be expressed as function of time:

$$\frac{\mathrm{d}D}{\mathrm{d}t} = -\frac{\sigma_\infty}{\psi_{\mathrm{D}}}\dot{\varepsilon}(D,t) \tag{26}$$

Equation (25) or (26) are the basic equations of isotropic damage growth for ductile deformation, creep and fatigue. The free energy per unit volume, ψ, depends on the constitutive model of the material and the particular loading mode in question; its general form, in the absence of inertial effects, is

$$\psi = \int \sigma_{ij}\,\mathrm{d}\varepsilon_{ij} - \gamma(D) \tag{27}$$

The partial derivative, ψ_{D}, is computed holding the temperature and strain constant [cf eqn (9)]. The term γ in eqn (27) is the surface energy of voids and discontinuities that arise due to damage growth per unit volume. The increment in strain $\mathrm{d}\varepsilon$ in general has three additive constituents: the instantaneous elastic and plastic strains, and the time-dependent creep strain. The constitutive models will be described individually in the following subsections for ductile deformation, creep and fatigue. The surface energy, which is common to all three, can be obtained under the following set of idealizations.

Consistent with the notion of isotropic damage, micro-defects within the damaged material are assumed to be spherical voids (of different sizes) which are distributed uniformly in space within the material volume. Consider a representative volume element, $V_0 = (4/3)\pi u^3$, in the shape of a sphere of radius u. Suppose there are n voids present in V_0, and a_i is the radius of the ith void ($i \in [1, n]$). If D is the isotropic damage on some cross-section of area S_0 within this volume, then, ignoring stress-amplification, the net surface area of defects on that cross-section is DS_0. If n is large, which can be ensured by a suitable choice of u, the volume of defects present in the sphere V_0 can be found by simple integration as $(4/3)\pi u^3 D$, which is also equal to the sum of the individual void volumes, $\Sigma_{i=1}^{n}\ (4/3)\pi a_i^3$. This gives

$$D = \frac{1}{u^3}\sum_{i=1}^{n} a_i^3 \tag{28}$$

If the force–displacement relation at the microscale is linear, and a void is formed when the stress on its impending boundary equals the local failure stress σ_{c}, the energy required to form the ith

void is $\Pi_i = \int_\eta (1/2)\sigma_c v \, d\eta$ where $d\eta$ is an elemental void boundary and v is the extension normal to the initial circular plane which finally expands to the spherical void. The surface energy required per unit growth of void surface area is $\partial\Pi/\partial\eta$; summing over all the n voids,

$$\gamma = \frac{\sigma_c \Sigma_{i=1}^{n} \pi a_i^3}{V_0} = \frac{\sigma_c \Sigma_{i=1}^{n} \pi a_i^3}{\frac{4}{3}\pi u^3} \tag{29}$$

which, with the help of eqn (28) becomes $\gamma = (3/4)\sigma_c D$.

It is assumed that σ_c is of the same order as the true fracture strength, σ_f (a more readily available material property) and in the sequel, γ is evaluated as

$$\gamma = \tfrac{3}{4}\sigma_f D \tag{30}$$

Equation (30) is a simple way of estimating the surface energy of formation of voids in terms of readily obtained quantities, pending availability of more accurate information regarding the number, shapes, sizes and interaction of the voids as a function of time.

4.1. Ductile deformation damage

The constitutive law for material behavior under uniaxial monotonic loading is described by the Ramberg–Osgood model, applied to the effective stress–actual strain relationship: $\varepsilon = (\sigma/E) + (\sigma/K)^M$ where the total strain (ε) is the sum of its elastic (ε_e) and plastic (ε_p) components; E is the undamaged elastic modulus, K is the undamaged strain hardening modulus and M is the hardening exponent. In general, damage initiates only after the accumulation of a threshold plastic strain, ε_0 (Lemaitre, 1985). The first term in eqn (27) takes the form (Bhattacharya and Ellingwood, 1996a):

$$\int_0^{\varepsilon} \sigma \, d\varepsilon' = \int_0^{\varepsilon_{e0}} E\varepsilon'_e \, d\varepsilon'_e + \int_{\varepsilon_{e0}}^{\varepsilon_e} E(1-D)\varepsilon'_e \, d\varepsilon'_e + \int_0^{\varepsilon_0} K\varepsilon'^{1/M}_p \, d\varepsilon'_p + \int_{\varepsilon_0}^{\varepsilon_p} K(1-D)\varepsilon'^{1/M}_p \, d\varepsilon'_p \tag{31}$$

here the principle of strain equivalence has been applied to the elastic and plastic strains separately (with the assumption that the exponent m is not affected by damage), and ε_{e0} is the elastic strain that corresponds to the plastic strain ε_0, in consequence of applying the Ramberg–Osgood law.

Subtracting eqn (30) from eqn (31), and using eqn (27),

$$\psi_D = -\frac{K^2}{2E}[\varepsilon_p^{2/M} - \varepsilon_0^{2/M}] - \frac{K}{1+\frac{1}{M}}[\varepsilon_p^{1+(1/M)} - \varepsilon_0^{1+(1/M)}] - \frac{3}{4}\sigma_f \tag{32}$$

which is a function of the plastic, rather than the total, strain. The rate of damage growth with respect to the plastic strain can be written as [using eqn (25)],

$$\frac{dD}{d\varepsilon_p} = -\frac{\sigma_\infty}{\psi_D}\frac{d\varepsilon}{d\varepsilon_p} \tag{33}$$

where σ_∞ is given by $K(1-D)\varepsilon_p^{(1/M)}$ using the principle of strain equivalence.

Equation (33) must be solved numerically. An approximate closed-form solution to eqn (33) can be obtained by noting that $d\varepsilon/d\varepsilon_p = 1 + d\varepsilon_e/d\varepsilon_p$ is practically equal to 1 at all ε of interest in ductile damage accumulation (Bhattacharya, 1997), and that $K/(2E) \sim 0$ for most engineering alloys:

$$D = 1 - \frac{C_2}{\varepsilon_p^{1+(1/M)} + C_1} \tag{34}$$

with the initial condition $D = 0$ at $\varepsilon_p = \varepsilon_0$. The constants,

$$C_1 = \frac{3}{4}\left(1 + \frac{1}{M}\right)\frac{\sigma_f}{K} - \varepsilon_0^{1+(1/M)} \tag{35}$$

$$C_2 = C_1 + \varepsilon_0^{1+(1/M)} \tag{36}$$

It is emphasized that C_1 and C_2 are not undetermined material constants, but are functions of widely available monotonic stress–strain parameters. The value of the threshold plastic strain, ε_0, is close to zero for most engineering materials ($\varepsilon_0 \leqslant 0.02$ for five alloys reported in Lemaitre, 1985) and in the absence of other information it may conservatively be taken to be zero.

For most metals and alloys, the non-dimensional material constant, C_1, is greater than the plastic strain range of interest (up to the fracture ductility, ε_f) and for such situations, eqn (34) may be simplified to

$$D = 1 - \frac{C_2}{C_1} + \frac{C_2}{C_1^2}\varepsilon_p^{1+(1/M)}; \quad C_1 > \varepsilon_p^{1+(1/M)}, \quad \varepsilon_p > \varepsilon_0 \tag{37}$$

For those materials, which possess a marked ductility in the post-yielding zone (i.e., for those with large M) the above equation may be further simplified into a linear relation between damage and plastic strain:

$$D = 1 - \frac{C_2}{C_1} + \frac{C_2}{C_1^2}\varepsilon_p; \quad C_1 > \varepsilon_p, \quad M \to \infty \tag{38}$$

Equation (37) is of a similar algebraic form as Lemaitre's (1985) solution for uniaxial ductile damage:

$$D = D_c\left(\frac{\varepsilon_p^{(2a+M)/M} - \varepsilon_0^{(2a+M)/M}}{\varepsilon_f^{(2a+M)/M} - \varepsilon_0^{(2a+M/M)}}\right); \quad \varepsilon_p \geqslant \varepsilon_0 \tag{39}$$

which contains an unknown material parameter, a, and requires a prior estimation of the failure condition (ε_f, D_c) to be useful. The simplification of the present model for highly ductile materials [eqn (38)] also is comparable to Lemaitre's (1985) linear approximation obtained under the same assumption:

$$D = D_c\left(\frac{\varepsilon_p - \varepsilon_0}{\varepsilon_f - \varepsilon_0}\right); \quad \varepsilon_p \geqslant \varepsilon_0, \quad M \to \infty \tag{40}$$

Unlike the proposed model, however, eqn (40) cannot be used to predict failure.

Examples

Table 1 lists mechanical properties of five materials. Of these, the first four materials are used to validate the ductile damage growth model. The properties of SAE 4340 steel will be used later in the analysis of fatigue damage.

Figure 1 compares the ductile damage growth predicted from eqn (33) and its closed-form simplifications with published experimental results for SAE 1035 steel (Lemaitre, 1985). The approximate closed-form solution [eqn (34)] is virtually identical to that of the original differential eqn (33), and their agreement with the experimental data is quite accurate. The two additional simplifications of eqn (34), i.e., eqns (37) and (38), are also plotted in the figure to show the extent of errors involved. Figure 2 compares the predicted ductile damage growth [eqn (34)] in 2024-T3 aluminum alloy with experimental results from three different sources. Some variability can be observed in the data from three different sources for the same nominal grade of material, however, the prediction [eqn (34)] lies within the experimental scatter. The mean damage function from the statistical analyses of data from Woo and Li (1993) is used in the present comparison; their estimate of the scatter in ductile damage growth has been used in a recent stochastic formulation of damage growth (Bhattacharya and Ellingwood, 1996b).

The experimentally observed critical damage and failure strain (ε_f) for four materials are listed in Table 2 along with their sources; these sources, however, did not report tensile properties of the specimens used. The corresponding D_c predicted by eqn (34) at $\varepsilon_p = \varepsilon_f$, using parameters from Table 1, is presented in the last column. The quality of the agreement with experimental results achieved using independently obtained material properties indicates the validity of the proposed model. The predicted D_c for SAE 4340 steel could not be verified experimentally.

The strain, ε_p, was selected as the independent variable (instead of time) in the preceding development mainly to conform to the experimental data used in the validation; the time-dependent nature of ductile damage growth could be depicted as well by starting with eqn (26). It should also be noted that, unlike many of the existing methods, knowledge of the critical damage and failure time (or strain) is not required in the present model. Rather, the D_c obtained using one set of

Table 1
Monotonic material properties at room temperature

Material	Source	Treatment and condition	Form	E GPa	K MPa	M	σ_f MPa
SAE 1035	Le Roy et al. (1981)	Austentized and quenched	Round tensile	180	871	3.4	1200
2024 Al	Hansen and Schreyer (1994)	T3	×	74.5	680	5.5	435
SAE 4130	Boller and Seeger (1987)	Tempered and quenched	Hourglass	221	1117	15.9	1692
INCO 718	Boyer (1987)	Heat-treated	Sheet	207	1435	23.8	1312
SE 4340 steel	Endo and Morrow (1969)	Quenched/tempered	Hourglass	193.2	1601	15.2	1911

Fig. 1. Monotonic damage growth in SAE 1035 steel.

material data can be used to predict failure times under different loading conditions for the same material and temperature.

4.2. Creep damage

The principle of strain equivalence, applied to creep (strain rate), may be written as follows: a damaged volume of material under the applied stress σ shows the same creep strain rate as a comparable undamaged volume under the effective stress $\tilde{\sigma}$. The Bailey–Norton power law (Dowling, 1993) describing creep straining under constant applied stress, σ, at constant temperature θ can then be written as

$$\dot{\varepsilon}_{cr} = A\tilde{\sigma}^m t^{\phi} \tag{41}$$

where $\dot{\varepsilon}_{cr}$ = creep strain rate, and A, m, ϕ are empirical temperature-dependent constants.

At constant-stress creep, the time-independent elastic and plastic components of strain remain constant (i.e., $d\varepsilon = d\varepsilon_{cr}$), and consequently, eqn (27) can be written as,

$$\psi = \int_0^{\varepsilon_{cr}} \sigma \, d\varepsilon'_{cr} - \gamma = \sigma_{\infty}\varepsilon_{cr} - \frac{3}{4}\sigma_f D \tag{42}$$

where σ_{∞} is the constant far field applied stress acting normal to the surface. Hence,

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Fig. 2. Monotonic damage growth in 2024-T3 aluminum.

Table 2
Experimental and predicted D_c in ductile damage

Material	Experimental				Predicted D_c [eqn (34)]
	Source	ε_0	ε_f	D_c	
SAE 1035 steel (AFNOR XC 38)	Lemaitre (1985)	0	0.56	0.22	0.26
2024-T3 Al (AU4G1)	Lemaitre (1985)	0.02	0.25	0.23	0.25
	Chow and Wang (1987)	0	0.32	0.22	
	Woo and Li (1993)	0.0084 (mean)	0.25 (max)	0.22 (mean)	
SAE 4130 steel	Lemaitre (1985)	0.02	0.37	0.24	0.22
INCO 718	Lemaitre (1985)	0.02	0.29	0.24	0.26
SAE 4340	Endo and Morrow (1969)	—	0.84	—	0.46

$$\psi_D = -\tfrac{3}{4}\sigma_f \tag{43}$$

Equation (26) for uniaxial constant stress creep damage growth is, therefore,

$$\frac{dD}{dt} = \frac{4}{3}\frac{\sigma_\infty}{\sigma_f}\dot{\varepsilon}_{cr} = \frac{4}{3}\frac{A\phi\sigma_\infty^{m+1}}{\sigma_f(1-D)^m}t^{\phi-1} \tag{44}$$

where eqns (41) and (1) have been used. For steady-state creep ($\phi = 1$), the growth rate simplifies to

$$\frac{\mathrm{d}D}{\mathrm{d}t} = B\left(\frac{\sigma_\infty^{1+(1/m)}}{1-D}\right)^m \tag{45}$$

where $B = (4/3)A/\sigma_\mathrm{f}$ is a temperature dependent material constant. Equation (45) has almost the same form as Kachanov's kinetic equation [eqn (4)] because the numerical values of m (usually ranging between 4 and 12) renders the exponent of σ_∞ close to one. The present method therefore can derive from an analytical approach a long-established phenomenological model of creep damage growth and provide estimates of the phenomenological parameters.

Integrating eqn (44), creep damage as a function of time is

$$D(t) = 1-[(1-D_{t_0})^{m+1} - (4/3)(A/\sigma_\mathrm{f})(m+1)\sigma_\infty^{1+m}(t^\phi - t_0^\phi)]^{1/(1+m)} \tag{46}$$

where t_0 is the initial time. The initial damage, D_{t_0}, is the result of the plastic deformation introduced, in addition to any pre-existing damage (D_0), when the specimen is loaded from zero to σ_∞ at the beginning of the creep straining. Its value can be obtained by integrating eqn (33) to $\varepsilon_\mathrm{p} = (\sigma_\infty/K)^M$, using the relevant initial condition:

$$D_{t_0} = \int_{\varepsilon_0}^{(\sigma_\infty/K)^M} \frac{\mathrm{d}D}{\mathrm{d}\varepsilon_\mathrm{p}}\mathrm{d}\varepsilon_\mathrm{p}'; \quad D = D_0 \text{ at } \varepsilon_\mathrm{p}' = \varepsilon_0 \tag{47}$$

Assuming $t_0 = 0$, the time to failure is:

$$t_\mathrm{f} = \left[\frac{(1-D_{t_0})^{m+1} - (1-D_\mathrm{c})^{m+1}}{(4/3)(A/\sigma_\mathrm{f})(m+1)\sigma_\infty^{1+m}}\right]^{1/\phi} \tag{48}$$

If, under steady state creep ($\phi = 1$), D_{t_0} can be neglected and m can be considered large enough to make $(1-D_\mathrm{c})^{m+1} \sim 0$ (even when $D_\mathrm{c} \neq 1$),

$$D = 1-\left[1-\frac{t}{t_\mathrm{f}}\right]^{1/(1+m)} \tag{49}$$

which is identical to the form proposed by Kachanov (1986).

Examples

Figure 3 compares the predicted damage growth in Superalloy IN 100 at 1000°C under constant stress of 240 MPa, with published experimental results from Lemaitre (1992) where the first macrocrack (size not stated) is reported to develop at 90% of rupture time with $D_\mathrm{c} = 0.22$. Since the creep law and tensile parameters for the specimen were not reported, steady state creep ($\phi = 1$), virgin initial state ($t_0 = D_{t_0} = 0$) and a representative value of $m = 4$ are assumed. The accelerated nature of creep damage growth is apparent in the figure, and the shape of the predicted curve matches the experiment well.

Table 3 lists the creep law parameters at three different temperatures for ASTM A36 steel from two different sources; the variability in material properties for a given nominal grade of material

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Fig. 3. Creep damage growth in IN 100 superalloy.

Table 3
Creep law parameters and tensile properties for ASTM A36 steel

		Creep law parameters						Tensile properties			
		From Fields and Fields (1989)			From Harmathy (1967)			Brockenbrough and Johnston (1968)	From Fields and Fields (1989)		
θ		A			A			σ_f	E	K	
°C	°F	ksi, h	m	ϕ	ksi, h	m	ϕ	ksi	10^3 ksi	ksi	M
427	800	1.16×10^{-10}	4.4	0.39	2.44×10^{-12}	4.7	1	83	23.9	40.7	5.5
482	900	1.28×10^{-10}	6.4	0.53	1.45×10^{-10}	4.7	1	67	23.2	35.6	6.9
538	1000	3.74×10^{-10}	6.2	0.68	4.93×10^{-9}	4.7	1	54	22.5	26.5	9.1

is evident. The tensile parameters at the three temperatures are also listed; these are used to compute the initial creep damage, D_{t_0}. Using these parameters and assuming $D_0 = \varepsilon_0 = t_0 = 0$, eqns (47) and (48) are solved to predict the stresses that cause failure in 1000, 10,000 and 100,000

Table 4
Failure stresses due to creep in ASTM A36 steel

		Failure stress (ksi)					
		Predicted				Experimental	
t_f (hr)	θ (°F)	A, m, ϕ from Fields and Fields (1989) $D_c = 0.2$	$D_c = 1.0$	A, m, ϕ from Harmathy (1967) $D_c = 0.2$	$D_c = 1.0$	Brockenbrough and Merritt (1994)	Brockenbrough and Johnston (1968)
1000	800	35.4	41.2	35.1	37.4	38.0	25.8
	900	16.6	17.1	21.5	22.7	18.5	19.0
	1000	13.1	13.5	11.3	12.0	9.5	11.7
10,000	800	34.7	39.1	28.6	30.4	24.8	21.0
	900	14.1	14.5	14.5	15.4	12.4	13.8
	1000	10.5	10.9	7.6	8.0	6.3	6.8
100,000	800	33.5	36.8	20.6	21.7	16.0	—
	900	12.0	12.3	9.7	10.3	8.2	—
—	1000	8.5	8.7	5.1	5.4	4.2	—

h for each of the three different temperatures, which are shown in Table 4. The critical damage for A36 steel at the operating temperatures are unknown, and two different values (0.2 and 1.0) have been assumed. Table 4 also lists some experimental failure stresses from two different sources for the same nominal grade of material, but for which parameters (A, m, ϕ) are unknown. The agreement of the predicted failure stresses with the experimental results is better at shorter exposure times. Also, it should be noted that the (A, m, ϕ) from Fields and Fields (1989) pertain to test durations of 16 h and less; the (A, m, ϕ) from Harmathy (1967) (test durations unknown) better predict the failure stresses.

Table 5 lists creep law parameters at 1100°F (593°C) and tensile properties at room temperature

Table 5
Creep and tensile properties for type 316 stainless steel

					Tensile parameters				
	Creep law parameters (Garofalo et al., 1961)				σ_y (Davis, 1994)	σ_f (Davis, 1994)	E (Davis, 1994)	K (Boyer, 1987)	M (Boyer, 1987)
θ °C (°F)	A MPa, h	m	ϕ	test σ MPa	MPa	MPa	GPa	MPa	
RT	—	—	—	—	275	595	193	891.4	4.22
593 (1100)	2.32×10^{-20}	6.92	1	199–315	152	443.7	151.6	492.7	4.22

Table 6
Experimental and predicted failure times in creep of type 316 stainless steel at 593°C (1100°F)

σ_∞ (MPa)	D_{t_0} [eqn (47)]	Predicted t_f (h) [eqn (48)]		Experimental (Garofalo et al., 1961), h					
		D_c		t_2 (secondary)			t_R (rupture)		
		0.2	1.0	min	mean	max	min	mean	max
199	1.08×10^{-2}	841	1032	960	1283	1950	1267	1749	2437
218	1.69×10^{-2}	384	477	150	325	546	170	439	779
239	2.72×10^{-2}	167	211	60	77	99	76	105	132
262	4.29×10^{-2}	69	89	17	27	31	22	37	43
288	6.74×10^{-2}	25	34	4	9.4	15	6.6	13.3	20.4
315	1.04×10^{-1}	8	12	1.3	3.3	6.2	1.9	4.9	9.0

(RT) and 1100°F for type 316 stainless steel. Table 6 lists creep test results for the same material from Garofalo et al. (1961) for six stress levels at 1100°F. The wide scatter in t_2 (time to end of secondary stage) and in t_R (time to rupture) is evident. The predicted t_f and the corresponding D_{t_0}, obtained by solving eqns (47) and (48) (using values from Table 5 and assuming $t = D_0 = \varepsilon_0 = 0$), are presented in Table 6 for each stress level. As in Table 4, two possible values of the unknown D_c have been assumed (0.2 and 1.0). The CDM-based t_f is, by definition, less than t_R and is believed to lie between t_2 and t_R, but Garofalo et al. (1961) did not list the time to the occurrences of the first microcrack. For lower stresses, the predictions lie within the experimentally observed range of values, but the proposed model is prone to over-predict the time to failure for higher stresses. Due to the accelerated nature of creep damage growth, the point of failure is not very sensitive to the exact value of D_c.

4.3. Fatigue damage

Fatigue failure can occur (after a sufficient number of cycles) at load levels below the static monotonic failure stress, provided that the cyclic stress range exceeds the endurance limit. With each cycle, additional damage is introduced in the material, and the damage at the end of one cycle acts as the initial damage for the damage increment in the next cycle:

$$D_{i+1} = D_i + \Delta D_i, \quad \Delta D_i \geqslant 0, \quad i = 1, \ldots, N_I - 1 \tag{50}$$

where D_i is the damage at the end of the ith cycle, ΔD_i is the damage increment during the ith cycle and N_I is the cycles to macroscopic crack initiation (localization). Crack initiation occurs when the critical value for damage is reached:

$$D_{N_I-1} < D_c$$
$$D_{N_I} \geqslant D_c \tag{51}$$

Fig. 4. Stress–strain coordinates in hysteresis loop of cycle i.

In any given cycle (Fig. 4), the unloading portion of a hysteresis loop and compressive stresses are assumed not to contribute to damage. Consequently, only the reloading section above the endurance limit, S_e, in the positive stress region causes damage to increase (similar assumptions regarding fatigue damage increment are also found in Kachanov, 1986; and Lemaitre, 1984). Under these assumptions, the equation of fatigue damage accumulation in the ith loading cycle can be written as

$$\frac{\mathrm{d}D}{\mathrm{d}\Delta\varepsilon} = \begin{cases} -(\Delta\sigma_\infty + \sigma_{\min})/\psi_D(\Delta\varepsilon); & \Delta\sigma_\infty + \sigma_{\min} \geqslant S_e \geqslant 0, \quad \dot{\varepsilon} > 0 \\ 0; & \text{otherwise} \end{cases} \tag{52}$$

with the initial damage $D = D_{i-1}$, where D_{i-1} is the damage at the end of cycle $i-1$. $\Delta\varepsilon$ and $\Delta\sigma$ are the cyclic strain and stress ranges, respectively, and $(\varepsilon_{\min}, \sigma_{\min})$ are the lower loop-tip coordinates in the ε–σ system, all of which may vary from cycle to cycle. The free energy per unit volume for the ith cycle,

$$\psi = \int_{\Delta\varepsilon_{0_i}}^{\Delta\varepsilon} (\Delta\sigma + \sigma_{\min_i})\,\mathrm{d}\Delta\varepsilon' - \frac{3}{4}\sigma_f(D - D_{i-1}) \tag{53}$$

is computed (analogous to that in ductile damage) using the Ramberg–Osgood equation for the hysteresis loop: $\Delta\varepsilon = (\Delta\sigma/E) + 2(\Delta\sigma/2K')^{M'}$, where $\Delta\sigma$ is the effective stress range, and K', M' are the cyclic hardening modulus and the cyclic hardening exponent, respectively. The term $\Delta\varepsilon_{0_i}$ (Fig. 4) is the threshold plastic strain range of damage increment in cycle i (analogous to ε_0 in ductile damage). Using the principle of strain equivalence, assuming $\mathrm{d}D/\mathrm{d}\Delta\varepsilon \simeq \mathrm{d}D/\mathrm{d}\Delta\varepsilon_p$ and $K'/E \sim 0$

(which is valid for most engineering materials), the closed-form solution for fatigue damage accumulated at the end of the ith cycle is (Bhattacharya, 1997):

$$D_i = 1-(1-D_{i-1})\frac{\dfrac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{0_i}^{1+(1/M')}-\Delta\varepsilon_{\mathrm{p}1_i}^{1/M'}\Delta\varepsilon_{0_i}+C_i}{\dfrac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{\mathrm{p}m_i}^{1+(1/M')}-\Delta\varepsilon_{\mathrm{p}1_i}^{1/M'}\Delta\varepsilon_{\mathrm{p}m_i}+C_i} \tag{54}$$

where

$$C_i = \frac{3}{4}\frac{\sigma_\mathrm{f}}{K_1}-\frac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{0_i}^{1+(1/M')}+\Delta\varepsilon_{\mathrm{p}1_i}^{1/M'}\Delta\varepsilon_{0_i} \tag{55}$$

where $K_1 = 2^{1-1/M'}K'$, and the subscript p signifies the plastic component of strain range.

The recursive nature of eqn (54) can be used to express damage at the end of n cycles in terms of the initial damage, D_0:

$$D_n = 1-(1-D_0)\prod_{i=1}^{n}\frac{\dfrac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{0_i}^{1+(1/M')}-\Delta\varepsilon_{\mathrm{p}1_i}^{1/M'}\Delta\varepsilon_{0_i}+C_i}{\dfrac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{\mathrm{p}m_i}^{1+(1/M')}-\Delta\varepsilon_{\mathrm{p}1_i}^{1/M'}\Delta\varepsilon_{\mathrm{p}m_i}+C_i} \tag{56}$$

In the case of strain-controlled loading, the strain ranges $\Delta\varepsilon_{\mathrm{p}m_i}$, and $\Delta\varepsilon_{\mathrm{p}1i}$ are independent of i, giving the following simplification:

$$D_n = 1-(1-D_0)\left[\frac{\dfrac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{\mathrm{p}0}^{1+(1/M')}-\Delta\varepsilon_{\mathrm{p}1}^{1/M'}\Delta\varepsilon_{\mathrm{p}0}+C}{\dfrac{1}{1+\dfrac{1}{M'}}\Delta\varepsilon_{\mathrm{p}}^{1+(1/M')}-\Delta\varepsilon_{\mathrm{p}1}^{1/M'}\Delta\varepsilon_{\mathrm{p}}+C}\right]^n \tag{57}$$

The number of cycles to crack initiation can be predicted by eqn (56) [or eqn (57)] using the conditions (51), if D_c is known. The subsequent propagation phase can be dealt with using fracture mechanics (Dowling, 1993).

Examples

The cyclic material properties and crack growth law parameters used in the following fatigue

Table 7
Tensile, cyclic and crack growth properties

	Stress–strain parameters							Paris law parameters (Barsom and Rolfe, 1987; Dowling, 1993)			
Material	Source	E GPa	K' MPa	M'	σ_f MPa	S_e MPa	σ_y MPa	C mm/cycle	m	ΔK_{th} MPa $\sqrt{m}$	K_c MPa $\sqrt{m}$
A106 Gr–B steel (288°C in air)	Chopra et al. (1995)	196.5	1994	7.74	539	310	301	6.9×10^{-9}	3.0	6.0	66
SAE 4340 steel	Endo and Morrow (1969)	192.9	1812	7.1	1911	542	1180	5.4×10^{-11}	3.24	10	130

analyses for two alloys of steel are listed in Table 7. Figure 5 shows the results of fully-reversed strain-controlled fatigue tests on A106-Grade B steel at 288°C in air. The predicted N_I, obtained by inverting eqn (57) and assuming $D_c = 0.25$ in eqn (51), compares well with test data on number of cycles to initiation of a crack of length 0.18 mm (Majumdar et al., 1993). Figure 5 also plots the predicted total fatigue life, $N_T = N_I + N_P$, and compares it with three sets of experimental

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Fig. 5. Fatigue damage growth in A106-Gr B steel at 288°C in air.

results: number of cycles (N_{25}) to a 25% drop in the peak tensile stress, which corresponds to a 3 mm crack and is quite close to rupture, from (i) Chopra et al. (1995) and (ii) Chopra (1996), and (iii) the number of cycles to failure (Majumdar et al., 1993). The crack propagation life, N_P, is obtained by integrating the Paris–Erdogan law between the limits $a_s = (1/\pi)(\Delta K_{th}/2S_e)^2$ and $a_f = 6.35$ mm, subject to the condition $K_{max} \leqslant \min(K_c, \sqrt{E\sigma_y\delta_T})$, where $\delta_T = 0.04$ mm (Barsom and Rolfe, 1987). The predicted N_T lies within the experimental scatter.

Figure 6 compares the predicted N_I and N_T curves with experimental results on fully reversed strain-controlled fatigue of quenched and tempered SAE 4340 steel. The cyclic loading properties (Table 7) are taken from Endo and Morrow (1969) and the number of cycles to failure are taken from Topper and Morrow (1970) which refers to the same set of tests. The predicted N_I [eqns (51) and (57)], obtained assuming $D_c = 0.46$ (cf Table 2) agrees very well with the observed cycles to crack initiation (Dowling, 1993) which corresponds to the development of a 0.038 mm crack. However it should be noted that the steel in Dowling (1993), though of the same nominal grade, has different material properties (like yield and ultimate strengths) from the steel in Endo and Morrow (1969). Figure 6 also presents the predicted N_T curve, which exhibits good agreement with experimental results (Topper and Morrow, 1970). N_P is obtained with the help of the Paris–Erdogan law, modified to include crack-tip plasticity effects and subject to the same limits and conditions as above (Bhattacharya, 1997).

The proposed model, as seen from Figs 5 and 6, is able to project the general trend in strain-controlled fatigue behavior described by the Coffin–Manson law (Dowling, 1993): in low-cycle

Fig. 6. Fatigue damage growth in SAE 4340 steel at room temperature in air.

fatigue, the crack initiation period is negligible and most of the fatigue life is spent in crack propagation; while in high cycle fatigue, most of the life is taken up by crack initiation.

5. Conclusion

Structural deterioration under different modes of behavior has been modeled from basic principles of mechanics and thermodynamics. Assuming isotropic damage occurring close to equilibrium at constant temperature in the pre-localization stage, and idealizing discontinuities as uniformly distributed spherical voids inside a given volume, the present method is able to reproduce the phenomenological model of Kachanov (for creep), as well as the dissipation potential model of Lemaitre (for ductile flow). No arbitrary material constants have been introduced in the present formulation; rather, care has been taken to ensure that only well-documented material constants enter the equations.

Ductile damage predictions match published experimental data rather well. An expanded database of the critical damage parameters for different materials and temperatures would be desirable for further validation of the proposed creep damage growth model. Predictions for fatigue crack initiation agree with the notion that critical damage occurs with localization of defects. Preliminary investigations by the authors have indicated that the proposed approach also can capture load-sequencing effects under variable amplitude loading, which otherwise can only be accounted for empirically in selected cases. This will be the subject of a future publication.

Finally, structural damage growth is essentially a random phenomenon, which is evident from the scatter in the different sets of experimental results in this paper. The deterministic methods of modeling structural damage growth described here can therefore predict only the central tendency (mean or median) of the random damage growth. Knowledge of damage uncertainty is of singular importance in estimating safety of a degrading structure. Extensions of the present formulation into the stochastic domain are underway (Bhattacharya and Ellingwood, 1996b).

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