Journal of Computational and Applied Mathematics 232 (2009) 122–126

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Mathematical study of trade-off relations in logistics systems

Youhei Nanazawa [a,*], Hiroshi Suito [a], Hideo Kawarada [b]

[a] Graduate School of Environmental Sciences, Okayama University, 3-1-1, Tsushima-naka, Okayama, 700-8530, Japan

[b] Department of Logistics and Distribution, Ryutsu Keizai University, Hirahata 120, Ryuugasaki, Ibaraki, 301-8555, Japan

ARTICLE INFO

Article history:
Received 20 April 2007
Received in revised form 7 June 2008

Keywords:
Trade-off
Logistics
Multi-objective optimization problems
Pareto optimal solutions
Indifference curve
Non-dominated sorting
Sharing function

ABSTRACT

This paper presents a mathematical model of trade-off relations arising in third party logistics using Pareto optimal solutions for multi-objective optimization problems. The model defines an optimal set of distribution costs and service levels constituting a trade-off relation. An analogy to the concept of the indifference curve in the field of economics is discussed. Numerical experiments for a simplified problem are performed, demonstrating an increasing process of the utility of logistics.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

Third Party Logistics (3PL) are business solutions which are required to solve difficult problems involved in user enterprises related to logistic systems. Such problems of logistics systems are diverse in their characteristics, but their common point is a reduction of distribution costs. Their extreme reduction typically engenders reduced service levels. In other words, distribution costs and the service levels are mutually opposed in a trade-off relation. This study is intended to clarify this trade-off relation mathematically and thereby represent it similarly to an indifference curve of the field of economics. Here, respective distribution costs and service levels characterize a pair of goods in economic theory. The utility corresponds to profits obtained by constructing logistics systems: we call these profits the utility of logistics.

The mathematical discussion in this paper addresses replacement of the trade-off relation by multi-objective optimization problems (MOPs). Concretely, we define two objective functions: one representing distribution costs and one representing service levels. We realize a trade-off relation by optimizing the two functions simultaneously. Unlike single-objective problems, the MOPs have no unique solutions. For that reason, it is necessary in the first place to define the solutions for the MOPs. Some candidate MOP solutions are Pareto optimal solutions (POSs) [3] and Nash equilibrium solutions [4]. In this study, POSs are applied as the MOP solutions because of their analogy to concepts used in economics. The curve containing POSs represents logistics systems that yield a constant utility. The 3PL, as a business solution, indicates a point on this curve corresponding to demand of a user enterprise under the condition that the utility of logistics is a constant. Next, we examine how 3PL can respond to demand to obtain the upper rank utility of logistics. That is to say, the response is dependent on improvement of the logistics systems concerned into one with the upper rank of utility. We exemplify the situation mentioned above using a simple mathematical model to treat a trade-off relation with respect to arrival time and distribution costs as MOPs. The formulation is simple, but does not lose the essence of the claim included in this paper.

* Corresponding address: Department of Environmental and Mathematical Sciences, Okayama University, 3-1-1, Tsushima-naka, 700-8530, Okayama, Japan.

E-mail addresses: nanazawa@s.ems.okayama-u.ac.jp, nanazawa@tokai-u.jp (Y. Nanazawa), suito@ems.okayama-u.ac.jp (H. Suito), kawarada@rku.ac.jp (H. Kawarada).

0377-0427/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2008.10.035

Studies related to optimization of distribution costs for logistics systems can be found in [1,5]. Ombuki [7] presented multi-objective optimization problems that arise in routing problems. In this paper, we present a new strategy, which improves logistics systems through comparison of several Pareto optimal solutions obtained by varying a parameter of logistics systems.

In Section 2, the mathematical model and the solution definition are given. In Section 3, the MOP solution method is presented using genetic algorithms (GAs) with a non-dominated sorting and a sharing function for obtaining a solution with higher precision. Section 4 presents a model of a real problem and applies the methodology stated in Section 3. In Section 5, numerical results obtained in Section 4 are discussed using concepts that are rooted in logistics systems.

2. Multi-objective optimization problem

2.1. Definition of MOPs

MOPs find solutions by optimizing multiple functions simultaneously. Generally, a MOP is formulated as follows:

$$\text{Minimize/Maximize } f_i(\boldsymbol{x})\ (i = 1, \ldots, N) \quad \text{such that } \boldsymbol{x} \in \boldsymbol{X}, \tag{1}$$

where f_i, N, $\boldsymbol{x}$ and $\boldsymbol{X}$ respectively denote the i-th objective function, the number of objective functions, a vector of variables and a feasible set. We deal with MOPs without equality/inequality constraints for simplicity.

2.2. The MOP solutions

In a minimization problem, we say "$\boldsymbol{x}_1$ dominates $\boldsymbol{x}_2$"

$$\boldsymbol{x}_1 \sqsupset \boldsymbol{x}_2, \tag{2}$$

if the following conditions s are satisfied:

$$\begin{cases} \forall i, & f_i(\boldsymbol{x}_1) \leq f_i(\boldsymbol{x}_2) \\ \exists i, & f_i(\boldsymbol{x}_1) < f_i(\boldsymbol{x}_2)\,. \end{cases} \tag{3}$$

Using the definition described above, POSs are defined as a set of non-dominated solutions:

$$\left\{\boldsymbol{x}^* \mid \boldsymbol{x}^* \not\sqsupset\ \forall \boldsymbol{x} \in \boldsymbol{X}\right\}. \tag{4}$$

3. Numerical algorithm

In this paper, GAs [2–4,6] are used to obtain the POSs in the MOPs. This type of optimization algorithm mimics biological evolution. The GAs comprise three operations: selection, crossover, and mutation. This optimization's feature is using only the value of an objective function without a gradient. That value is called a fitness. The GAs operate primarily through selection of individuals according to their respective fitnesses. Crossover and mutation produce new individuals based on older individuals' information. An outline of GAs is summarized as follows:

Item 1: Initialize population, $g = 0$.
Item 2: Evaluate the fitness of each in the population.
Item 3: If $g >$ maximum generation, go to Item 9.
Item 4: Selection according to fitness.
Item 5: Crossover.
Item 6: Mutation.
Item 7: $g = g + 1$.
Item 8: Go to Item 2.
Item 9: End this outline.

For details, we refer the reader to [2,3]. However, in MOPs, several criteria are assigned, not a single fitness. Therefore, to evaluate the individuals, we must build another kind of fitness, called dummy fitness. In the following, we explain the algorithm for giving dummy fitness, particularly addressing the evaluation method referring to [3]:

Step 1: Let r be a variable that fixes the ranks of individuals; its initial value is 1.
Step 2: Compute values of objective functions for all individuals.
Step 3: If the dummy fitness values for all individuals are set up, go to the next step of the GA. If not, go to (4).
Step 4: Repeat the method stated in Section 2.2 for individuals, except for those individuals for which dummy fitness is already set up. Separate all individuals into those with and without non-dominated ones. Then let the rank of non-dominated individuals be r and denote their set as front r. This division method is called non-dominated sorting.

Step 5: Give dummy fitness to individuals belonging to front r; herein, if r is 1, the value is 1 and if r is larger than 1, the value is the minimum dummy fitness in individuals belonging to front $r-1$.
Step 6: Compute an adjustment value of i-th individual belonging to front r using the following equation:

$$m_i = \sum_{j=1}^{M} Sh\left(d\left(i,j\right)\right), \tag{5}$$

where M represents the number of individuals belonging to front r. The distance between i-th and j-th individuals belonging to front r is $d\left(i,j\right)$. In this paper, the distance between individuals is defined using a Euclidean norm given by the values of dimensionless objective functions, which are defined by the quotients of each objective function to the maximum of all individual objective functions belonging to front r. The value of $Sh\left(d\right)$ is determined using a sharing function, which is represented as

$$Sh\left(d\left(i,j\right)\right) = \begin{cases} 1 - \dfrac{d\left(i,j\right)}{\sigma_{\text{share}}} & \text{if } d\left(i,j\right) < \sigma_{\text{share}} \\ 0 & \text{if } d\left(i,j\right) \geq \sigma_{\text{share}}, \end{cases} \tag{6}$$

where σ_{share} is set as the desired value of an interval separating individuals. In other words, the adjustment value is determined in proportion to the number of individuals with the same rank located in the neighborhood. Actually, σ_{share} is

$$\sigma_{\text{share}} = \frac{d_{\max}}{h}, \tag{7}$$

where $d_{\max}$ is the maximum value of the distance between individuals belonging to the same rank; set $h = 2$.
Step 7: Divide the dummy fitness of the i-th individual by m_i to adjust the dummy fitness.
Step 8: Add 1 to the value of r and return to (2).

In this paper, we record individuals with rank 1 arising in all generations. They are separated by non-dominated sorting. Then, the resultant non-dominated solutions are adopted as POSs.

4. Target problem

4.1. Modeling of the problem

We set the following problem as a modeling of the trade-off relation:
"We wish to carry a cargo to a destination up to an appointed time with minimum consumption of fuel. We attain that target by running a truck at low speeds to suppress fuel consumption. We wish to find the optimal speed of the truck at each interval of distance between the starting point and the destination."

Here, the suppression of fuel consumption indicates the reduction of distribution costs. The closer approximation of the appointed time indicates the improvement of the service level.

4.2. Formulation of the problem

A multi-objective minimization problem is defined as follows.

$$\underset{\boldsymbol{v}\in\boldsymbol{V}}{\text{Minimize}}\ \boldsymbol{f}(\boldsymbol{v}) = (f_1(\boldsymbol{v}), f_2(\boldsymbol{v})) \tag{8}$$

$$\begin{cases} f_1(\boldsymbol{v}) = \left| T - \displaystyle\sum_{i=1}^{L} \frac{x_i}{v_i} \right| \\ f_2(\boldsymbol{v}) = \displaystyle\sum_{i=1}^{L} \frac{x_i}{g\left(v_i\right)}. \end{cases}$$

An objective function to represent the arrival of the cargo at the destination up to the appointed time as closely as possible is f_1, while f_2 represents the amount of fuel consumed by transportation. $f_1(\boldsymbol{v})$ and $f_2(\boldsymbol{v})$ correspond to objective functions appearing in step 2 in Section 3. Parameter L is the number of intervals between the starting point and the destination. The required total time is T, i.e., the truck starts at $t = 0$ and is appointed to arrive at the destination at $t = T$. In addition, v_i is the truck speed over interval i, which is limited as $V_{\min}$ to $V_{\max}$. Let $\boldsymbol{V}$ be $\{\boldsymbol{v} = (v_1, v_2, \ldots, v_L) \mid V_{\min} \leq v_i \leq V_{\max}\}$; $g\left(v\right)$ represents a distance for which a truck can travel by consuming a unit amount of fuel, so

$$g\left(v\right) = a\left\{1 - \frac{(v - V^*)^2}{\left(V_{\max} - V_{\min}\right)^2}\right\}, \quad \text{where } V_{\min} < V^* < V_{\max}. \tag{9}$$

The maximum value of $g\left(v\right)$ is a. A speed of truck V^* is set to give the lowest fuel cost in this transportation.

Table 1
Parameters of the problem.

Parameter	Value	Parameter	V alue
L	3	T	6 h
x_1	10 km	V^*	80 km/h
x_2	500 km	$V_{\min}$	20 km/h
x_3	100 km	$V_{\max}$	120 km/h
a	10 km/l		

Table 2
Parameters of GA.

Parameter	Value
Dimension	3
Population size	100
Max. generation	400
Bit length	20
Crossover rate	1.0
Mutation rate	0.05

Table 3
Operators of GA.

Operator type	Name
Selection	Roulette
Crossover	One-point
Mutation	Bit inversion

Table 4
Member of Pareto optimal solutions.

	f_1	f_2	v_1	v_2	v_3
A	0.564	84.87	104.44	92.99	91.67
B	1.735	71.96	66.23	78.48	82.44
C	1.856	71.48	103.47	78.35	72.60
D	5.654	62.00	34.49	52.66	53.50

5. Numerical results

Parameters used in numerical experiments are shown in Tables 1–3. The POSs obtained are shown in Fig. 1. Points on the curve are POSs. The horizontal axis represents the difference between the appointed time and the arrival time using f_1. The vertical axis represents the total fuel consumption by transportation using f_2. In this case, 448 points exist in POSs. Points A–D in Fig. 1 designate examples of POSs, the detailed data for which are shown in Table 4. We can see that the points A–D satisfy the POS condition. Furthermore, the points indicated in Fig. 1, other than A–D, are also members of POSs. In an actual problem, the set of POSs is produced through numerical computation. Then it is rather effective that a user enterprise can choose a solution from the set. The merit of applying POSs is that a user enterprise can gain flexibility in decision-making.

When we set $T = 9$ and $V^* = 40, 60, 80, 100$, we obtain the POSs shown in Fig. 2. These various V^* can indicate modifications of logistics systems. We see that POSs resulting from $V^* = 60$ are superior to POSs resulting from $V^* = 40$.

6. Discussion

This paper presents an examination of MOPs with respect to arrival time and distribution costs at freight transportation as an expression of the trade-off relation arising in logistics systems. The POSs obtained in this model constituting an indifference curve of the utility of logistics suggest effective choices for enterprises. Recalling the four points A–D located on the indifference curve discussed in Section 5, let us consider a case in which a user enterprise is currently operating logistics systems under a condition corresponding to A. In such a case, 3PL can suggest appropriate changes from A to B, C, or D, in accordance with the demands of user enterprises.

On the other hand, changing parameter V^*, as discussed in Section 5, alters the fuel expenses of logistics systems. For example, changing parameter V^* means changing the truck size. By varying V^*, the maximum utilities of logistics are obtainable, which indicates the relative achievement of a better logistics system.

Various constraints, such as truck speed limitations at each interval, driver burdens, etc., on the logistic system and more realistic situations should be addressed in future works.

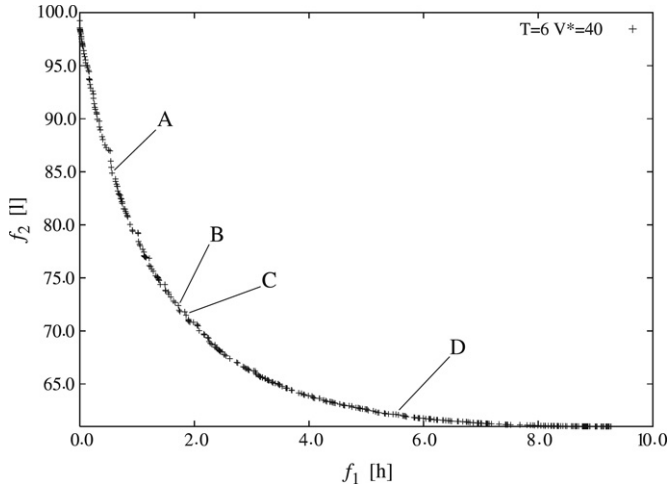


Fig. 1. Pareto optimal solutions.

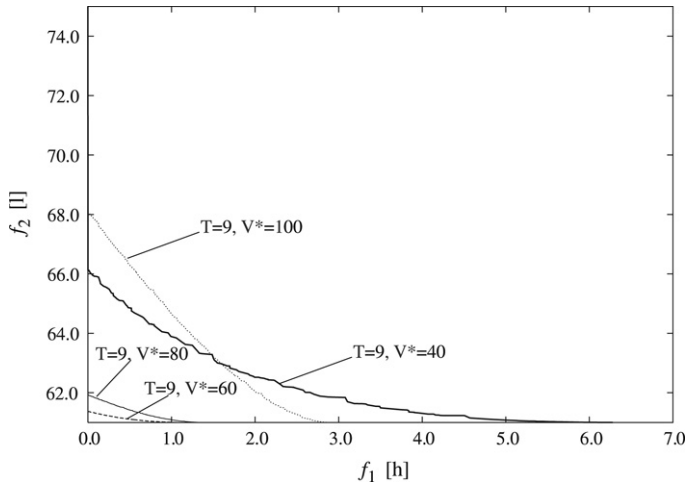


Fig. 2. Modification of logistic systems.

References

[1] A.A. Assad, Modeling and implementation issues in vehicle routing, in: B.L. Golden, A.A. Assad (Eds.), Vehicle Routing: Methods and Studies, North-Holland, 1988, pp. 7–45.
[2] D.E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, 1989.
[3] N. Marco, S. Lanteri, J.-A. Desideri, J. Périaux, A parallel genetic algorithm for multi-objective optimization in computational fluid dynamics, in: Evolutionary Algorithms in Engineering and Computer Science, John Wiley and Sons, 1999, pp. 445–456.
[4] M. Sefrioui, J. Périaux, Nash genetic algorithms: Examples and applications, in: IEEE International Conference on Evolutionary Computation, vol. 1, 2000, pp. 509–516.
[5] O. Bräysy, M. Gendreau, Vehicle routing problem with time windows, part i: Route construction and local search algorithms, Transportation Science 39 (1) (2005) 104–118.
[6] Y. Nanazawa, H. Suito, T. Ueda, M. Minami, Numerical algorithm using compartment model analysis for estimating permeability in renal tissue, Transactions of the Japan Society for Industrial and Applied Mathematics 16 (4) (2006) 435–452 (in Japanese).
[7] B. Ombuki, B.J. Ross, F. Hanshar, Multi-objective genetic algorithms for vehicle routing problem with time windows, Applied Intelligence 24 (1) (2006) 17–30.