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Pricing vulnerable power exchange options in an intensity based framework

Puneet Pasricha, Anubha Goel *

Department of Mathematics, Indian Institute of Technology Delhi, Hauz Khas, New Delhi 110016, India

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In this article, we study the valuation of a European vulnerable power exchange option in an intensity based framework. We assume that the default by the counter-party is the time of the first jump of a doubly stochastic Poisson process whose intensity is modeled by a jump–diffusion process. The dynamics of the two assets are assumed to be driven by correlated jump–diffusion processes. All the three processes are assumed to be correlated in continuous part as well as in the jump part. In the proposed framework, employing the measure-change technique, we obtain the explicit formula for the price of the power exchange option with counter-party risk. Furthermore, sensitivity analysis is given to illustrate the effects of counterparty risk on the price of the option and effect of various parameters on the option prices.

1. Introduction

Over the counter (OTC) markets form a significant proportion of the financial markets. Although OTC markets operate with hardly any rules and are less transparent unlike the organized exchanges, which allows better transaction enforcement and stricter security, these markets still have experienced tremendous growth in recent years. Derivatives products are traded actively in over-the-counter (OTC) markets by many financial institutions. Unlike the transactions on organized exchanges, the transactions on OTC have a counterparty risk that the other party may not honor its contractual obligations and hence these markets do not guaranty the promised payments between the two parties. Specifically, since the financial crisis, the counterparty credit risk of derivative products has become one of the major concern of the investors.

For modeling the credit default risk, two categories of models exist in the literature namely firm value models (structural models) and reduced form models (intensity-based models). Firm value models consider Merton (1976) [1] as the base model, which gives a mechanism of default in terms of the relationship between the assets and the liabilities at maturity time T. This basic model has further been extended by incorporating other factors like stochastic interest rates, default at any time, etc. [2]. On the other hand, reduced-form models do not specify the actual mechanism of default but model the default as a non-negative random variable with distribution depending on the economic co-variables. A detailed description of well known reduced form models can be found in [3–5].

For pricing of vulnerable options, the pioneering work is by Johnson and Stulz (1987) [6]. They assumed that the option is the only liability of the counterparty and also considered the correlation between the underlying asset and the assets of the counterparty. Later, Jarrow and Turnbull [3] followed the reduced form approach of credit risk for pricing of vulnerable options under the assumption that the underlying asset and the default intensity of the counterparty are independent. Since then, the pricing problem of vulnerable options has been studied by many researchers [7–10]. More recently, Yoon and Kim

* Corresponding author.
E-mail addresses: maz148097@maths.iitd.ac.in (P. Pasricha), maz148331@maths.iitd.ac.in (A. Goel).

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(2015) [11] studied the pricing of vulnerable European options using double Mellin transforms with the assumption that the interest rates are stochastic. Jeon et al. (2016) [12] investigated the closed formula of the pricing of vulnerable geometric Asian options. Jeon et al. (2017) [13] studied vulnerable path-dependent options using double Melling transforms. Wang et al. (2017) [14] used stochastic volatility model to price vulnerable options. Wang et al. (2017) [15] considered the valuation of the vulnerable American put options assuming that a jump–diffusion model governs the dynamics of the assets.

Power exchange options are the generalization of exchange options [16,17] and power options [18]. Margrabe (1978) [17] investigates the exchange options and their valuation. The exchange options are derivative products that allow the holder of the option to exchange an asset for another on the maturity of the option. Fischer (1978) [16] also studies the valuation of exchange options considering the scenario when the exercise price is the same as the price of an un-traded asset. Tompkins (2000) [18] discusses power options and their applications to hedge nonlinear risks. Power options and exchange options have many practical and useful applications.

Blenman and Clark (2005) [19], first time explored power exchange options as a generalization of exchange options and power options. They obtained a closed form expression for the value of power exchange options assuming that geometric Brownian motion governs the asset price dynamics. Wang (2016) [20] extended the work of Blenman and Clark (2005) [19] and considered a model with correlated jump risk in order to price the power exchange options. He proposed a jump–diffusion process with jump risk being divided into systematic and idiosyncratic components. He considered the correlation between the assets using a common jump process and correlated Brownian motion. He obtained a closed form solution for power exchange options. Li et al. (2018) [21] obtained the pricing formulas of exchange option using the change of numeraire method. Wang et al. (2017) [22] studied the pricing of power exchange options with counter-party risk in a structural framework of credit risk. They modeled the assets of counter-party risk as a jump–diffusion process and defined default when the value of counter-party's asset falls below a threshold.

In this article, we study the pricing of European power exchange options with counter-party risk following the idea of Su and Wang (2012) [10] in an intensity based framework. We model the default of the counter-party as the time of the first jump of a doubly stochastic Poisson process whose intensity process is modeled as a jump–diffusion process since many unexpected and rare event may severely affect the intensity of default. The prices of the two assets are assumed to be driven by correlated jump–diffusion processes. Also, we assume the recovery rate to be a constant, i.e., a pre-specified fraction of the payoff will be paid at maturity if the counterparty defaults. Using the measure change technique, we obtain an explicit formula for the price of the vulnerable European power exchange option. Moreover, the sensitivity analysis is also presented to study the effect of various parameters in the proposed model on option prices.

Counter-party risk (credit risk) is the risk of the lender that may arise from a borrower not being able to meet its debt obligations. One modeling challenge for pricing vulnerable options is the additional credit risk process, which complicates the mathematical tractability of the models. Most of the models in the literature on the pricing of vulnerable options have focussed on structural models of credit risk. However, due to the analytical tractability and ease of implementation and calibration, the reduced form models are more popular among the practitioners. The main contribution of this article is the reduced form modeling of credit risk to obtain the price of power exchange options. To the best of our knowledge, this is the first time that the reduced form model has been used to model the default in the context of the valuation of power exchange options. Moreover, jumps have been considered in all the stochastic processes governing the dynamics of the assets as well as the intensity of the default process.

The remainder of this paper is organized as follows. Section 2 presents the basic model setup. In Section 3, we give the explicit pricing formula of vulnerable power exchange options. Section 4 gives the sensitivity analysis with respect to different parameters in the proposed model. Section 5 concludes the paper.

2. Model setup

We begin with a finite time horizon $T > 0$. Assume that the filtered probability space $(\Omega, \mathcal{F}, Q, \mathcal{F}_{t\in[0,T]})$ models the uncertainty in the economy and E denotes the expectation with respect to the risk neutral measure Q.

Under the risk neutral measure Q, suppose the asset price dynamics for asset i, $i = 1, 2$ are given as

$$S_{i,t} = S_{i,0} \exp\left\{(r - \frac{1}{2}\sigma_i^2 - k_i^*)t + \sigma_i W_{it} + \delta_i \sum_{k=1}^{N_t} Z_k + \sum_{k=1}^{N_{i,t}} Z_{i,k}\right\} \tag{1}$$

where r is the risk free interest rate, $\sigma_i, i = 1, 2$ are the volatilities of the underlying assets. W_{1t}, W_{2t} are two correlated Brownian motions with correlation coefficient ρ_{12}. Let $S_{1,0} > 0, S_{2,0} > 0$ are initial asset prices. The common process $\{N_t, t \geq 0\}$ that reflects the jumps growing out of systematic events such as financial crisis, which affects all the entities in the financial system and is assumed to be a Poisson process with arrival rate ν. When the common jump arrives, the jump size is controlled by $Z_k, k = 1, 2, \ldots$, which is normally distributed with mean μ and variance γ^2. We also assume that $\{Z_k, k = 1, 2, \ldots\}$ are independently and identically distributed. To capture the differences in the effects of common jump components on asset price and default intensity, we use $\delta_i, i = 1, 2$. When idiosyncratic jumps happen, the corresponding jump size is assumed to be controlled by $Z_{i,k}, i = 1, 2$ which is normally distributed with mean $\mu_i, i = 1, 2$ and variance $\gamma_i^2, i = 1, 2$ and with pdf $f_i(x), i = 1, 2$. It is further assumed that $B_{1t}, B_{2t}, Z_k, Z_{1,k}$ and $Z_{2,k}$ are mutually independent and independent of all Poisson jumps.

The term k^* in the dynamics of the assets is the compensator such that the discounted asset price is a martingale under Q, and is given by

$$k^* = \nu(e^{\delta_1\mu+\frac{1}{2}\delta_1^2\gamma^2} - 1) + \nu_1(e^{\mu_1+\frac{1}{2}\gamma_1^2} - 1).$$

In this article, we are considering the reduced-form modeling approach of credit risk to model the default risk of the counterparty. In line with the reduced form approach, here the default time of the writer of the option is modeled as a first jump time of a doubly stochastic Poisson process (also known as Cox process) M_t with the intensity process λ_t. The default time of the option writer is then given by

$$\tau = \inf\{t \geq 0 : M_t > 0\}. \tag{2}$$

We assume that the intensity process λ_t of M_t is modeled as a jump–diffusion process which is a combination of Brownian motion and a jump process in the following form

$$d\lambda_t = k(\theta - \lambda_t)dt + \sigma_3 dW_{3,t} + \delta_3 Z_t dN_t + Z_{3,u} dN_{3,t}, \tag{3}$$

where k, θ, σ_3 are positive constants and δ_3 capture the effects of common jump component on default intensity. W_{3t} is a standard Brownian motion. Let $Z_{3,k}$ are i.i.d and are normally distributed with mean μ_3 and variance γ_3^2 and with pdf $f_3(x)$. Hence, all the stochastic processes are exposed to common jump risk and affected by the common jump component. Moreover, suppose that N_t, Z_t are independent of B_{1t}, B_{2t}, B_{3t}. Finally, let the covariance matrix of (W_{1t}, W_{2t}, W_{3t}) be given by

$$cov(W_{1t}, W_{2t}, W_{3t}) = \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{pmatrix} t.$$

3. Valuation of vulnerable power exchange options

In this section, we derive a closed form expression for the price of European power exchange option with credit risk. A power exchange option is a European option to exchange the power value $\gamma_1 S_1^{\beta_1}$ of one asset to the power value $\gamma_2 S_2^{\beta_2}$ of another asset. We assume $\gamma_1 = \gamma_2 = 1$. We assume that a constant fraction w (the recovery rate) of the payoff (of default-free power exchange option) is paid at maturity if the writer of the option defaults. Let $\mathcal{F}_{i,t} = \sigma\{S_{i,s} : 0 \leq s \leq T\}, i = 1, 2$ and $\mathcal{G}_t = \sigma\{\lambda_s : 0 \leq s \leq T\}$ and $\mathcal{H}_t = \sigma\{I_{\{\tau \leq s\}} : s \leq t\}$. Let $\mathcal{F}_t = \mathcal{F}_{1,t} \bigcup \mathcal{F}_{2,t} \bigcup \mathcal{G}_t \bigcup \mathcal{H}_t$ and $\mathcal{A}_t = \mathcal{F}_{1,T} \bigcup \mathcal{F}_{2,T} \bigcup \mathcal{G}_T \bigcup \mathcal{H}_t$. Further, let $G_T = \{N_T = n, N_{1T} = n_1, N_{2,T} = n_2\}$. Let $C^*(0, T)$ denote price of the vulnerable European power exchange option at time 0 with maturity T. Therefore, by the risk neutral pricing theorem, $C^*(0, T)$ is given by

$$\begin{aligned} C^*(0, T) &= e^{-rT} E[w(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ I_{\{\tau \leq T\}} + (S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ I_{\{\tau > T\}} \mid \mathcal{F}_0] \quad (4)\\ &= we^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0] + (1 - w)e^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ I_{\{\tau > T\}} \mid \mathcal{F}_0] \\ &= we^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0] + (1 - w)e^{-rT} E[E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ I_{\{\tau > T\}} \mid \mathcal{A}_0] \mid \mathcal{F}_0] \\ &= we^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0] + (1 - w)e^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ E[I_{\{\tau > T\}} \mid \mathcal{A}_0] \mid \mathcal{F}_0] \\ &= we^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0] + (1 - w)e^{-rT} I_{\{\tau > 0\}} E[e^{-\int_0^T \lambda_s ds} (S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0] \\ &= I_1 + I_2, \quad (5) \end{aligned}$$

where I_1 and I_2 are given by

$$I_1 = we^{-rT} E[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0], \tag{6}$$

$$I_2 = (1 - w)e^{-rT} I_{\{\tau > 0\}} E[e^{-\int_0^T \lambda_s ds} (S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0]. \tag{7}$$

Now, we calculate I_1 and I_2 in the following propositions.

Proposition 3.1.

$$I_2 = (1 - w)e^{-rT} I_{\{\tau > 0\}} X(0, T) E^{\lambda}(S_{2T}^{\beta_2}) \sum_{n=0}^{\infty} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \hat{Q}(G_T) \left[e^{R_1 + \frac{1}{2}V_1} \Phi(\frac{R_1 + V_1}{\sqrt{V_1}}) - \Phi(\frac{R_1}{\sqrt{V_1}}) \right]. \tag{8}$$

where $X(0, T)$ is given in Eq. (11) and $E^{\lambda}(S_{2T}^{\beta_2})$ is given in Eq. (18). $\Phi(\cdot)$ is the CDF of a standard normal random variable.

$$R_1 = \ln(S_{1,0}^{\beta_1}) + \beta_1 \tilde{\Lambda}_{1,T} - \ln(S_{2,0}^{\beta_2}) - \beta_2 \tilde{\Lambda}_{2,T} + \sum_{k=1}^{n} (\beta_1\delta_1 - \beta\delta_2)\tilde{\mu} + \sum_{k=1}^{n1} \beta_1 \tilde{\mu}_1 - \sum_{k=1}^{n2} \beta_2 \tilde{\mu}_2$$

$$V_1 = \sigma_1^2 \beta_1^2 T + \sigma_2^2 \beta_2^2 T - 2\rho_{12}\beta_1\beta_2\sigma_1\sigma_2 T + n(\beta_1\delta_1 - \beta_2\delta_2)^2\gamma^2 + n_1\beta_1^2\gamma_1^2 + n_2\beta_2^2\gamma_2^2.$$

Proof. To find I_2, we need to calculate $E[e^{-\int_0^T \lambda_s ds}(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0]$. For this, we introduce a new measure Q^λ equivalent to Q by the Radon–Nikodým derivative

$$\frac{dQ^\lambda}{dQ} = \frac{e^{-\int_0^T \lambda_s ds}}{E[e^{-\int_0^T \lambda_s ds}]}. \tag{9}$$

Using Fubini's theorem in Eq. (3), we have

$$\begin{aligned} -\int_0^T \lambda_s ds &= -\theta T - \frac{(\lambda_0-\theta)}{\kappa}[1-e^{-\kappa T}] - \int_0^T \frac{\sigma_3}{\kappa}[1-e^{-\kappa(T-u)}]dW_{3u} \\ &\quad - \int_0^T \frac{\delta_3 Z_u}{\kappa}[1-e^{-\kappa(T-u)}]dN_u - \int_0^T \frac{Z_{3,u}}{\kappa}[1-e^{-\kappa(T-u)}]dN_{3u}. \end{aligned} \tag{10}$$

Let $A(u,T,k) = [1-e^{-k(T-u)}]$ and $X(0,T) = E[e^{-\int_0^T \lambda_s ds} \mid \mathcal{F}_0]$, then we have

$$\begin{aligned} X(0,T) &= \exp\{-\theta T - \frac{(\lambda_0-\theta)}{\kappa}A(0,T,\kappa) + \frac{1}{2}\int_0^T \frac{\sigma_3^2}{\kappa^2}A^2(u,T,\kappa)du + \\ &\quad + \nu\int_0^T\int_{-\infty}^{\infty}[e^{-\frac{\delta_3 A(u,T,\kappa)}{\kappa}x} - 1]f(x)dxdu + \nu_3\int_0^T\int_{-\infty}^{\infty}[e^{-\frac{x}{\kappa}A(u,T,\kappa)} - 1]f_3(x)dxdu\} \end{aligned} \tag{11}$$

Hence, using Eqs. (10) and (11) in Eq. (9), we have

$$\begin{aligned} \frac{dQ^{(\lambda)}}{dQ} &= \exp\{-\int_0^T \frac{\sigma_3}{\kappa}A(u,T,\kappa)dW_{3u} - \int_0^T \frac{\delta_3 A(u,T,\kappa)}{\kappa}Z_u dN_u - \int_0^T \frac{A(u,T,\kappa)}{\kappa}Z_{3,u}dN_{3u} \\ &\quad -\frac{1}{2}\int_0^T \frac{\sigma_3^2}{\kappa^2}A^2(u,T,\kappa)du - \nu\int_0^T\int_{-\infty}^{\infty}[e^{-\frac{\delta_3 A(u,T,\kappa)}{\kappa}x} - 1]f(x)dxdu \\ &\quad -\nu_3\int_0^T\int_{-\infty}^{\infty}[e^{-\frac{x}{\kappa}A(u,T,\kappa)} - 1]f_3(x)dxdu\} \end{aligned}$$

Under the probability measure defined in Eq. (9), we have the following results.

(i) By Girsanav's theorem, we have that $\widehat{W}_{1t}$, $\widehat{W}_{2t}$ and $\widehat{W}_{3t}$ are standard Brownian motions under Q^λ such that

$$\begin{aligned} \widehat{W}_{1t} &= W_{1t} + \rho_{13}\int_0^t \frac{\sigma_3}{k}A(u,T,k)du, \\ \widehat{W}_{2t} &= W_{2t} + \rho_{23}\int_0^t \frac{\sigma_3}{k}A(u,T,k)du, \\ \widehat{W}_{3t} &= W_{3t} + \int_0^t \frac{\sigma_3}{k}A(u,T,k)du. \end{aligned}$$

Also, the covariance matrix of $(\widehat{W}_{1t}, \widehat{W}_{2t}, \widehat{W}_{3t})$ is same as that of (W_{1t}, W_{2t}, W_{3t})

(ii) Using Theorem $T10$ (page 241) in [23], we observe that $\{N_t, t \geq 0\}$ and $\{N_{3t}, t \geq 0\}$ are Poisson processes with respective intensities

$$\begin{aligned} \hat{\nu}_t &= \nu_t e^{-\frac{\delta_3 A(u,T,\kappa)}{\kappa}\mu + \frac{1}{2}(\frac{\delta_3 A(u,T,\kappa)}{\kappa})^2\gamma^2} \\ \hat{\nu}_{3,t} &= \nu_{3,t} e^{\frac{-A(t,T,\kappa)}{\kappa}\mu_3 + \frac{A^2(t,T,\kappa)}{2\kappa^2}\gamma_3^2} \end{aligned}$$

where $\nu_t = \nu$ and $\nu_{3,t} = \nu_3$.

(iii) Using Theorem $T10$ (page 241) in [23], we observe that Z_t and $Z_{3,t}$ are normal random variables, i.e.,

$$\begin{aligned} &Z_t \sim N(\hat{\mu}_t, \gamma^2) \\ &Z_{3,t} \sim N(\hat{\mu}_{3t}, \gamma_3^2) \end{aligned}$$

where $\hat{\mu}_t = \mu - \frac{\delta_3 A(u,T,\kappa)}{\kappa}\gamma^2$ and $\hat{\mu}_{3t} = \mu_3 - \frac{A(t,T,\kappa)}{\kappa}\gamma_3^2$.

(iv) $Z_{1,t}, Z_{2,t}, N_{1,t}$ and $N_{2,t}$ maintain the same distributions under $Q^{(\lambda)}$ and Q.

Now, I_2 can be calculated under Q^λ.

$$
\begin{aligned}
I_2 &= (1-w)e^{-rT} I_{\{\tau>0\}} E[e^{-\int_0^T \lambda_s ds} \mid \mathcal{F}_0] E[\frac{e^{-\int_0^T \lambda_s ds}}{E[e^{-\int_0^T \lambda_s ds}]} (S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ | \mathcal{F}_0] \\
&= (1-w)e^{-rT} I_{\{\tau>0\}} X(0,T) E^\lambda[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ | \mathcal{F}_0], \qquad (12)
\end{aligned}
$$

where $E^\lambda[.]$ is the expectation under Q^λ.

Now, we have to find $E^\lambda[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ | \mathcal{F}_0]$. The asset price dynamics $S_{i,t}$, $i = 1, 2$ can be obtained under Q^λ by replacing W_{it} by $\widehat{W}_{it}$ and considering the jump processes with new intensities as follows

$$
\begin{aligned}
S_{i,T} &= S_{i,0} \exp\left\{ (r - \frac{1}{2}\sigma_i^2)T - \hat{k}_i^* + \sigma_i(\widehat{W}_{iT} - \rho_{i3} \int_0^T \frac{\sigma_3}{k} A(u,T,k)du) + \delta_i \sum_{k=1}^{N_T} Z_k + \sum_{k=1}^{N_{i,T}} Z_{i,k} \right\} \\
&= S_{i0} \exp\left\{ \Lambda_{i,T} + \sigma_i \widehat{W}_{iT} + \delta_i \sum_{k=1}^{N_T} Z_k + \sum_{k=1}^{N_{i,T}} Z_{i,k} \right\}, \qquad (13)
\end{aligned}
$$

where for $i = 1, 2$, we define

$$
\Lambda_{iT} = (r - \frac{1}{2}\sigma_i^2)T - \hat{k}^* - \int_0^T \rho_{i3}\sigma_i \frac{\sigma_3}{k} A(u,T,k)du \qquad (14)
$$

$$
\hat{k}_i^* = \int_0^T \hat{\nu}(e^{\delta_i\hat{\mu} + \frac{1}{2}\delta_i^2\gamma^2} - 1) + \int_0^T \hat{\nu}_i(e^{\hat{\mu}_i + \frac{1}{2}\gamma_i^2} - 1) \qquad (15)
$$

We will calculate $E^\lambda[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ \mid \mathcal{F}_0]$ by introducing a new measure as follows:

$$
\begin{aligned}
E^\lambda[(S_{1,T}^{\beta_1} - S_{2,T}^{\beta_2})^+ | \mathcal{F}_0] &= E^\lambda[S_{2T}^{\beta_2}] E^\lambda \left[\frac{S_{2T}^{\beta_2}}{E^\lambda[S_{2T}^{\beta_2}]} \left(\frac{S_{1T}^{\beta_1}}{S_{2T}^{\beta_2}} - 1 \right)^+ | \mathcal{F}_0 \right] \\
&= E^\lambda[S_{2T}^{\beta_2}] \widehat{E} \left[\left(\frac{S_{1T}^{\beta_1}}{S_{2T}^{\beta_2}} - 1 \right)^+ | \mathcal{F}_0 \right], \qquad (16)
\end{aligned}
$$

where $\widehat{E}$ is expectation with respect to the new measure $\widehat{Q}$ equivalent to measure Q^λ given by

$$
\frac{d\widehat{Q}}{dQ^\lambda} = \frac{S_{2T}^{\beta_2}}{E^\lambda[S_{2T}^{\beta_2}]}. \qquad (17)
$$

From Eq. (13), we have

$$
\begin{aligned}
E^\lambda[S_{2,T}^{\beta_2}] &= S_{2,0}^{\beta_2} \exp\{\beta_2 \Lambda_{2,T} + \frac{1}{2}\sigma_2^2\beta_2^2 T + \int_0^T \hat{\nu}_u [e^{\delta_2\beta_2\hat{\mu}_u + \frac{1}{2}\delta_2^2\beta_2^2\gamma^2} - 1]du \\
&\quad + \int_0^T \nu_2[e^{\beta_2\mu_2 + \frac{1}{2}\beta_2^2\gamma_2^2} - 1]du\} \qquad (18)
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\frac{d\widehat{Q}}{dQ^\lambda} &= \exp\{\beta_2\sigma_2\widehat{W}_{2,T} + \beta_2\delta_2 \sum_{k=1}^{N_T} Z_k + \sum_{k=1}^{N_{2,T}} Z_{2,k} - \frac{1}{2}\sigma_2^2\beta_2^2 T \\
&\quad - \int_0^T \hat{\nu}_u[e^{\delta_2\beta_2\hat{\mu}_u + \frac{1}{2}\delta_2^2\beta_2^2\gamma^2} - 1]du - \int_0^T \nu_2[e^{\beta_2\mu_2 + \frac{1}{2}\beta_2^2\gamma_2^2} - 1]du\}
\end{aligned}
$$

Under the probability measure defined in Eq. (17), we have the following results.

(i) By Girsanav's theorem, we have that $\tilde{W}_{1,t}$, $\tilde{W}_{2,t}$ are standard Brownian motions under $\hat{Q}$ such that

$$
\begin{aligned}
\tilde{W}_{1,t} &= \widehat{W}_{1,t} - \rho_{12}\sigma_2\beta_2 t, \\
\tilde{W}_{2,t} &= \widehat{W}_{2,t} - \sigma_2\beta_2 t.
\end{aligned}
$$

Also, the correlation coefficient between $(\tilde{W}_{1,t}, \tilde{W}_{2,t})$ is ρ_{12}.

(ii) Using Theorem $T10$ (page 241) in [23], we observe that $\{N_t, t \geq 0\}$ and $\{N_{2t}, t \geq 0\}$ are Poisson processes with respective intensities

$$\tilde{\nu}_t = \hat{\nu}_t e^{\delta_2\beta_2\hat{\mu}_t + \frac{1}{2}\delta_2^2\beta_2^2\gamma^2}$$
$$\tilde{\nu}_{2,t} = \nu_2 e^{\beta_2\mu_2 + \frac{\beta_2^2}{2}\gamma_2^2}.$$

(iii) Using Theorem $T10$ (page 241) in [23], we observe that Z_t and $Z_{2,t}$ are normal random variables, i.e.,

$$Z_t \sim N(\tilde{\mu}_t, \gamma^2)$$
$$Z_{2,t} \sim N(\tilde{\mu}_{2t}, \gamma_2^2)$$

where $\tilde{\mu}_t = \hat{\mu}_t + \beta_2\delta_2\gamma^2$, $\tilde{\mu}_{2t} = \mu_2 + \beta_2\gamma_2^2$

(iv) $Z_{1,t}, N_{1,t}$ maintain the same distributions under $\hat{Q}$ and $Q^{(\lambda)}$.

The asset price dynamics $S_{i,t}$, $i = 1, 2$ can be obtained under $\hat{Q}$ as follows

$$S_{i,T} = S_{i,0}\exp\left\{\tilde{\Lambda}_{i,T} + \sigma_i\tilde{W}_{iT} + \delta_i\sum_{k=1}^{N_T} Z_k + \sum_{k=1}^{N_{i,T}} Z_{i,k}\right\} \tag{19}$$

where for $i = 1, 2$, we define

$$\tilde{\Lambda}_{1T} = (r - \frac{1}{2}\sigma_1^2)T - \tilde{k}_1 - \int_0^T \rho_{13}\sigma_1\frac{\sigma_3}{k}A(u, T, k)du + \rho_{12}\sigma_1\sigma_2\beta_2 T \tag{20}$$

$$\tilde{\Lambda}_{2T} = (r - \frac{1}{2}\sigma_2^2)T - \tilde{k}_2 - \int_0^T \rho_{13}\sigma_2\frac{\sigma_3}{k}A(u, T, k)du + \sigma_2^2\beta_2 T \tag{21}$$

$$\tilde{k}_1 = \int_0^T \nu_1(e^{\mu_1 + \frac{1}{2}\gamma_1^2} - 1)du + \int_0^T \tilde{\nu}_u(e^{\delta_1\tilde{\mu} + \frac{1}{2}\delta_1^2\gamma^2} - 1)du$$

$$\tilde{k}_2 = \int_0^T \tilde{\nu}_{2u}(e^{\tilde{\mu}_{2u} + \frac{1}{2}\gamma_2^2} - 1)du + \int_0^T \tilde{\nu}_u(e^{\delta_2\tilde{\mu} + \frac{1}{2}\delta_2^2\gamma^2} - 1)du$$

To obtain a pricing formula, we find it by conditioning over the exact number of Poisson jumps, i.e., G_T. The probability that it takes these values is given by

$$\hat{Q}(G_T) = \frac{(\int_0^T \tilde{\nu}_u du)^n (\int_0^T \tilde{\nu}_{1u} du)^{n_1} (\int_0^T \tilde{\nu}_{2u} du)^{n_2}}{n! n_1! n_2!} e^{-\int_0^T(\tilde{\nu}_u + \tilde{\nu}_{1u} + \tilde{\nu}_{3u})du}$$

From Eq. (19), we have

$$\ln(\frac{S_{1T}^{\beta_1}}{S_{2T}^{\beta_2}}) = \ln(S_{1,0}^{\beta_1}) - \ln(S_{2,0}^{\beta_2}) + \beta_1\tilde{\Lambda}_{1,T} - \beta_2\tilde{\Lambda}_{2,T}$$
$$+ \beta_1\sigma_1\tilde{W}_{1T} - \beta_2\sigma_2\tilde{W}_{2T} + (\beta_1\delta_1 - \beta_2\delta_2)\sum_{k=1}^{n} Z_k + \sum_{k=1}^{n_1}\beta_1 Z_{1,k} - \sum_{k=1}^{n_2}\beta_2 Z_{2,k}\}$$

Define the following variables

$$R_1 = \ln(S_{1,0}^{\beta_1}) + \beta_1\tilde{\Lambda}_{1,T} - \ln(S_{2,0}^{\beta_2}) - \beta_2\tilde{\Lambda}_{2,T} + \sum_{k=1}^{n}(\beta_1\delta_1 - \beta\delta_2)\tilde{\mu} + \sum_{k=1}^{n1}\beta_1\tilde{\mu}_1 - \sum_{k=1}^{n2}\beta_2\tilde{\mu}_2$$
$$V_1 = \sigma_1^2\beta_1^2 T + \sigma_2^2\beta_2^2 T - 2\rho_{12}\beta_1\beta_2\sigma_1\sigma_2 T + n(\beta_1\delta_1 - \beta_2\delta_2)^2\gamma^2 + n_1\beta_1^2\gamma_1^2 + n_2\beta_2^2\gamma_2^2$$

The above expressions show that $\ln(\frac{S_{1T}^{\beta_1}}{S_{2T}^{\beta_2}})$ is a normal random variable with mean R_1 and variance V_1. Therefore, we have

$$\widehat{E}\left[\left(\frac{S_{1T}^{\beta_1}}{S_{2T}^{\beta_2}} - 1\right)^+\right] = e^{R_1 + \frac{1}{2}V_1} N(\frac{R_1 + V_1}{\sqrt{V_1}}) - N(\frac{R_1}{\sqrt{V_1}}) \tag{22}$$

Hence, the value of the integral I_2 is given by Eq. (23). □

In order to calculate I_1, we follow the same steps as in Proposition 3.1 by changing measure from Q to $\widehat{Q}$ without going to Q^{λ}. We have the following result.

Table 1
Values of the parameters in the base case.

Parameters	Values	Parameters	Values
$S_1(0)$	100	$S_2(0)$	100
σ_1	0.2	σ_2	0.2
σ_3	0.25	w	0.4
ρ_{12}	0.4	ρ_{13}	0.6
ρ_{23}	0.6	θ	0.02
k	0.2	λ_0	0.5
ν	0.5	r	0.02
β_1	1.2	β_2	1.2
μ_1	0.02	μ_2	0.02
μ_3	0.02	γ^2	0.1
γ_1^2	0.1	γ_2^2	0.1
γ_3^2	0.02	δ_1	0.8
δ_2	0.8	δ_3	0.6

Proposition 3.2.

$$I_1 = we^{-rT}E(S_{2T}^{\beta_2})\sum_{n=0}^{\infty}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\hat{Q}(G_T)\left[e^{R_2+\frac{1}{2}V_2}N(\frac{R_2+V_2}{\sqrt{V_2}})-N(\frac{R_2}{\sqrt{V_2}})\right]. \tag{23}$$

where

$$R_2 = \ln(S_{1,0}^{\beta_1})+\beta_1\Gamma_{1,T}-\ln(S_{2,0}^{\beta_2})-\beta_2\Gamma_{2,T}+\sum_{k=1}^{n}(\beta_1\delta_1-\beta\delta_2)(\mu+\beta_2\delta_2\gamma^2)$$
$$+\sum_{k=1}^{n_1}\beta_1(\mu_1)-\sum_{k=1}^{n_2}\beta_2(\mu_2+\beta_2\gamma^2)$$

$$V_2 = \sigma_1^2\beta_1^2T+\sigma_2^2\beta_2^2T-2\rho_{12}\beta_1\beta_2\sigma_1\sigma_2T+n(\beta_1\delta_1-\beta_2\delta_2)^2\gamma^2+n_1\beta_1^2\gamma_1^2+n_2\beta_2^2\gamma_2^2.$$

$$E[S_{2,T}^{\beta_2}] = S_{2,0}^{\beta_2}\exp\{\beta_2\Gamma_{2,T}+\frac{1}{2}\sigma_2^2\beta_2^2T+\nu(e^{\delta_2\beta_2\mu+\frac{1}{2}\delta_2^2\beta_2^2\gamma^2}-1)T$$
$$+\nu_2(e^{\beta_2\mu_2+\frac{1}{2}\beta_2^2\gamma_2^2}-1)T\} \tag{24}$$

$$\Gamma_{1T} = (r-\frac{1}{2}\sigma_1^2-K_1)T+\rho_{12}\sigma_1\sigma_2\beta_2T \tag{25}$$

$$\Gamma_{2T} = (r-\frac{1}{2}\sigma_2^2-K_2)T+\sigma_2^2\beta_2T \tag{26}$$

$$K_1 = \nu_1(e^{\mu_1+\frac{1}{2}\gamma_1^2}-1)+\check{\nu}(e^{\delta_1(\mu+\beta_2\delta_2\gamma^2)+\frac{1}{2}\delta_1^2\gamma^2}-1) \tag{27}$$

$$K_2 = \check{\nu}_2(e^{(\mu_2+\beta_2\gamma_2^2)+\frac{1}{2}\gamma_2^2}-1)+\check{\nu}(e^{\delta_2(\mu+\beta_2\delta_2\gamma^2)+\frac{1}{2}\delta_2^2\gamma^2}-1) \tag{28}$$

$$\check{\nu} = \nu\exp(\delta_2\beta_2\mu+\frac{1}{2}\delta_2^2\beta_2^2\gamma^2) \tag{29}$$

$$\check{\nu_2} = \nu_2\exp(\beta_2\mu_2+\frac{1}{2}\beta_2^2\gamma_2^2) \tag{30}$$

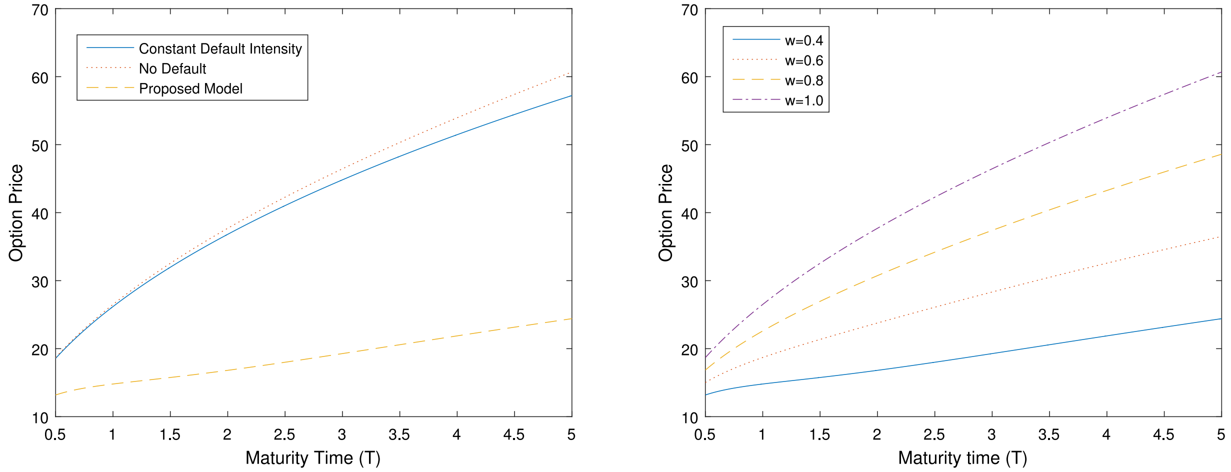
$$\check{Q}(G_T) = \frac{(\int_0^T\check{\nu}_u du)^n(\int_0^T\nu_1 du)^{n_1}(\int_0^T\check{\nu}_{2u}du)^{n_2}}{n!n_1!n_2!}e^{-\int_0^T(\check{\nu}_u+\nu_1+\check{\nu}_{3u})du} \tag{31}$$

Combining the above propositions, the price of the vulnerable European power exchange option at time 0 is $C^*(0,T)=I_1+I_2$ where I_1 and I_2 are given the previous propositions.

4. Numerical results

In this section, we give the sensitivity analysis in order to explore the impact of different parameters on option prices, when the default of the counter-party is modeled in an intensity based framework. The values of the parameters in the base case are listed in Table 1 and are chosen according to the previous literature. For performing the numerical calculations, the value of only one parameter is changed at one time while the values of the other parameters are kept the same as in the base case.

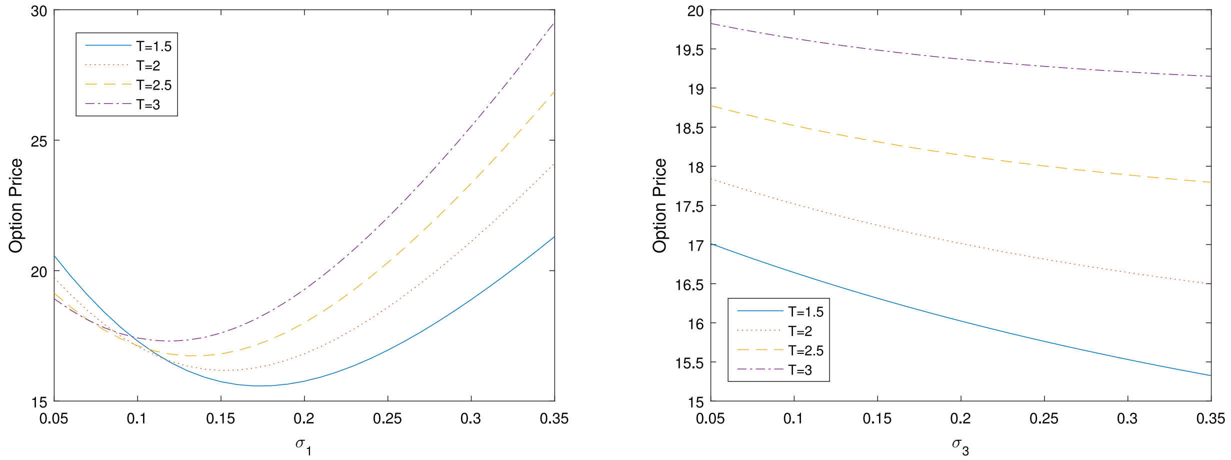
Fig. 1(a) plots option prices against time to maturity for three different scenarios namely no default case, vulnerable with constant default intensity and vulnerable under the proposed framework. From Fig. 1(a), we observe that option prices



(a) Price of Options for different models (proposed model, constant default intensity and model with no default) against the time of maturity T

(b) Price of Options against the time of maturity for different recovery rates w of counter-party

Fig. 1. Price of options against the maturity time R and recovery rate w.



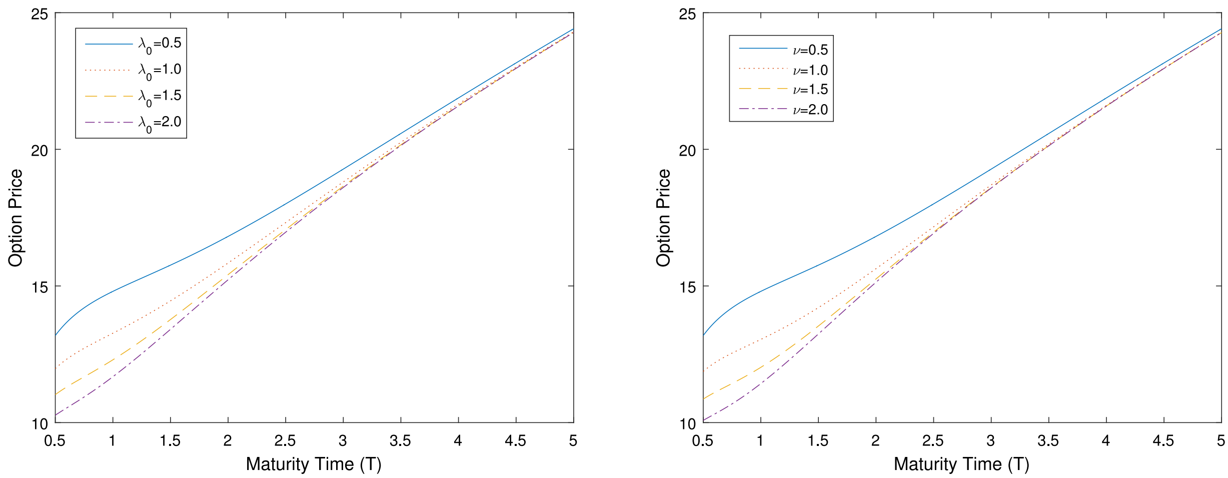
(a) Price of Options against σ_1 for maturity time T=1.5, 2, 2.5, 3

(b) Price of Options against σ_3 for maturity time T=1.5, 2, 2.5, 3

Fig. 2. Price of options against the volatility of asset price and default intensity.

increase with an increase in the time to maturity. This agrees with the fact that option prices are increasing function of time to maturity. Also, the option prices are higher when there is no risk of default by the counterparty as compared to the cases with the risk of default. Further, the option prices decrease when we consider the default intensity to be a stochastic process as compared to the case when it is a constant. The possible reason is that the default intensity of a counterparty is not necessarily a constant but time-varying since as time passes, one would have new information, beyond mere survival, that would bear on the credit quality of the counterparty. From Fig. 1(b), we observe that the option prices increase with an increase in the recovery rate w since the expected payoff at T increases as the recovery rate increases.

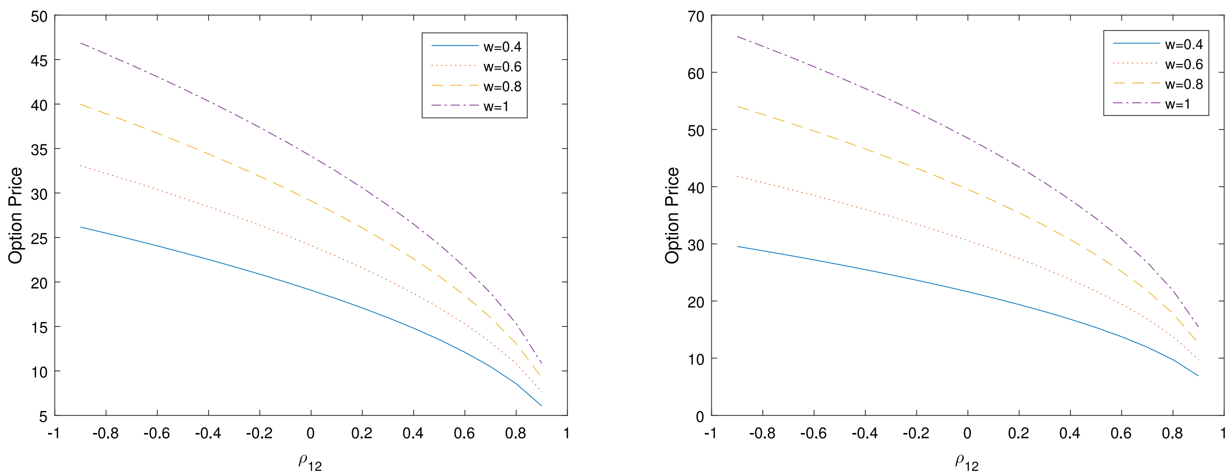
Figs. 2(a) and 2(b) show the option prices obtained by the proposed model against the volatility of asset price S_{1t} and volatility of the default intensity for different maturities, $T = 1.5, 2, 2.5, 3$. From Fig. 2(a), we observe that option prices decrease for smaller values of σ_1 and then increase with the increase in the value of the volatility σ_1 from 0.2 to 0.6. Also, we observe a U-shaped curve for all maturities. From Fig. 2(b), we observe that option prices are decreasing function of σ_3 for all T. The reason for this observation is: as the volatility of default intensity increases, the probability of default increases and hence option prices decrease.



(a) Price of Options against maturity time T for different values of initial intensity of default λ_0

(b) Price of Options against maturity time T for different values of rate ν of Poisson process

Fig. 3. Price of options against the initial intensity of default and rate of Poisson process driving intensity process.



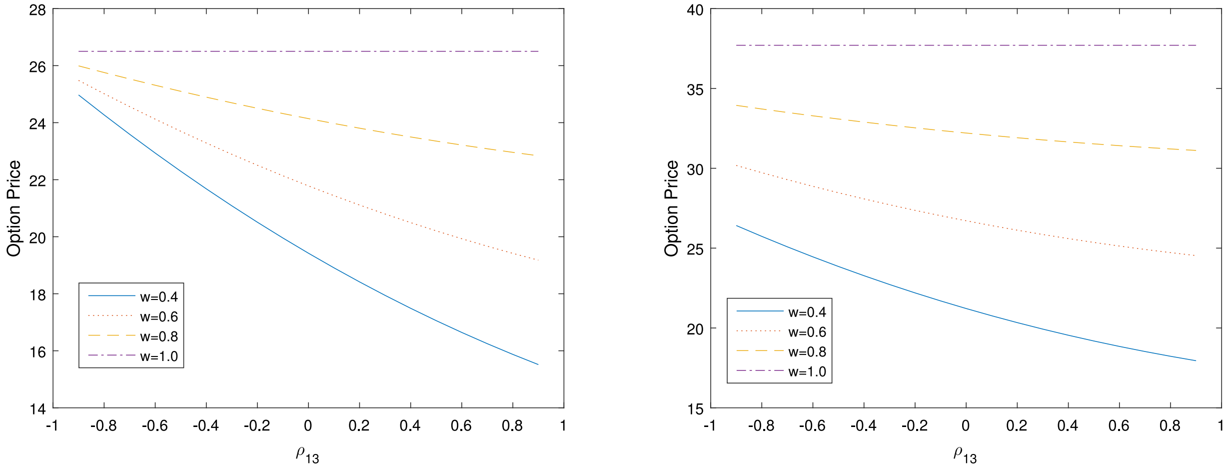
(a) Price of Options against ρ_{12} for maturity time T=1

(b) Price of Options against ρ_{12} for maturity time T=2

Fig. 4. Price of options against the correlation coefficient ρ_{12} between asset prices.

Figs. 3(a) and 3(b) show the option prices against the initial intensity of default and the arrival rate of the Poisson process governing the intensity of default respectively. The option prices decrease with an increase in both λ_0 and ν. The higher the value of initial intensity λ_0 or ν, the higher is the probability of default and hence option prices decrease.

Figs. 4(a) and 4(b) show the option prices against the ρ_{12}, i.e., the correlation between the asset prices for $T = 1$ and $T = 2$ respectively and for different values of recovery rates. The option prices are decreasing with respect to the increase in the value of ρ_{12} and option price is higher for higher recovery rate and given ρ_{12}. We observe that the option prices decrease with the increase in the value of the correlation coefficient. This observation is because of the fact that when ρ_{12} is less than zero, it is more likely that the values S_1 and S_2 may move in the opposite directions and hence the distance between S_1 and S_2 may increase at the maturity and hence higher the option price. Similarly, if $\rho > 0$, the prices are more likely to move in the same direction and hence option value is lesser than the prices when $\rho < 0$. Figs. 5(a) and 5(b) show the option prices against the ρ_{13}, i.e. correlation between the diffusion part of asset price S_{1t} and the diffusion part of default intensity for $T = 1$ and $T = 2$ respectively and for different values of recovery rates. For $w = 1$, the power exchange option is without credit risk, and hence ρ_{13} plays no role and hence price is constant. For $w \neq 1$, we observe that option prices decrease with the increase in the value of ρ_{13}.



(a) Price of Options against ρ_{13} for maturity time T=1

(b) Price of Options against ρ_{13} for maturity time T=2

Fig. 5. Price of options against the correlation coefficient ρ_{13} between asset prices.

5. Conclusion

In this article, we studied the pricing of power exchange option with counter-party risk in an intensity based credit risk model. We assumed the default of the counter-party to be the first jump time of the Cox process whose intensity is modeled as a jump–diffusion process. Using the change of measure technique, we obtained the explicit formula for the price of power exchange options. Finally, we studied the sensitivity analysis of the power exchange option prices with various parameters of the intensity of default, e.g. initial default intensity, recovery rate, correlation coefficients, etc.

References

[1] Robert C. Merton, On the pricing of corporate debt: The risk structure of interest rates, J. Finance 29 (2) (1974) 449–470.
[2] Fischer Black, John C. Cox, Valuing corporate securities: Some effects of bond indenture provisions, J. Finance 31 (2) (1976) 351–367.
[3] Robert A. Jarrow, Stuart M. Turnbull, Pricing derivatives on financial securities subject to credit risk, J. Finance 50 (1) (1995) 53–85.
[4] Darrell Duffie, Kenneth J. Singleton, Credit Risk: Pricing, Measurement, and Management, Princeton University Press, 2012.
[5] John C. Hull, Alan D. White, Valuing credit default swaps I: No counterparty default risk, J. Deriv. 8 (1) (2000) 29–40.
[6] Herb Johnson, Rene Stulz, The pricing of options with default risk, J. Finance 42 (2) (1987) 267–280.
[7] Lung-Fu Chang, Mao-Wei Hung, Valuation of vulnerable American options with correlated credit risk, Rev. Deriv. Res. 9 (2) (2006) 137–165.
[8] Peter Klein, Michael Inglis, Pricing vulnerable European options when the options payoff can increase the risk of financial distress, J. Bank. Financ. 25 (5) (2001) 993–1012.
[9] Gechun Liang, Xuemin Ren, The credit risk and pricing of OTC options, Asia-Pacific Financial Mark. 14 (1) (2007) 45–68.
[10] Xiaonan Su, Wensheng Wang, Pricing options with credit risk in a reduced form model, J. Korean Stat. Soc. 41 (4) (2012) 437–444.
[11] Ji-Hun Yoon, Jeong-Hoon Kim, The pricing of vulnerable options with double Mellin transforms, J. Math. Anal. Appl. 422 (2) (2015) 838–857.
[12] Junkee Jeon, Ji-Hun Yoon, Myungjoo Kang, Valuing vulnerable geometric Asian options, Comput. Math. Appl. 71 (2) (2016) 676–691.
[13] Junkee Jeon, Ji-Hun Yoon, Myungjoo Kang, Pricing vulnerable path-dependent options using integral transforms, J. Comput. Appl. Math. 313 (2017) 259–272.
[14] Guanying Wang, Xingchun Wang, Ke Zhou, Pricing vulnerable options with stochastic volatility, Physica A 485 (2017) 91–103.
[15] Guanying Wang, Xingchun Wang, Zhongyi Liu, Pricing vulnerable American put options under jump–diffusion processes, Probab. Engrg. Inform. Sci. 31 (2) (2017) 121–138.
[16] Stanley Fischer, Call option pricing when the exercise price is uncertain, and the valuation of index bonds, J. Finance 33 (1) (1978) 169–176.
[17] William Margrabe, The value of an option to exchange one asset for another, J. Finance 33 (1) (1978) 177–186.
[18] R.G. Tompkins, Power options: Hedging nonlinear risks, J. Risk 2 (2000) 29–46.
[19] Lloyd P. Blenman, Steven P. Clark, Power exchange options, Finance Res. Lett. 2 (2) (2005) 97–106.
[20] Xingchun Wang, Pricing power exchange options with correlated jump risk, Finance Res. Lett. 19 (2016) 90–97.
[21] Wenhan Li, Lixia Liu, Guiwen Lv, Cuixiang Li, Exchange option pricing in jump-diffusion models based on esscher transform, Comm. Statist. Theory Methods (2018) 1–12, pages.
[22] Xingchun Wang, Shiyu Song, Yongjin Wang, The valuation of power exchange options with counterparty risk and jump risk, J. Futures Mark. 37 (5) (2017) 499–521.
[23] Pierre Brémaud, Point Processes and Queues: Martingale Dynamics, Springer-Verlag, 1981.