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Robust stability criteria for systems with interval time-varying delay and nonlinear perturbations

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ABSTRACT

This paper considers the robust stability for a class of linear systems with interval time-varying delay and nonlinear perturbations. A Lyapunov–Krasovskii functional, which takes the range information of the time-varying delay into account, is proposed to analyze the stability. A new approach is introduced for estimating the upper bound on the time derivative of the Lyapunov–Krasovskii functional. On the basis of the estimation and by utilizing free-weighting matrices, new delay-range-dependent stability criteria are established in terms of linear matrix inequalities (LMIs). Numerical examples are given to show the effectiveness of the proposed approach.

1. Introduction

During the past few decades, considerable attention has been paid to the stability of time-delay systems (see, e.g., [1–4], and references therein). Usually, the range of delays considered in most of the existing references is from zero to an upper bound [5]. In practice, however, the delay may vary in a range for which the lower bound is not restricted to being zero. A typical example with interval time delay is the networked control system, which has been widely studied in the recent literature (see, e.g., [6,7]). With the development of networked control technology, many efforts have been made to investigate the stability of systems with interval time-varying delay (see [8–17]).

Since delay-dependent criteria are generally less conservative than delay-independent ones [4], many researchers have focused on delay-dependent stability. Many significant results have been reported in the recent literature [18,1,2,19,6,3,20–22,8–11,23,24,12–14,5,25,4,15–17,26]. For example, a novel Lyapunov–Krasovskii functional was introduced in [9]. An augmented Lyapunov–Krasovskii functional approach was developed in [11,13]. A Jensen integral inequality approach was employed in [10,24,12,14,17]. A novel piecewise analysis method was proposed in [15]. Delay-range-dependent stability was investigated in [21] by using the free-weighting matrix approach [22,25]. The stability problem of discrete-time systems with interval time-varying delay was studied in [19,16].

In practice, real systems usually present some uncertainties due to environmental noise, uncertain or slowly varying parameters, etc. Therefore, the stability problem of time-delay systems with nonlinear perturbations has received increasing attention (see, e.g., [18,20,26,27]). A model transformation method was used in [18]. A bounding technique for some cross terms was proposed in [23]. A descriptor model transformation together with a decomposition technique using the delay

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term matrix was employed in [20]. Recently, a less conservative delay-dependent stability criterion was provided in [26] by employing the free-weighting matrix approach. Robust stabilization for nonlinear discrete-time systems was studied in [27]. In the above references, reducing the conservatism of the existing stability criteria is a central issue. As we know, a bounding technique [23] or model transformation [1] may increase the conservatism. The free-weighting matrix method, by contrast, is helpful for reducing the conservatism of stability criteria [25]. On the other hand, choosing an appropriate Lyapunov–Krasovskii functional and estimating the upper bound of its time derivative are very important in deriving the stability criteria.

In this paper, we deal with the delay-dependent stability problem for a class of linear systems with nonlinear perturbations and interval time-varying delay. We first introduce a new Lyapunov–Krasovskii functional by taking the range information of the delay into account. The delay-dependent stability of systems is then analyzed by using the functional. An approach is proposed in estimating the upper bound of the time derivative of the functional. New delay-range-dependent stability criteria are obtained by introducing free-weighting matrices and free-weighting parameters. The proposed stability criteria are formulated in terms of a set of linear matrix inequalities (LMIs). Finally, two numerical examples are given to show the effectiveness of the proposed approach.

Notations. R^n denotes the n-dimensional Euclidean space. The superscript "T" stands for matrix transposition. $X > Y$ (respectively, $X \geq Y$), where X and Y are real symmetric matrices, means that the matrix $X - Y$ is positive definite (respectively, positive semi-definite). I is an identity matrix with appropriate dimension. In symmetric block matrices, we use an asterisk ($*$) to represent a term that is induced by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following system with a time-varying state delay and nonlinear perturbations:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h(t)) + f(x(t), t) + g(x(t-h(t)), t) \\ x(t) = \phi(t), \quad \forall t \in [-h_M, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector; A and A_h are known matrices in $R^{n\times n}$. The delay $h(t)$ is time varying and satisfies

$$0 \leq h_m \leq h(t) \leq h_M, \qquad \dot{h}(t) \leq h_d, \tag{2}$$

where h_m and h_M are constants representing respectively the lower and upper bounds of the delay, h_d is a positive constant. The initial condition $\phi(t)$ is a continuous vector-valued function. Moreover, the functions $f(x(t), t)$ and $g(x(t-h(t)), t)$ are unknown and denote the nonlinear perturbations with respect to the current state $x(t)$ and delayed state $x(t-h(t))$, respectively. They satisfy that $f(0, t) = 0$, $g(0, t) = 0$ and

$$f^T(x(t), t) f(x(t), t) \leq \alpha^2 x^T(t) F^T F x(t), \tag{3}$$

$$g^T(x(t-h(t)), t) g(x(t-h(t)), t) \leq \beta^2 x^T(t-h(t)) G^T G x(t-h(t)), \tag{4}$$

where $\alpha \geq 0$ and $\beta \geq 0$ are known scalars, F and G are known constant matrices.

In this paper, we investigate the stability problem of system (1) with the interval time-varying delay satisfying (2) and the nonlinear perturbations $f(x(t), t)$ and $g(x(t-h(t)), t)$ satisfying (3) and (4). Our main objective is to derive new delay-range-dependent stability conditions under which system (1) is asymptotically stable.

3. Main results

In this section, we present new delay-range-dependent stability conditions for system (1) with the delay satisfying (2) and the perturbations satisfying (3) and (4).

Theorem 1. *System (1) subject to (2)–(4) is asymptotically stable for given $0 \leq h_m \leq h_M$ and h_d if there exist scalars $\varepsilon_1 \geq 0$, $\varepsilon_2 \geq 0$ and matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$, $X \geq 0$, $Y \geq 0$, $L^T = \begin{bmatrix} L_1^T & L_2^T & L_3^T & L_4^T \end{bmatrix}$, $M^T = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}$ and $N^T = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix}$ of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Phi + h_m X + \rho Y & M - L & -N & \sqrt{h_m} S^T R_1 & \sqrt{\rho} S^T R_2 \\ * & -Q_1 & 0 & 0 & 0 \\ * & * & -Q_3 & 0 & 0 \\ * & * & * & -R_1 & 0 \\ * & * & * & * & -R_2 \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} X & L \\ * & R_1 \end{bmatrix} \geq 0, \qquad \begin{bmatrix} Y & M \\ * & R_2 \end{bmatrix} \geq 0, \qquad \begin{bmatrix} Y & N \\ * & R_2 \end{bmatrix} \geq 0, \tag{6}$$

where $\rho = h_M - h_m$, $S = \begin{bmatrix} A & A_h & I & I \end{bmatrix}$,

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P + L_3^T & P + L_4^T \\ * & \Phi_{22} & N_3^T - M_3^T & N_4^T - M_4^T \\ * & * & -\varepsilon_1 I & 0 \\ * & * & * & -\varepsilon_2 I \end{bmatrix}, \tag{7}$$

$$\Phi_{11} = PA + A^T P + L_1 + L_1^T + Q_1 + Q_2 + Q_3 + \varepsilon_1 \alpha^2 F^T F,$$

$$\Phi_{12} = PA_h + L_2^T + N_1 - M_1,$$

$$\Phi_{22} = -(1 - h_d)Q_2 + N_2 + N_2^T - M_2 - M_2^T + \varepsilon_2 \beta^2 G^T G.$$

Proof. Choose a Lyapunov–Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{8}$$

where

$$V_1(t) = x(t)^T P x(t),$$

$$V_2(t) = \int_{t-h_m}^{t} x^T(s) Q_1 x(s) \mathrm{d}s + \int_{t-h(t)}^{t} x^T(s) Q_2 x(s) \mathrm{d}s + \int_{t-h_M}^{t} x^T(s) Q_3 x(s) \mathrm{d}s,$$

$$V_3(t) = \int_{-h_m}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta,$$

$$V_4(t) = \int_{-h_M}^{-h_m} \int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta.$$

Calculating the time derivatives of $V_i(t)$, $i = 1, \ldots, 4$, along the trajectory of system (1) yields

$$\dot{V}_1(t) = 2x^T(t) P [A x(t) + A_h x(t - h(t)) + f(x(t), t) + g(x(t - h(t)), t)], \tag{9}$$

$$\begin{aligned} \dot{V}_2(t) &= x^T(t)[Q_1 + Q_2 + Q_3]x(t) - (1 - \dot{h}(t)) x^T(t - h(t)) Q_2 x(t - h(t)) \\ &\quad - x^T(t - h_m) Q_1 x(t - h_m) - x^T(t - h_M) Q_3 x(t - h_M) \\ &\le x^T(t)[Q_1 + Q_2 + Q_3]x(t) - (1 - h_d) x^T(t - h(t)) Q_2 x(t - h(t)) \\ &\quad - x^T(t - h_m) Q_1 x(t - h_m) - x^T(t - h_M) Q_3 x(t - h_M) \end{aligned} \tag{10}$$

$$\dot{V}_3(t) = h_m \dot{x}^T(t) R_1 \dot{x}(t) - \int_{t-h_m}^{t} \dot{x}^T(s) R_1 \dot{x}(s) \mathrm{d}s, \tag{11}$$

$$\dot{V}_4(t) = \rho \dot{x}^T(t) R_2 \dot{x}(t) - \int_{t-h(t)}^{t-h_m} \dot{x}^T(s) R_2 \dot{x}(s) \mathrm{d}s - \int_{t-h_M}^{t-h(t)} \dot{x}^T(s) R_2 \dot{x}(s) \mathrm{d}s. \tag{12}$$

Define $\xi_1^T(t) = \begin{bmatrix} x^T(t) & x^T(t - h(t)) & f^T(x(t), t) & g^T(x(t - h(t), t)) \end{bmatrix}$. Then, the following equations hold for any matrices L, M and N with appropriate dimensions:

$$2\xi_1^T(t) L \left[x(t) - x(t - h_m) - \int_{t-h_m}^{t} \dot{x}(s) \mathrm{d}s \right] = 0, \tag{13}$$

$$2\xi_1^T(t) M \left[x(t - h_m) - x(t - h(t)) - \int_{t-h(t)}^{t-h_m} \dot{x}(s) \mathrm{d}s \right] = 0, \tag{14}$$

$$2\xi_1^T(t) N \left[x(t - h(t)) - x(t - h_M) - \int_{t-h_M}^{t-h(t)} \dot{x}(s) \mathrm{d}s \right] = 0. \tag{15}$$

Moreover, for matrices X and Y with appropriate dimensions, we have

$$h_m \xi_1^T(t) X \xi_1(t) - \int_{t-h_m}^{t} \xi_1^T(t) X \xi_1(t) \mathrm{d}s = 0, \tag{16}$$

$$\rho \xi_1^T(t) Y \xi_1(t) - \int_{t-h(t)}^{t-h_m} \xi_1^T(t) Y \xi_1(t) \mathrm{d}s - \int_{t-h_M}^{t-h(t)} \xi_1^T(t) Y \xi_1(t) \mathrm{d}s = 0. \tag{17}$$

On the other hand, for any scalars $\varepsilon_1 \geq 0$, $\varepsilon_2 \geq 0$, it follows from (3) and (4) that

$$\varepsilon_1[\alpha^2 x^T(t)F^TFx(t) - f^T(x(t), t)f(x(t), t)] \geq 0, \tag{18}$$

$$\varepsilon_2[\beta^2 x^T(t-h(t))G^TGx(t-h(t)) - g^T(x(t-h(t)), t)g(x(t-h(t)), t)] \geq 0. \tag{19}$$

Adding the terms on the left sides of (13)–(19) to the sum of $V_i(t)$, $i = 1, \ldots, 4$, yields

$$\begin{aligned}
\dot{V}(t) \leq\ & 2x^T(t)P[Ax(t) + A_h x(t-h(t)) + f(x(t), t) + g(x(t-h(t)), t)] + x^T(t)[Q_1 + Q_2 + Q_3]x(t) \\
& - (1-h_d)x^T(t-h(t))Q_2x(t-h(t)) - x^T(t-h_m)Q_1x(t-h_m) - x^T(t-h_M)Q_3x(t-h_M) \\
& + h_m\dot{x}^T(t)R_1\dot{x}(t) - \int_{t-h_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s + \rho\dot{x}^T(t)R_2\dot{x}(t) - \int_{t-h(t)}^{t-h_m}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s - \int_{t-h_M}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s \\
& + 2\xi_1^T(t)L\left[x(t) - x(t-h_m) - \int_{t-h_m}^{t}\dot{x}(s)\mathrm{d}s\right] + 2\xi_1^T(t)M\left[x(t-h_m) - x(t-h(t)) - \int_{t-h(t)}^{t-h_m}\dot{x}(s)\mathrm{d}s\right] \\
& + 2\xi_1^T(t)N\left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)}\dot{x}(s)\mathrm{d}s\right] + h_m\xi_1^T(t)X\xi_1(t) - \int_{t-h_m}^{t}\xi_1^T(t)X\xi_1(t)\mathrm{d}s \\
& + \rho\xi_1^T(t)Y\xi_1(t) - \int_{t-h(t)}^{t-h_m}\xi_1^T(t)Y\xi_1(t)\mathrm{d}s - \int_{t-h_M}^{t-h(t)}\xi_1^T(t)Y\xi_1(t)\mathrm{d}s + \varepsilon_1[\alpha^2x^T(t)F^TFx(t) \\
& - f^T(x(t), t)f(x(t), t)] + \varepsilon_2[\beta^2x^T(t-h(t))G^TGx(t-h(t)) - g^T(x(t-h(t)), t)g(x(t-h(t)), t)] \\
=\ & \xi_3^T(t)\left[\Theta + h_m\bar{S}^TR_1\bar{S} + \rho\bar{S}^TR_2\bar{S}\right]\xi_3(t) - \int_{t-h_m}^{t}\xi_2^T(t,s)\begin{bmatrix} X & L \\ L^T & R_1\end{bmatrix}\xi_2(t,s)\mathrm{d}s \\
& - \int_{t-h(t)}^{t-h_m}\xi_2^T(t,s)\begin{bmatrix} Y & M \\ M^T & R_2\end{bmatrix}\xi_2(t,s)\mathrm{d}s - \int_{t-h_M}^{t-h(t)}\xi_2^T(t,s)\begin{bmatrix} Y & N \\ N^T & R_2\end{bmatrix}\xi_2(t,s)\mathrm{d}s,
\end{aligned} \tag{20}$$

where

$$\xi_3^T(t) = \begin{bmatrix}\xi_1^T(t) & x^T(t-h_m) & x^T(t-h_M)\end{bmatrix}, \qquad \xi_2^T(t,s) = \begin{bmatrix}\xi_1^T(t) & \dot{x}^T(s)\end{bmatrix},$$

and

$$\Theta = \begin{bmatrix} \Phi + h_mX + \rho Y & M - L & -N \\ * & -Q_1 & 0 \\ * & * & -Q_3 \end{bmatrix}, \qquad \bar{S} = \begin{bmatrix} S & 0 & 0 \end{bmatrix}.$$

Applying the Schur complement, we know that $\Theta + h_m\bar{S}^TR_1\bar{S} + \rho\bar{S}^TR_2\bar{S} < 0$ is equivalent to (5). Thus, if (5) and (6) hold, then (20) implies that there exists a scalar $\delta > 0$ such that $\dot{V}(t) \leq -\delta\|x(t)\|^2$ [3]. Therefore, system (1) is asymptotically stable under the conditions of Theorem 1. □

Remark 1. In the proof of Theorem 1, we introduce a new estimation on the upper bound of the time derivative of $V(t)$. More precisely, the identity equalities (16) and (17) are employed. In contrast, the following inequalities (here the $\zeta(t)$, Z_1, Z_2, N, S and M are defined in [21]):

$$h_M\zeta^T(t)NZ_1^{-1}N^T\zeta(t) - \int_{t-h(t)}^{t}\zeta^T(t)NZ_1^{-1}N^T\zeta(t)\mathrm{d}s \geq 0,$$

$$\rho\zeta^T(t)S(Z_1+Z_2)^{-1}S^T\zeta(t) - \int_{t-h_M}^{t-h(t)}\zeta^T(t)S(Z_1+Z_2)^{-1}S^T\zeta(t)\mathrm{d}s \geq 0$$

and

$$\rho\zeta^T(t)MZ_2^{-1}M^T\zeta(t) - \int_{t-h(t)}^{t-h_m}\zeta^T(t)MZ_2^{-1}M^T\zeta(t)\mathrm{d}s \geq 0$$

were employed in [21]; and $h(t)$, $h_M - h(t)$ and $h(t) - h_m$ were enlarged to h_M, ρ and ρ, respectively. It is easy to see that their treatment is more conservative than the expression in the proof of Theorem 1.

Remark 2. In some existing literature, for example [26], the term of $\int_{t-h_M}^{t}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ in the time derivative of $V(t)$ was often estimated as $\int_{t-h(t)}^{t}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ and the term $\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ was ignored. This treatment, as shown in Example 2, may lead to conservativeness. In the proof of Theorem 1, the term $\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s$ is retained.

Remark 3. The range of varying delay considered in many existing references is from 0 to an upper bound. For example, [18] investigated the stability problem for system (1) in the case of $h_m = 0$. When $h_m = 0$, i.e., the time-varying delay satisfies

$$0 \le h(t) \le h_M, \qquad \dot{h}(t) \le h_d, \tag{21}$$

the robust stability problem of system (1) is reduced to the problems discussed in [20] or [26]. Following the same lines as in the proof of Theorem 1, we can obtain the following result.

Theorem 2. *System (1) subject to (3)–(4) and (21) is asymptotically stable for given $h_M > 0$ and $h_d > 0$ if there exist scalars $\varepsilon_1 \ge 0$, $\varepsilon_2 \ge 0$ and matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z \ge 0$, $R > 0$, $M^T = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}$ and $N^T = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix}$ of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Phi + h_M Z & -N & \sqrt{h_M} S^T R \\ * & -Q_2 & 0 \\ * & * & -R \end{bmatrix} < 0, \qquad \begin{bmatrix} Z & M \\ * & R \end{bmatrix} \ge 0, \qquad \begin{bmatrix} Z & N \\ * & R \end{bmatrix} \ge 0, \tag{22}$$

where $S = \begin{bmatrix} A & A_h & I & I \end{bmatrix}$,

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P + M_3^T & P + M_4^T \\ * & \Phi_{22} & N_3^T - M_3^T & N_4^T - M_4^T \\ * & * & -\varepsilon_1 I & 0 \\ * & * & * & -\varepsilon_2 I \end{bmatrix}, \tag{23}$$

with

$$\Phi_{11} = PA + A^T P + M_1 + M_1^T + Q_1 + Q_2 + \varepsilon_1 \alpha^2 F^T F,$$
$$\Phi_{12} = PA_h + M_2^T + N_1 - M_1,$$
$$\Phi_{22} = -(1 - h_d)Q_1 + N_2 + N_2^T - M_2 - M_2^T + \varepsilon_2 \beta^2 G^T G.$$

Remark 4. If there is no perturbation, that is, $f(x(t), t) = 0$, $g(x(t - h(t)), t) = 0$, then the stability problem of system (1) is reduced to analyzing the stability of the system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t - h(t)) \\ x(t) = \phi(t), \quad \forall t \in [-h_M, 0]. \end{cases} \tag{24}$$

This problem has been widely studied in the recent literature (see, e.g., [21,8–14,17]). For system (24), we have the following conclusion, which can be obtained directly from Theorem 1.

Corollary 1. *System (24) subject to (2) is asymptotically stable for given $0 \le h_m \le h_M$ and h_d if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$, $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \ge 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} \ge 0$, $L^T = \begin{bmatrix} L_1^T & L_2^T \end{bmatrix}$, $M^T = \begin{bmatrix} M_1^T & M_2^T \end{bmatrix}$ and $N^T = \begin{bmatrix} N_1^T & N_2^T \end{bmatrix}$ of appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & M_1 - L_1 & -N_1 & \sqrt{h_m} A^T R_1 & \sqrt{\rho} A^T R_2 \\ * & \Psi_{22} & M_2 - L_2 & -N_2 & \sqrt{h_m} A_h^T R_1 & \sqrt{\rho} A_h^T R_2 \\ * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & -Q_3 & 0 & 0 \\ * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & -R_2 \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} X & L \\ * & R_1 \end{bmatrix} \ge 0, \qquad \begin{bmatrix} Y & M \\ * & R_2 \end{bmatrix} \ge 0, \qquad \begin{bmatrix} Y & N \\ * & R_2 \end{bmatrix} \ge 0, \tag{26}$$

where $\rho = h_M - h_m$,

$$\Psi_{11} = PA + A^T P + L_1 + L_1^T + Q_1 + Q_2 + Q_3 + h_m X_{11} + \rho Y_{11},$$
$$\Psi_{12} = PA_h + L_2^T + N_1 - M_1 + h_m X_{12} + \rho Y_{12},$$
$$\Psi_{22} = -(1 - h_d)Q_2 + N_2 + N_2^T - M_2 - M_2^T + h_m X_{22} + \rho Y_{22}.$$

When the information of h_d is unknown, for system (24) the following result can be obtained directly from Corollary 1 by setting $Q_2 = 0$.

Table 1
Allowable upper bound of h_M for different h_d, α and β.

α and β	$\alpha = 0,\ \beta = 0.1$			$\alpha = 0.1,\ \beta = 0.1$		
h_d	$h_d = 0.5$	$h_d = 0.9$	$h_d = 1.1$	$h_d = 0.5$	$h_d = 0.9$	$h_d = 1.1$
Theorem 2, [18]	0.546	0.279	–	0.495	0.255	–
Theorem 1, [20]	0.674	–	–	0.571	–	–
Theorem 1, [26]	1.142	0.738	0.735	1.009	0.714	0.714
Theorem 2	1.442	1.280	1.280	1.284	1.209	1.209

Corollary 2. *System (24) subject to (2) is asymptotically stable for given* $0 \le h_m \le h_M$ *and unknown* h_d *if there exist matrices* $P > 0, Q_1 > 0, Q_3 > 0, R_1 > 0, R_2 > 0, X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \ge 0, Y = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} \ge 0, L^T = \begin{bmatrix} L_1^T & L_2^T \end{bmatrix}, M^T = \begin{bmatrix} M_1^T & M_2^T \end{bmatrix}$ *and* $N^T = \begin{bmatrix} N_1^T & N_2^T \end{bmatrix}$ *of appropriate dimensions such that (26) and the following LMI holds:*

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & M_1 - L_1 & -N_1 & \sqrt{h_m} A^T R_1 & \sqrt{\rho} A^T R_2 \\ * & \Upsilon_{22} & M_2 - L_2 & -N_2 & \sqrt{h_m} A_h^T R_1 & \sqrt{\rho} A_h^T R_2 \\ * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & -Q_3 & 0 & 0 \\ * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & -R_2 \end{bmatrix} < 0, \tag{27}$$

where $\rho = h_M - h_m$,

$$\Upsilon_{11} = PA + A^T P + L_1 + L_1^T + Q_1 + Q_3 + h_m X_{11} + \rho Y_{11},$$
$$\Upsilon_{12} = PA_h + L_2^T + N_1 - M_1 + h_m X_{12} + \rho Y_{12},$$
$$\Upsilon_{22} = N_2 + N_2^T - M_2 - M_2^T + h_m X_{22} + \rho Y_{22}.$$

4. Numerical examples

In this section, we give two numerical examples in order to compare several existing criteria and those obtained in this paper. All the numerical results are calculated via the LMI toolbox of MATLAB.

Example 1 (*[20]*). Consider the system described by (1) with the following parameters:

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \qquad A_h = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \qquad F = G = I. \tag{28}$$

Assume that the nonlinear perturbations $f(x(t), t)$ and $g(x(t - h(t)), t)$ satisfy (3) and (4) respectively and the delay $h(t)$ satisfies (21). Now we calculate the allowable upper bound of h_M that guarantees the robust stability of system (1) under different α and β listed in Table 1. On the basis of the stability criteria given in [18,20,26] and Theorem 2 in this paper, computational results are obtained and these are shown in Table 1. From the table, it can be seen that Theorem 2 provides much less conservative results than others. Moreover, when $h_d \ge 1$, the stability criteria proposed in [18,20] cannot be applied to check the robust stability of system (1).

Example 2 (*[21]*). Consider the system (24) with

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_h = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{29}$$

Assume the time delay satisfies (2). For given h_m and h_d, we calculate the allowable upper bound of h_M that guarantees the asymptotical stability of system (24). Using different methods, computational results are obtained and these are listed in Table 2. It is seen that the stability criteria in [12] and Corollary 1 in this paper are not covered by each other, although they are less conservative than those in [21,10,11].

For unknown h_d, the comparison of allowable upper bounds of h_M is listed in Table 3. It is seen that the stability criteria in [9] and Corollary 2 are not covered by each other, although they are less conservative than those in [21,14]. Note that, for unknown h_d, the results in [15] are better than those for Corollary 2 in this example. However, the criteria in [15] are complex and involve more matrix variables than ours. Moreover, it seems that Corollary 2 can give less conservative results when the lower bound of the varying delay, i.e., h_m, is small.

5. Conclusion

We have studied the robust stability problem for a class of linear systems with interval time-varying delay and nonlinear perturbations. An appropriate Lyapunov–Krasovskii functional is proposed for deriving the delay-range-dependent stability

Table 2
Allowable upper bound of h_M for various h_m and h_d.

h_m	$h_m = 0$		$h_m = 1$		$h_m = 2$	
h_d	$h_d = 0.5$	$h_d = 0.9$	$h_d = 0.5$	$h_d = 0.9$	$h_d = 0.5$	$h_d = 0.9$
Theorem 1, [21]	2.04	1.37	2.07	1.74	2.43	2.43
Theorem 1, [10]	2.04	1.37	2.07	1.74	2.43	2.43
Theorem 1, [11]	2.04	1.37	2.07	1.74	2.43	2.43
Corollary 2, [24]	2.33	1.87	–	–	–	–
Theorem 1, [12]	2.08	1.66	2.15	2.12	2.71	2.71
Corollary 1	2.33	1.87	2.33	2.07	2.61	2.61

Table 3
Allowable upper bound of h_M for various h_m and unknown h_d.

h_m	0	0.5	1	1.5	2	3	4
Theorem 1, [21]	1.34	1.47	1.74	2.06	2.43	3.22	4.06
Proposition 2, [9]	1.34	1.51	1.80	2.14	2.52	3.33	4.18
Corollary 2, [14]	1.34	1.49	1.76	2.08	2.44	3.22	4.06
Theorem 1, [15]	1.98	2.05	2.16	2.37	2.64	–	–
Corollary 2	1.86	1.90	2.06	2.31	2.61	3.31	4.09

criteria. A new approach, different from the existing analysis methods, is introduced in estimating the upper bound on the difference of Lyapunov functions without ignoring any useful terms. Numerical examples have illustrated the effectiveness of the proposed method.

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