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## Route planning for a mixed delivery system in long distance transportation and comparison with pure delivery systems

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### ABSTRACT

The long distance routing problems are divided into three kinds of pure delivery systems where every order is allocated the same distribution strategy. Pure delivery systems have been generally, independently and widely studied. This research provides a solution to help pallet and package delivery companies in decision making, considering a mixed delivery system to improve the use of resources. It returns the route planning after allocating to each order the distribution strategy that best fits to the global scene and proves if a mixed delivery system achieves best results than a pure one and under which circumstances.

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### 1. Introduction

For [1], an efficient freight transportation industry is the key to facilitating the required movement of raw materials and finished goods. It involves maintaining the availability of intermediate materials and providing fast and reliable delivery of the final product, supports production, trade and consumption activities. Freight transportation is a major element of the economy and needs to adapt to the currently changing economic trends such as just-in-time production, Internet based

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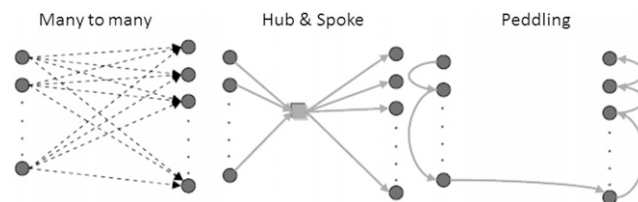


Fig. 1. Pure distribution strategies. Adapted from [2].

electronic businesses (e-commerce) and business-to-customer environments where distributors and retailers are being eliminated.

From an operational point of view, trucking services are classified as TL (Truckload) and LTL (Less-Than-Truckload). Truckload offers door-to-door transportation where a truck is assigned to each customer. In LTL, several customers are served simultaneously by using the same truck. The LTL trucking service can be characterized by the type of network, line operations network and load planning network.

Different types of distribution strategies apply, depending upon the features of the network and the trucking service. For [2] there are three types of pure distribution strategies in long distance transportation: many to many, hub&spoke and peddling as it is shown in Fig. 1.

Decision makers have to calculate the route planning where every order follows the same distribution strategy. In order to improve the use of resources and minimize the total distance a mixed delivery system is proposed where each order is allocated one of the previous adopted strategies in specific conditions and satisfying several constraints. Additionally, a methodology is developed to calculate the route planning, and finally, this research looks into the features that characterize the election of the distribution strategy.

## 2. Literature review

The many to many and peddling routes bear relation to the most common truck dispatching problem, that is known as Vehicle Routing Problem and it is an optimization problem that returns how to serve a set of customers geographically diffused around a central depot using a fleet of vehicles. It is one of the most widely studied topics in the field of operations research. For several authors, such as [3], the first research about the VRP was introduced by [4]. Since it appeared, several varieties of the original problem have arisen such as Capacitated Vehicle Routing Problem (CVRP), Multidepot Vehicle Routing Problem (MVRP) or Vehicle Routing Problem with Time Windows (VRPTW) and commonly they deal with local distribution. In the case of long distance carrier problems the literature about the Vehicle Routing Problem is not widely studied and it is more related with hub&spoke distribution strategies.

Fig. 2 shows several pure distributions solutions and how they have evolved to mixed delivery systems. In this figure, *empty* refers to minimization of unloaded trucks distance and *tight times* to a close schedule that allows little time.

The first time hub&spoke network was studied was in the late 1960s [5]; however, it was not until the late 1980s that this was considered to be an important aspect of research. The first formal studies were introduced by [6,7] and a large number of new possibilities have emerged. Usually the problem is settled as a pure distribution strategy, as the following emblematic studies [8–11] or [12] show.

There are few researches that introduce features to evolve a pure distribution into a mixed distribution strategy; for example [13–16] were the first to allow direct shipments; although somewhat uncommon, some authors allow stopovers to be made between the origin and the hub and between the hub and the destination. The first such research was [17], which proposed only a hub and allowed two stopovers as maximum. In [18], the number of stopovers was unlimited. Another feature of this research paper is that it included the waiting time. Other studies that include time as a constraint are [19,20].

Other perspective is provided by the Location Routing Problem which appeared in the late 1970s. This combines the (HLP) to solve the hub&spoke and the Classical Vehicle Routing Problem (VRP). The last way to interpret the problem is as a Multidepot Vehicle Routing Problem (MDVRP), where the nodes are allocated to the depots and then the routes are planned as in [21–24]. This paper is based on this perspective and more exactly in [25], which considers only a hub and allows direct shipments and stopovers in all the links. The problem was solved dividing the problem into three Capacitated Vehicle Routing Problems and previously the algorithm decided if an order was allocated to the hub or not. The author asserts that in this case a mixed delivery system is more efficient than a pure one.

## 3. Problem description

This research objective is to find the least distance to serve all the orders of the customers selecting the distribution strategy under different boundary conditions and constraints values. The main features are explained as follows: every order consists of two operations placed in different locations; one is the picking in the origin node and the other one is the delivery in the destination node. The distance between both of them is over 150 km and below 1500 km.

	Distribution Strategies	Features	Problem	Studies
PURE	Hub & Spoke	LTL	HLAP	[5]–[12]
	Many to many Direct Shipments	FTL Tight times	Empty	[30] – [32]
	Peddling	LTL	VRP	[3], [4]
MIXED	Hub&Spoke Direct Shipments	LTL, FTL	Modified HLAP	[13]–[16]
	Hub&Spoke Peddling	LTL	Modified HLAP	[17],[18]
			LRP	[22]
			MVRP	[21], [23], [24], [26]
	Direct Shipment Peddling	FTL LTL	VRP	[33] – [36]
	Hub&Spoke Direct Shipments Peddling	LTL, FTL	MVRP	[25]

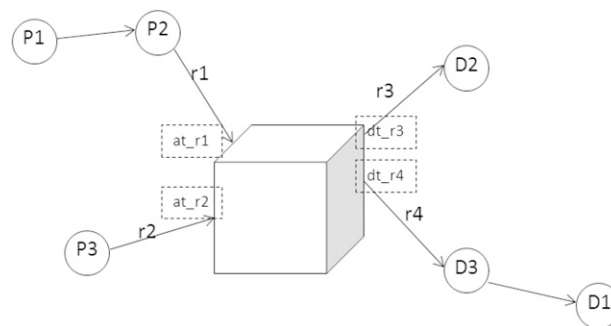
LTL, FTL: Less than truckload, full truckload

HLAP: Hub location allocation Problem

MVRP: Multidepot Vehicle Routing Problem

LRP: Location Routing Problem

**Fig. 2.** Pure distribution strategies evolution to mixed distribution strategies (see Refs. [3–18,21–26,30–36]).



**Fig. 3.** Arrival and departure times of routes to and from only a hub and relation to ensure feasibility.

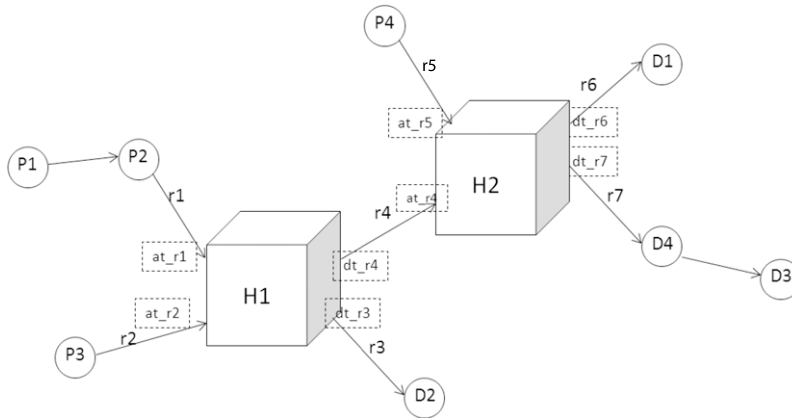
**Table 1**

Time parameters to ensure feasibility.

Id order	Arrival time hub $i$	Departure time hub $i$	Feasibility	
1(P1, D1)	$AT1H1 = at_{r1}$	$DTH11 = dt_{r4}$	$AT1H1 < DTH11$	$at_{r1} < \min(dt_{r4}, dt_{r3})$
2(P2, D2)	$AT2H1 = at_{r1}$	$DTH12 = dt_{r3}$	$AT2H1 < DTH12$	
3(P3, D3)	$AT3H1 = at_{r2}$	$DTH13 = dt_{r4}$	$AT3H1 < DTH13$	$at_{r2} < dt_{r4}$

The orders' nodes must be served inside the boundaries of their corresponding time windows, although the waiting time is allowed on certain defined occasions. The difference between the origin's time windows and destinations time window of an order is set in two ways, very tight or over 24 h. The difference between the low and high boundaries of every nodes time window is always the same.

There are two unlinked hubs used to manage freight. They do not take capacity restrictions and are open 24 h a day. For each order allocated to only a hub the first arrival moment to the hub and the last departure time from the hub is set to ensure that the service quality is satisfied. In the image of Fig. 3 there are four routes:  $r_1$  groups the pickup nodes of orders 1 and 2 to the hub and it arrives at the moment called  $at_{r1}$ ,  $r_2$  goes from the pickup node of order 3 and it arrives to the hub at time  $at_{r2}$ , on the other side, there are two routes that go from the hub to delivery nodes of the orders, route  $r_3$  must leave the hub in  $dt_{r3}$  to arrive to delivery node of order 2 on time and  $r_4$  in  $dt_{r4}$  to satisfy delivery time windows of orders 1 and 3. So for each order allocated to a hub two time parameters are introduced that are shown in Table 1. These parameters are the *Arrival time of the order  $O$  to hub  $i$*   $ATOH_i$  and *Departure time from hub  $i$  to the order  $O$*   $DTH_{iO}$ , and the value for each one is shown in Table 1. A solution is feasible if all conditions of the fourth column are satisfied.



**Fig. 4.** Arrival and departure times of routes to and from two hubs and relation to ensure feasibility.

**Table 2**

Time parameters to ensure feasibility with two hubs.

Id order	Arrival time hub $i$	Departure time hub $i$	Feasibility
1(P1, D1)	$AT1H1 = at_{r1}$	$DTH21 = dt_{r6}$	$AT1H1 < DTH21$ $at_{r1} < \min(dt_{r4}, dt_{r3})$ $dt_{r4} < at_{r4}$
2(P2, D2)	$AT2H1 = at_{r1}$	$DTH12 = dt_{r3}$	$AT2H1 < DTH12$ $at_{r4} < \min(dt_{r6}, dt_{r7})$ $at_{r1} < \min(dt_{r6}, dt_{r7}, dt_{r3})$
3(P3, D3)	$AT3H1 = at_{r2}$	$DTH23 = dt_{r7}$	$AT3H1 < DTH23$ $at_{r2} < dt_{r4}$ $dt_{r4} < at_{r4}$ $at_{r4} < \min(dt_{r6}, dt_{r7})$ $at_{r2} < \min(dt_{r6}, dt_{r7})$
4(P4, D4)	$AT4H2 = at_{r5}$	$DTH24 = dt_{r7}$	$AT4H2 < DTH24$ $at_{r6} < dt_{r7}$

If the order is allocated two hubs, the complexity to check the feasibility increases. In this case, four time parameters are introduced, *Arrival time of the order  $O$  to hub  $i$   $ATOH_i$  and to the hub  $j$   $ATOH_j$  and Departure time from hub  $i$ , to the order  $O$   $DTHiO$  and from hub  $j$  to the order  $O$   $DTHjO$* . In Table 2, these parameters are shown and their respective values for the example in Fig. 4. As in the previous case, a solution is feasible if all conditions in the fourth column of this table are achieved.

In order to take (LTL) service and the pallet as criteria to measure capacity, the size considered for the orders is relatively small, between 1 and 15 pallets, where vehicle pallet capacity is under 33. It has been taken as an average speed of 70 km/h where the duration of stopovers is included; this data was considered in [26].

## 4. Solution description

### 4.1. General solution

In Fig. 5 is shown the general scheme of the algorithm. The inputs are the set of orders, the features of the hubs and other required data. First, to each order is allocated a distribution strategy according to the selected alternative described in 4.2. Afterwards, an iterative process starts and ends when a number of iterations is reached or a feasible solution is found. In each iteration, a route planning is calculated and the algorithm proves the feasibility of the solution (the orders are served on time) and improve the previous ones modifying the strategy of some orders.

The objective is to design the delivery routes from a pickup node to a delivery node of an order after allocating the distribution for each one. Customer demands are deterministic and known. The time required to travel between customers including the hubs is deterministic. All the vehicles have the same capacity. The notations, objective function and decision variables are as follows:

Sets:

H: set of nodes that are hubs

O: set of all orders

R: set of nodes that are pickups

E: set of nodes that are deliveries

$\alpha$ : kind of edges

Parameters:

$c_{ij}$ : distance between point  $i$  and  $j$ ,  $i, j \in H \cup R \cup E$ .

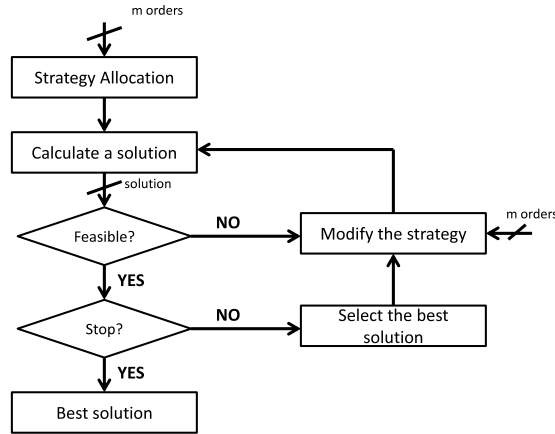


Fig. 5. Scheme of the algorithm.

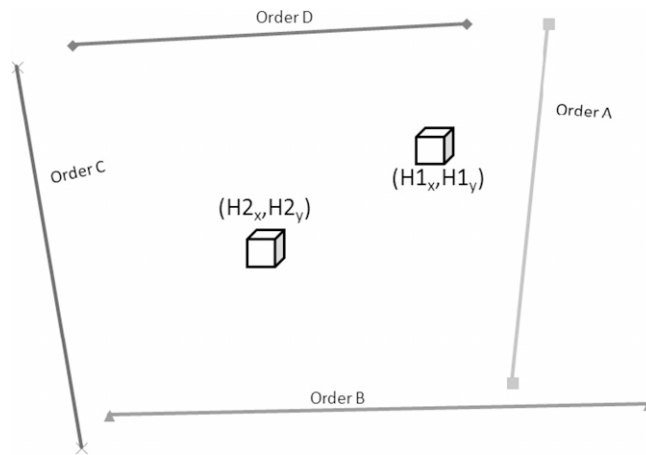


Fig. 6. Rule to allocate direct distribution strategy.

Decision variables:

$X_{ij}^\alpha$ : 1, if point  $i$  immediately precedes point  $j$  on edge of kind  $\alpha$ , ( $\alpha = 0, i, j \in R \cup E$ ;  $\alpha = 1, i, j \in R \cup H$ ;  $\alpha = 2, i, j \in H$ ;  $\alpha = 3, i, j \in H \cup E$ ; 0, otherwise)

The objective function of the problem is to minimize the total travel distance according to the expression (1).

$$\min \left( \sum_{i=1}^H \sum_{j=1}^H X_{ij}^2 \times c_{ij} + \sum_{i=1}^R \sum_{j=1}^E X_{ij}^0 \times c_{ij} + \sum_{i=1}^R \sum_{j=1}^{R \cup H} X_{ij}^1 \times c_{ij} + \sum_{i=1}^{E \cup H} \sum_{j=1}^E X_{ij}^3 \times c_{ij} \right). \quad (1)$$

#### 4.2. Strategy allocation

The algorithm starts allocating the distribution strategy to each order. In order to compare pure distribution strategies with mixed delivery systems, this phase considers four alternatives to assign the distribution strategy. In the first one, no order is allocated to any hub; in the second one, every order is allocated at least to one hub; in the third one, every order is allocated at least to one hub but in this case, the assignment can be modified in the iterative process allowing to appear direct shipments; the last alternative decides the assignment of one, two or no hubs to each order according to a rule described in Fig. 6. This rule allocates the direct strategy to an order if it satisfies the following restrictions: the locations of the pickup node and the delivery node follow one of the four orders' schemes in Fig. 6, (A, B, C, D); the time difference between the pickup and the delivery node of an order is under 24 h and the distance between these nodes is not greater than a specific value; otherwise, it allocates at least one hub.

### 4.3. Route planning

After allocating the distribution strategy to each order, the algorithm creates new sets of orders,  $S_i$ , where  $i \in \{0, 6\}$  and they are established as follows:

- $S_0$ : Contains orders with direct shipment distribution strategy assigned, so the pickup and the delivery node are the same as the original orders' nodes.
- $S_1$ : This set contains the orders that are allocated to the nearest hub to the origin of the order, for example the hub called H1 in Fig. 6. These orders are constituted by the original pickup node of the order and as destination the hub H1.
- $S_2$ : This set contains the orders that are allocated to the nearest hub to the destination of the order, for example the hub called H1 in Fig. 6. These orders are constituted by the original destination node of the order and H1 as the pickup node.
- $S_3$ : Is similar to  $S_1$ , but in this case it considers the hub H2 as the destination of the order.
- $S_4$ : Is similar to  $S_2$ , but in this case it considers the hub H2 as the origin of the order. For example, the hub called H1 in Fig. 6. The contained orders are constituted by the original pickup node of the order and as destination the hub H1.
- $S_5$ : This set contains the orders that are allocated two hubs. The origin of the order is H1 and the destination of the order is H2, being the nearest to the original pickup and delivery nodes of the order and are not the same.
- $S_6$ : Similar to  $S_5$ , the differences are that the pickup node is H2 and the delivery node is H1.

Besides, while these sets are being constituted the time windows of the new nodes and the previous time parameters ( $ATOHi$ ,  $DTHiO$ ) must be calculated to ensure the feasibility of the solution. When only a hub is allocated, the lower boundary of the time window is the arrival time from the origin and the upper boundary of the time window is the departure time to arrive to the destination on time.

When the previous steps have concluded, the algorithm is ready to start the calculus. The route planning executes 7 Vehicle Routing Problem – VRP that are based on ant colony optimization metaheuristic. Each one of the VRPs belongs to each one of the previous sets. They are executed in an independent way, so they can be parallelized in order to reduce the process time. When all of them have finished, the algorithm checks the feasibility.

The VRPs have some particular features (how to consider the destination or the origin if it is a hub or a customer node, the calculus of the parameter times, etc.): however, they have several common characteristics, so the following description is applicable to the previous sets. They contain two algorithms based on *Ant Colony Optimization* metaheuristic, called ACO-I and ACO-II. This metaheuristic belongs to the population new metaheuristic algorithms to solve hard combinatory optimization problems which are demonstrated by ants' behavior. ACO algorithms are stochastic constructive procedures. The problem constraints are built into the ants constructive heuristic. In most application ants build feasible solutions; however, as in this case it is necessary to let them return infeasible solutions. This technique uses two types of information to construct the next solutions. One is the *pheromone trail*,  $\tau$ , ( $\tau_i$  if associated with nodes,  $\tau_{ij}$  if associated with links), which encodes a long-term memory about the entire ant search process and is updated by the ants themselves. The second data is the *heuristic information*,  $\eta$ , ( $\eta_i$  if associated with nodes,  $\eta_{ij}$  if associated with links). It represents a priori information or run time information. These values are used by the ants heuristic rule to make probabilistic decisions on how to make the next choice.

Every set is made up of two ACOS, ACO-I is used to choose the first order to be introduced in a newly created route, heuristic information prioritizes the orders with longest distance between the pickup and the delivery node. The pseudo-random-proportional rule is shown in the expression (2) and it is used to build the solution.

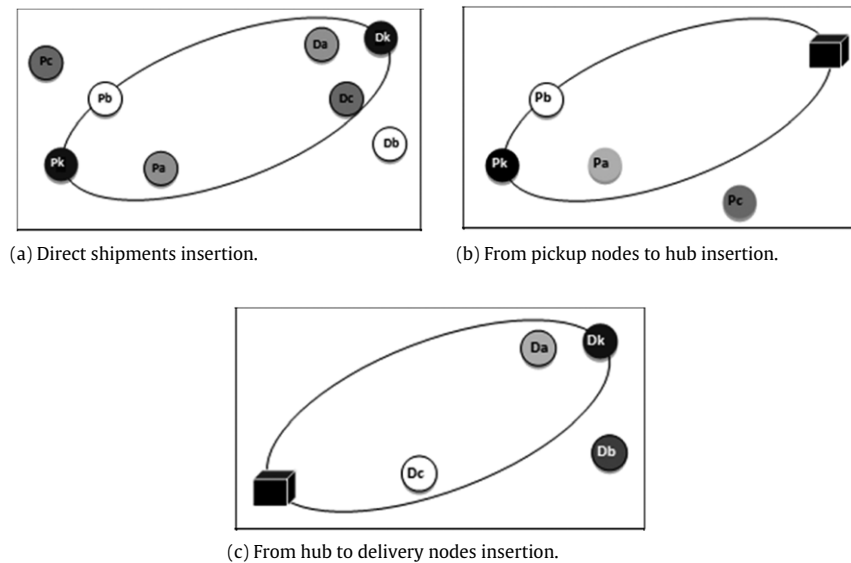
$$\text{if } (q \leq q_0) P_o^r = \begin{cases} 1, & \text{if } o = \underset{o \in \text{rest\_orders}(r)}{\text{argmax}} ((\tau\_first_o^r)^{\alpha_{first}} \cdot (\eta\_first_o^r)^{\beta_{first}}) \\ 0, & \text{otherwise} \end{cases}$$

$$\text{else, } P_o^r = \begin{cases} \frac{((\tau\_first_o^r)^{\alpha_{first}} \cdot (\eta\_first_o^r)^{\beta_{first}})}{\sum_{o \in \text{rest\_orders}(r)} ((\tau\_first_o^r)^{\alpha_{first}} \cdot (\eta\_first_o^r)^{\beta_{first}})}, & \text{if } o \in \text{rest\_orders}(r) \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

When a route is created, the ACO-II starts. It is an iterative process that selects the rest orders to introduce in a route until it is not possible or the stop condition is reached. This ACO-II uses the pseudo random probabilistic rule in expression (3). In this case the heuristic information is the available capacity after inserting an order in the route.

$$\text{if } (q \leq q_0) P_{ok}^r = \begin{cases} 1, & \text{if } k = \underset{k \in \text{rest\_orders}(r)}{\text{argmax}} ((\tau\_rest_{ok}^r)^{\alpha_{rest}} \cdot (\eta\_rest_{ok}^r)^{\beta_{rest}}) \\ 0, & \text{otherwise} \end{cases}$$

$$\text{else, } P_{ok}^r = \begin{cases} \frac{((\tau\_rest_{ok}^r)^{\alpha_{rest}} \cdot (\eta\_rest_{ok}^r)^{\beta_{rest}})}{\sum_{k \in \text{rest\_orders}(r)} ((\tau\_rest_{ok}^r)^{\alpha_{rest}} \cdot (\eta\_rest_{ok}^r)^{\beta_{rest}})}, & \text{if } k \in \text{rest\_orders}(r) \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$



**Fig. 7.** Rule to allocate direct distribution strategy.

**Table 3**

Time parameters to ensure feasibility with two hubs.

Geographical dispersion. Number of zones	Difference time windows	Waiting time	Strategy	Number of orders
3	Broad	Allowed	Direct	50
21	Tight	Not allowed	H&S Modified	100
	Mixed		Rule	
			Hub&spoke	

In both ACOs the initial pheromone value is the same  $\tau_0 = \frac{1}{n \cdot \text{Obj}}$  where  $n$  is the total number of orders and  $\text{Obj}$  is the value of the first solution. In order to be faster, a matrix of practicable group of orders is introduced in a route where they must satisfy the following criteria: the addition of the load must be under the capacity of the vehicle, the time windows of every order must be satisfied and finally, the nodes of the new order must be inside the ellipse where the extremes nodes of the major axis are the nodes of the first order introduced in a route as is shown in Fig. 7.

After the solution is calculated and if it is feasible, the pheromone levels of each ACO are updated following the next expression,  $\tau = (1 - \varphi) \cdot \tau + \varphi \cdot \tau_0$ ,  $0 \leq \varphi \leq 1$ . When all the ants of an iteration have been calculated, the global pheromone value is updated, as it is shown in the next expression  $\tau = (1 - \rho) \cdot \tau + \rho \cdot (\frac{1}{\text{best\_Obj}})$ ,  $0 \leq \rho \leq 1$  and it is checked the feasibility of the solution. Those orders that do not satisfy any parameter restriction or have not been grouped, modify the distribution strategy and the algorithm starts again generating the sets of orders.

## 5. Computational experiments

### 5.1. Experimental data

The tests were developed under different assumptions to prove how different parameters influence on results. These parameters are: the geographical dispersion, the difference time between the pickup and the delivery node and if waiting time is allowed. Besides, it was considered to prove each distribution strategy and different sizes of the problem (see Table 3).

For the tests was taken the Iberian Peninsula, without considering Portugal. The number of zones for the geographical dispersion is 3 and 21. These amounts were considered because the first one is the least value to introduce a few of dispersion (with only two zones all orders would be direct shipments) and the second one considering one district code per zone and taking into account several districts codes when the zone is peripheral. For the difference time between the pickup and the delivery node of an order, was considered to be broad, tight or a mixed one.

The locations of the hubs are Alcobendas in Madrid and Plaza in Zaragoza because these are two crucial logistic points in Spain. The distance is calculated using the haversine formula which provides the shortest distance over the Earth's surface (see [27–30]).

The instances are randomly generated. Time window in the pickup node starts and ends always on the same moment, however in the delivery node it depends on the case: if it is broad, it starts and ends always on the same moment too; if it is tight, it depends on the distance; if it is mixed, some of them start and end on the same moment and others depend on the distance.



N° Orders	Time Window	Waiting	3 Zones					21 Zones				
			D-M	D-H	HS-M	HS-H	M-H	D-M	D-H	HS-M	HS-H	M-H
5	Broad	Yes	1.29	1.29	1.04	1.04	1.00	0.98	1.01	1.08	1.12	1.03
	Broad	NO	1.46	1.47	1.03	1.04	1.00	1.07	1.12	1.08	1.12	1.04
	Tight	Yes	0.95	0.95	1.29	1.30	1.00	1.00	1.00	1.34	1.34	1.00
	Tight	NO	0.95	0.95	1.44	1.44	1.00	1.00	1.00	1.34	1.34	1.00
	Mixed	Yes	0.85	0.85	1.17	1.17	1.00	0.99	0.92	1.24	1.15	0.92
	Mixed	NO	1.04	1.04	1.14	1.14	1.00	1.06	1.06	1.24	1.24	1.00
25	Broad	Yes	0.93	0.93	1.00	0.99	0.99	0.95	0.98	1.03	1.08	1.04
	Broad	NO	1.76	1.77	1.01	1.01	1.01	1.16	1.21	1.02	1.06	1.04
	Tight	Yes	0.91	0.92	1.28	1.29	1.01	1.01	1.01	1.24	1.24	1.00
	Tight	NO	1.02	1.00	1.77	1.73	0.98	1.00	1.00	1.28	1.28	1.00
	Mixed	Yes	0.88	0.87	1.16	1.15	0.99	0.98	1.02	1.18	1.22	1.04
	Mixed	NO	1.25	1.27	1.43	1.45	1.01	1.02	1.03	1.24	1.25	1.01
50	Broad	Yes	0.87	0.84	0.97	0.94	0.97	0.95	0.97	0.97	0.99	1.02
	Broad	NO	1.78	1.80	0.97	0.99	1.01	1.39	1.39	1.00	1.00	1.00
	Tight	Yes	0.93	0.91	1.35	1.32	0.98	1.00	1.02	1.26	1.28	1.02
	Tight	NO	1.04	1.03	1.91	1.89	0.99	1.01	1.01	1.36	1.35	0.99
	Mixed	Yes	0.81	0.79	1.20	1.16	0.97	0.96	1.02	1.17	1.24	1.06
	Mixed	NO	1.26	1.27	1.32	1.33	1.01	1.03	1.05	1.22	1.25	1.02
100	Broad	Yes	0.96	0.97	0.99	1.00	1.01	0.97	0.97	0.98	0.99	1.01
	Broad	NO	2.00	1.96	0.98	0.96	0.98	1.41	1.38	1.06	1.04	0.98
	Tight	Yes	0.87	0.89	1.34	1.36	1.02	1.02	1.04	1.17	1.19	1.02
	Tight	NO	0.92	0.91	2.03	2.01	0.99	1.01	1.02	1.31	1.32	1.01
	Mixed	Yes	0.81	0.81	1.22	1.22	1.00	0.90	0.91	1.14	1.16	1.01
	Mixed	NO	1.22	1.31	1.54	1.65	1.07	1.12	1.11	1.27	1.26	0.99
250	Broad	Yes	0.89	0.90	0.99	1.00	1.01	1.03	1.05	1.09	1.11	1.01
	Broad	NO	1.90	1.89	0.93	0.92	0.99	1.53	1.53	1.15	1.15	1.00
	Tight	Yes	0.89	0.88	1.32	1.32	1.00	1.01	1.02	1.28	1.30	1.02
	Tight	NO	0.94	0.95	2.13	2.18	1.02	1.01	1.02	1.45	1.46	1.01
	Mixed	Yes	0.75	0.77	1.14	1.18	1.03	0.82	0.88	1.19	1.28	1.08
	Mixed	NO	1.37	1.49	1.45	1.59	1.09	1.09	1.09	1.31	1.31	1.00

Ratio < 0,80

0,80 ≤ Ratio < 1,00

1,00 ≤ Ratio < 1,20

1,20 ≤ Ratio < 2,00

2,00 ≤ Ratio

Fig. 8. Relationship between pure and mixed delivery systems.

The number of pallets for each order is randomly generated from 1 to 15.

## 5.2. Results and discussions

The algorithm described was coded in Java and run on a PC Intel Core i3 at 2.3 GHz with 4 GB of RAM with Windows 7. It was necessary to estimate the value of the set of parameters that influences the three ACOs implemented in the algorithm. The parameter values for the three ACOs  $\alpha = 1$ ,  $\beta = 2$  and  $\phi = \rho = 0 : 1$ .

The main results of the experiments are shown in the table of Fig. 8. In order to compare the cost of the pure and mixed delivery systems is introduced a ratio. This relationship is shown as the cost of pure delivery system divided by the cost of mixed delivery systems. There are two parts, one for a dispersion of 3 zones and the other one for a dispersion of 21 zones. In both of them the ratios considered are the following: 'D-M', 'D-H' relate the pure distribution 'Direct Shipment' represented with the letter 'D'; 'M' and 'H' symbolize the mixed distribution strategy 'Hub & Spoke Modified' and 'Heuristic' consecutively. The following ratios are 'HS-M' and 'HS-H' that are similar to the previous ones, but in this case the pure distribution strategy is 'Hub&Spoke' which appears with the expression 'HS'. Finally, the last ratio 'M-H' compares the two mixed distribution strategies to evaluate which returns best solutions.

The table has been categorized in five intervals depending on the value of the ratio. When the value is under 0.8 means pure strategies are better than mixed delivery systems and it takes a red color. For values between 0.8 and the 1.20 is considered that the results are very similar. If it is under 1 the pure distribution strategies are better than mixed and takes a light red color and if it is over 1 is the contrary and it takes a light green color. When the value is between 1.20 and 2.00 it takes a dark green and when it is over 2.00 takes a light blue and these values mean mixed distribution strategies are better than a pure one. The graph of Fig. 9 explains the meaning of these results. Most results are grouped over a value a little greater than 1, so it is a slightly right skewed distribution and it means that in general mixed distribution returns best results.

## 6. Conclusions

In this paper is solved a less than truckload problem where the distribution strategy for every order is selected and the route planning is calculated minimizing the total distance under different assumptions. In order to compare when a mixed distribution strategy is better than a pure one, several tests were carried out. The results of this research show that if the dispersion of the nodes is reduced, direct shipment under specific time constraints is frequently better than a mixed one, because the percentage of coincidences is large. In the case of hub&spoke pure, it becomes better when the time constraints are more relaxed because the possibility of grouping orders increases. Besides, the results make clear that if the difference time is very tight the direct shipment is usually the best solution and if the dispersion of the nodes is large the best solution are the mixed strategies. When it is considered a mixed distribution strategy, the percentage of direct shipments is greater if the difference time between the windows is tight while it is very reduced if the difference time is wide.



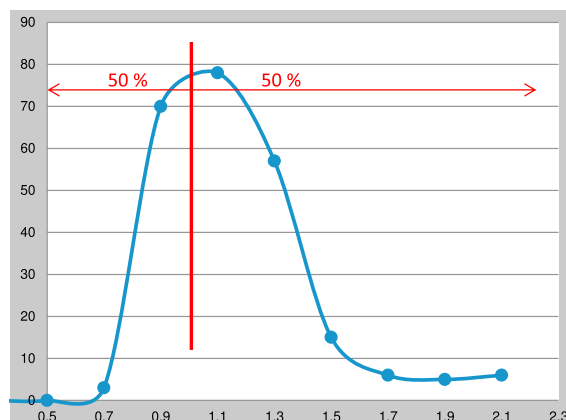


Fig. 9. Results distribution.

On the other hand, if the rule and the hub&spoke modified alternatives are compared, it shows that the costs of both solutions are very similar but the rule converges before to the solution.

Future researches could include how to locate the hubs or select them between a set of possibilities. Other alternatives are to use other metaheuristics or a combination of metaheuristics techniques for each one of the phases, trying to routing first and allocating second. The amount of varieties of the problem can be extensively studied yet.

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