



An adjustable approach to fuzzy soft set based decision making

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ABSTRACT

Molodtsov's soft set theory was originally proposed as a general mathematical tool for dealing with uncertainty. Recently, decision making based on (fuzzy) soft sets has found paramount importance. This paper aims to give deeper insights into decision making based on fuzzy soft sets. We discuss the validity of the Roy–Maji method and show its true limitations. We point out that the choice value designed for the crisp case is no longer fit to solve decision making problems involving fuzzy soft sets. By means of level soft sets, we present an adjustable approach to fuzzy soft set based decision making and give some illustrative examples. Moreover, the weighted fuzzy soft set is introduced and its application to decision making is also investigated.

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1. Introduction

The real world is full of uncertainty, imprecision and vagueness. Actually most of the concepts we are meeting in everyday life are vague rather than precise. But classical mathematics, on the other hand, requires that all mathematical notions must be exact, otherwise precise reasoning would be impossible. In recent years, scientists and engineers have become interested in modelling vagueness because many practical problems emerging in multitude within fields such as biology, economics, engineering, environmental science, social science, and medical science involve data containing various types of uncertainties.

To solve these problems, classical mathematical tools may not be successfully used. While a wide range of theories such as probability theory, fuzzy set theory [1], intuitionistic fuzzy set theory [2], rough set theory [3], vague set theory [4] and the interval mathematics [5] are well known and often useful mathematical approaches to modelling vagueness, each of these theories has its inherent difficulties, which are pointed out in [6]. In 1999, Molodtsov [6] initiated *soft set theory* as a new mathematical tool for dealing with uncertainties which is free from the difficulties affecting existing methods. This theory has proven useful in many different fields such as the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory [7,6].

Up to the present, research on soft sets has been very active and many important results have been achieved in the theoretical aspect. Maji et al. introduced several algebraic operations in soft set theory and published a detailed theoretical study on soft sets [8]. The same authors also extended crisp soft sets to *fuzzy soft sets* [9]. Aktaş and Çağman [10] initiated *soft groups* and showed that fuzzy groups can be viewed as a special case of the soft groups. Jun [11] applied soft sets to the

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theory of BCK/BCI-algebras, and introduced the concept of soft BCK/BCI-algebras. Jun and Park [12] reported applications of soft sets in ideal theory of BCK/BCI-algebras. Feng et al. [13] defined soft semirings and several related notions to establish a connection between soft sets and semirings.

At the same time, there has been some progress concerning practical applications of soft set theory, especially the use of soft sets in decision making. Maji et al. [14] first applied soft sets to solve the decision making problem that is based on the concept of knowledge reduction in the theory of rough sets [15]. Chen et al. [16] presented a new definition of soft set parametrization reduction so as to improve the soft set based decision making in [14]. They also pointed out the basic difference between parametrization reduction of soft sets and attributes reduction in rough set theory.

All the above mentioned studies in decision making problems were based on crisp soft sets. The problem of decision making in an imprecise environment, however, has found paramount importance in recent years. To cope with fuzzy soft set based decision making problems, Roy and Maji [17] presented a novel method of object recognition from an imprecise multi-observer data. It is worth noting that in crisp soft set based decision making, the most important notion is the so-called *choice value* that precisely equals the number of good attributes possessed by an object [14]. Therefore, it is simply reasonable to select the object with the maximum choice value as the optimal alternative. Not so surprisingly, the situation becomes more complicated when we consider decision making that involves fuzzy soft sets. The “fuzzy choice value” of an object is the sum total of all membership values with respect to different attributes; hence it does not represent the number of fair attributes possessed by that object. To address this issue, instead of using choice values the Roy–Maji method in [17] builds upon a series of new concepts. More specifically, it involves the construction of comparison table from the resultant fuzzy soft set and the optimal decision is taken based on the maximum *score* computed from the comparison table. This is completely different from the way of decision making based on crisp soft sets as in [14,16]. On the other hand, Kong et al. [18] argued that the Roy–Maji method using scores was incorrect and they presented a revised algorithm. In fact, they implicitly followed the idea in crisp soft set based decision making, and assumed that the object with the maximum “fuzzy choice value” should be selected as the optimum candidate in [18]. The divergence of opinions in [17,18] is whether scores or choice values should be used as the criterion for making the optimum decision.

In this study, we try to investigate the fuzzy soft set based decision making problems in [17,18] more deeply. We first point out that the concept of choice values designed for crisp soft sets is not fit to solve decision making problems involving fuzzy soft sets; hence Roy and Maji’s algorithm in [17] is more suitable than the “fuzzy choice value” based revised algorithm in [18] for solving fuzzy soft set based decision making problems. Moreover, we present a novel approach to fuzzy soft set based decision making problems by using *level soft sets*. By considering different types of thresholds, we can derive different level soft sets from the original fuzzy soft set. In general, the final optimal decisions based on different level soft sets could be different. Thus the newly proposed approach is in fact an adjustable method which captures an important feature for decision making in an imprecise environment: some of these problems are essentially humanistic and thus subjective in nature (e.g. human understanding and vision systems); there actually does not exist a unique or uniform criterion for evaluating the alternatives. Moreover, this new method can be successfully applied to some decision making problems that cannot be solved by using the methods in [17,18]. It also involves relatively less computations than the Roy–Maji method in [17]. This makes our new proposal not only more suitable but more feasible for dealing with the real-life application.

The remainder of this paper is organized as follows. Section 2 recalls the basic concepts of soft sets and fuzzy soft sets. Section 3 examines the fuzzy soft set based decision problems in detail. In Section 4, we present an adjustable approach by means of level soft sets. Different kinds of level soft sets such as t -level soft sets, mid-level soft sets and top-level soft sets are considered and illustrated by some concrete examples. Section 5 is devoted to weighted fuzzy soft sets with applications to decision making problems. Finally, conclusions are given in Section 6.

2. Soft sets and fuzzy soft sets

In this section, we recall the basic notions of soft sets and fuzzy soft sets. Let U be an initial universe of objects and E_U (simply denoted by E) the set of parameters in relation to objects in U . Parameters are often attributes, characteristics, or properties of objects. Let $\mathcal{P}(U)$ denote the power set of U and $A \subseteq E$. Following [10,8], the concept of soft sets is defined as follows.

Definition 2.1 ([10]). A pair (F, A) is called a *soft set* over U , where F is a mapping given by $F : A \rightarrow \mathcal{P}(U)$.

By definition, a soft set (F, A) over the universe U can be regarded as a parameterized family of subsets of the universe U , which gives an approximate (soft) description of the objects in U . As pointed in [6], for any parameter $\epsilon \in A$, the subset $F(\epsilon) \subseteq U$ may be considered as the set of ϵ -approximate elements in the soft set (F, A) . It is worth noting that $F(\epsilon)$ may be arbitrary: some of them may be empty, and some may have nonempty intersection [6]. For illustration, Molodtsov considered several examples in [6]. Similar examples were also discussed in [8,10].

Maji et al. [9] initiated the study on hybrid structures involving both fuzzy sets and soft sets. They introduced in [9] the notion of fuzzy soft sets, which can be seen as a fuzzy generalization of (crisp) soft sets.

Definition 2.2 ([17]). Let $\mathcal{F}(U)$ be the set of all fuzzy subsets in a universe U . Let E be a set of parameters and $A \subseteq E$. A pair $(\tilde{F}, A)$ is called a *fuzzy soft set* over U , where $\tilde{F}$ is a mapping given by $\tilde{F} : A \rightarrow \mathcal{F}(U)$.

Table 1Tabular representation of the fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$.

U	e_1	e_2	e_3
h_1	0.4	1.0	0.5
h_2	0.6	0.5	0.6
h_3	0.5	0.5	0.8
h_4	0.9	0.5	0.2
h_5	0.3	0.7	0.9

In the above definition, fuzzy subsets in the universe U are used as substitutes for the crisp subsets of U . Hence it is easy to see that every (classical) soft set may be considered as a fuzzy soft set. Generally speaking, $\tilde{F}(\epsilon)$ is a fuzzy subset in U and it is called the *fuzzy approximate value set* of the parameter ϵ . Following the standard notations, $\tilde{F}(\epsilon)$ can be written as $\tilde{F}(\epsilon) = \{(x, \tilde{F}(\epsilon)(x)) : x \in U\}$.

It is well known that the notion of fuzzy sets provides a convenient tool for representing vague concepts by allowing partial memberships. In the definition of a fuzzy soft set, fuzzy subsets are used as substitutes for the crisp subsets. Hence every soft set may be considered as a fuzzy soft set. In addition, by analogy with soft sets, one easily sees that every fuzzy soft set can be viewed as an (fuzzy) information system and be represented by a data table with entries belonging to the unit interval $[0, 1]$. For illustration, we consider the following example.

Example 2.3. Suppose that there are five houses in the universe

$$U = \{h_1, h_2, h_3, h_4, h_5\}$$

and the set of parameters is given by $E = \{e_1, e_2, e_3, e_4, e_5\}$, where e_i stand for “beautiful”, “modern”, “cheap”, “in good repair” and “in green surroundings” respectively. Let $A = \{e_1, e_2, e_3\} \subset E$ be consisting of the parameters that Mr. X is interested in buying a house. That means out of available houses in U , Mr. X want to buy the house which qualifies with the attributes in A to the utmost extent. Now all the available information on houses under consideration can be formulated as a fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$ describing “attractiveness of houses” that Mr. X is going to buy. Table 1 gives the tabular representation of the fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$. We can view the fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$ as the collection of the following fuzzy approximations:

$$\tilde{F}(e_1) = \text{beautiful houses} = \{(h_1, 0.4), (h_2, 0.6), (h_3, 0.5), (h_4, 0.9), (h_5, 0.3)\},$$

$$\tilde{F}(e_2) = \text{modern houses} = \{(h_1, 1.0), (h_2, 0.5), (h_3, 0.5), (h_4, 0.5), (h_5, 0.7)\},$$

$$\tilde{F}(e_3) = \text{cheap houses} = \{(h_1, 0.5), (h_2, 0.6), (h_3, 0.8), (h_4, 0.2), (h_5, 0.9)\}.$$

3. Fuzzy soft set based decision making

We begin this section with a novel algorithm designed for solving fuzzy soft set based decision making problems, which was presented in [17].

3.1. Roy and Maji's original algorithm using scores

Algorithm 1 ([17]).

1. Input the fuzzy soft sets (F, A) , (G, B) and (H, C) .
2. Input the parameter set P as observed by the observer.
3. Compute the corresponding resultant fuzzy soft set (S, P) from the fuzzy soft sets (F, A) , (G, B) , (H, C) and place it in tabular form.
4. Construct the comparison table of the fuzzy soft set (S, P) and compute r_i and t_i for o_i , $\forall i$.
5. Compute the score s_i of o_i , $\forall i$.
6. The decision is o_k if $s_k = \max_i s_i$.
7. If k has more than one value then any one of o_k may be chosen.

Roy and Maji [17] pointed out that the object recognition problem may be viewed as a multi-observer decision making problem, where the final identification of the object is based on the set of inputs from different observers who provide the overall object characterization in terms of diverse sets of parameters. The above Algorithm 1 gives solutions to the recognition problem by means of fuzzy soft sets. This method involves construction of comparison table from the resultant fuzzy soft set and the final optimal decision is taken based on the maximum score computed from the comparison table.

The comparison table is a square table in which rows and columns both are labelled by the object names $o_1, o_2, \dots, o_n$ of the universe, and the entries c_{ij} indicate the number of parameters for which the membership value of o_i exceeds or equals the membership value of o_j . Clearly, $0 \leq c_{ij} \leq m$ and $c_{ii} = m$ ($\forall i, j$), where m is the number of parameters. The row-sum r_i of an object o_i is computed by

$$r_i = \sum_{j=1}^n c_{ij}.$$

Table 2Tabular representation of the resultant fuzzy soft set (S, P) with choice values.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	Choice value (c_i)
o_1	0.3	0.1	0.4	0.4	0.1	0.1	0.5	$c_1 = 1.9$
o_2	0.3	0.3	0.5	0.1	0.3	0.1	0.5	$c_2 = 2.1$
o_3	0.4	0.3	0.5	0.1	0.3	0.1	0.6	$c_3 = 2.3$
o_4	0.7	0.4	0.2	0.1	0.2	0.1	0.3	$c_4 = 2.0$
o_5	0.2	0.5	0.2	0.3	0.5	0.5	0.4	$c_5 = 2.6$
o_6	0.3	0.5	0.2	0.2	0.4	0.3	0.3	$c_6 = 2.2$

Table 3Comparison table of the fuzzy soft set (S, P) .

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	7	4	2	4	4	4
o_2	6	7	5	5	3	3
o_3	6	7	7	5	3	3
o_4	4	4	4	7	2	3
o_5	3	4	4	6	7	6
o_6	4	5	4	6	3	7

Table 4Score table of the fuzzy soft set (S, P) .

	Row-sum (r_i)	Column-sum (t_j)	Score (s_i)
o_1	25	30	-5
o_2	29	31	-2
o_3	31	26	5
o_4	24	33	-9
o_5	30	22	8
o_6	29	26	3

Similarly the column-sum t_j of an object o_j is computed by

$$t_j = \sum_{i=1}^n c_{ij}.$$

Finally, the score s_i of an object o_i is defined by $s_i = r_i - t_i$.

The basic idea of Algorithm 1 was illustrated in [17] by a concrete example (see Section 4 of [17] for details). To maintain self-containment of the present paper, we sketch out that example here. Let $U = \{o_1, o_2, \dots, o_6\}$ be the universe of objects. The tabular representation of the resultant fuzzy soft set (S, P) (obtained in Step 3) with choice values is given by Table 2. The comparison table of the fuzzy soft set (S, P) is shown as in Table 3. Then we compute the row-sum r_i , column-sum t_i and the score s_i for each o_i . As a result, we have Table 4, namely the score table of (S, P) . From Table 4, it is clear that o_5 has the maximum score $s_5 = 8$; hence by Algorithm 1, the optimal decision is to select o_5 .

3.2. A counterexample given in [18]

It is worth noting that in this example, the maximum choice value (see [14] for the definition of choice values) is $c_5 = 2.6$, which also belongs to o_5 , the object with the maximum score (see Table 2). Kong et al. [18] paid attention to this fact and addressed that the illustrating example in [17] is only a special case. In [18] a counterexample was also given to illustrate that the optimal choice could not be obtained in general by using Roy and Maji's algorithm.

Here we also make a sketch of the counterexample in [18]. For clearness, we denote by (S', P') the fuzzy soft set in [18]. The tabular representation of (S', P') with choice values and scores is as in Table 5. From Table 5, one can see that the maximum score is $s_3 = 6$, scored by o_3 . Thus according to Algorithm 1, the optimal decision is in favor of selecting o_3 , the object with the maximum score. On the other hand, Table 5 shows that o_6 is the object with the maximum choice value. Thus Kong et al. [18] thought that o_6 should be the optimal alternative because it has the maximum choice value $c_6 = 2.1$. In view of these facts, Kong et al. [18] concluded that Algorithm 1 was incorrect.

Moreover, Kong et al. [18] revised Algorithm 1 by redefining

$$c_{ij} = \sum_{k=1}^m (f_{ik} - f_{jk})$$

and

$$r_i = \sum_{j=1}^n c_{ij}$$

Table 5Tabular representation of the fuzzy soft set (S', P') with choice values and scores.

U	e_1	e_2	e_3	e_4	e_5	Choice value (c_i)	Score (s_i)
o_1	0.1	0.5	0.3	0.4	0.3	$c_1 = 1.6$	$s_1 = 0$
o_2	0.3	0.5	0.2	0.3	0.6	$c_2 = 1.9$	$s_2 = 3$
o_3	0.1	0.7	0.4	0.5	0.1	$c_3 = 1.8$	$s_3 = 6$
o_4	0.7	0.2	0.2	0.2	0.3	$c_4 = 1.6$	$s_4 = -6$
o_5	0.2	0.6	0.3	0.2	0.3	$c_5 = 1.6$	$s_5 = 1$
o_6	0.9	0.2	0.1	0.1	0.8	$c_6 = 2.1$	$s_6 = -4$

where f_{ik} is the membership value of object o_i for the k th parameter; m is the number of parameters; n is the number of objects.

In the revised definitions above, we can further deduce

$$c_{ij} = \sum_{k=1}^m f_{ik} - \sum_{k=1}^m f_{jk} = c_i - c_j.$$

That is, the new definition of c_{ij} is the difference in choice value between objects o_i and o_j . Clearly, it follows that the new definition of r_i indicates the total difference in choice value between objects o_i and all the other members in the universe U .

From the above discussion, we arrive at a conclusion: this revised version of [Algorithm 1](#) is actually based on the maximum choice value instead of the maximum score in making decisions.

3.3. Validity of the Roy–Maji method and its limitations

We can show that [Algorithm 1](#) is actually a useful method for selecting the optimal alternative in decision making problems based on fuzzy soft sets, while the counterexample given in [18] is not sufficient for concluding that the Roy–Maji method is incorrect.

As was mentioned above, it is implicit in [18] that the object with the maximum choice value (instead of the maximum score as in [17]) should be selected as the optimum candidate. Note that for decision making problems using crisp soft sets, the choice value of an object precisely represents the number of (good) attributes possessed by that object. Hence it is reasonable to select the object with the maximum choice value as the optimal alternative. This idea in fact goes back to Maji et al. [14].

In decision making problems involving fuzzy soft sets, the situation becomes somewhat different and much more complicated. In particular, it is worth noting that the so-called choice value considered in [18] should be exactly interpreted as the “fuzzy choice value”, which is just the sum total of all membership values with respect to different attributes. Thus for a fuzzy soft set, this choice value no longer represents the number of (good) attributes possessed by that object. It can be seen, on the other hand, as a synthesized measure to estimate each object via a fusion of all attributes. However, this direct addition of all the membership values with respect to different attributes is not always reasonable. Consider for instance the set of round and coarse objects. While it may be reasonable to compare the roundness of two objects, it is not equally reasonable to add the degree of roundness of an object to the degree to which that object is coarse; in a certain sense this is just like, for instance, the addition of height and weight.

To solve this difficulty associated with fuzzy soft sets, rather than using the concept of choice values designed for crisp soft sets, we can compare the membership values of two objects with respect to a common attribute to determine which one relatively possesses that attribute. In fact, this idea is implemented in [17] by introducing the concept of a comparison table and a measure called the score of an object. Based on these notions, [Algorithm 1](#) can be successfully used in many cases to select the optimal objects in fuzzy soft set based decision making problems. Moreover, it is worth noting that in the special case of crisp soft set based decision making the choice values and scores are equivalent for selecting the optimal alternatives.

The following simple example will illustrate that it is more reasonable to use scores rather than “fuzzy choice value” as the criterion in fuzzy soft set based decision making. Let (G, B) be a fuzzy soft set and [Table 6](#) the tabular representation of it with choice values and scores. From [Table 6](#), we can see the choice values of o_1 and o_2 are almost equal, but the scores are different widely. In general, we often choose o_2 as the optimal alternative because the scores indicate that o_2 relatively possesses more fair attributes than o_1 . If we make the optimal decision by using choice values as in [18] the optimal alternative will be o_1 ; but this is inconsistent with the above intuition. Therefore Roy and Maji’s [Algorithm 1](#) is more suitable than the choice value based method presented in [18] for solving fuzzy soft set based decision making problems.

It should be noted that the Roy–Maji method using scores still has some inherent limitations. First, there exist some fuzzy soft set based decision problems in which [Algorithm 1](#) may not be successfully used to find the optimal decision. To illustrate this limitation, we consider the following example. Let (S_1, P_1) be a fuzzy soft set and [Table 7](#) the tabular representation of it with choice values and scores. From [Table 7](#), it is clear that all these objects have the same score (i.e., $s_1 = s_2 = s_3 = 0$). Hence by using [Algorithm 1](#) we could hardly arrive at the final optimal decision, whence any one of them might be selected as the optimal candidate. In addition, it is worth noting that all these objects also have the same

Table 6Tabular representation of the fuzzy soft set (G, B) with choice values and scores.

U	e_1	e_2	e_3	e_4	e_5	Choice value (c_i)	Score (s_i)
o_1	0.9	0.1	0.2	0.1	0.3	$c_1 = 1.6$	$s_1 = -3$
o_2	0.19	0.3	0.4	0.3	0.4	$c_2 = 1.59$	$s_2 = 3$

Table 7Tabular representation of the fuzzy soft set (S_1, P_1) with choice values and scores.

U	e_1	e_2	e_3	e_4	Choice value (c_i)	Score (s_i)
o_1	0.9	0.7	0.3	0.2	$c_1 = 2.1$	$s_1 = 0$
o_2	0.8	0.6	0.4	0.3	$c_2 = 2.1$	$s_2 = 0$
o_3	0.7	0.5	0.5	0.4	$c_3 = 2.1$	$s_3 = 0$

choice value (i.e., $c_1 = c_2 = c_3 = 2.1$). The second drawback of the Roy–Maji method is due to its computational complexity. It is easy to see that Algorithm 1 requires too many calculations, especially when the decision problem is associated with a large parameter set or a great number of objects. Thus sometimes it is not very efficient to select the optimal object by using the Roy–Maji method.

4. An adjustable approach using level soft sets

Like most of the decision making problems, fuzzy soft set based decision making involves the evaluation of all the objects which are decision alternatives. Some of these problems are essentially humanistic and thus subjective in nature (e.g. human understanding and vision systems). In general, there actually does not exist a unique or uniform criterion for the evaluation of decision alternatives. In fact, the divergence of opinions in [17,18] is whether scores or choice values should be used to estimate decision alternatives and to determine the optimal objects.

4.1. Level soft sets of fuzzy soft sets

To overcome all the above difficulties, we present an adjustable approach to fuzzy soft set based decision making problems. This proposal is based on the following novel concept called level soft sets.

Definition 4.1. Let $\mathfrak{S} = (\tilde{F}, A)$ be a fuzzy soft set over U , where $A \subseteq E$ and E is the parameter set. For $t \in [0, 1]$, the t -level soft set of the fuzzy soft set $\mathfrak{S}$ is a crisp soft set $L(\mathfrak{S}; t) = (F_t, A)$ defined by

$$F_t(a) = L(\tilde{F}(a); t) = \{x \in U : \tilde{F}(a)(x) \geq t\},$$

for all $a \in A$.

In this definition, $t \in [0, 1]$ can be viewed as a given threshold on membership values. For real-life applications of fuzzy soft set based decision making, usually these thresholds are in advance chosen by decision makers and represent their requirements on “membership levels”.

It is easy to see that level soft sets are defined in terms of level sets of a fuzzy set, which is an important notion in fuzzy set theory. If we regard fuzzy soft sets as extensions of fuzzy sets from the soft set theoretical viewpoint, we can say in a certain sense that level soft sets are “soft generalizations” of classical level sets. To illustrate this, let us reconsider the fuzzy soft set $(\tilde{F}, A)$ in Example 2.3.

Example 4.2. (The 0.5-level Soft Set of the Fuzzy Soft Set $\mathfrak{S}$.) As was mentioned above, the fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$ in Example 2.3 can be viewed as the following collection of fuzzy approximations:

$$\tilde{F}(e_1) = \text{beautiful houses} = \{(h_1, 0.4), (h_2, 0.6), (h_3, 0.5), (h_4, 0.9), (h_5, 0.3)\},$$

$$\tilde{F}(e_2) = \text{modern houses} = \{(h_1, 1.0), (h_2, 0.5), (h_3, 0.5), (h_4, 0.5), (h_5, 0.7)\},$$

$$\tilde{F}(e_3) = \text{cheap houses} = \{(h_1, 0.5), (h_2, 0.6), (h_3, 0.8), (h_4, 0.2), (h_5, 0.9)\}.$$

By taking $t = 0.5$, we obtain the 0.5-level sets of the fuzzy sets $\tilde{F}(e_1)$, $\tilde{F}(e_2)$ and $\tilde{F}(e_3)$ as follows:

$$L(\tilde{F}(e_1), 0.5) = \{h_2, h_3, h_4\},$$

$$L(\tilde{F}(e_2), 0.5) = \{h_1, h_2, h_3, h_4, h_5\},$$

$$L(\tilde{F}(e_3), 0.5) = \{h_1, h_2, h_3, h_5\}.$$

Then the 0.5-level soft set of $\mathfrak{S} = (\tilde{F}, A)$ is a soft set $L(\mathfrak{S}; 0.5) = (F_{0.5}, A)$, where the set-valued mapping $F_{0.5} : A \rightarrow \mathcal{P}(U)$ is defined by $F_{0.5}(e_i) = L(\tilde{F}(e_i), 0.5)$ for $i = 1, 2, 3$. That is to say, the approximate value sets of the level soft set $L(\mathfrak{S}; 0.5)$ are the 0.5-level set of the fuzzy set $\tilde{F}(e_i)$ for every parameter $e_i \in A$. Table 8 gives the tabular representation of the level soft set $L(\mathfrak{S}; 0.5)$.

Table 8Tabular representation of the level soft set $L(\mathfrak{S}; 0.5)$.

U	e_1	e_2	e_3
h_1	0	1	1
h_2	1	1	1
h_3	1	1	1
h_4	1	1	0
h_5	0	1	1

Table 9Tabular representation of the mid-level soft set $L((S_1, P_1); \text{mid})$ with choice values.

U	e_1	e_2	e_3	e_4	Choice value (c_i)
o_1	1	1	0	0	$c_1 = 2$
o_2	1	1	1	1	$c_2 = 4$
o_3	0	0	1	1	$c_3 = 2$

4.2. Level soft sets with respect to a threshold fuzzy set

In the definition of t -level soft sets, the level (or threshold) assigned to each parameter is always the constant value $t \in [0, 1]$. But in some decision making problems, it may happen that decision makers would like to impose different thresholds on different decision parameters. To cope with such problems, we can use a function instead of a constant number as the threshold on membership values.

Definition 4.3. Let $\mathfrak{S} = (\tilde{F}, A)$ be a fuzzy soft set over U , where $A \subseteq E$ and E is the parameter set. Let $\lambda : A \rightarrow [0, 1]$ be a fuzzy set in A which is called a *threshold fuzzy set*. The level soft set of the fuzzy soft set $\mathfrak{S}$ with respect to the fuzzy set λ is a crisp soft set $L(\mathfrak{S}; \lambda) = (F_\lambda, A)$ defined by

$$F_\lambda(a) = L(\tilde{F}(a); \lambda(a)) = \{x \in U : \tilde{F}(a)(x) \geq \lambda(a)\},$$

for all $a \in A$.

It is clear that level soft sets with respect to a fuzzy set generalize t -level soft sets by substituting a function on the parameter set A , namely a fuzzy set $\lambda : A \rightarrow [0, 1]$, for a constant $t \in [0, 1]$.

Let $\hat{t}$ denote the constant fuzzy set in A given by $\hat{t}(a) = t$ for all $a \in A$. Then we immediately have $L(\mathfrak{S}; \hat{t}) = L(\mathfrak{S}; t)$; that is, the level soft set with respect to the constant fuzzy set $\hat{t}$ coincides with the t -level soft set.

In order to better understand the above idea, let us consider the following implementations.

Example 4.4 (The Mid-level Soft Set of a Fuzzy Soft Set). Let $\mathfrak{S} = (\tilde{F}, A)$ be a fuzzy soft set over a finite universe U , where $A \subseteq E$ and E is the parameter set. Based on the fuzzy soft set $\mathfrak{S} = (F, A)$, we can define a fuzzy set $\text{mid}_{\mathfrak{S}} : A \rightarrow [0, 1]$ by

$$\text{mid}_{\mathfrak{S}}(a) = \frac{1}{|U|} \sum_{x \in U} \tilde{F}(a)(x),$$

for all $a \in A$. The fuzzy set $\text{mid}_{\mathfrak{S}}$ is called the *mid-threshold* of the fuzzy soft set $\mathfrak{S}$. In addition, the level soft set of $\mathfrak{S}$ with respect to the mid-threshold fuzzy set $\text{mid}_{\mathfrak{S}}$, namely $L(\mathfrak{S}; \text{mid}_{\mathfrak{S}})$ is called the *mid-level soft set* of $\mathfrak{S}$ and simply denoted $L(\mathfrak{S}; \text{mid})$. In what follows the *mid-level decision rule* will mean using the mid-threshold and considering the mid-level soft set in fuzzy soft set based decision making.

For a concrete example of mid-level soft sets, let us reconsider the fuzzy soft set (S_1, P_1) with its tabular representation given by Table 7. It is clear that the mid-threshold of (S_1, P_1) is a fuzzy set

$$\text{mid}_{(S_1, P_1)} = \{(e_1, 0.8), (e_2, 0.6), (e_3, 0.4), (e_4, 0.3)\},$$

and the mid-level soft set of (S_1, P_1) is a soft set $L((S_1, P_1); \text{mid})$ with its tabular representation given by Table 9.

Example 4.5 (The Top-level Soft Set of a Fuzzy Soft Set). Let $\mathfrak{S} = (\tilde{F}, A)$ be a fuzzy soft set over a finite universe U , where $A \subseteq E$ and E is the parameter set. Based on the fuzzy soft set $\mathfrak{S} = (F, A)$, we can define a fuzzy set $\text{max}_{\mathfrak{S}} : A \rightarrow [0, 1]$ by

$$\text{max}_{\mathfrak{S}}(a) = \max_{x \in U} \tilde{F}(a)(x),$$

for all $a \in A$. The fuzzy set $\text{max}_{\mathfrak{S}}$ is called the *max-threshold* of the fuzzy soft set $\mathfrak{S}$. In addition, the level soft set of $\mathfrak{S}$ with respect to the max-threshold $\text{max}_{\mathfrak{S}}$, namely $L(\mathfrak{S}; \text{max}_{\mathfrak{S}})$ is called the *top-level soft set* of $\mathfrak{S}$ and simply denoted $L(\mathfrak{S}; \text{max})$. In what follows the *top-level decision rule* will mean using the max-threshold and considering the top-level soft set in fuzzy soft set based decision making.

Table 10Tabular representation of the top-level soft set $L((S_1, P_1); \max)$ with choice values.

U	e_1	e_2	e_3	e_4	Choice value (c_i)
o_1	1	1	0	0	$c_1 = 2$
o_2	0	0	0	0	$c_2 = 0$
o_3	0	0	1	1	$c_3 = 2$

For a concrete example of top-level soft sets, we also reconsider the fuzzy soft set (S_1, P_1) with its tabular representation given by Table 7. It is clear that the max-threshold of (S_1, P_1) is a fuzzy set

$$\widetilde{\max}_{(S_1, P_1)} = \{(e_1, 0.9), (e_2, 0.7), (e_3, 0.5), (e_4, 0.4)\},$$

and the top-level soft set of (S_1, P_1) is a soft set $L((S_1, P_1); \max)$ with its tabular representation given by Table 10.

4.3. An adjustable approach based on level soft sets

Algorithm 2. 1. Input the (resultant) fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$.

2. Input a threshold fuzzy set $\lambda : A \rightarrow [0, 1]$ (or give a threshold value $t \in [0, 1]$; or choose the mid-level decision rule; or choose the top-level decision rule) for decision making.
3. Compute the level soft set $L(\mathfrak{S}; \lambda)$ of $\mathfrak{S}$ with respect to the threshold fuzzy set λ (or the t -level soft set $L(\mathfrak{S}; t)$; or the mid-level soft set $L(\mathfrak{S}; \text{mid})$; or the top-level soft set $L(\mathfrak{S}; \max)$).
4. Present the level soft set $L(\mathfrak{S}; \lambda)$ (or $L(\mathfrak{S}; t)$; or $L(\mathfrak{S}; \text{mid})$; or $L(\mathfrak{S}; \max)$) in tabular form and compute the choice value c_i of o_i , $\forall i$.
5. The optimal decision is to select o_k if $c_k = \max_i c_i$.
6. If k has more than one value then any one of o_k may be chosen.

Remark 4.6. In the last step of the above algorithm, one may go back to the second step and change the threshold (or decision rule) that he once used so as to adjust the final optimal decision, especially when there are too many “optimal choices” to be chosen.

The primary motivation for designing Algorithm 2 is to solve fuzzy soft set based decision making problem by using level soft sets initiated in this study. This novel idea is completely different from the existing methods presented in [17,18]. Level soft sets build bridges between fuzzy soft sets and crisp soft sets. By using level soft sets, we need not treat fuzzy soft sets directly in decision making but only deal with the crisp soft sets derived from them after choosing certain thresholds or decision strategies such as the mid-level or the top-level decision rules. Hence in this new approach to fuzzy soft set based decision making, the problem concerning reasonableness of “fuzzy choice values” simply does not arise. Intuitively, the choice value of an object in a level soft set can be interpreted as the number of fair attributes which belong to that object in the sense of exceeding the given “membership levels”.

The advantages of Algorithm 2 are mainly twofold. First, it is easy to see that this algorithm involves relatively less computations than Algorithm 1. Instead of constructing comparison tables and calculating scores as in Algorithm 1, we only have to consider (classical) choice values of objects in level soft sets. The reason for this simplicity is that in Algorithm 2 we actually deal with crisp soft sets rather than the initial fuzzy soft sets, by considering certain membership levels. Second, Algorithm 2 can be seen as an adjustable approach to fuzzy soft set based decision making because the final optimal decision is in relation to the thresholds on membership values or in other words, the decision criteria used by decision makers. For instance, if we choose the top-level decision rule in the second step of Algorithm 2, we shall consider the choice value of each object in the top-level soft set $L(\mathfrak{S}; \max)$; if another decision criterion such as the mid-level decision rule is used, we shall consider choice values in the mid-level soft set $L(\mathfrak{S}; \text{mid})$. In general, the choice value of an object in $L(\mathfrak{S}; \text{mid})$ need not coincide with the value in $L(\mathfrak{S}; \max)$. Consequently, the optimal objects determined by the mid-level decision rule may be different from those selected according to the top-level rule. As was mentioned above, many decision making problems are essentially humanistic and subjective in nature; hence for decision making in an imprecise environment, there actually does not exist a unique or uniform criterion. This adjustable feature makes Algorithm 2 not only efficient but more appropriate for many real-world applications.

To illustrate the basic idea of Algorithm 2, we apply it to some fuzzy soft set based decision making problems. First, let us consider the decision making problem in [17] that involves the fuzzy soft set (S, P) with tabular representation as in Table 2. Assume that the decision maker intends to select the object fulfilling the highest standard in most aspects as his optimal choice. Clearly, it is reasonable to use the top-level decision rule for decision making in such cases. For the fuzzy soft set (S, P) , we have the max-threshold fuzzy set $\widetilde{\max}_{(S, P)}$ that can be written as follows:

$$\widetilde{\max}_{(S, P)} = \begin{pmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ 0.7 & 0.5 & 0.5 & 0.4 & 0.5 & 0.5 & 0.6 \end{pmatrix}.$$

Table 11Tabular representation of the top-level soft set $L((S, P); \max)$ with choice values.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	Choice value (c_i)
o_1	0	0	0	1	0	0	0	$c_1 = 1$
o_2	0	0	1	0	0	0	0	$c_2 = 1$
o_3	0	0	1	0	0	0	1	$c_3 = 2$
o_4	1	0	0	0	0	0	0	$c_4 = 1$
o_5	0	1	0	0	1	1	0	$c_5 = 3$
o_6	0	1	0	0	0	0	0	$c_6 = 1$

Table 12Tabular representation of the top-level soft set $L((S', P'); \max)$ with choice values.

U	e_1	e_2	e_3	e_4	e_5	Choice value (c_i)
o_1	0	0	0	0	0	$c_1 = 0$
o_2	0	0	0	0	0	$c_2 = 0$
o_3	0	1	1	1	0	$c_3 = 3$
o_4	0	0	0	0	0	$c_4 = 0$
o_5	0	0	0	0	0	$c_5 = 0$
o_6	1	0	0	0	1	$c_6 = 2$

The top-level soft set of (S, P) is a soft set $L((S, P); \max)$ with its tabular representation given by Table 11. From the choice value listed in Table 11, it is clear that the optimal decision is to select o_5 because it has the maximum choice value $c_5 = 3$. Note that this result is the same as that obtained by using Algorithm 1 (see Table 4 and the discussion in Section 3.1 for details).

Next, we revisit the decision making problems discussed in [18]. At this time the top-level decision rule will be used again. For the fuzzy soft set (S', P') , we have the max-threshold fuzzy set $\widetilde{\max}_{(S', P')}$ that can be written as follows:

$$\widetilde{\max}_{(S', P')} = \begin{pmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \\ 0.9 & 0.7 & 0.4 & 0.5 & 0.8 \end{pmatrix}.$$

The top-level soft set of (S', P') is a soft set $L((S', P'); \max)$ with its tabular representation given by Table 12. From Table 12, it is clear that the optimal decision is to select o_3 because it has the maximum choice value $c_3 = 3$; this optimal decision also coincides with the result given by Algorithm 1 in [17] (see Table 5 and the discussion in Section 3.2 for details).

In the end, let us reconsider the decision making problem involving the fuzzy soft set (S_1, P_1) with its tabular representation given by Table 7. Although this problem cannot be solved by Algorithm 1 or the “fuzzy choice value” based method presented in [18], we shall be able to cope with it by Algorithm 2 because of its adjustable feature. Specifically, if we deal with this problem by the top-level decision rule, we shall use the max-threshold $\widetilde{\max}_{(S_1, P_1)}$ and thus obtain the top-level soft set $L((S_1, P_1); \max)$ with tabular representation as in Table 10. From Table 10, it is clear that the maximum choice value is $c_1 = c_3 = 2$; hence either o_1 or o_3 could be selected as the optimal alternative. At the same time, if we deal with the same problem by the mid-level decision rule instead, we shall use the mid-threshold $\text{mid}_{(S_1, P_1)}$ and thus obtain the mid-level soft set $L((S_1, P_1); \text{mid})$ with its tabular representation as in Table 9. From Table 9, it follows that the maximum choice value is $c_2 = 4$ and so the optimal decision is to select o_2 .

5. Weighted fuzzy soft set based decision making

In [19], Lin asked a very fundamental and interesting question: Should a membership function be regarded as the only characteristic function of a fuzzy set? While answering this question by himself in [19], Lin defined a new theory of mathematical analysis, namely the theory of W -softsets (means weighted soft sets). Following Lin's style, Maji et al. [14] defined the weighted table of a soft set. A weighted table of a soft set is presented by having $d_{ij} = w_j \times h_{ij}$ instead of 0 and 1 only, where h_{ij} are the entries in the table of the soft set and w_j are the weights of the attribute e_j . The weighted choice value of an object o_i is $\bar{c}_i$, given by $\bar{c}_i = \sum_j d_{ij}$. By imposing weights on choice parameters, a revised algorithm for arriving at the final optimal decisions was established in [14]. Following this line of exploration, we introduce in this section the notion of weighted fuzzy soft sets and discuss its applications to decision making problems.

Definition 5.1. Let $\mathcal{F}(U)$ be the set of all fuzzy sets in the universe U . Let E be a set of parameters and $A \subseteq E$. A weighted fuzzy soft set is a triple $\mathfrak{T} = (F, A, w)$ where (F, A) is a fuzzy soft set over U , and $w : A \rightarrow [0, 1]$ is a weight function specifying the weight $w_j = w(e_j)$ for each attribute $e_j \in A$.

By definition, every fuzzy soft set can be considered as a weighted fuzzy soft set. Clearly, the notion of weighted fuzzy soft sets provides a mathematical framework for modelling and analyzing the decision making problems in which all the choice parameters may not be of equal importance. These differences between the importance of parameters are characterized by the weight function in a weighted fuzzy soft set.

Table 13Tabular representation of the weighted fuzzy soft set $(\tilde{F}, A, w)$.

U	$e_1, w_1 = 0.9$	$e_2, w_2 = 0.6$	$e_3, w_3 = 0.6$
h_1	0.4	1.0	0.5
h_2	0.6	0.5	0.6
h_3	0.5	0.5	0.8
h_4	0.9	0.5	0.2
h_5	0.3	0.7	0.9

Table 14Tabular representation of the level soft set $L(\tilde{F}, A; w)$ with weighted choice values.

U	$e_1, w_1 = 0.9$	$e_2, w_2 = 0.6$	$e_3, w_3 = 0.6$	Weighted choice value ($\bar{c}_i$)
h_1	0	1	0	$\bar{c}_1 = 0.6$
h_2	0	0	1	$\bar{c}_2 = 0.6$
h_3	0	0	1	$\bar{c}_3 = 0.6$
h_4	1	0	0	$\bar{c}_4 = 0.9$
h_5	0	1	1	$\bar{c}_5 = 1.2$

As one may expect, a revised version of Algorithm 2 can be developed to deal with the decision making problems based on weighted fuzzy soft sets (see Algorithm 3). In the revised algorithm, we take the weights of parameters into consideration and compute the weighted choice values $\bar{c}_i$ instead of choice values c_i . Note that for a weighted fuzzy soft set $\mathfrak{T} = (\tilde{F}, A, w)$ the weight function $w : A \rightarrow [0, 1]$ can be taken as a threshold fuzzy set, which means that one can consider the level soft set $L(\tilde{F}, A; w)$. This will be called decision making based on the *weight function decision rule* in what follows. Sometimes it is justifiable to use this decision rule since the decision maker may require higher “membership levels” on the parameters that he puts on more emphasis. For instance, in the purchase of a house, one usually requires a higher standard for the aspect that he thinks more important.

Algorithm 3. 1. Input a weighted fuzzy soft set $\mathfrak{T} = (\tilde{F}, A, w)$.

2. Input a threshold fuzzy set $\lambda : A \rightarrow [0, 1]$ (or give a threshold value $t \in [0, 1]$; or choose the mid-level decision rule; or choose the top-level decision rule; or choose the weight function decision rule) for decision making.

3. Compute the level soft set $L(\tilde{F}, A; \lambda)$ of $\mathfrak{T}$ with respect to the threshold fuzzy set λ (or the t -level soft set $L(\tilde{F}, A; t)$; or the mid-level soft set $L(\tilde{F}, A; \text{mid})$; or the top-level soft set $L(\tilde{F}, A; \text{max})$; or the level soft set $L(\tilde{F}, A; w)$).

4. Present the level soft set $L(\tilde{F}, A; \lambda)$ (or $L(\tilde{F}, A; t)$; or $L(\tilde{F}, A; \text{mid})$; or $L(\tilde{F}, A; \text{max})$; or $L(\tilde{F}, A; w)$) in tabular form and compute the weighted choice value $\bar{c}_i$ of o_i , $\forall i$.

5. The optimal decision is to select o_k if $\bar{c}_k = \max_i \bar{c}_i$.

6. If k has more than one value then any one of o_k may be chosen.

Note that in the last step of Algorithm 3, if too many “optimal choices” are obtained, one can also go back to the second step and change the threshold (or decision rule) previously used so as to adjust the final optimal decision.

To illustrate the above idea, we reconsider Example 2.3 presented at the end of Section 2. Now assume that Mr. X have imposed the following weights for the parameters in A : for the parameter “beautiful”, $w_1 = 0.9$; for the parameter “modern”, $w_2 = 0.6$; for the parameter “cheap”, $w_3 = 0.6$. Thus we have a weight function $w : A \rightarrow [0, 1]$ and the fuzzy soft set $\mathfrak{S} = (\tilde{F}, A)$ in Example 2.3 is changed into a weighted fuzzy soft set $(\tilde{F}, A, w)$ with its tabular representation as in Table 13. Now let us just follow the weight function decision rule. Thus the weight function w is taken as the threshold and as a result we have the level soft set $L(\tilde{F}, A; w)$ with its tabular representation shown by Table 14. From Table 14, it is clear that the maximum weighted choice value is $\bar{c}_5 = 1.2$ and so the optimal decision is to select h_5 . Therefore Mr. X should buy h_5 as the best house after specifying weights for different parameters.

As an adjustable approach, one can also use other rules in the above decision making problem and in general the final optimal decision will change accordingly. For instance, if the top-level decision rule is used, we consider the top-level soft set $L(\tilde{F}, A; \text{max})$ and thus the corresponding optimal decision is to select h_4 since it has the maximum weighted choice value $\bar{c}_4 = 0.9$.

6. Conclusion

In this study, we have revisited the fuzzy soft set based decision making problems in [17,18]. We found the choice value designed for crisp soft set based decision making in [14] is not fit to solve decision making problems involving fuzzy soft sets. So the Roy–Maji method using scores in [17] is generally a useful method, whereas the counterexample given by [18] is based on an improper understanding; hence it is not sufficient for showing that the Roy–Maji method is incorrect.

At the same time, we pointed out that the Roy–Maji method has some limitations. We have proposed an adjustable approach by using level soft sets and illustrated this novel method with some concrete examples. This new proposal proves to

be not only more suitable but more feasible for some real-life applications of decision making in an imprecise environment. Moreover, we introduced the weighted fuzzy soft set and investigated its application to decision making.

To extend this work, one can apply level soft sets to other practical applications based on fuzzy soft sets, or discuss how to cope with fuzzy soft set based decision making problems under incomplete information.

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