Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational and Applied Mathematics 207 (2007) 64 – 72
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
www.elsevier.com/locate/cam

Harmonic wave generation in non linear thermoelasticity by variational iteration method and Adomian's method

N.H. Sweilam*

Department of Mathematics, Faculty of Science, Cairo University, Giza, Egypt

Received 28 February 2006; received in revised form 28 April 2006

Abstract

This paper applies the variational iteration method and Adomian's decomposition method to solve numerically the harmonic wave generation in a nonlinear, one-dimensional elastic half-space model subjected initially to a prescribed harmonic displacement. The results show that the variational iteration method is much easier, more convenient, and more stable and efficient than Adomian decomposition method.

MSC: 74B20; 65M99

Keywords: Nonlinear coupled thermoelasticity; Adomian's decomposition method; Variational iteration method

0. Introduction

Wave generation in nonlinear thermoelasticity problems has gained a considerable interest for its importance in understanding the nature of interaction between the elastic and thermal fields as well as for its applications. Much effort was paid on existence, uniqueness and stability of the solution of the problem, see Refs. [2,24,26] and the references cited therein; variational principle and Hamilton principle were established [15–17], and various numerical techniques were appeared [18,25]. Recently much attention has been devoted to numerical methods, which do not require discretization of space–time variables or linearization of the nonlinear equations, among which the variational iteration method (VIM) suggested in [6–14] shows its remarkable merits over others. The method was successfully applied to a nonlinear one dimensional coupled equations in thermoelasticity [28], revealing the method is very convenient, efficient and accurate. The basic idea of variational iteration method is to construct a correction functional with a general Lagrange multiplier which can be identified optimally via variational theory. Adomian's decomposition method (ADM) is to split the given equation into linear and nonlinear parts, invert the highest-order derivative operator contained in the linear operator in both sides, calculate Adomian's polynomials, and finally find the successive terms of the series solution by recurrent relation using Adomian's polynomials (see [1,4,5,19–23,27,29,30]). The aim of the present work is to apply VIM and ADM to solve a real-life problem that exhibits nonlinear coupling between the mechanical and thermal fields and to

* Tel.: +20 2 5676551; fax: +20 2 5728843.
E-mail address: n_sweilam@yahoo.com.

0377-0427/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2006.07.013

produce one-dimensional wave solutions in case of thermoelastic half-space subjected initially to a prescribed harmonic displacement.

1. The model problem

The aim of this paper is to solve the following system of nonlinear coupled one-dimensional partial differential equations [2]:

$$u_{tt} - u_{xx}(1 + 2\gamma u_x + 3\delta u_x^2) = \beta_1\theta_x + \beta_2(u_x\theta)_x, \tag{1.1}$$

$$\left(\theta - au_x - \tfrac{1}{2}bu_x^2\right)_t - ((1 + \alpha u_x)\theta_x)_x = 0, \quad 0 \leqslant x < \infty \tag{1.2}$$

under the following initial conditions:

$$u(x, 0) = \theta(x, 0) = u_0(1 - \cos(x)), \quad u_t(x, 0) = \theta_t(x, 0) = 0, \tag{1.3}$$

and the boundary conditions

$$u(0, t) = \theta(0, t) = 0, \quad u_t(0, t) = \theta_t(0, t) = 0, \tag{1.4}$$

in the case of a thermoelastic half-space subjected initially to a mechanical disturbance.

The symbols $u = u(x, t)$ and $\theta = \theta(x, t)$ in (1.1), (1.3) denote, respectively, the dimensionless elastic displacement and the dimensionless temperature, x and t are the spatial coordinate and the time. The constants involved in Eqs. (1.1), (1.3) have obvious physical significance and will be assumed to have the following order of magnitude:

$$\gamma = \mathrm{O}(1), \quad \delta = \mathrm{O}(1 \text{ to } 10^{-1}), \quad \beta_1 = \mathrm{O}(10^{-3}), \quad \beta_2 = \mathrm{O}(10^{-3}),$$

$$a = \mathrm{O}(10^{-1}), \quad b = \mathrm{O}(10^{-1}), \quad \alpha = \mathrm{O}(1) \quad \text{and} \quad u_0 = \mathrm{O}(10^{-3}).$$

2. Variational iteration method

Consider the following nonhomogeneous, nonlinear system of partial differential equations:

$$L_1u(x, t) + N_1(u(x, t), \theta(x, t)) = f(x, t), \tag{2.1}$$

$$L_2\theta(x, t) + N_2(u(x, t), \theta(x, t)) = g(x, t), \tag{2.2}$$

where L_1, L_2 are linear differential operators with respect to time, N_1, N_2 are nonlinear operators and $f(x, t), g(x, t)$ are given functions.

According to the variational iteration method, we can construct correct functionals as follows:

$$u_{n+1}(x, t) = u_n(x, t) + \int_0^t \lambda_1(\tau)[L_1u_n(x, \tau) + N_1(\tilde{u}_n(x, \tau), \tilde{\theta}_n(x, \tau)) - f(x, \tau)]\,\mathrm{d}\tau, \tag{2.3}$$

$$\theta_{n+1}(x, t) = \theta_n(x, t) + \int_0^t \lambda_2(\tau)[L_2\theta_n(x, \tau) + N_2(\tilde{u}_n(x, \tau), \tilde{\theta}_n(x, \tau)) - g(x, \tau)]\,\mathrm{d}\tau, \tag{2.4}$$

where λ_1 and λ_2 are general Lagrange multipliers, which can be identified optimally via variational theory [6,7]. The second term on the right-hand side in (2.3) and (2.4) is called the corrections and the subscript n denotes the nth order approximation, $\tilde{u}_n$ and $\tilde{\theta}_n$ are restricted variations. We can assume that the above correctional functionals are stationary (i.e., $\delta u_{n+1} = 0$ and $\delta\theta_{n+1} = 0$), then the Lagrange multipliers can be identified. Now we can start with the given initial approximation and by the above iteration formulas we can obtain the approximate solutions.

2.1. Implementation of VIM to the model problem

According to the variational iteration method and after some manipulation of Eqs. (1.1)–(1.3), the correct functionals are as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda_1(\tau)[u_{ntt} - \hat{u}_{nxx} - \beta_1\hat{\theta}_{nx} - \hat{u}_{nxx} \cdot (2\gamma\hat{u}_{nx} + 3\delta\hat{u}_{nx}^2 + \beta_2\hat{\theta}_n) - \beta_2\hat{u}_{nx}\hat{\theta}_{nx}]\,\mathrm{d}\tau, \tag{2.5}$$

$$\theta_{n+1}(x,t) = \theta_n(x,t) + \int_0^t \lambda_2(\tau)[\theta_{nt} - \widehat{\theta}\,\theta_{nxx} - a\hat{u}_{nxt} - b\hat{u}_{nx}\hat{u}_{nxt} - \alpha\hat{u}_{nxx}\hat{\theta}_{nx} - \alpha\hat{u}_{nx}\hat{\theta}_{nxx}]\,\mathrm{d}\tau, \tag{2.6}$$

where $\hat{u}_n$ and $\hat{\theta}_n$ are considered as a restricted variation, i.e., $\delta\hat{u}_n = 0$ and $\delta\hat{\theta}_n = 0$. Making the above correction functionals stationary, and using the initial conditions (1.3):

$$\begin{aligned}\delta u_{n+1}(x,t) &= \delta u_n(x,t) + \delta\int_0^t \lambda_1(\tau)[u_{ntt} - \hat{u}_{nxx} - \beta_1\hat{\theta}_{nx} - \hat{u}_{nxx} \cdot (2\gamma\hat{u}_{nx} + 3\delta\hat{u}_{nx}^2 + \beta_2\hat{\theta}_n) - \beta_2\hat{u}_{nx}\hat{\theta}_{nx}]\,\mathrm{d}\tau \\ &= \delta u_n(x,t) + \lambda_1\delta\dot{u}_n|_{\lambda=t} - \dot{\lambda}_1\delta u_n|_{\lambda=t} + \int_0^t \ddot{\lambda}_1(\tau)\delta u_n\,\mathrm{d}\tau = 0,\end{aligned}$$

$$\begin{aligned}\delta\theta_{n+1}(x,t) &= \delta\theta_n(x,t) + \delta\int_0^t \lambda_2(\tau)[\theta_{nt} - \hat{\theta}_{nxx} - a\hat{u}_{nxt} - b\hat{u}_{nx}\hat{u}_{nxt} - \alpha\hat{u}_{nxx}\hat{\theta}_{nx} - \alpha\hat{u}_{nx}\hat{\theta}_{nxx}]\,\mathrm{d}\tau \\ &= \delta\theta_n(x,t) + \lambda_2\delta\theta_n|_{\tau=t} + \int_0^t (-\dot{\lambda}_2)\delta\theta_n\,\mathrm{d}\tau = 0\end{aligned}$$

we obtain the following stationary conditions:

$$1 - \dot{\lambda}_1(\tau)|_{\tau=t} = 0, \quad \ddot{\lambda}_1(\tau) = 0, \quad \lambda_1(\tau)|_{\tau=t} = 0, \tag{2.7}$$

$$1 + \lambda_2(\tau)|_{\tau=t} = 0, \quad \dot{\lambda}_2(\tau) = 0. \tag{2.8}$$

The solution of equations (2.7), (2.8) are

$$\lambda_1(\tau) = \tau - t, \quad \lambda_2(\tau) = -1. \tag{2.9}$$

By substitution of the identified Lagrange multipliers into Eqs. (2.3)–(2.4) we have the following iteration relations:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t (\tau - t)[u_{ntt} - u_{nxx} - \beta_1\theta_{nx} - u_{nxx} \cdot (2\gamma u_{nx} + 3\delta u_{nx}^2 + \beta_2\theta_n) - \beta_2 u_{nx}\theta_{nx}]\,\mathrm{d}\tau, \tag{2.10}$$

$$\theta_{n+1}(x,t) = \theta_n(x,t) - \int_0^t [\theta_{nt} - \theta_{nxx} - au_{nxt} - bu_{nx}u_{nxt} - \alpha u_{nxx}\theta_{nx} - \alpha u_{nx}\theta_{nxx}]\,\mathrm{d}\tau. \tag{2.11}$$

In an algorithmic form, the VIM can be expressed and implemented in nonlinear coupling in thermoelasticity models as follows:

Algorithm 1. Let n be the iteration index, set a suitable value for the tolerance (Tol)

Step 1: Compute the initial approximations $u_0 = u(x,0)$ and $\theta_0 = u(x,0)$ given by (1.3), set $n = 0$.
Step 2: Use the calculated values of u_n and θ_n to compute u_{n+1} from (2.10).

Step 3: Define $u_n = u_{n+1}$.
Step 4: Use the calculated values of u_n and θ_n to compute θ_{n+1} from (2.11).
Step 5: If $\max |u_{n+1} - u_n| < \text{Tol}$ and $\max |\theta_{n+1} - \theta_n| < \text{Tol}$ stop, otherwise continue.
Step 6: Set $u_{n+1} = u_n$.
Step 7: Set $n := n + 1$ and return to step 2.

2.2. Numerical experiments

The algorithm presented above was tested in order to put evidence the effects of nonlinearity and of the different material constants involving in the equations. Using the above algorithm we can obtain directly the other components as

$$\begin{aligned}
u_1(x,t) &= 0.001(1-\cos(x)) + t^2(2.5 \times 10^{-8}\cos(2x) - 3.0 \times 10^{-10}\cos(3x) \\
&\quad + \cos(x)(0.0010000253 \times 10^{-6}\sin(x)) + 0.000025\sin(x)), \\
\theta_1(x,t) &= 0.001(1-\cos(x)) - (t\cos(x)(-0.001 - 2.0 \times 10^{-6}\sin(x))).
\end{aligned}$$

The rest of the components of the iteration formulas (2.10), (2.11) were obtained in the same manner using the Mathematica Package. The behavior of the solutions obtained by VIM is shown for different time values in Figs. 1–6 in Appendix A. The numerical results show that the iterative steps number 2–7 in the above algorithm can be done in three steps only for the model problem where $\text{Tol} = 10^{-5}$.

3. Description of the Adomian's decomposition method

To illustrate the basic concepts of the Adomian's decomposition method for solving the above system (1.1)–(1.2), first we rewrite it in the following operator form:

$$L_{tt}u = L_{xx}u + \beta_1 L_x\theta + M(u,\theta), \tag{3.1}$$

$$L_t\theta = L_{xx}\theta + aL_{xt}u + N(u,\theta), \tag{3.2}$$

where the notations

$$L_t = \frac{\partial}{\partial t}, \quad L_{tt} = \frac{\partial^2}{\partial t^2}, \quad L_x = \frac{\partial}{\partial x}, \quad L_{xx} = \frac{\partial^2}{\partial x^2} \quad \text{and} \quad L_{xt} = \frac{\partial}{\partial x \partial t}$$

symbolize the linear differential operators. The nonlinear operators $M(u,\theta)$ and $N(u,\theta)$ are defined by

$$M(u,\theta) = u_{xx}(2\gamma u_x + 3\delta u_x^2 + \beta_2\theta) + \beta_2 u_x\theta_x,$$

$$N(u,\theta) = bu_x u_{xt} + \alpha u_{xx}\theta_x + \alpha u_x\theta_{xx}.$$

By using the inverse operators and (1.3), we can write (3.1)–(3.2) in the following form:

$$u(x,t) = u(x,0) + L_{tt}^{-1}(L_{xx}u + \beta_1 L_x\theta) + L_{tt}^{-1}M(u,\theta), \tag{3.3}$$

$$\theta(x,t) = \theta(x,0) + L_t^{-1}(L_{xx}\theta + aL_{xt}u) + L_t^{-1}N(u,\theta), \tag{3.4}$$

where the inverse operators are defined by

$$L_{tt}^{-1}(.) = \int_0^t \int_0^t (.)\,\mathrm{d}t\,\mathrm{d}t, \quad L_t^{-1}(.) = \int_0^t (.)\,\mathrm{d}t.$$

The solutions $u(x,t)$ and $\theta(x,t)$ can be decomposed by an infinite series (see [3]) as follows:

$$u(x,t) = \sum_{i=0}^{\infty} u_i(x,t), \quad \theta(x,t) = \sum_{i=0}^{\infty} \theta_i(x,t), \tag{3.5}$$

where $u_i(x,t)$ and $\theta_i(x,t)$ are the components of $u(x,t)$ and $\theta(x,t)$ that will be elegantly determined. The nonlinear terms $M(u,\theta)$ and $N(u,\theta)$ are defined by the following infinite series:

$$M(u,\theta)=\sum_{m=0}^{\infty} A_m, \quad N(u,\theta)=\sum_{m=0}^{\infty} B_m, \tag{3.6}$$

where A_m and B_m are called Adomian polynomials (see [3]) and defined by

$$A_m=\frac{1}{m!}\left[\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} M\left(\sum_{i=0}^{m}\lambda^i u_i, \sum_{i=0}^{m}\lambda^i \theta_i\right)\right]_{\lambda=0}, \quad i\geqslant 0, \tag{3.7}$$

$$B_m=\frac{1}{m!}\left[\frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N\left(\sum_{i=0}^{m}\lambda^i u_i, \sum_{i=0}^{m}\lambda^i \theta_i\right)\right]_{\lambda=0}, \quad i\geqslant 0. \tag{3.8}$$

The components u_i and θ_i, $i\geqslant 0$, can be defined by the following recursive relationships:

$$u_0(x,t)=u(x,0), \quad \theta_0(x,t)=\theta(x,0),$$

$$u_{n+1}(x,t)=L_{tt}^{-1}(u_{nxx}+\beta_1\theta_{nxx})+L_{tt}^{-1}(A_n), \tag{3.9}$$

$$\theta_{n+1}(x,t)=L_{t}^{-1}(\theta_{nxx}+au_{nxt})+L_{t}^{-1}(B_n). \tag{3.10}$$

This will enable us to determine the components u_n and θ_n recurrently. For numerical comparisons purpose, we construct the solutions $u(x,t)$ and $\theta(x,t)$ as follows:

$$\operatorname*{Lim}_{n\to\infty} \Psi_n = u(x,t), \quad \operatorname*{Lim}_{n\to\infty} \Theta_n=\theta(x,t), \tag{3.11}$$

with the recurrence relation (3.9) and (3.10), where

$$\Psi_n(x,t)=\sum_{i=0}^{n-1} u_i(x,t), \quad \Theta_n(x,t)=\sum_{i=0}^{n-1}\theta_i(x,t), \quad i\geqslant 0.$$

Moreover, the decomposition series solutions (3.5) are generally convergent very rapidly in real physical problems (see [19,5]). The convergence of the decomposition series has investigated by several authors [3,20,23,29].

In an algorithmic form, the ADM can be expressed and implemented in nonlinear coupling in thermoelasticity models as follows:

Algorithm 2. Let n be the iteration index, set a suitable value for the tolerance (Tol)

Step 1: Compute the initial approximations $u_0=u(x,0)$ and $\theta_0=u(x,0)$ given by (1.3), set $n=0$.

Step 2: Compute the Adomian polynomials A_n and B_n from (3.7) and (3.8), respectively.

Step 3: Use the calculated values of u_n and θ_n to compute u_{n+1} from (3.9).

Step 4: Define $u_n=u_{n+1}$.

Step 5: Use the calculated values of u_n and θ_n to compute θ_{n+1} from (3.10).

Step 6: If max $|u_{n+1}-u_n|<$ Tol and max $|\theta_{n+1}-\theta_n|<$ Tol stop, otherwise continue.

Step 7: Set $u_{n+1}=u_n$.

Step 8: Set $n:=n+1$ and return to step 2.

3.1. Numerical experiment

Algorithm 2 was tested in order to put evidence the effects of nonlinearity and of the different material constants involving in the equations. Using Algorithm 2 we can obtain directly the other components as follows:

$$A_0=u_{0xx}(2\gamma u_{0x}+3\delta u_{0x}^2+\beta_2\theta_0)+\beta_2 u_{0x}\theta_{0x},$$

$$B_0=b\,u_{0x}u_{0xt}+\alpha u_{0xx}\theta_{0x}+\alpha u_{0x}\theta_{0xx},$$

$$A_1 = u_{1xx}(2\gamma u_{0x} + 3\delta u_{0x}^2 + \beta_2\theta_0) + u_{0xx}(2\gamma u_{1x} + 6\delta u_{0x}u_{1x} + \beta_2\theta_1) + \beta_2 u_{1x}\theta_{0x} + \beta_2 u_{0x}\theta_{1x},$$

$$B_1 = b(u_{1x}u_{0xt} + u_{0x}u_{1xt}) + \alpha(u_{1xx}\theta_{0x} + u_{0xx}\theta_{1x} + u_{1x}\theta_{0xx} + u_{0x}\theta_{1xx}).$$

Then the first components of the solution are computed and given in the following form:

$$u_1(x,t) = t^2(2.5\times 10^{-8}\cos(2x) - 3.0\times 10^{-10}\cos(3x) + \cos(x)(0.0010000253\times 10^{-6}\sin(x)) + 0.000025\sin(x)),$$
$$\theta_1(x,t) = (t\cos(x)(0.001 + 2.0\times 10^{-6}\sin(x))).$$

The rest of components of the iteration formulas (3.9), (3.10) were obtained in the same manner using the Mathematica Package. The numerical results show that the iterative steps 2–8 in the above algorithm can be done in three steps only for the model problem where $\mathrm{Tol} = 10^{-5}$. The numerical results of the ADM are of the same order as VIM which presented in Figs. 1–6 in Appendix A. However, many terms can be calculated in order to achieve a high level of accuracy of the decomposition method.

4. Comparison between VIM and ADM

It can be seen from the computation process that:

(1) When we begin with the initial conditions as initial approximation in VIM, the correction functional can be easily constructed by a general Lagrange multiplier, and the multiplier can be optimally identified by variational theory.
(2) Comparison of VIM with the ADM reveals that although the numerical results of both methods when applied to the above system (1.1), (1.3) are nearly the same to some tolerance, the approximations obtained by VIM converge faster to the solution than those of ADM.
(3) The main advantage of VIM is to overcome the difficulty arising in calculating Adomian's polynomials in the ADM which it is, in general, very big terms and the consuming time to compute it is big, so it needs a large computer memory and time.

5. Conclusions

The variational iteration method and the Adomian decomposition technique were used to find numerical solutions of a model problem in nonlinear one-dimensional thermoelasticity with given initial conditions and no exact solution is available. It may be concluded that the two methods are very powerful and efficient techniques in finding an acceptable solution for wide classes of nonlinear problems. Also, it can be noted that there are many advantage of these methods, the main advantages are the fast convergence to the solutions, does not require discretizations of space and time variables, no need to solve nonlinear system of equations as in finite element methods and finite difference methods, then, no necessity of large computer memory. The numerical experiments show that the variational iteration method is easier and faster than the Adomian decomposition.

Appendix A

In this part, numerical calculations for the mechanical displacement and for the temperature distribution in the media are carried out, the following results are obtained by using VIM the same results can be calculated by ADM.

The Figs. 1 and 2 show the distributions of the mechanical displacement and the temperature as a function of the distance for the different time values. It is noted that the curves at the wave front becomes larger as time increasing.

The Figs. 3 and 4 show the effect of the linear thermoelastic coupling constant β_1 on the distributions of the mechanical displacement and the temperature for two different values of β as a function of time at $x = 100$.

Fig. 5 shows the approximate mechanical displacement solution at $u(x,t)$ at $x \in (0, 15)$ and $t \in (0, 50)$. Fig. 6 shows the approximate temperature $\theta(x,t)$ at $x \in (0, 15)$ and $t \in (0, 50)$.

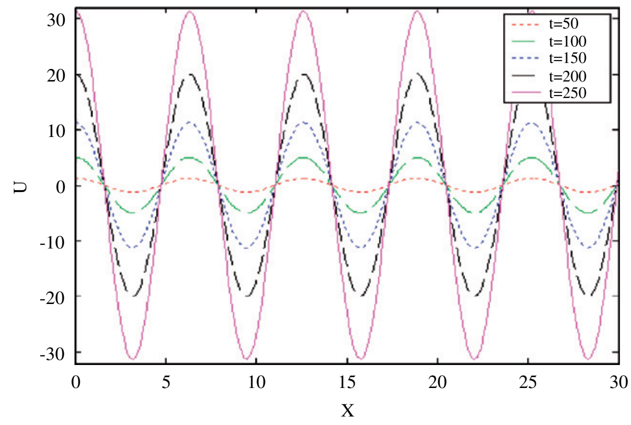

Fig. 1. The displacement at different times $t = 50,\ 100,\ 150,\ 200,\ 250$. $\beta_1 = \beta_2 = 0.05$, $a = b = 0.5$, $\delta = 0.8$, $\alpha = 1$, $\gamma = 1$, $x = 0$ to 30.

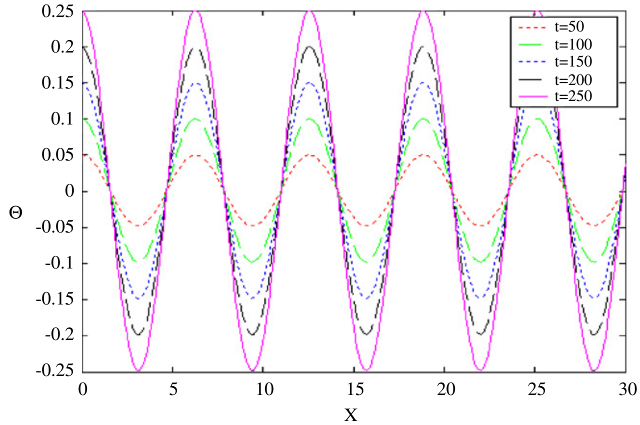

Fig. 2. The temperature at different times $t = 50,\ 100,\ 150,\ 200,\ 250$. $\beta_1 = \beta_2 = 0.05$, $a = b = 0.5$, $\delta = 0.8$, $\alpha = 1$, $\gamma = 1$, $x = 0$ to 30.

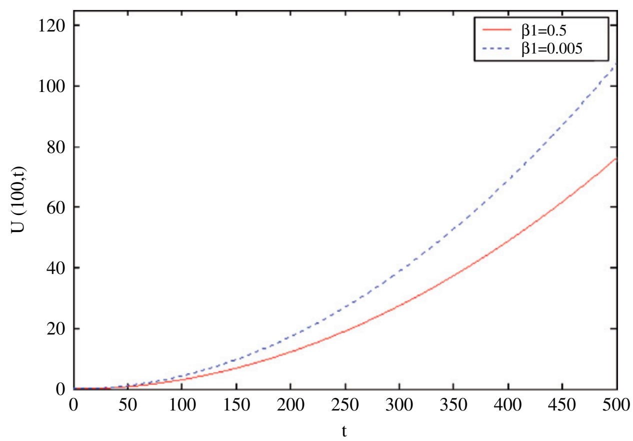

Fig. 3. The displacement as a function of time for two values of β_1 at $x = 100$, $\delta = 0.8$, $a = b = 0.5$, $\alpha = 1$, $\gamma = 1$.

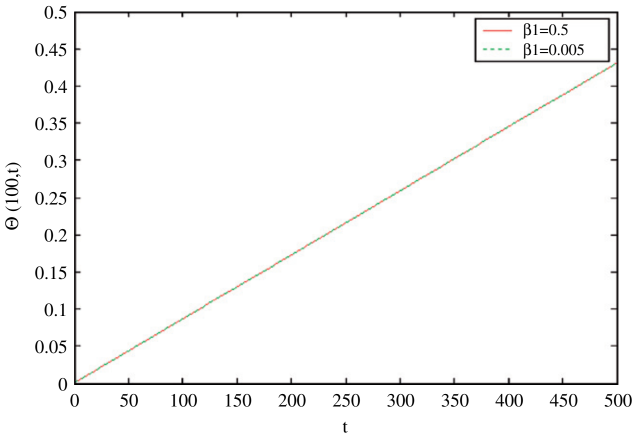

Fig. 4. The temperature as a function of time for two values of β_1 at $x = 100$, $\delta = 0.8$, $a = b = 0.5$, $\alpha = 1$, $\gamma = 1$.

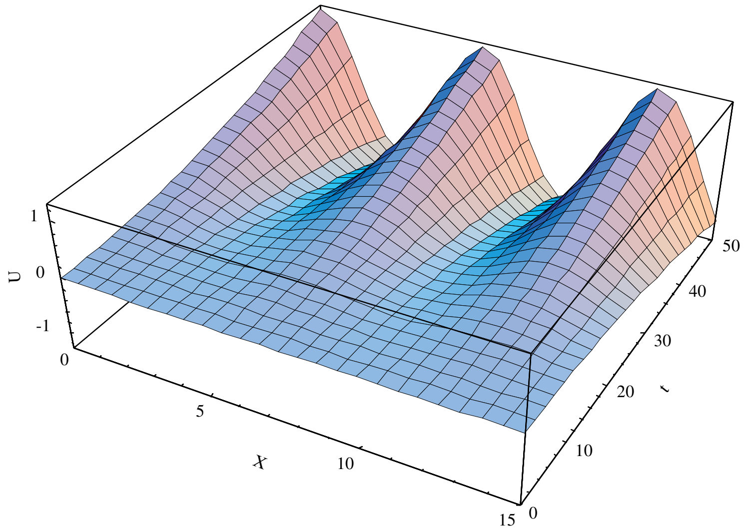

Fig. 5. The approximate solution $u(x, t)$ at $x \in (0, 15)$ and $t \in (0, 50)$.

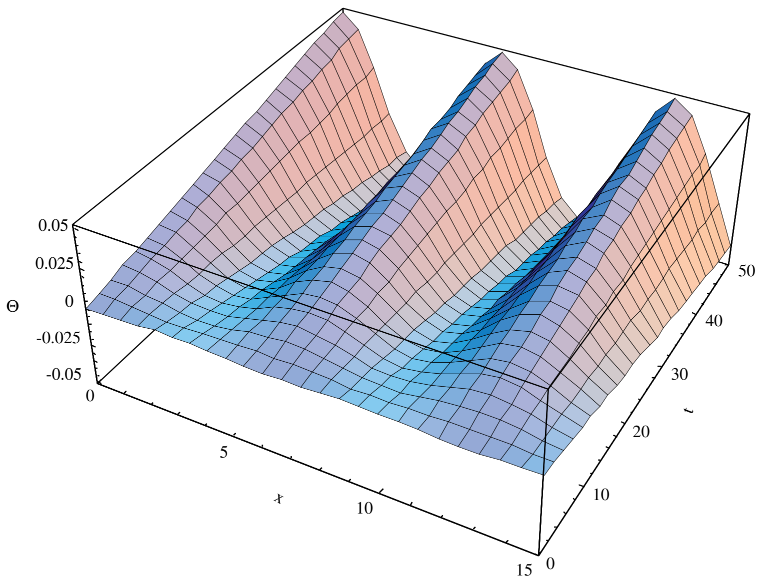

Fig. 6. The approximate solution $\theta(x, t)$ at $x \in (0, 15)$ and $t \in (0, 50)$.

References

[1] S. Abbasbandy, M.T. Darvishi, A numerical solution of Burger's equation by modified Adomian method, Appl. Math. Comput. 163 (2005) 1265–1272.
[2] A.N. Abd-ella, A.F. Ghaleb, G.A. Maugin, Harmonic wave generation in nonlinear thermoelasticity, Internat. J. Eng. Sci. 32 (1994) 1103–1116.
[3] G. Adomian, Nonlinear Stochastic Systems and Applications to Physics, Kluwer Academic Publishers, Dordrecht, 1989.
[4] E. Babolian, J. Biazar, On the order of convergence of Adomain method, Appl. Math. Comput. 130 (2002) 383–387.
[5] S.M. El-Sayed, D. Kaya, On the numerical solution of the system of two-dimensional Burger's equations by the decomposition method, Appl. Math. Comput. 158 (2004) 101–109.
[6] J.H. He, Some asymptotic methods for strongly nonlinear equations, Internat. J. Modern Phys. B 20 (10) (2006) 1141–1199.
[7] J.H. He, Non-perturbative methods for strongly nonlinear problems, dissertation.de-Verlag im Internet GmbH, Berlin, 2006.
[8] J.H. He, X.H. Wu, Construction of solitary solution and compaction-like solution by variational iteration method, Chaos, Solitons & Fractals 29 (1) (2006) 108–113.
[9] J.H. He, Y.Q. Wan, Q. Guo, An iteration formulation for normalized diode characteristics, Internat. J. Circuit Theory Appl. 32 (6) (2004) 629–632.
[10] J.H. He, A simple perturbation approach to Blasius equation, Applied Math. Comput. 140 (2–3) (2003) 217–222.
[11] J.H. He, Variational iteration method for autonomous ordinary differential systems, Appl. Math. Comput. 114 (2–3) (2000) 115–123.
[12] J.H. He, Variational iteration method—a kind of non-linear analytical technique: some examples, Internat. J. Non-linear Mech. 34 (4) (1999) 699–708.
[13] J.H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media, Comput. Methods Appl. Mech. Eng. 167 (1–2) (1998) 57–68.
[14] J.H. He, Approximate solution of nonlinear differential equations with convolution product nonlinearities, Comput. Methods Appl. Mech. Eng. 167 (1–2) (1998) 69–73.
[15] J.H. He, A generalized variational principle in micromorphic thermoelasticity, Mech. Res. Comm. 32 (1) (2005) 93–98.
[16] J.H. He, Generalized variational principles for thermopiezoelectricity, Arch. Appl. Mech. 72 (4–5) (2002) 248–256.
[17] J.H. He, Hamilton principle and generalized variational principles of linear thermopiezoelectricity, J. Appl. Mech. Tran. ASME 68 (4) (2001) 666–667.
[18] S. Jiang, Numerical solution for the Cauchy problem in nonlinear 1-D-thermoelasticity, Computing 44 (1990) 147–158.
[19] D. Kaya, S.M. El-Sayed, On the solution of the couples Schrodinger-KdV equation by the decomposition method, Phys. Lett. A 313 (2003) 82–88.
[20] D. Kaya, I.E. Inan, A convergence analysis of the ADM and an application, Appl. Math. Comput. 161 (2005) 1015–1025.
[21] D. Kaya, A. Yokus, A decomposition method for finding solitary and periodic solutions for a coupled higher-dimensional Burgers equations, Appl. Math. Comput. 164 (2005) 857–864.
[22] D. Lesnic, Decomposition methods for non-linear, non-characteristic Cauchy heat problems, Comm. Nonlinear Sci. Numer. Simulation 10 (2005) 581–596.
[23] D. Lesnic, Convergence of Adomain's decomposition method: periodic temperatures, Comput. Math. Appl. 44 (2002) 13–34.
[24] C.A. De Moura, A linear uncoupling numerical scheme for the nonlinear coupled thermodynamics equations, in: V. Pereyra, A. Reinoze (Eds.), Lecture Notes in Mathematics, vol. 1005, Berlin, Springer, 1983, pp. 204–211.
[25] E.K. Rawy, L. Iskandar, A.F. Ghaleb, Numerical solution for a nonlinear, one dimensional problem of thermoelasticity, J. Comput. Appl. Math. 100 (1998) 53–76.
[26] M. Slemrod, Global existence, uniqueness and asymptotic stability of classical solutions in onedimensional non-linear thermoelasticity, Arch. Rational Mech. Anal. 76 (1981) 97–133.
[27] A. Soufyane, M. Boulmalf, Solution of linear and nonlinear parabolic equations by the decomposition method, Appl. Math. Comput. 162 (2005) 687–693.
[28] N.H. Sweilam, M.M. Khader, Variational iteration method for one dimensional nonlinear thermoelasticity, Chaos, Solitons & Fractals, in press, doi: 10.1016/j.chaos.2005.11.028.
[29] P. Vadas, S. Olek, Convergence and accuracy of Adomain's decomposition method for the solution of Lorenz equations, Internat. J. Heat and Mass Transfer 43 (2000) 1715–1734.
[30] A.M. Wazwaz, Necessary conditions for the appearance of noise terms in decomposition solution series, Appl. Math. Comput. 81 (1997) 265–274.