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Multi-harmonic modelling of motional magnetic field problems using a hybrid finite element–boundary element discretisation☆

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Abstract

This paper deals with the numerical steady-state analysis of motional magnetic field problems in the frequency domain. An original method for taking into account an arbitrary periodic movement in a two-dimensional or three-dimensional hybrid finite element–boundary element model is proposed. It is elaborated in detail for a general two-dimensional eddy current problem and validated by means of a simple test case. The latter concerns a conducting pendulum that swings back and forth in the magnetic field of a permanent magnet. The time-periodic problem is solved by means of the proposed multi-harmonic method, using both a hybrid model and a finite element model. The obtained waveforms are shown to converge to each other and to those obtained with a time-domain approach.

Keywords: Harmonic balance; Finite element method; Boundary element method; Hybrid method; Magnetic fields; Eddy currents; Motion

1. Introduction

Steady-state finite element (FE) analyses of dynamic electromagnetic field problems can be carried out either in the frequency domain or in the time domain. The former approach, adopting the harmonic balance finite element (HBFE) method [9], is not widely followed for nonlinear motional

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problems as the nonlinearity and the movement are difficult to account for in an efficient and general way. Furthermore, it leads to a very large system of algebraic equations. As a consequence, the time-domain approach (also referred to as time-stepping) is usually preferred, in spite of the large number of time steps required to run first through the possibly long transient and then through a fundamental period of the system at quasi-steady-state.

In two recent papers by the authors, the nonlinear and motional aspects in the HBFE method have been addressed in a novel way. In [5] an easy-to-implement method for assembling and solving the governing harmonic balance (HB) system of nonlinear algebraic equations considering an arbitrary magnetic constitutive law has been proposed. In [6] the authors have presented an original method to account for rotational movement in the two-dimensional HBFE modelling of rotating machines.

In this paper we show that the latter method can be easily generalised to an arbitrary movement if a hybrid finite element–boundary element (FEBE) model is used. The FE discretisation is adopted in all saturable and current carrying (non-deforming) parts in the model, while the boundary element (BE) method is used for considering the deformable regions.

The application of such hybrid models is well established for problems with a fixed geometry (for static, harmonic and time-stepping simulation, e.g. [4,8]) and for motional problems (with time-stepping, e.g. [7]). However, to the best of our knowledge, a general multi-harmonic approach for a hybrid model with movement has not yet been presented.

The proposed method is elaborated and validated considering a two-dimensional linear eddy current problem. Its extension to nonlinear problems and to three-dimensional problems (with various formulations) is straightforward.

2. Two-dimensional time-periodic eddy current problem without movement

We consider a time-periodic linear eddy current problem in $\mathbb{R}^2$. In the domain Ω_s, the time-periodic current density $\underline{j} = j_s(x, y, t)\,\underline{1}_z$ is given, whereas in the conducting domain Ω_c eddy currents may develop. The rest of $\mathbb{R}^2$ is taken by free space. The domains Ω_s and Ω_c are stationary in space for now. The extension to an eddy current problem with movement follows in Section 3.

The governing differential equations and constitutive laws are

$$\operatorname{curl}\underline{h} = \underline{j} \quad \operatorname{div}\underline{b} = 0 \quad \text{and} \quad \underline{h} = v\underline{b} \text{ in } \mathbb{R}^2, \tag{1}$$

$$\operatorname{curl}\underline{e} = -\partial_t\,\underline{b} \quad \text{and} \quad \underline{j} = \sigma\,\underline{e} \text{ in } \Omega_c, \tag{2}$$

where the z-components of the magnetic field $\underline{h}(x, y, t)$ and the magnetic induction $\underline{b}(x, y, t)$ vanish, as well as the x- and y-components of the current density $\underline{j}(x, y, t)$ and the electrical field $\underline{e}(x, y, t)$. The magnetic reluctivity v and the electrical conductivity σ are constant scalars.

The eddy current problem is formulated in terms of, e.g., the magnetic vector potential $\underline{a} = a(x, y, t)\underline{1}_z$. For any continuous potential, (1) and (2) are satisfied on account of

$$\underline{b} = \operatorname{curl}\underline{a} = \underline{1}_z \times \operatorname{grad} a \quad \text{and} \quad \underline{e} = -\partial_t\,\underline{a} - \operatorname{grad}\phi, \tag{3}$$

where ϕ is the electric scalar potential. If no external voltage is applied to the domain Ω_c, the term $-\operatorname{grad}\phi$ can be omitted.

Finally, Ampère's law (1) written in terms of $a(x,y,t)$ is to be resolved:

$$\operatorname{div}(\nu \operatorname{grad} a) - \sigma\partial_t a + j_\mathrm{s} = 0. \tag{4}$$

In order to do so numerically, we consider either a FE or a hybrid FEBE space discretisation, which is to be combined with either a time-stepping or a multi-harmonic approach.

2.1. Finite element model

We consider a domain Ω in $\mathbb{R}^2$ that comprises the domains Ω_s and Ω_c. On its boundary Γ, sufficiently distant from Ω_s and Ω_c, some conditions are explicitly or implicitly imposed.

The weak form of (4) reads

$$\int_\Omega \nu \operatorname{grad} a \operatorname{grad} a' \,\mathrm{d}\Omega + \int_{\Omega_\mathrm{c}} \sigma\partial_t a a' \,\mathrm{d}\Omega = \int_{\Omega_\mathrm{s}} j_\mathrm{s} a' \,\mathrm{d}\Omega + \oint_\Gamma \nu \frac{\partial a}{\partial n} a' \,\mathrm{d}\Gamma, \tag{5}$$

where the tangential magnetic field on Γ, $h_t = \nu\,\partial a/\partial n = \nu \underline{n} \operatorname{grad} a$ (with $\underline{n}$ the inward unit normal on Γ) is given, and the test function a' is continuous in Ω.

A time-invariant space discretisation of Ω, consisting of e.g. first-order triangular finite elements, leads to the definition of $\#a$ basis functions $\alpha_j(x,y)$ for the vector potential $a(x,y,t)$:

$$a(x,y,t) = \sum_{j=1}^{\#a} a_j(t)\,\alpha_j(x,y)\,. \tag{6}$$

By employing the $\#a$ basis functions $\alpha_i(x,y)$ as test functions as well, (5) leads to a system of $\#a$ algebraic and first-order ordinary differential equations:

$$\mathbf{SA} + \mathbf{L}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A} = \mathbf{J}(t) + \mathbf{J}_\Gamma \quad \text{with} \quad \mathbf{A}(t) = [a_1(t)\cdots a_{\#a}(t)]^\mathrm{T}. \tag{7}$$

$\mathbf{S}$ and $\mathbf{L}$ are time-invariant $\#a \times \#a$ matrices, and the $\#a \times 1$ column matrices $\mathbf{J}(t)$ and $\mathbf{J}_\Gamma$ follow from the imposed current density in Ω_s and the tangential field on Γ respectively. Their components are given by

$$S_{i,j} = \int_\Omega \nu \operatorname{grad}\alpha_i \operatorname{grad}\alpha_j \,\mathrm{d}\Omega, \quad L_{i,j} = \int_{\Omega_\mathrm{c}} \sigma\alpha_i\alpha_j \,\mathrm{d}\Omega, \tag{8}$$

$$J_i = \int_{\Omega_\mathrm{s}} j_\mathrm{s}\alpha_i \,\mathrm{d}\Omega \quad \text{and} \quad J_{\Gamma i} = \oint_\Gamma \nu \frac{\partial a}{\partial n}\alpha_i \,\mathrm{d}\Gamma. \tag{9}$$

2.2. Hybrid FEBE method

In the free space (of reluctivity ν_0) exterior to Ω, the potential a can be calculated from an equivalent current density q on Γ [4,8]:

$$a = \frac{1}{\nu_0}\oint_\Gamma q\,G \,\mathrm{d}\Gamma \quad \text{with} \quad G = \frac{1}{2\pi}\ln(1/r), \tag{10}$$

where $G(r)$ is the Green function for the 2D Laplace equation, r being the distance between a source point (on Γ) and an observation point (in $\mathbb{R}^2 \setminus \Omega$).

The tangential magnetic field on the boundary Γ is given by

$$\nu \frac{\partial a}{\partial n} = \frac{1}{2} q + \oint_\Gamma q \frac{\partial G}{\partial n} \, \mathrm{d}\Gamma. \tag{11}$$

On the basis of a time-invariant spatial discretisation of Γ, consisting of, e.g. line segments, the equivalent current density $q(\xi, t)$ is written in terms of $\#q$ basis functions $\beta_l(\xi)$:

$$q(\xi,t) = \sum_{l=1}^{\#q} q_l(t)\, \beta_l(\xi) \quad \text{and} \quad \mathbf{Q}(t) = [q_1(t) \cdots q_{\#q}(t)]^{\mathrm{T}}, \tag{12}$$

the time-dependent coefficients $q_l(t)$ of which are assembled in the column matrix $\mathbf{Q}(t)$.

By using (11) in the FE equations (7) and weighing (10) with the $\#q$ basis functions $\beta_k(\xi)$, the system of algebraic and differential equations of the hybrid model is obtained:

$$\begin{bmatrix} \mathbf{S} & \mathbf{C} \\ \mathbf{D}^{\mathrm{T}} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{Q} \end{bmatrix} + \begin{bmatrix} \mathbf{L} & 0 \\ 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathbf{A} \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ 0 \end{bmatrix}. \tag{13}$$

The time-invariant $\#a \times \#q$ matrices $\mathbf{C}$ and $\mathbf{D}$, and the time-invariant $\#q \times \#q$ matrix $\mathbf{M}$ are given by

$$C_{i,l} = \oint_\Gamma \alpha_i \left(\frac{1}{2} \beta_l + \oint_\Gamma \beta_l \frac{\partial G}{\partial n} \, \mathrm{d}\Gamma \right) \mathrm{d}\Gamma, \quad D_{j,k} = \oint_\Gamma \alpha_j \beta_k \, \mathrm{d}\Gamma, \tag{14}$$

and

$$M_{k,l} = \frac{1}{\nu_0} \oint_\Gamma \beta_k \left(\oint_\Gamma \beta_l G \, \mathrm{d}\Gamma \right) \mathrm{d}\Gamma. \tag{15}$$

2.3. HB method

The potential $a(x,y,t)$ and the current densities $j(x,y,t)$ and $q(\xi,t)$ vary periodically in time, with fundamental frequency f and period $T = 1/f$. The multi-harmonic time discretisation consists in approximating them by a truncated Fourier series comprising a dc-term and n_f frequencies f_m $(1 \leqslant m \leqslant n_f)$, the latter being nonzero multiples of f. The corresponding $2n_f + 1$ orthonormal basis functions are

$$H_0(t) = 1, \; H_{2m-1}(t) = \sqrt{2}\cos(2\pi f_m t), \; H_{2m}(t) = -\sqrt{2}\sin(2\pi f_m t), \tag{16}$$

with

$$\frac{1}{T} \int_0^{\mathrm{T}} H_\kappa(t) H_\lambda(t) \, \mathrm{d}t = \delta_{\kappa\lambda} \quad (0 \leqslant \kappa, \lambda \leqslant 2n_f). \tag{17}$$

The complete discretisation of $a(x,y,t)$ and $q(\xi,t)$ may thus be written as

$$\mathbf{A}(t) = \sum_{\lambda=0}^{2n_f} \mathbf{A}^{(\lambda)} H_\lambda(t) \quad \text{and} \quad \mathbf{Q}(t) = \sum_{\lambda=0}^{2n_f} \mathbf{Q}^{(\lambda)} H_\lambda(t), \tag{18}$$

where the superscript (λ) denotes the λth harmonic component.

By weighing (13) over the fundamental period $[0,T]$ with the $2n_f+1$ basis function $H_\kappa(t)$, i.e.,

$$\frac{1}{T}\int_0^{\mathrm{T}} H_\kappa(t)\left(\begin{bmatrix}\mathbf{S} & \mathbf{C}\\ \mathbf{D}^{\mathrm{T}} & \mathbf{M}\end{bmatrix}\begin{bmatrix}\mathbf{A}\\ \mathbf{Q}\end{bmatrix}+\begin{bmatrix}\mathbf{L} & 0\\ 0 & 0\end{bmatrix}\frac{\mathrm{d}}{\mathrm{d}\,t}\begin{bmatrix}\mathbf{A}\\ \mathbf{Q}\end{bmatrix}-\begin{bmatrix}\mathbf{J}\\ 0\end{bmatrix}\right)\mathrm{d}t=0, \tag{19}$$

a system of $(2n_f+1)(\#a+\#q)$ algebraic equations is obtained:

$$\begin{bmatrix}\mathbf{S}_{\mathrm{H}}+\mathbf{L}_{\mathrm{H}} & \mathbf{C}_{\mathrm{H}}\\ \mathbf{D}_{\mathrm{H}}^{\mathrm{T}} & \mathbf{M}_{\mathrm{H}}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{\mathrm{H}}\\ \mathbf{Q}_{\mathrm{H}}\end{bmatrix}=\begin{bmatrix}\mathbf{J}_{H}\\ 0\end{bmatrix}, \tag{20}$$

with

$$\text{with } \mathbf{A}_{\mathrm{H}}=[\mathbf{A}^{(0)^{\mathrm{T}}}\cdots\mathbf{A}^{(2n_f)^{\mathrm{T}}}]^{\mathrm{T}} \quad \text{and} \quad \mathbf{Q}_{\mathrm{H}}=[\mathbf{Q}^{(0)^{\mathrm{T}}}\cdots\mathbf{Q}^{(2n_f)^{\mathrm{T}}}]^{\mathrm{T}}. \tag{21}$$

The column matrix $\mathbf{J}_{\mathrm{H}}$ contains the harmonic components of the imposed current excitation:

$$\mathbf{J}_{\mathrm{H}}=[\mathbf{J}^{(0)^{\mathrm{T}}}\cdots\mathbf{J}^{(2n_f)^{\mathrm{T}}}]^{\mathrm{T}} \quad \text{with} \quad \mathbf{J}^{(\kappa)}=\frac{1}{T}\int_0^{\mathrm{T}} H_\kappa(t)\,\mathbf{J}(t)\,\mathrm{d}t. \tag{22}$$

Thanks to the orthonormality of the time-harmonic basis functions, see (17), the matrices $\mathbf{S}_{\mathrm{H}}$, $\mathbf{C}_{\mathrm{H}}$, $\mathbf{D}_{\mathrm{H}}$ and $\mathbf{M}_{\mathrm{H}}$ have a block diagonal structure, the $2n_f+1$ blocks on their diagonal being $\mathbf{S}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{M}$, respectively. The particular block structure of $\mathbf{L}_{\mathrm{H}}$ is given by

$$\mathbf{L}^{(\kappa,\lambda)}=\begin{cases}2\pi f_m\mathbf{L} & \text{if } \kappa=2m \text{ and } \lambda=2m-1, \quad 1\leqslant m\leqslant n_f\\ -2\pi f_m\mathbf{L} & \text{if } \kappa=2m-1 \text{ and } \lambda=2m, \quad 0\leqslant\kappa,\lambda\leqslant 2n_f\\ 0 & \text{in all other cases,}\end{cases} \tag{23}$$

The different frequencies are not coupled in (20). The two components of each nonzero frequency are coupled due to the presence of induced currents ($\mathbf{L}_{\mathrm{H}}$).

In the general *nonlinear* case, considering an arbitrary magnetic constitutive law in Ω, the block structure of $\mathbf{S}_{\mathrm{H}}$ is completely filled, and all frequency components in (20) are coupled. The matrices $\mathbf{L}_{\mathrm{H}}$, $\mathbf{C}_{\mathrm{H}}$, $\mathbf{D}_{\mathrm{H}}$ and $\mathbf{M}_{\mathrm{H}}$ are not affected by the nonlinearity.

The nonlinear case is treated in more detail in [5]. It is shown that the system of nonlinear HBFE equations can be easily solved by means of the Newton–Raphson method, provided the assembly of the linearised systems is done through the differential reluctivity tensor. Note that this approach clearly differs from the "classical" HBFE approach, in which the reluctivity harmonics are explicitly considered and the nonlinear system of equations is solved using an iterative method other than the Newton–Raphson method [3,9].

As the nonlinearity does not interfere with the treatment of the movement, as presented hereafter, we will not further consider it.

3. Two-dimensional eddy current problem with movement

We now consider a two-dimensional linear eddy current problem with imposed movement. It comprises several solid (nondeformable) current carrying and/or permeable bodies which are in relative motion with respect to each other. Eqs. (1)–(4), and in particular, Faraday's law (2), remain valid if they are solved in different reference frames, each tied to a (conducting) body [2].

In each solid body, a FE discretisation which is time-invariant with respect to the local reference frame may be adopted. This way, their contribution to the matrices $\mathbf{S}$ and $\mathbf{L}$ in (7) is time-invariant as well.

The deforming free space between the moving bodies can be modelled with the FE method or the BE method. The former case is mainly restricted to the modelling of rotation in rotating electrical machines using the so-called moving band, which is a single thin layer of elements in the airgap connecting the fixed stator mesh to the fixed rotor mesh [2]. Whenever the rotor angle changes in time, the discretisation of the moving band has to be updated. The contribution of the moving band to $\mathbf{S}$ thus depends on the rotor position $\theta(t)$, which can be readily accounted for in the time-stepping approach. In the multi-harmonic approach, the moving band contribution to $\mathbf{S}_{\mathrm{H}}$ is evaluated numerically by considering a sufficiently great number of time instants t_i and rotor positions $\theta(t_i)$ in a fundamental period [6]:

$$\mathbf{S}^{(\kappa,\lambda)} = \frac{1}{T}\int_0^T H_\kappa(t)\,H_\lambda(t)\,\mathbf{S}(t)\,\mathrm{d}t \approx \frac{1}{n}\sum_{i=1}^{n} H_\kappa(t_i)\,H_\lambda(t_i)\mathbf{S}(\theta(t_i)). \tag{24}$$

In general, the periodic movement causes all frequency components to be linked, as is also the case with magnetic saturation.

The moving band technique is quite convenient for modelling rotational movements, but is not easily extendible to other types of movement (e.g. translation) in two-dimensional or three-dimensional models. Obviously, a hybrid FEBE discretisation offers the sought flexibility. Hereby the FE domain Ω is confined to the moving nondeformable domains, and its contour Γ consists of the boundaries that are in relative motion. Due to the double integrals in (14)–(15) which contain the Green's function G or its gradient, the matrices $\mathbf{M}$ and $\mathbf{C}$ vary in time, while $\mathbf{S}$, $\mathbf{L}$ and $\mathbf{D}$ are constant. The multi-harmonic approach now requires the numerical evaluation of $\mathbf{M}_{\mathrm{H}}$ and $\mathbf{C}_{\mathrm{H}}$.

4. Example

As an application example, we consider a magnetic conducting pendulum oscillating in the magnetic field of a permanent magnet. Calculations are carried out using either the proposed multi-harmonic method or time-stepping, and using either a hybrid FEBE discretisation or a FE discretisation. The respective results are compared to each other. A short discussion on the computational cost of the different simulations follows.

4.1. Model

The pendulum consists of a massive block (100 mm $\times$ 100 mm, relative permeability $\mu/\mu_0 = \nu_0/\nu = 500$, conductivity $\sigma = 10^6$ S/m) suspended by a 1 m long nonmagnetic nonconducting rod (see Fig. 1). An oscillation $\theta(t) = \hat{\theta}\sin(2\pi f t)$, with frequency $f = 0.5$ Hz and amplitude $\hat{\theta} = 10°$, is imposed. In the vertical position, θ equals zero. For the fundamental frequency f, the penetration depth equals $1/\sqrt{\pi f \mu\sigma} = 31.83$ mm.

The (nonconducting) permanent magnet has a constant vertical remanent induction of $b_{\mathrm{r}} = 1$ T. The consitutive law $\underline{h} = \nu_0\underline{b} - \nu_0 b_{\mathrm{r}}\,\underline{1}_y$ is adopted.

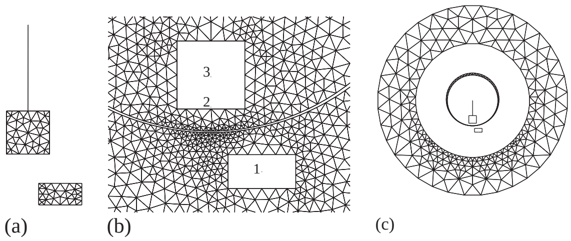

Fig. 1. (a) Discretisation of the pendulum block and the permanent magnet; (b) discretisation around the pendulum and the magnet, and the position of points 1, 2 and 3; (c) discretisation of the moving band and the circular ring.

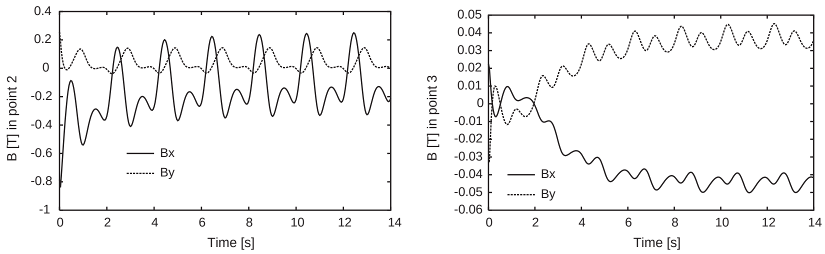

Fig. 2. Transient of the induction (x- and y-components in the local coordinate system) in points 2 and 3 (the position of which is indicated in Fig. 1b).

The discretisation of the pendulum block and the magnet is the same in the FE model and the FEBE model (see Fig. 1a). In the FE model, a moving band allows the rotation of the pendulum (Figs. 1b and c), and the free space extending to infinity is approximately accounted for by means of a transformation method. The latter method consists in mapping the outer circular ring shown in Fig. 1c onto the infinite exterior space [1].

4.2. Calculation results

First a time-stepping simulation (adopting the backward Euler method, with $\Delta t = T/100$) is carried out with the FE model and the FEBE model. The initial conditions are $a(x, y, t = 0) = 0$ and $q(\xi, t = 0) = 0$.

Some results obtained with the FE model are presented in Figs. 2 and 3. Apparently ten periods suffice to reach steady-state with a reasonable precision. Harmonic balance calculations are then performed with both the FE model and the hybrid model. The dc-term, the fundamental frequency $f = 0.5$ Hz and up to five harmonics kf $(2 \leqslant k \leqslant 5)$ are taken into account. The matrices $\mathbf{S}_\mathrm{H}$, $\mathbf{M}_\mathrm{H}$ and $\mathbf{C}_\mathrm{H}$ are evaluated numerically by considering 100 time instants and pendulum positions in a fundamental period. Depending on the number of nonzero frequencies adopted, the simulations are denoted HB1 to HB5. In HB3, e.g., 0, 0.5, 1 and 1.5 Hz are taken into account.

In Fig. 4 some induction waveforms obtained with the HB method are compared with those obtained with time-stepping. In Fig. 5 some frequency components of the flux pattern (in the two reference frames, fixed to either magnet or pendulum), obtained with the FE model, are shown.

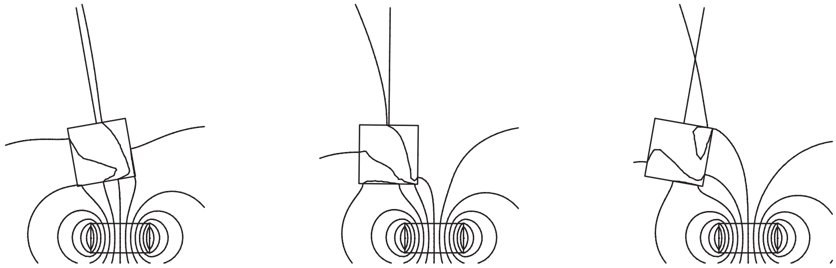

Fig. 3. Flux pattern at three instants in a fundamental period at steady-state. From left to right: $t=(k+0.25)T$, $(k+0.50)T$ and $(k+0.75)T$, where k is an integer.

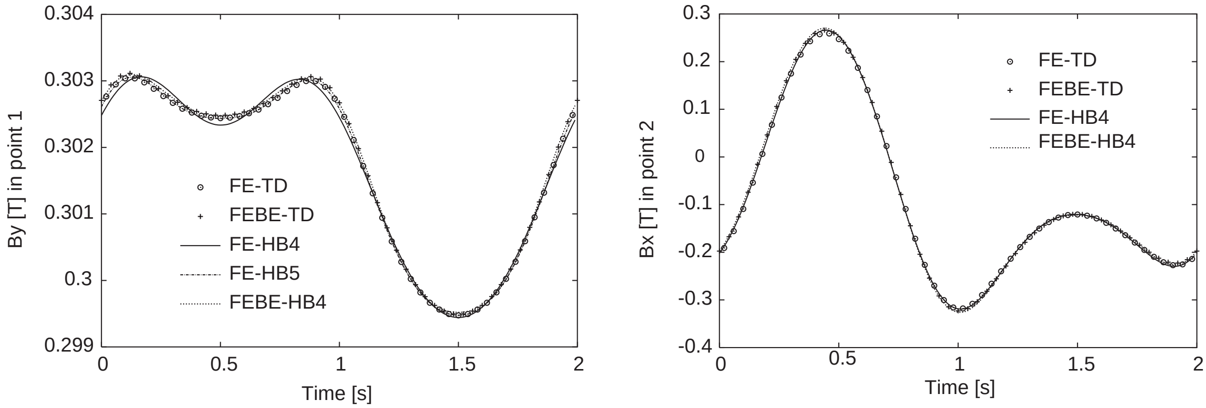

Fig. 4. Steady-state induction waveforms in points 1 and 2 obtained with time-stepping (TD) and (HB).

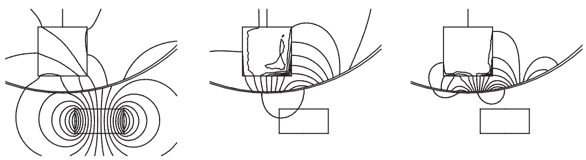

Fig. 5. Frequency components of the flux pattern in reference frames fixed to either magnet or pendulum. From left to right: dc-component, $-\sqrt{2}\sin(2\pi 0.5\mathrm{Hz}t)$ and $-\sqrt{2}\sin(2\pi 1.5\mathrm{Hz}t)$-component.

The convergence of the HB waveforms as the number of considered harmonics increases, is evidenced in Fig. 6. Note that FE and FEBE results converge independently to the correct solution. The HB method produces reasonably accurate results with only 4 or 5 nonzero frequencies.

4.3. Computational cost

A sufficiently fine space and time discretisation is adopted so as to allow a good agreement between the results. In the FE model, the second-order spatial discretisation of $a(x,y,t)$ requires

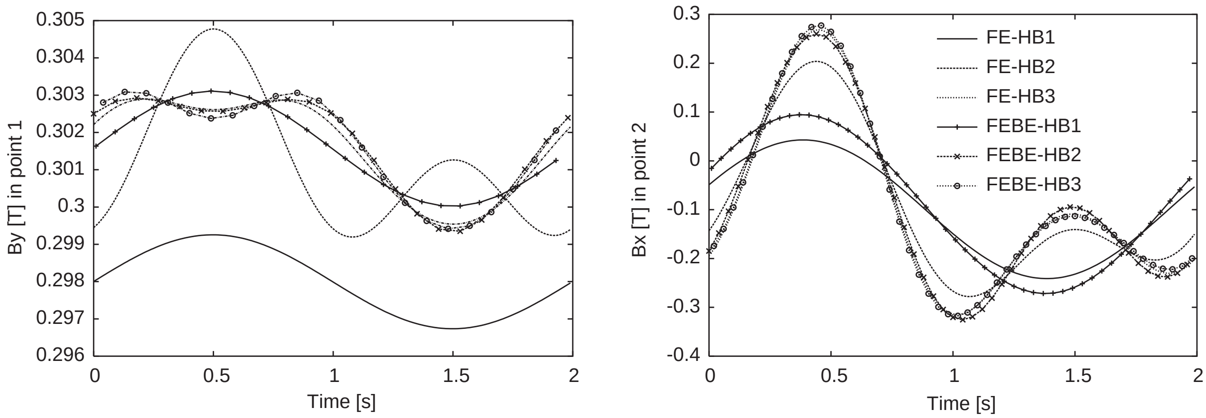

Fig. 6. Induction waveforms in points 1 and 2 obtained with the HB method (same legend left and right).

a total of 7295° of freedom, whereas the second-order discretisation of $a(x,y,t)$ and $q(\xi,t)$ in the hybrid FEBE model results in only 363° of freedom.

Adopting the time-domain approach with either the FE or the hybrid model, the average computation time per time step is 2.5 and 1.3 s, respectively, on a Pentium III 750 MHz. When using the FE model, the resolution of the system of algebraic equations (by means of GMRES, [6]) constitutes 80% of the computation time, compared to only 15% when using the hybrid FEBE model. In the latter case, the assembly of the system matrix is particularly expensive as it requires the numerical evaluation of the matrices $\mathbf{M}$ and $\mathbf{C}$. The time-invariant part of these matrices is evaluated only once, thus halving the average computation time per time step (1.3 s instead of 2.8 s).

In order to reach quasi-steady-state, some 30 periods are time-stepped, yielding a total computation time of 7400 and 3950 s, respectively.

The multi-harmonic calculations FE-HB1 to FE-HB5 take 34, 81, 152, 232 and 330 s, respectively, 80% of which is due to the resolution of the system. The calculation times for FEBE-HB1 to FEBE-HB4 are 162, 201, 244 and 295 s, respectively. Here, at least 90% of the computation time is devoted to the evaluation of $\mathbf{M}_{\mathrm{H}}$ and $\mathbf{C}_{\mathrm{H}}$. (The systems of algebraic equations are again solved by means of GMRES.)

For this particular eddy current problem, the multi-harmonic approach is a very interesting alternative to plain time-stepping as a limited number of harmonics is required. The hybrid discretisation results in approximately the same computation time as the FE discretisation, but has the great advantage that an arbitrary periodic movement can be imposed.

5. Conclusions

A novel method for dealing with an arbitrary imposed movement in a time-periodic magnetic field problem has been presented. Herein a FE discretisation of the moving solid bodies and a BE discretisation of the deforming air space is combined with a multi-harmonic time discretisation.

The algebra of the method has been elaborated for a linear two-dimensional eddy current problem. The implementation of the multi-harmonic method is relatively simple and the extension to other types of time-periodic magnetic field problems with movement is straightforward.

The method has been successfully applied to a test case. Time-stepping results obtained with a FE model and a hybrid model have served as a reference.

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