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On a nonlinear nonautonomous predator–prey model with diffusion and distributed delay

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Abstract

In this paper, a nonlinear nonautonomous predator–prey model with diffusion and continuous distributed delay is studied, where all the parameters are time-dependent. The system, which is composed of two patches, has two species: the prey can diffuse between two patches, but the predator is confined to one patch. We first discuss the uniform persistence and global asymptotic stability of the model; after that, by constructing a suitable Lyapunov functional, some sufficient conditions for the existence of a unique almost periodic solution of the system are obtained. An example shows the feasibility of our main results.

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Keywords: Nonlinear; Nonautonomous; Predator–prey; Continuous distributed delay; Almost periodic solution; Diffusion; Globally asymptotic stability

1. Introduction

For predator–prey models without time delay and without diffusion between patches, concerning their qualitative properties, especially the properties with sound ecological meanings, such as boundedness, stability, permanence and existence of periodic solutions, many good results have already been obtained

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and collected in some monographs (e.g., [12,14]), while further scientific researches suggest that time delay effect or diffusion between patches exists in many ecological systems. Time delay effect refers to the dynamics of a predator being related to the predation in the past. Moreover, due to the spatial heterogeneity and unbalanced food resources, the migration phenomena of biological species can often occur between heterogeneous spatial environments and patches. Because these new topics have great ecological significance (see [5,9,16,18,20,22]), in recent years, scientists have paid considerable attention to them.

Song and Chen [21] proposed a predator–prey model which includes not only the dispersal processes but also some of the past states of the system, that is, the system

$$\begin{aligned}\dot{x}_1 &= x_1(a_1(t) - b_1(t)x_1 - c(t)y) + D_1(t)(x_2 - x_1),\\ \dot{x}_2 &= x_2(a_2(t) - b_2(t)x_2) + D_2(t)(x_1 - x_2),\\ \dot{y} &= y(-d(t) + e(t)x_1 - q(t)y - \beta(t)\int_{-\tau}^{0} K(s)y(t+s)\mathrm{d}s),\end{aligned} \tag{1.1}$$

where $x_1(t)$ and $y(t)$ represent the population density of prey species x and predator species y in patch 1, and $x_2(t)$ is the density of prey species x in patch 2. Predator species y is confined to patch 1, while the prey species x can diffuse between two patches. $D_i(t)$ $(i = 1, 2)$ are strictly positive functions and denote the dispersal rate of species x in the ith patch $(i = 1, 2)$, $K(s) \geqslant 0$ on $[-\tau, 0]$ is a piecewise continuous and normalized function such that $\int_{-\tau}^{0} K(s)\,\mathrm{d}s = 1$. In [21], the authors proved that system (1.1) with the initial condition $\Phi \in C([-\tau, 0]; \mathbb{R}_+^3)$, $\Phi(0) > 0$ is uniformly persistent under some appropriate conditions and obtained some sufficient conditions for the global stability of the system. Recently, by using the coincidence degree theory, Zhang and Wang [26] and Chen et al. [9] had investigated the condition which ensured the existence of a positive periodic solution of system (1.1). However, all of above three papers have not dealt with the almost periodic case. As we know, the predator–prey interactions in the real world are affected by many factors and undergo all kinds of perturbation, among which many are periodic ones (for example, those due to seasonal effects of weather, food supply, mating habits, hunting or harvesting seasons, etc.). When the periods of the periodic perturbations are rationally dependent, the system sustains periodic perturbations while if the periods are rationally independent, the effect on the system caused by the periodic perturbations is not periodic but quasi periodic or generally almost periodic. In this sense, when we study the interactions between the predator–prey model, it is more appropriate to assume that the parameters in the model system are almost periodic in the time t. It then naturally leads one to ask: What is the condition to ensure that the almost periodic system (1.1) admits a unique almost periodic solution?

On the other hand, in 1973, Ayala et al. [2] conducted experiments on fruit fly dynamics to test the validity of ten models of competitions. One of the models accounting best for the experimental results is given by

$$\begin{aligned}\dot{x}_1 &= r_1x_1\left(1 - \left(\frac{x_1}{K_1}\right)^{\theta_1} - \alpha_{12}\frac{x_2}{K_2}\right),\\ \dot{x}_2 &= r_2x_2\left(1 - \left(\frac{x_2}{K_2}\right)^{\theta_2} - \alpha_{21}\frac{x_1}{K_1}\right).\end{aligned} \tag{1.2}$$

In order to fit data in their experiments and to yield significantly more accurate results, Gilpin and Ayala [16,17] claimed that a slightly more complicated model was needed and proposed the following competition model:

$$\dot{x}_i = r_i x_i \left(1 - \left(\frac{x_i}{K_i}\right)^{\theta_i} - \sum_{j=1, j\neq i}^{n} b_{ij} \frac{x_j}{K_j}\right), \quad i = 1, 2, \ldots, n, \tag{1.3}$$

where x_i is the population density of the ith species, r_i is the intrinsic exponential growth rate of the ith species, K_i is the environment-carrying capacity of species i in the absence of competition, θ_i provides a nonlinear measure of intra-specific interference, and b_{ij} provides a measure of interspecific interference. Recently, Fan and Wang [13] argued that the nonautonomous case is more realistic and by using the coincidence degree theory, they investigated the periodic solution of the nonautonomous system (1.3). As was pointed out by Berryman [4], the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. And so, in [19], Li and Lu introduced the following nonlinear prey-competition model

$$\dot{x}_i = \begin{cases} x_i[b_i(t) - \sum_{j=1}^{m} a_{ij}(t) x_j^{\alpha_{ij}} + \sum_{j=m+1}^{n} a_{ij}(t) x_j^{\alpha_{ij}}], & 1 \leqslant i \leqslant m, \\ x_i \left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t) x_j^{\alpha_{ij}} \right], & m+1 \leqslant i \leqslant n, \end{cases} \tag{1.4}$$

where the coefficients $b_i(t),\ a_{ij}(t)\ (i, j=1, 2, .., n)$ are T-periodic continuous functions, $x_i\,(1 \leqslant i \leqslant m)$ are the density of predator species, $x_i\,(m+1 \leqslant i \leqslant n)$ are the density of prey species, $\alpha_{ij} > 0\ (i, j=1, 2, \ldots, n)$ are positive constants. Using the differential inequality theorem and the V-function method, they obtained some sufficient conditions for the existence of unique global asymptotic stability T-periodic solution of the system. For more works on nonlinear population dynamics, one could refer to [27,7,11] and the reference cited therein. However, to this day, no scholar considers the influence of the diffusion on nonlinear predator–prey system.

Stimulated by the works of [9,21,26,19,27], in this paper, we consider the following nonlinear delay diffusion predator–prey system

$$\begin{aligned} \dot{x}_1 &= x_1(a_1(t) - b_1(t)x_1^{\alpha_1} - c(t)y^{\beta_1}) + D_1(t)(x_2 - x_1), \\ \dot{x}_2 &= x_2(a_2(t) - b_2(t)x_2^{\alpha_2}) + D_2(t)(x_1 - x_2), \\ \dot{y} &= y\left(-d(t) + e(t)x_1^{\alpha_3} - q(t)y^{\beta_2} - p(t)\int_{-\tau}^{0} K(s)y^{\beta_3}(t+s)\,\mathrm{d}s\right), \end{aligned} \tag{1.5}$$

where x_1 and y are the population density of prey species x and predator species y in patch 1, respectively, and x_2 is the density of species x in patch 2. Species y is confined to patch 1 while species x can diffuse between two patches. $\alpha_i, \beta_i, i = 1, 2, 3$ are all positive constants. $D_i(t)(i = 1, 2)$ are strictly positive functions and denote the dispersal rate of species x in the ith patch $(i=1, 2)$, $p(t)y(t)\int_{-\tau}^{0} K(s)y^{\beta_3}(t+s)\,\mathrm{d}s$ represents the influence of the past state of species y. For more background and biological adjustment, one could refer to [19,21,26] and the reference cited therein. Obviously, system (1.1) is the special case of system (1.5). To the best of the author's knowledge, this is the first time such a system is proposed. The aim of this paper is as follows:

(1) obtaining sufficient condition which guarantees the permanence of system (1.5),
(2) obtaining sufficient condition which guarantees the global asymptotic stability of system (1.5),
(3) for almost periodic case, obtaining sufficient condition which guarantees the existence of the unique almost periodic solution of system (1.5).

Now we let $f^l = \inf_{t\in R} f(t)$ and $f^u = \sup_{t\in R} f(t)$ for a continuous and bounded function.
In system (1.5), we always assume

(H_1) $a_i(t), b_i(t), D_i(t)$ $(i=1,2)$, $c(t), d(t), e(t), q(t)$ and $p(t)$ are continuous and strictly positive functions, which satisfy

$$\min\{a_i^l, b_i^l, c^l, e^l, p^l, D_i^l, d^l, q^l\} > 0,$$
$$\max\{a_i^u, b_i^u, c^u, e^u, p^u, D_i^u, d^u, q^u\} < +\infty.$$

(H_2) $K(s) \geqslant 0$ on $[-\tau, 0]$, $(0 \leqslant \tau < \infty)$, and $K(s)$ is a piecewise continuous and normalized function such that $\int_{-\tau}^{0} K(s)\,\mathrm{d}s = 1$.

We adopt the following notations and concepts throughout this paper. Let $x = (x_1, x_2, y) \in \mathbb{R}^3_+ = \{x \in \mathbb{R}^3, x_i \geqslant 0\ (i=1,2), y \geqslant 0\}$. Denote $x > 0$ if $x \in \operatorname{Int}\mathbb{R}^3_+$. For ecological reasons, we consider system (1.5) only in $\operatorname{Int}\mathbb{R}^3_+$. Let $C^+ = C([-\tau, 0]; \mathbb{R}^3_+)$ denote the Banach space of all nonnegative continuous functions with

$$||\Phi|| = \sup_{s\in[-\tau,0]} |\Phi(s)| \quad \text{for } \Phi \in C^+.$$

Then, if we choose the initial function space of system (1.5) to be C^+, it is easy to see that, for any $\Phi = (\Phi_1, \Phi_2, \Phi_3) \in C^+$ and $\Phi(0) > 0$, there exists $\alpha \in (0, +\infty)$ and a unique solution $x(t, \Phi)$ of system (1.5) on $[-\tau, \alpha)$, which remains positive for all $t \in [0, \alpha)$; such solutions of system (1.5) are called positive solutions. Hence, in the rest of this paper, we always assume that

$$\Phi \in C^+, \quad \Phi(0) > 0. \tag{1.6}$$

This paper is organized as follows. In Section 2, we examine the dynamics of the general case of (1.5) and establish sufficient criteria for boundedness, permanence and globally asymptotic stability. In Section 3, we will explore the existence and uniqueness of positive almost periodic solution of (1.5) when the parameters in (1.5) are continuous, almost periodic functions. An example illustrates the feasibility of main results. For more recent works on periodic solution and stability of the population dynamic system, one could refer to [8,6,7,10,23,24]. For more work concerned with the influence of time delay (due to stage structure or gestation) and diffusion on population dynamics, one could refer to [23,24,1,15].

2. General nonautonomous case

In this section, we shall explore the dynamics of system (1.5) and present some results including the boundedness, uniform persistent and the globally asymptotic stability of the system.

Definition 2.1. System (1.5) is said to be uniformly persistent if there exists a compact region $D \subseteq \mathrm{Int}\,\mathbb{R}^3_+$ such that every solution $x(t)=(x_1(t), x_2(t), y(t))^{\mathrm{T}}$ of system (1.5) with initial condition (1.6) eventually enters and remains in region D.

Lemma 2.1. *If $a>0, b>0$ and $\dot{x} \geqslant (\leqslant) b-ax$, when $t \geqslant 0$ and $x(0)>0$, we have*

$$x(t) \geqslant (\leqslant) \frac{b}{a}\left[1+\left(\frac{ax(0)}{b}-1\right)\mathrm{e}^{-at}\right].$$

Proof. From $\dot{x} \geqslant b-ax$, it follows

$$\frac{\mathrm{d}(x\mathrm{e}^{at})}{\mathrm{d}t} \geqslant b\mathrm{e}^{at}.$$

By integrating the above inequality from 0 to t, it follows

$$x(t)\mathrm{e}^{at}-x(0) \geqslant \frac{b}{a}(\mathrm{e}^{at}-1)$$

or

$$x(t) \geqslant x(0)\mathrm{e}^{-at}+\frac{b}{a}(1-\mathrm{e}^{-at}).$$

That is

$$x(t) \geqslant \frac{b}{a}\left[1+\left(\frac{ax(0)}{b}-1\right)\mathrm{e}^{-at}\right].$$

Similar to the above analysis, one could prove the case $\dot{x} \leqslant b-ax$. □

Lemma 2.2. *If $a>0, b>0$ and $\dot{x} \geqslant (\leqslant) x(b-ax^{\alpha})$, where α is a positive constant, when $t \geqslant 0$ and $x(0)>0$, we have*

$$x(t) \geqslant (\leqslant) \left(\frac{b}{a}\right)^{1/\alpha}\left[1+\left(\frac{bx^{-\alpha}(0)}{a}-1\right)\mathrm{e}^{-b\alpha t}\right]^{-1/\alpha}.$$

Proof. From $\dot{x} \geqslant x(b-ax^{\alpha})$, it follows

$$-\frac{\mathrm{d}(x^{-\alpha})}{\mathrm{d}t} \geqslant (bx^{-\alpha}-a)\alpha$$

or

$$\frac{\mathrm{d}(x^{-\alpha})}{\mathrm{d}t} \leqslant -b\alpha x^{-\alpha}+a\alpha.$$

From Lemma 2.1 and the above inequality, it follows

$$x^{-\alpha}(t) \leqslant \frac{a}{b}\left[1+\left(\frac{bx^{-\alpha}(0)}{a}-1\right)\mathrm{e}^{-b\alpha t}\right], \quad t \geqslant 0,$$

and so,

$$x(t) \geqslant \left(\frac{b}{a}\right)^{1/\alpha} \left[1 + \left(\frac{bx^{-\alpha}(0)}{a} - 1\right) \mathrm{e}^{-b\alpha t}\right]^{-1/\alpha}.$$

Similarly, we can prove the case $\dot{x} \leqslant x(b - ax^{\alpha})$. □

Lemma 2.3. *Let* $x(t) = (x_1(t), x_2(t), y(t))^{\mathrm{T}}$ *be any solution of system* (1.5) *with the initial conditions* (1.6). *Then there exists a* $T > 0$ *such that*

$$x_i(t) \leqslant M_1^{\varepsilon}, \quad (i = 1, 2), \quad y(t) \leqslant M_2^{\varepsilon} \quad for \ t \geqslant T, \tag{2.1}$$

where

$$\begin{aligned} &M_1^{\varepsilon} > M_1^*, && M_2^{\varepsilon} > M_2^*, \\ &M_1^* = \max\left\{\left(\frac{a_1^u}{b_1^l}\right)^{1/\alpha_1}, \left(\frac{a_2^u}{b_2^l}\right)^{1/\alpha_2}\right\}, && M_2^* = \left(\frac{e^u (M_1^*)^{\alpha_3}}{q^l}\right)^{1/\beta_2}. \end{aligned} \tag{2.2}$$

Proof. We define

$$V(t) = \max\{x_1(t), x_2(t)\}.$$

Calculating the upper right derivative of V along the positive solution of system (1.5), we have the following possibilities:

(P_1) If $x_1(t) > x_2(t)$ or $x_1(t) = x_2(t)$ and $\dot{x}_1(t) \geqslant \dot{x}_2(t)$,

$$\begin{aligned} D^+V(t) &= \dot{x}_1 = x_1(a_1(t) - b_1(t)x_1^{\alpha_1} - c(t)y^{\beta_1}) + D_1(t)(x_2 - x_1) \\ &\leqslant x_1(a_1^u - b_1^l x_1^{\alpha_1}). \end{aligned}$$

Then by Lemma 2.2, for arbitrary small positive constant ε, there exists $T_1' > 0$ such that

$$V(t) \leqslant \left(\frac{a_1^u}{b_1^l}\right)^{1/\alpha_1} + \varepsilon \text{ for } t \geqslant T_1'.$$

(P_2) If $x_1(t) < x_2(t)$ or $x_1(t) = x_2(t)$ and $\dot{x}_1(t) \leqslant \dot{x}_2(t)$,

$$\begin{aligned} D^+V(t) &= \dot{x}_2(t) = x_2(a_2(t) - b_2(t)x_2^{\alpha_2}) + D_2(t)(x_1 - x_2) \\ &\leqslant x_2(a_2^u - b_2^l x_2^{\alpha_2}). \end{aligned}$$

Then by Lemma 2.2, for above $\varepsilon > 0$, there exists $T_2' > 0$ such that

$$V(t) \leqslant \left(\frac{a_2^u}{b_2^l}\right)^{1/\alpha_2} + \varepsilon \text{ for } t \geqslant T_2'.$$

Now let $T' = \max\{T_1', T_2'\}$ and $M_1^* = \max\{(\frac{a_1^u}{b_1^l})^{1/\alpha_1}, (\frac{a_2^u}{b_2^l})^{1/\alpha_2}\}$, then one has

$$V(t) = \max\{x_1(t), x_2(t)\} \leqslant M_1^* + \varepsilon \triangleq M_1^\varepsilon \text{ for } t \geqslant T'.$$

From the third equation of system (1.5), one has

$$\dot{y} \leqslant y(e^u (M_1^\varepsilon)^{\alpha_3} - q^l y^{\beta_2}), \quad t \geqslant T' \tag{2.3}$$

From Lemma 2.2, there exists a large enough $T'' > T' > 0$ such that for $t \geqslant T''$, every solution $y(t)$ of system (1.5) with initial condition (1.6) satisfies

$$y(t) \leqslant \left(\frac{e^u (M_1^\varepsilon)^{\alpha_3}}{q^l}\right)^{1/\beta_2} + \varepsilon.$$

The above inequality implies that there exists a positive integer N, such that for $t \geqslant T''$,

$$y(t) \leqslant \left(\frac{e^u (M_1^*)^{\alpha_3}}{q^l}\right)^{1/\beta_2} + N\varepsilon \triangleq M_2^\varepsilon. \tag{2.4}$$

If we take $T = T''$, then the conclusion of Lemma 2.3 follows. □

Theorem 2.1. *Suppose that*

(H$_3$) $a_1^l > c^u (M_2^*)^{\beta_1}$,
(H$_4$) $e^l (m_1^*)^{\alpha_3} > d^u + p^u (M_2^*)^{\beta_3}$.

Then system (1.5) *is uniformly persistent, i.e., there exist* $T^* > T$ *and* $m_i^* > 0$, $(i = 1, 2)$ *such that*

$$m_1^* \leqslant x_i(t) \leqslant M_1^\varepsilon \quad (i = 1, 2), \quad m_2^* \leqslant y(t) \leqslant M_2^\varepsilon \ for \ t \geqslant T^*, \tag{2.5}$$

where M_i^ε $(i = 1, 2)$ *are defined by* (2.2) *and*

$$m_1^* = \frac{1}{2} \min \left\{ \left(\frac{a_1^l - c^u (M_2^*)^{\beta_1}}{b_1^u}\right)^{1/\alpha_1}, \left(\frac{a_2^l}{b_2^u}\right)^{1/\alpha_2} \right\},$$

$$m_2^* = \frac{1}{2}\left(\frac{e^l (m_1^*)^{\alpha_3} - d^u - \beta^u (M_2^*)^{\beta_3}}{q^u}\right)^{1/\beta_2}. \tag{2.6}$$

Proof. We define

$$V_1(t) = \min\{x_1(t), x_2(t)\}.$$

Calculating the lower right derivative of V_1 along the positive solution of system (1.5), we have the following possibilities:

(Q$_1$) If $x_1(t) < x_2(t)$ or $x_1(t) = x_2(t)$ and $\dot{x}_1(t) \leqslant \dot{x}_2(t)$,

$$\begin{aligned} D_+ V_1(t) = \dot{x}_1 &= x_1(a_1(t) - b_1(t) x_1^{\alpha_1} - c(t) y^{\beta_1}) + D_1(t)(x_2 - x_1) \\ &\geqslant x_1(a_1^l - c^u (M_2^\varepsilon)^{\beta_1} - b_1^u x_1^{\alpha_1}) \text{ for } t \geqslant T. \end{aligned}$$

Then by Lemma 2.2, for above $\varepsilon > 0$, there exists $T_3 > T > 0$, such that

$$V_1(t) \geqslant \left(\frac{a_1^u - c^u (M_2^\varepsilon)^{\beta_1}}{b_1^u}\right)^{1/\alpha_1} - \varepsilon \text{ for } t \geqslant T_3.$$

The above inequality implies that there exists a positive integer N_1, such that for

$$V_1(t) \geqslant \left(\frac{a_1^u - c^u (M_2^*)^{\beta_1}}{b_1^u}\right)^{1/\alpha_1} - N_1\varepsilon \text{ for } t \geqslant T_3.$$

Noting that ε is an arbitrary small positive number, we could choose ε small enough, such that

$$N_1\varepsilon < \frac{1}{2}\left(\frac{a_1^l - c^u (M_2^*)^{\beta_1}}{b_1^u}\right)^{1/\alpha_1}.$$

And so

$$V_1(t) \geqslant \frac{1}{2}\left(\frac{a_1^l - c^u (M_2^*)^{\beta_1}}{b_1^u}\right)^{1/\alpha_1} \geqslant m_1^* \text{ for } t \geqslant T_3. \tag{2.7}$$

(Q_2) If $x_1(t) > x_2(t)$ or $x_1(t) = x_2(t)$ and $\dot{x}_1(t) \geqslant \dot{x}_2(t)$,

$$\begin{aligned} D_+ V_1(t) &= \dot{x}_2 = x_2(a_2(t) - b_2(t)x_2^{\alpha_2}) + D_2(t)(x_1 - x_2) \\ &\geqslant x_2(a_2^l - b_2^u x_2^{\alpha_2}). \end{aligned}$$

Then by Lemma 2.2, for above $\varepsilon > 0$, there exists $T_4 > 0$, such that

$$V_1(t) \geqslant \left(\frac{a_2^l}{b_2^u}\right)^{1/\alpha_2} - \varepsilon \text{ for } t \geqslant T_4.$$

Noting that ε is an arbitrary small positive number, we could take ε small enough, such that

$$\varepsilon < \frac{1}{2}\left(\frac{a_2^l}{b_2^u}\right)^{1/\alpha_2}.$$

And so

$$V_1(t) \geqslant \frac{1}{2}\left(\frac{a_2^l}{b_2^u}\right)^{1/\alpha_2} \geqslant m_1^* \text{ for } t \geqslant T_4. \tag{2.8}$$

Now let $T_5 = \max\{T_3, T_4\}$, then

$$V_1(t) = \min\{x_1(t), x_2(t)\} \geqslant m_1^* \text{ for } t \geqslant T_5. \tag{2.9}$$

From the third equation of system (1.5) and (2.9), one has

$$\dot{y} \geqslant y(-d^u + e^l (m_1^*)^{\alpha_3} - p^u (M_2^\varepsilon)^{\beta_3} - q^u y^{\beta_2}), \quad t \geqslant T_5 + \tau. \tag{2.10}$$

From Lemma 2.2, there exists a large enough $T_6 > T_5 + \tau > 0$ such that for $t \geqslant T_6$, every solution $y(t)$ of system (1.5) with initial condition (1.6) satisfies

$$y(t) \geqslant \left(\frac{e^l (m_1^*)^{\alpha_3} - d^u - p^u (M_2^\varepsilon)^{\beta_3}}{q^l} \right)^{1/\beta_2} - \varepsilon.$$

From this, we know that there exists a positive integer N_2, such that for $t \geqslant T_6$,

$$y(t) \geqslant \left(\frac{e^l (m_1^*)^{\alpha_3} - d^u - p^u (M_2^*)^{\beta_3}}{q^l} \right)^{1/\beta_2} - N_2 \varepsilon. \tag{2.11}$$

Noting that ε is an arbitrary small positive number, we could choose ε that was small enough, such that

$$N_2 \varepsilon < \frac{1}{2} \left(\frac{e^l (m_1^*)^{\alpha_3} - d^u - p^u (M_2^\varepsilon)^{\beta_3}}{q^l} \right)^{1/\beta_2}.$$

Then,

$$y(t) \geqslant \frac{1}{2} \left(\frac{e^l (m_1^*)^{\alpha_3} - d^u - p^u (M_2^*)^{\beta_3}}{q^l} \right)^{1/\beta_2} = m_2^*. \tag{2.12}$$

If we take $T^* = T_6$, then the conclusion of Theorem 2.1 follows. □

Definition 2.2. A bounded positive solution $x(t) = (x_1(t), x_2(t), y(t))^{\mathrm{T}}$ of (1.5) is said to be globally asymptotically stable if for any other positive bounded solution $\hat{x}(t) = (\hat{x}_1(t), \hat{x}_2(t), \hat{y}(t))^{\mathrm{T}}$ of (1.5), the following equality holds:

$$\lim_{t \to +\infty} \left[\sum_{j=1}^{2} |x_j(t) - \hat{x}_j(t)| + |y(t) - \hat{y}(t)| \right] = 0.$$

The following lemma is from [3], and will be employed in establishing the globally asymptotic stability of (1.5).

Lemma 2.4. *Let h be a real number and f be a nonnegative function defined on $[h; +\infty)$ such that f is integrable on $[h; +\infty)$ and is uniformly continuous on $[h; +\infty)$, then* $\lim_{t \to +\infty} f(t) = 0$.

Noting that the method used in [21] to prove the stability property of system (1.1) could not apply to nonlinear case. Here we will adopt the idea of Zhao and Chen [27] and Chen and Shi [7] to prove the global attractivity of the positive solution of system (1.5). Suppose $X(t) = (x_1(t), x_2(t), y(t))^{\mathrm{T}}$ is a positive solution of system (1.5) with coefficients satisfying conditions of Theorem 2.1, then there exist $T > 0$ such that

$$m_1^* \leqslant x_i(t) \leqslant M_1^\varepsilon \quad (i = 1, 2), \qquad m_2^* \leqslant y(t) \leqslant M_2^\varepsilon \quad \text{for } t \geqslant T.$$

Now take c to be any positive constant such that $0 < c \leqslant \min\{m_1^*, m_2^*\}$. Making the change of variable $u_i(t) = x_i(t)/c$, $v(t) = y(t)/c$. Then system (1.5) is transformed to

$$\begin{aligned}
\dot{u}_1 &= u_1(a_1(t) - b_1(t)c^{\alpha_1}u_1^{\alpha_1} - c(t)c^{\beta_1}v^{\beta_1}) + D_1(t)(u_2 - u_1),\\
\dot{u}_2 &= u_2(a_2(t) - b_2(t)c^{\alpha_2}u_2^{\alpha_2}) + D_2(t)(u_1 - u_2),\\
\dot{v} &= v(-d(t) + e(t)c^{\alpha_3}u_1^{\alpha_3} - q(t)c^{\beta_2}v^{\beta_2} - p(t)c^{\beta_3}\int_{-\tau}^{0} K(s)v^{\beta_3}(t+s)\,\mathrm{d}s).
\end{aligned} \tag{2.13}$$

Apparently, system (2.13) is equivalent to system (1.5), which implies that (2.13) is permanent under the conditions of Theorem 2.1.

Theorem 2.2. *In addition to* (H_1)–(H_4), *assume further that*

(H_5) $\alpha_1 \geqslant \max\{\alpha_3, 1\}$, $\alpha_2 \geqslant 1$, $\beta_2 \geqslant \max\{\beta_1, \beta_3\}$.
(H_6) *There exists positive constants* ρ_1, ρ_2, ρ_3 *and* c $(0 < c \leqslant \min\{m_1^*, m_2^*\})$ *such that*

$$\min_{t\in R}\{A_1(t), A_2(t), A_3(t)\} > 0,$$

where

$$\begin{aligned}
A_1(t) &= \rho_1 b_1(t)c^{\alpha_1} - \rho_3 c^{\alpha_3}e(t) - \rho_2\frac{cD_2(t)}{m_1^*},\\
A_2(t) &= \rho_2 c^{\alpha_2}b_2(t) - \rho_1\frac{cD_1(t)}{m_1^*},\\
A_3(t) &= \rho_3 c^{\beta_2}q(t) - \rho_1 c(t)c^{\beta_1} - \rho_3 c^{\beta_3}\int_{-\tau}^{0} K(s)p(t-s)\,\mathrm{d}s.
\end{aligned}$$

Then system (1.5) *with initial condition* (1.6) *is globally asymptotic stable*.

Proof. To finish the proof of Theorem 2.2, we only need to prove that system (2.13) is globally asymptotically stable.

Let $X(t) = (u_1(t), u_2(t), v(t))^{\mathrm{T}}$ and $\hat{X}(t) = (\hat{u}_1(t), \hat{u}_2(t), \hat{v}(t))^{\mathrm{T}}$ be any two positive solutions of (2.13). It follows from Theorem 2.1 and the relation of systems (1.5), (2.13) that there exists a large enough $T > 0$, M_i^ε and m_i^* (defined by (2.1) and (2.6), respectively) such that for all $t \geqslant T$,

$$0 < \frac{m_1^*}{c} \leqslant u_i(t), \hat{u}_i(t) \leqslant \frac{M_1^\varepsilon}{c}, 0 < \frac{m_2^*}{c} \leqslant v(t), \hat{v}(t) \leqslant \frac{M_2^\varepsilon}{c}. \tag{2.14}$$

Consider a Lyapunov functional defined by

$$\begin{aligned}
V(t) = {} & \sum_{j=1}^{2}\rho_j|\ln\{u_j(t)\} - \ln\{\hat{u}_j(t)\}| + \rho_3|\ln\{v(t)\} - \ln\{\hat{v}(t)\}|\\
& + \rho_3 c^{\beta_3}\int_{-\tau}^{0}\int_{t+s}^{t} K(s)p(\theta - s)|v^{\beta_3}(\theta) - \hat{v}^{\beta_3}(\theta)|\,\mathrm{d}\theta\,\mathrm{d}s, \quad t \geqslant T.
\end{aligned} \tag{2.15}$$

Now we calculate and estimate the upper right derivative of $V(t)$ along the solutions of system (1.2),

$$\begin{aligned}D^+V(t) \leqslant & -\rho_1 b_1(t)c^{\alpha_1}|u_1^{\alpha_1}-\hat{u}_1^{\alpha_1}| + \rho_1 c(t)c^{\beta_1}|v^{\beta_1}-\hat{v}^{\beta_1}| \\ & - \rho_2 c^{\alpha_2} b_2(t)|u_2^{\alpha_2}-\hat{u}_2^{\alpha_2}| + \rho_1\overline{D}_1(t) + \rho_2\overline{D}_2(t) \\ & + \rho_3 c^{\alpha_3} e(t)|u_1^{\alpha_3}-\hat{u}_1^{\alpha_3}| - \rho_3 c^{\beta_2} q(t)|v^{\beta_2}-\hat{v}^{\beta_2}| \\ & + c^{\beta_3}\rho_3 p(t)\int_{-\tau}^0 K(s)|v^{\beta_3}(t+s)-\hat{v}^{\beta_3}(t+s)|\,\mathrm{d}s \\ & + \rho_3 c^{\beta_3}\int_{-\tau}^0 K(s)p(t-s)|v^{\beta_3}(t)-\hat{v}^{\beta_3}(t)|\,\mathrm{d}s \\ & - c^{\beta_3}\rho_3 p(t)\int_{-\tau}^0 K(s)|v^{\beta_3}(t+s)-\hat{v}^{\beta_3}(t+s)|\,\mathrm{d}s,\end{aligned}$$

where

$$\overline{D}_1(t) = \begin{cases} D_1(t)\left(\frac{u_2(t)}{u_1(t)} - \frac{\hat{u}_2(t)}{\hat{u}_1(t)}\right), & u_1(t)\geqslant \hat{u}_1(t), \\ D_1(t)\left(\frac{\hat{u}_2(t)}{\hat{u}_1(t)} - \frac{u_2(t)}{u_1(t)}\right), & u_1(t) < \hat{u}_1(t). \end{cases}$$

$$\overline{D}_2(t) = \begin{cases} D_2(t)\left(\frac{u_1(t)}{u_2(t)} - \frac{\hat{u}_1(t)}{\hat{u}_2(t)}\right), & u_2(t)\geqslant \hat{u}_2(t), \\ D_2(t)\left(\frac{\hat{u}_1(t)}{\hat{u}_2(t)} - \frac{u_1(t)}{u_2(t)}\right), & u_2(t) < \hat{u}_2(t). \end{cases}$$

Similar to the analysis of [6, p. 39], we have

$$\overline{D}_1(t) \leqslant \frac{cD_1(t)}{m_1^*}|u_2(t)-\hat{u}_2(t)|, \quad \overline{D}_2(t) \leqslant \frac{cD_2(t)}{m_1^*}|u_1(t)-\hat{u}_1(t)|. \tag{2.16}$$

From $u_i(t) = x_i(t)/c$, we know that $u_i(t)\geqslant 1$, $\hat{u}_i(t)\geqslant 1$, $(i=1,2)$. Since when $a\geqslant 1$, $a\geqslant b$ and $x>0$, $y=a^x-b^x$ is increasing function, for $\alpha_1\geqslant \max\{\alpha_3, 1\}$ we get

$$\begin{aligned}&|u_1^{\alpha_3}-\hat{u}_1^{\alpha_3}| \leqslant |u_1^{\alpha_1}-\hat{u}_1^{\alpha_1}|, \\ &|u_1-\hat{u}_1| \leqslant |u_1^{\alpha_1}-\hat{u}_1^{\alpha_1}|.\end{aligned} \tag{2.17}$$

And for $\alpha_2\geqslant 1$, we have

$$|u_2-\hat{u}_2| \leqslant |u_2^{\alpha_2}-\hat{u}_2^{\alpha_2}|. \tag{2.18}$$

Also, from $v(t) = y(t)/c \geqslant 1$, $\beta_2 \geqslant \max\{\beta_1, \beta_3\}$ it follows

$$\begin{aligned}&|v^{\beta_1}-\hat{v}^{\beta_1}| \leqslant |v^{\beta_2}-\hat{v}^{\beta_2}|, \\ &|v^{\beta_3}-\hat{v}^{\beta_3}| \leqslant |v^{\beta_2}-\hat{v}^{\beta_2}|.\end{aligned} \tag{2.19}$$

By applying (2.16)–(2.19), it follows

$$\begin{aligned} D^+V(t) \leqslant & -\left(\rho_1 b_1(t)c^{\alpha_1} - \rho_3 c^{\alpha_3} e(t) - \rho_2 \frac{cD_2(t)}{m_1^*}\right)|u_1^{\alpha_1} - \hat{u}_1^{\alpha_1}| \\ & - \left(\rho_2 c^{\alpha_2} b_2(t) - \rho_1 \frac{cD_1(t)}{m_1^*}\right)|u_2^{\alpha_2} - \hat{u}_2^{\alpha_2}| \\ & - \left(\rho_3 c^{\beta_2} q(t) - \rho_1 c(t) c^{\beta_1} - \rho_3 c^{\beta_3} \int_{-\tau}^{0} K(s) p(t-s)\,\mathrm{d}s\right)|v^{\beta_2} - \hat{v}^{\beta_2}|. \end{aligned}$$

From the conditions (H_6) of Theorem 2.2, it follows that there exists a positive constant $\alpha > 0$ and large enough $T > 0$ such that

$$D^+V(t) \leqslant -\alpha\left(\sum_{j=1}^{2} |u_j^{\alpha_j}(t) - \hat{u}_j^{\alpha_j}(t)| + |v^{\beta_2}(t) - \hat{v}^{\beta_2}(t)|\right), \quad t \geqslant T. \tag{2.20}$$

Integrating both sides of (2.20) from T to t produces

$$V(t) + \alpha \int_T^t \left(\sum_{j=1}^{2} |u_j^{\alpha_j}(s) - \hat{u}_j^{\alpha_j}(s)| + |v^{\beta_2}(s) - \hat{v}^{\beta_2}(s)|\right) \mathrm{d}s \leqslant V(T) < +\infty, \quad t \geqslant T.$$

Then

$$\int_T^t \left(\sum_{j=1}^{2} |u_j^{\alpha_j}(s) - \hat{u}_j^{\alpha_j}(s)| + |v^{\beta_2}(s) - \hat{v}^{\beta_2}(s)|\right) \mathrm{d}s \leqslant \alpha^{-1} V(T) < +\infty, \quad t \geqslant T.$$

Hence,

$$\sum_{j=1}^{2} |u_j^{\alpha_j}(t) - \hat{u}_j^{\alpha_j}(t)| + |v^{\beta_2}(t) - \hat{v}^{\beta_2}(t)| \in L^1([T, +\infty)).$$

The boundedness of $u_i(t)$ and $v(t)$ and the ultimate boundedness of $\hat{u}_i(t)$ and $\hat{v}(t)$ imply that $u_i(t)$, $\hat{u}_i(t)$, $v(t)$ and $\hat{v}(t)$ all have bounded derivatives for $t \geqslant T$ (from the equations satisfied by them). Then it follows that $\sum_{j=1}^{2} |u_j^{\alpha_j}(t) - \hat{u}_j^{\alpha_j}(t)| + |v^{\beta_2}(t) - \hat{v}^{\beta_2}(t)|$ is uniformly continuous on $[T, +\infty)$. By Lemma 2.4, we have

$$\lim_{t \to +\infty} \left(\sum_{j=1}^{2} |u_j^{\alpha_j}(t) - \hat{u}_j^{\alpha_j}(t)| + |v^{\beta_2}(t) - \hat{v}^{\beta_2}(t)|\right) = 0.$$

From this, it easily follows

$$\lim_{t \to +\infty} |u_j(t) - \hat{u}_j(t)| = 0, \quad j = 1, 2.$$

$$\lim_{t \to +\infty} |v(t) - \hat{v}(t)| = 0.$$

The proof of Theorem 2.2 is complete. □

3. Almost periodic case

As we point out in the introduction section "it is more appropriate to assume that the parameters in the model system are almost periodic in the time t", so this section is devoted to the study of the almost periodic solution of system (1.5), and we assume

(H_7) $a_i(t), b_i(t), D_i(t)$ $(i = 1, 2)$; $d(t), e(t), q(t)$ and $p(t)$ are continuous positive almost periodic functions.

Obviously, condition (H_7) holds implying that condition (H_1) holds.

Theoretically, one can investigate the existence and uniqueness of almost periodic solutions for functional differential equations by using Lyapunov functional as follows [25, p. 388]:

Let $C = C([-r, 0], \mathbb{R}^n)$, $H \in \mathbb{R}_+$ or $H = +\infty$. Denote $C_H = \{\varphi : \varphi \in C, |\varphi| < H\}$, $|\varphi| = \sup_{\theta\in[-r,0]} |\varphi(\theta)|$.

Consider the system

$$\dot{x}(t) = f(t, x_t), \tag{3.1}$$

where $f(t, \phi)$ is continuous in $(t, \phi) \in \mathbb{R} \times C_H$ and almost periodic in t uniformly for $\phi \in C_H$, $C_H \subseteq C$, $\forall\alpha > 0$, $\exists L(\alpha) > 0$ such that $|f(t, \varphi)| \leqslant L(\alpha)$ as $t \in \mathbb{R}$, $\varphi \in C_\alpha$.

To investigate the almost periodic solution of system (3.1), we introduce the associate product system of system (3.1)

$$\dot{x}(t) = f(t, x_t), \dot{y}(t) = f(t, y_t). \tag{3.2}$$

Lemma 3.1. *Suppose that for* $t \geqslant 0$, $\phi, \psi \in C_H$, *there exists a continuous Lyapunov functional* $V(t, \phi, \psi)$ *which has the following properties*:

(I) $u(|\phi - \psi|) \leqslant V(t, \phi, \psi) \leqslant v(|\phi - \psi|)$, *where* $u(s)$ *and* $v(s)$ *are continuous nondecreasing functions, and* $u(s) \to 0$ *as* $s \to 0$.
(II) $|V(t, \phi_1, \psi_1) - V(t, \phi_2, \psi_2)| \leqslant L(|(\phi_1 - \phi_2) - (\psi_1 - \psi_2)|)$, *where L is a positive constant.*
(III) $\dot{V}_{(3.2)}(t, \phi, \psi) \leqslant -\lambda V(t, \phi, \psi)$, *where* λ *is a positive constant.*
Moreover, one assumes that (3.1) *has a solution* $x(t, \sigma, \varphi)$ *such that* $|x_t(\sigma, \varphi)| \leqslant \mathrm{H}_1$ *for* $t \geqslant \sigma \geqslant 0$, $\mathrm{H} > \mathrm{H}_1 > 0$. *Then system* (3.1) *has a unique almost periodic solution which is uniformly asymptotically stable*.

According to Lemma 3.1, we first obtained a sufficient condition which guarantees the existence of a bounded solution of (1.5), and then constructed an adaptive Lyapunov functional for (1.5).

Lemma 3.2. *Assume that* (H_2)–(H_7) *hold, then there exists a bounded solution* $x(t) = (x_1(t), x_2(t), y(t))^{\mathrm{T}}$ *of initial problem* (1.5)–(1.6) *with*

$$0 < m_1^* \leqslant x_i(t) \leqslant M_1^*, \quad i = 1, 2, \quad 0 < m_2^* \leqslant y(t) \leqslant M_2^*, \quad t \in \mathbb{R}.$$

The proofs of Lemma 3.2 are standard and similar to that of [22, Lemma 2], we therefore omit them here.

Here we state the main results of this section.

Theorem 3.1. *If* (H_2)–(H_7) *hold, then almost periodic system* (1.5)–(1.6) *have a unique positive almost periodic solution which is globally asymptotically stable.*

Proof. From Theorem 2.1, (2.13) and Lemma 3.2 we know that the following system:

$$\begin{aligned}
\dot{u}_1 &= a_1(t) - D_1(t) - b_1(t)c^{\alpha_1}\mathrm{e}^{\alpha_1 u_1} - c(t)c^{\beta_1}\mathrm{e}^{\beta_1 u_3} + D_1(t)\mathrm{e}^{u_2-u_1},\\
\dot{u}_2 &= a_2(t) - D_2(t) - b_2(t)c^{\alpha_2}\mathrm{e}^{\alpha_2 u_2} + D_2(t)\mathrm{e}^{u_1-u_2},\\
\dot{v} &= -d(t) + e(t)c^{\alpha_3}\mathrm{e}^{\alpha_3 u_1} - q(t)c^{\beta_2}\mathrm{e}^{\beta_2 v(t)} - p(t)c^{\beta_3}\int_{-\tau}^{0} K(s)\mathrm{e}^{\beta_3 v(s)}\,\mathrm{d}s
\end{aligned} \tag{3.3}$$

has a bounded solution $U(t) = (u_1(t), u_2(t), u_3(t))^{\mathrm{T}}$ on $\mathbb{R}$ satisfying

$$\ln\left\{\frac{m_1^*}{c}\right\} \leqslant u_i(t) \leqslant \ln\left\{\frac{M_1^\varepsilon}{c}\right\}, \quad i = 1, 2.$$
$$\ln\left\{\frac{m_2^*}{c}\right\} \leqslant v(t) \leqslant \ln\left\{\frac{M_2^\varepsilon}{c}\right\}, \quad (t \in \mathbb{R}).$$

Consider the associated product system of (3.3)

$$\begin{aligned}
\dot{u}_1 &= a_1(t) - D_1(t) - b_1(t)c^{\alpha_1}\mathrm{e}^{\alpha_1 u_1} - c(t)c^{\beta_1}\mathrm{e}^{\beta_1 u_3} + D_1(t)\mathrm{e}^{u_2-u_1},\\
\dot{u}_2 &= a_2(t) - D_2(t) - b_2(t)c^{\alpha_2}\mathrm{e}^{\alpha_2 u_2} + D_2(t)\mathrm{e}^{u_1-u_2},\\
\dot{v} &= -d(t) + e(t)c^{\alpha_3}\mathrm{e}^{\alpha_3 u_1} - q(t)c^{\beta_2}\mathrm{e}^{\beta_2 v} - p(t)c^{\beta_3}\int_{-\tau}^{0} K(s)\mathrm{e}^{\beta_3 v(s)}\,\mathrm{d}s,\\
\dot{x}_1 &= a_1(t) - D_1(t) - b_1(t)c^{\alpha_1}\mathrm{e}^{\alpha_1 x_1} - c(t)c^{\beta_1}\mathrm{e}^{\beta_1 x_3} + D_1(t)\mathrm{e}^{x_2-x_1},\\
\dot{x}_2 &= a_2(t) - D_2(t) - b_2(t)c^{\alpha_2}\mathrm{e}^{\alpha_2 x_2} + D_2(t)\mathrm{e}^{x_1-x_2},\\
\dot{y} &= -d(t) + e(t)c^{\alpha_3}\mathrm{e}^{\alpha_3 x_1} - q(t)c^{\beta_2}\mathrm{e}^{\beta_2 y} - p(t)c^{\beta_3}\int_{-\tau}^{0} K(s)\mathrm{e}^{\beta_3 y(s)}\,\mathrm{d}s.
\end{aligned} \tag{3.4}$$

Construct a Lyapunov functional $V(t) = V(t, x_t, y_t)$ as follows:

$$\begin{aligned}
V(t) &= \sum_{j=1}^{2} \rho_j |u_j(t) - x_j(t)| + \rho_3 |v(t) - y(t)|\\
&\quad + \rho_3 c^{\beta_3}\int_{-\tau}^{0}\int_{t+s}^{t} K(s)p(\theta - s)|\exp\{\beta_3 v(\theta)\} - \exp\{\beta_3 y(\theta)\}|\,\mathrm{d}\theta\,\mathrm{d}s, \quad t \geqslant 0.
\end{aligned} \tag{3.5}$$

It is easy to know that conditions (I) and (II) of Lemma 3.1 are satisfied. By direct computation, similar to that of the analysis of Theorem 2.2, we have

$$\begin{aligned}
D^+V(t) \leqslant &-\left(\rho_1 b_1(t)c^{\alpha_1} - \rho_3 c^{\alpha_3}e(t) - \rho_2\frac{cD_2(t)}{m_1^*}\right)|\mathrm{e}^{\alpha_1 u_1} - \mathrm{e}^{\alpha_1 x_1}|\\
&-\left(\rho_2 c^{\alpha_2} b_2(t) - \rho_1\frac{cD_1(t)}{m_1^*}\right)|\mathrm{e}^{\alpha_2 u_2} - \mathrm{e}^{\alpha_2 x_2}|\\
&-\left(\rho_3 c^{\beta_2} q(t) - \rho_1 c(t)c^{\beta_1} - \rho_3 c^{\beta_3}\int_{-\tau}^{0} K(s)p(t-s)\,\mathrm{d}s\right)|\mathrm{e}^{\beta_2 v} - \mathrm{e}^{\beta_2 y}|.
\end{aligned}$$

Then similar to the analysis of [27, p. 575], under the assumption of Theorem 3.1, one could deduce that

$$D^+V(t) \leqslant -\eta_1 \left[\sum_{j=1}^{2} \rho_j |x_j(t) - u_j(t)| + \rho_3 |v(t) - y(t)| \right],$$

where η_1 is a positive constant. So, from the definition of $V(t)$ it immediately follows that there exists a positive constant γ such that

$$\dot{V}_{(3.4)} \leqslant -\gamma V(t), \quad t \in \mathbb{R}. \tag{3.6}$$

From Lemma 3.1, there exists a unique positive almost periodic solution $U(t) = (u_1(t), u_2(t), u_3(t))^{\mathrm{T}}$ of Eq. (3.3) which is uniformly asymptotic stable, which means that there exists a unique positive almost periodic solution $x(t) = (\mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}, \mathrm{e}^{u_3(t)})^{\mathrm{T}}$ of Eq. (1.5). This ends the proof of Theorem 3.1. □

Example 3.1. Consider the following example:

$$\dot{x}_1 = x_1 \left(4 - 2x_1 - \left(\frac{11}{8} + \frac{\sin \sqrt{2}t}{8} \right) y \right) + \left(2 + \frac{\cos \sqrt{3}t}{2} \right) (x_2 - x_1),$$
$$\dot{x}_2 = x_2 (5 + \sin 2t - 3x_2) + \left(\frac{3}{32} + \frac{\sin \sqrt{5}t}{64} \right) (x_1 - x_2),$$

$$\dot{y} = y \left(- \left(\frac{3}{16} + \frac{\cos \sqrt{7}t}{16} \right) + \frac{5}{4} x_1 - \left(2 - \frac{\sin \sqrt{11}t}{4} \right) y \right.$$
$$\left. - \left(\frac{3}{32} + \frac{\cos \sqrt{13}t}{32} \right) \int_{-\tau}^{0} K(s) y(t+s)\, \mathrm{d}s \right), \tag{3.7}$$

where $K(s)$ satisfies condition (H_2). In this case, $\alpha_i = 1$, $\beta_i = 1$, $i = 1, 2, 3$, need not make the change of variable $u_i = x_i/c$, $v = y/c$, and so it is easy to examine that the coefficients of system (3.1) satisfy all assumptions in Theorems 2.1, 2.2 and 3.1. Thus, system (3.1) is permanent; also, system (3.1) has a unique almost periodic solution which is globally asymptotically stable.

4. Conclusion

In this paper, a nonlinear nonautonomous predator–prey model with dispersion as well as continuous time delay is considered. Attentions are paid to the topic such as persistence, global attractivity and the existence of an unique positive almost periodic solution of the system. Some interesting results are obtained, which can be seen as the generalization of the main results of [21]. Those results have further application on population dynamics.

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References

[1] W.G. Aiello, H.I. Freedman, A time-delay model of single-species growth with stage structure, Math. Biosci. 101 (2) (1990) 139–153.
[2] F.J. Ayala, M.E. Gilpin, J.G. Eherenfeld, Competition between species: theoretical models and experimental tests, Theor. Popul. Biol. 4 (1973) 331–356.
[3] I. Barbălat, Systems d'equations differential d'oscillations nonlinearies, Rev. Roumaine Math. Pure Appl. 4 (2) (1959) 267–270.
[4] A.A. Berryman, The origins and evolution of predator–prey theory, Ecology 75 (1992) 1530–1535.
[5] L.S. Chen, Mathematical Models and Methods in Ecology, Science Press, Beijing, 1988 (in Chinese).
[6] F.D. Chen, Positive periodic solutions of neutral Lotka–Volterra system with feedback control, Appl. Math. Comput., in press.
[7] F.D. Chen, S.J. Lin, Periodicity in a Logistic type system with several delays, Comput. Math. Appl. 48 (1–2) (2004) 35–44.
[8] F.D. Chen, F.X. Lin, X.X. Chen, Sufficient conditions for the existence positive periodic solutions of a class of neutral delay models with feedback control, Appl. Math. Comput. 158 (1) (2004) 45–68.
[9] S.H. Chen, F. Wang, T. Young, Existence of positive periodic solution for nonautonomous predator–prey system with diffusion and time delay, J. Comput. Appl. Math. 159 (2003) 375–386.
[10] F.D. Chen, et al., Periodicity in a food-limited population model with toxicants and state dependent delays, J. Math. Anal. Appl. 288 (1) (2003) 132–142.
[11] F.D. Chen et al., Positive periodic solutions of a class of non-autonomous single species population model with delays and feedback control, Acta Math. Sin., in press.
[12] L. Edelstein-Keshet, Mathematical Models in Biology, Random House, New York, 1988.
[13] M. Fan, K. Wang, Global periodic solutions of a generalized n-species Gilpin–Ayala competition model, Comput. Math. Appl. 40 (2000) 1141–1151.
[14] C.H. Feng, On the existence and uniqueness of almost periodic solutions for delay Logistic equations, Appl. Math. Comput. 136 (2003) 487–494.
[15] U. Foryś, Hopf bifurcation in Marchuk's model of immune reactions, Math. Comput. Modelling 34 (7–8) (2001) 725–735.
[16] M.E. Gilpin, F.J. Ayala, Global models of growth and competition, Proc. Nat. Acad. Sci. USA 70 (1973) 3590–3593.
[17] M.E. Gilpin, F.J. Ayala, Schoener's model and Drosophila competition, Theor. Popul. Biol. 9 (1976) 12–14.
[18] S.A. Levin, Dispersion and population interactions, Am. Nat. 108 (1974) 207–228.
[19] C.R. Li, S.J. Lu, The qualitative analysis of N-species periodic coefficient, nonlinear relation, prey-competition systems, Appl. Math-JCU 12 (2) (1997) 147–156 (in Chinese).
[20] N. MacDonald, Time Lags in Biological Models, Lecture Notes in Biomathematics, vol. 27, Springer, New York, 1978.
[21] X.Y. Song, L.S. Chen, Persistence and global stability for nonautonomous predator–prey system with diffusion and time delay, Comput. Math. Appl. 35 (6) (1998) 33–40.
[22] X.Y. Song, L.S. Chen, Persistence and periodic orbits for two-species predator–prey system with diffusion, Can. Appl. Math. Quart. 6 (3) (1998) 233–244.

[23] R. Xu, M.A. Rui, J. Chaplain, F.A. Davidson, Periodic solutions for a delayed predator–prey model of prey dispersal in two-patch environments, Nonlinear Anal. Real World Appl. 5 (1) (2004) 183–206.
[24] R. Xu, M.A. Rui, J. Chaplain, F.A. Davidson, Periodic solution of a Lotka–Volterra predator–prey model with dispersion and time delays, Appl. Math. Comput. 148 (2) (2004) 537–560.
[25] Z.X. Zheng, Theory of Functional Differential Equations, Anhui Education Press, 1994.
[26] Z.Q. Zhang, Z.C. Wang, Periodic solutions for nonautonomous predator–prey system with diffusion and time delay, Hiroshima Math. J. 31 (3) (2001) 371–381.
[27] J.D. Zhao, W.C. Chen, The qualitative analysis of N-species nonlinear prey-competition systems, Appl. Math. Comput. 149 (2) (2004) 567–576.