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Numerical solutions of nonlinear evolution equations using variational iteration method

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Abstract

The variational iteration method is used to solve three kinds of nonlinear partial differential equations, coupled nonlinear reaction diffusion equations, Hirota–Satsuma coupled KdV system and Drinefel'd–Sokolov–Wilson equations. Numerical solutions obtained by the variational iteration method are compared with the exact solutions, revealing that the obtained solutions are of high accuracy. He's variational iteration method is introduced to overcome the difficulty arising in calculating Adomian polynomial in Adomian method. The method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in mathematical physics.

MSC: 76M30; 78M30; 35A15

Keywords: Variational iteration method; Nonlinear partial differential equations; Lagrange multiplier

1. Introduction

Large varieties of physical, chemical, and biological phenomena are governed by nonlinear evolution equations. The importance of obtaining the exact solutions, if available, of those nonlinear equations facilitates the verification of numerical solvers and aids in the stability analysis of solutions. Analytical solutions to nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. Many authors mainly had paid attention to study solutions of nonlinear equations by using various methods, such as, inverse scattering method [3], homotopy perturbation method [13], modified Lindstedt–Poincaré method [15,21], and variational iteration methods [1,4,5,18,19,23–26]. The reaction diffusion equations (RDEs) have recently attracted considerable attention, partly due to their occurrence in many fields of science, in physics as well as in chemistry or biology, partly due to their interesting features and rich variety of properties of their solutions [30]. The evaluations of dynamics quantities are governed by nonlinear partial differential equation. The processes of diffusion and reaction each play essential roles in the dynamics of many systems, e.g., in many systems [6,28,29].

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The variational iteration method was first proposed by He [9–12] and was successfully applied to autonomous ordinary differential equations by He [14], to nonlinear polycrystalline solids [22], and other fields. The combination of a perturbation method, variational iteration method, method of variation of constants and averaging method established an approximate solution of one degree of freedom weakly nonlinear system in [7]. The variational iteration method has many merits and has much advantages over the tanh method [27].

The motivation of this paper is to extend the analysis of the variational iteration method proposed by He [2,9–12,16,17] to solve three different types of nonlinear equations, namely, the coupled nonlinear RDEs, Hirota–Satsuma coupled KdV system, and Drinefel'd–Sokolov–Wilson equations.

The rest of this paper is arranged as follows: In Section 2, we simply provide the mathematical framework of the variational iteration method. In Section 3, in order to illustrate the method, three models in physics are investigated. Finally, some conclusion and discussion are provided.

2. Methods and its applications

For the purpose of illustration of the methodology to the proposed method, using variational iteration method, we begin by considering a differential equation in the formal form,

$$Lu + Nu = g(x), \tag{1}$$

where L is a linear operator, N a nonlinear operator, and $g(x)$ an inhomogeneous term.

According to the variational iteration method, we can construct a correct functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\{Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\}\,\mathrm{d}\tau, \tag{2}$$

where λ is a general Lagrangian multiplier [9–12], which can be identified optimally via the variational theory, the subscript n denotes the nth order approximation, $\tilde{u}_n$ is considered as a restricted variation [9–12], or see then He's monographs [16,17] i.e., $\delta\tilde{u}_n = 0$.

3. Applications

In this section, the extended variational iteration method is used to find approximate solutions of three kinds of equations in mathematical physics, namely, the coupled nonlinear RDEs, Hirota–Satsuma coupled KdV system, and Drinefel'd–Sokolov–Wilson equations, and compared with that obtained by other methods.

3.1. Example 1: coupled nonlinear RDEs

Let us first consider the coupled nonlinear RDE [20]

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(A\frac{\partial u}{\partial x}\right) + Cu + Gu, \tag{3}$$

where

$$A = A(u), \quad C = C(u), \quad G = G(u), \quad u = \begin{bmatrix} u \\ v \end{bmatrix}, \quad A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \quad G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}. \tag{4}$$

Eq. (3), of the type known as RDE, has attracted considerable attention because of its various applications in science and its interesting features from a mathematical point of view.

Consider the set of coupled equations describing the evaluation of two interacting populations by choosing

$$u = \begin{bmatrix} u \\ v \end{bmatrix}, \quad A = \begin{bmatrix} k \\ k \end{bmatrix}, \quad C = \begin{bmatrix} uv \\ u^2 \end{bmatrix}, \quad G = \begin{bmatrix} -Bu \\ Bu \end{bmatrix}. \tag{5}$$

From Eq. (5), we get the following system of PDEs as:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + u^2 v - Bu, \tag{6}$$

$$\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2} - u^2 v + Bu, \tag{7}$$

where B is a constant and k is the diffusion coefficient for the concentration of u and v with the initial conditions [20]

$$u(x,0) = A_1 + A_1\tanh(cx), \quad v(x,0) = A_0' - A_1\tanh(cx), \tag{8}$$

where $A_1 = 1$ and $A_0' = 1.5$.

To solve the system of Eqs. (6), (7) by means of variational iteration method, we construct a correctional functional which reads

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda_1\{u_{nt} - ku_{nxx} - \widetilde{u_n^2 v_n} + Bu_n\}\,\mathrm{d}\tau, \tag{9}$$

$$v_{n+1}(x,t) = v_n(x,t) + \int_0^t \lambda_2\{v_{nt} - kv_{nxx} + \widetilde{u_n^2 v_n} - Bu_n\}\,\mathrm{d}\tau, \tag{10}$$

where λ_1 and λ_2 are general Lagrangian multipliers are to be determined later, and $\widetilde{u_n^2 v_n}$, denotes restricted variations i.e., $\delta\,\widetilde{u_n^2 v_n} = 0$. With the aid of the above correction functional stationary, we obtain

$$\lambda_1'(\tau) = 0, \quad \lambda_2'(\tau) = 0, \tag{11a}$$

$$1 + \lambda_1(\tau)\rfloor_{\tau=t} = 0, \quad 1 + \lambda_2(\tau)\rfloor_{\tau=t} = 0. \tag{11b}$$

Eqs. (11a) are called Lagrange–Euler equations, and Eqs. (11b), one natural boundary conditions.

The Lagrange multipliers, therefore, can be identified as $\lambda_1 = \lambda_2 = -1$, and the following variational iteration formula are given by:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \{u_{nt} - ku_{nxx} - u_n^2 v_n + Bu_n\}\,\mathrm{d}\tau, \tag{12}$$

$$v_{n+1}(x,t) = v_n(x,t) - \int_0^t \{v_{nt} - kv_{nxx} + u_n^2 v_n - Bu_n\}\,\mathrm{d}\tau. \tag{13}$$

We start with initial approximations $u(x,0)$ and $v(x,0)$ given by Eq. (8) with the above iteration formula (12), (13), we get

$$u_1(x,t) = 1 + \tanh cx + \frac{t}{2} - 2tc^2\tanh cx + 2tc^2\tanh^3 cx + t\tanh cx - \frac{t}{2}\tanh^2 cx - t\tanh^3 cx, \tag{14}$$

$$v_1(x,t) = \frac{3}{2} - \tanh cx - \frac{t}{2} + 2tc^2\tanh cx - 2tc^2\tanh^3 cx - t\tanh cx + \frac{t}{2}\tanh^2 cx + t\tanh^3 cx, \tag{15}$$

$$\begin{aligned} u_2(x,t) = {} & \left\{1 + \frac{t}{2} - 0.03t^4 - 0.04t^3 + 0.25t^2 - \frac{t^2c^2}{2}\right\} + \tanh(cx)\{-2tc^2 + 0.25t^2 + 0.33t^3c^2 - 0.4t^3 \\ & - 0.187t^4 + 0.37t^4c^2 + 8t^2c^4 - 5t^2c^2 + 1 + t\} \\ & + \tanh^2(cx)\left\{-\frac{3}{2}t^2 - 1.08t^3 - 0.28t^4 + 1.5t^4c^2 - 0.67t^3c^4\right. \\ & \left. + 2.67t^3c^2 - 1.5t^4c^3t^4 + 3t^2c^2 - 0.5t\right\} \end{aligned}$$

$$
\begin{aligned}
&+ \tanh^3(cx)\{2tc^2 - 0.17t^3 - 1.7t^2 + 3.33t^3c^2 + 2t^4c^6 \\
&+ 0.312t^4 - 4t^3c^4 + 3.37t^4c^2 - 3t^4c^4 - 20t^2c^4 + 14t^2c^2 - t\} \\
&+ \tanh^4(cx)\{-5.33t^3c^2 + 2.29t^3 + 1.03t^4 + 4.5t^4c^4 - 4.5t^4c^2 + 1.25t^2 + 1.3t^3c^4 - 2.5t^2c^2\} \\
&+ \tanh^5(cx)\left\{\frac{3}{2}t^3 - 9t^2c^2 + \frac{3t^2}{2} + 0.187t^4 - 7.66t^3c^2 + 7.99t^3c^4\right. \\
&\left.-6t^4c^6 - 3.37t^4c^2 + 9t^4c^4 + 12t^2c^4\right\} \\
&+ \tanh^6(cx)\{-1.166t^3 - 1.09t^4 + 2.66t^3c^2 - 0.66t^3c^4 + 4.5t^4c^2 - 4.5t^4c^4\} \\
&+ \tanh^7(cx)\{4t^3c^4 - 0.99t^3 - 5.6t^4 - 9t^4c^4 + 6t^4c^6 + 4t^3c^2 + 4.125t^4c^2\} \\
&+ \tanh^8(cx)\left\{-\frac{3}{2}t^4c^2 + \frac{3}{2}t^4c^4 + 0.375t^4\right\} + \tanh^9(cx)\{0.25t^4 - 2t^4c^6 - 1.5t^4c^2 + 3t^4c^4\}, \quad (16)
\end{aligned}
$$

$$
\begin{aligned}
v_2(x,t) = &\left\{1.5 - \frac{t}{2} + 0.03t^4 + 0.04t^3 - 0.25t^2 + \frac{t^2c^2}{2}\right\} \\
&+ \tanh(cx)\{2tc^2 - 0.25t^2 - 0.33t^3c^2 + 0.4t^3 + 0.187t^4 - 0.37t^4c^2 - 8t^2c^4 + 5t^2c^2 - 1 - t\} \\
&+ \tanh^2(cx)\left\{\frac{3}{2}t^2 + 1.08t^3 + 0.28t^4 - 1.5t^4c^2 + 0.67t^3c^4\right. \\
&\left.-2.67t^3c^2 + 1.5t^4c^3t^4 - 3t^2c^2 + 0.5t\right\} \\
&+ \tanh^3(cx)\{-2tc^2 + 0.17t^3 + 1.7t^2 - 3.33t^3c^2 - 2t^4c^6 \\
&- 0.312t^4 + 4t^3c^4 - 3.37t^4c^2 + 3t^4c^4 + 20t^2c^4 - 14t^2c^2 + t\} \\
&+ \tanh^4(cx)\{5.33t^3c^2 - 2.29t^3 - 1.03t^4 - 4.5t^4c^4 + 4.5t^4c^2 - 1.25t^2 - 1.3t^3c^4 + 2.5t^2c^2\} \\
&+ \tanh^5(cx)\left\{-\frac{3}{2}t^3 + 9t^2c^2 - \frac{3t^2}{2} - 0.187t^4 + 7.66t^3c^2\right. \\
&\left.- 7.99t^3c^4 + 6t^4c^6 + 3.37t^4c^2 - 9t^4c^4 - 12t^2c^4\right\} \\
&+ \tanh^6(cx)\{1.166t^3 + 1.09t^4 - 2.66t^3c^2 + 0.66t^3c^4 - 4.5t^4c^2 + 4.5t^4c^4\} \\
&+ \tanh^7(cx)\{-4t^3c^4 + 0.99t^3 + 5.6t^4 + 9t^4c^4 - 6t^4c^6 - 4t^3c^2 - 4.125t^4c^2\} \\
&+ \tanh^8(cx)\left\{\frac{3}{2}t^4c^2 - \frac{3}{2}t^4c^4 - 0.375t^4\right\} + \tanh^9(cx)\{-0.25t^4 + 2t^4c^6 + 1.5t^4c^2 - 3t^4c^4\}. \quad (17)
\end{aligned}
$$

The remaining components of $u_n(x,t)$ and $v_n(x,t)$ can be completely determined such that each term is determined by the previous terms using Eqs. (12), (13).

It is to be noted that the exact solutions of $u(x,t)$ and $v(x,t)$ are given by [20]

$$u(x,t) = A_1(1 + \tanh(c(x - vt))), \quad v(x,t) = A_0' - A_1 \tanh(c(x - vt)), \tag{18}$$

where c and v are constants.

The behavior of the approximate solutions obtained by variational iteration method with the exact solutions (Eqs. (18)) for different values of times is shown in Figs. 1(a) and (b). The comparison shows that the two solutions obtained are in excellent agreement.

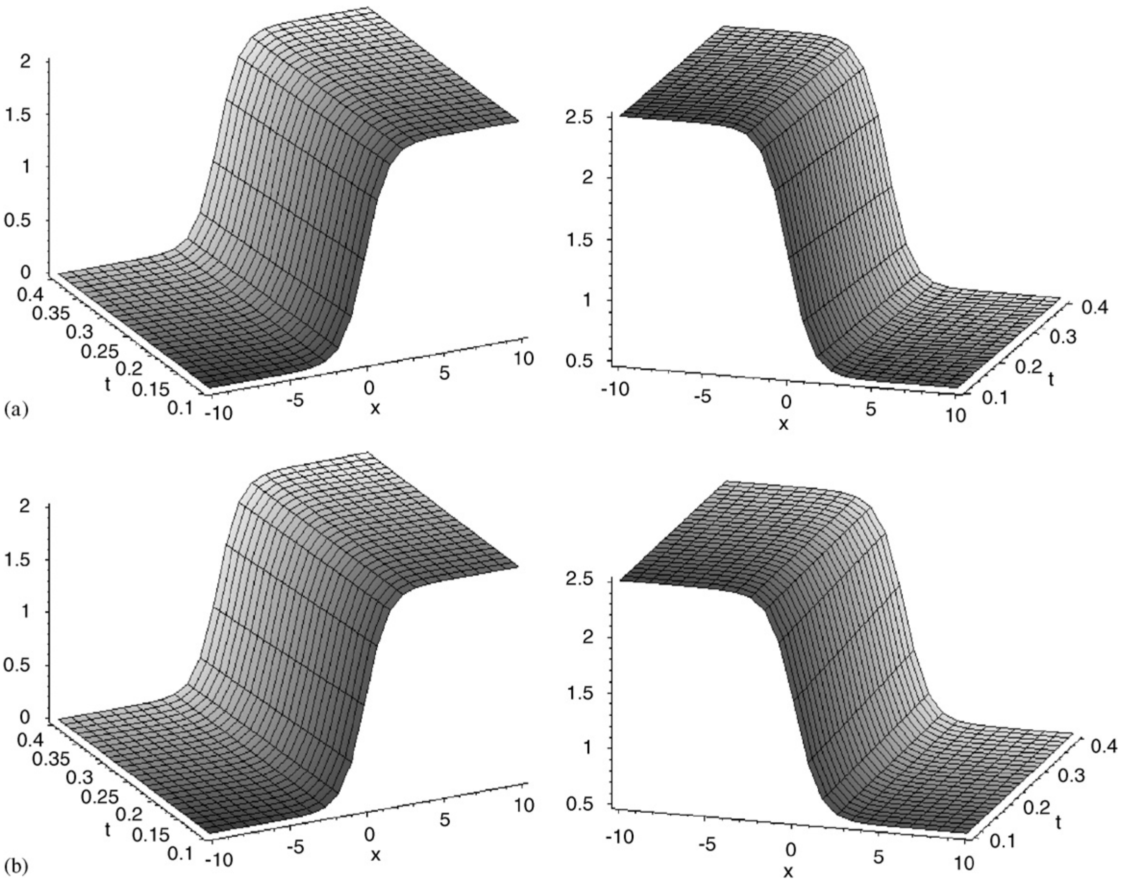

Fig. 1. (a) The numerical solution of $u(x, t)$ obtained by variational iteration method with different values of time with the fixed values of $c = v = 1$. The numerical solution of $v(x, t)$ obtained by variational iteration method with different values of time with the fixed values of $c = v = 1$. (b) The exact solution of $u(x, t)$ (Eq. (18)) with different values of time with the fixed values of $c = v = 1$. The exact solution of $v(x, t)$ (Eq. (18)) with different values of time with the fixed values of $c = v = 1$.

3.2. Example 2: Hirota–Satsuma coupled KdV System

A second instructive example is the model introduced by [8]

$$u_t - \tfrac{1}{2}u_{xxx} + 3uu_x - 3(vw)_x = 0, \tag{19}$$

$$v_t + v_{xxx} - 3uv_x = 0, \tag{20}$$

$$w_t + w_{xxx} - 3uw_x = 0, \tag{21}$$

with initial conditions [8]

$$u(x, 0) = \tfrac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(kx), \tag{22}$$

$$v(x, 0) = \frac{-4k^2 C_0(\beta + k^2)}{3C_1^2} + \frac{4k^2(\beta + k^2)}{3C_1} \tanh(kx), \tag{23}$$

$$w(x, 0) = C_0 + C_1 \tanh(kx), \tag{24}$$

where C_0, β, and C_1 are constants.

In the same manner, to solve Eqs. (19)–(21) by means of variational iteration method, we construct a correctional functional which reads

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda_1\left\{u_{nt}-\frac{1}{2}u_{nxxx}+3\widetilde{u_n u_{nx}}-3(\widetilde{v_n w_n})_x\right\}\mathrm{d}\tau, \tag{25}$$

$$v_{n+1}(x,t)=v_n(x,t)+\int_0^t \lambda_2\{v_{nt}+v_{nxxx}-3\widetilde{u_n v_{nx}}\}\,\mathrm{d}\tau, \tag{26}$$

$$w_{n+1}(x,t)=w_n(x,t)+\int_0^t \lambda_3\{w_{nt}+w_{nxxx}-3\widetilde{u_n w_{nx}}\}\,\mathrm{d}\tau, \tag{27}$$

where λ_1, λ_2, and λ_3 are general Lagrangian multipliers, and $\widetilde{u_n u_{nx}}$, $(\widetilde{v_n w_n})_x$, $\widetilde{u_n v_{nx}}$, and $\widetilde{u_n w_{nx}}$ denotes restricted variations i.e., $\delta\widetilde{u_n u_{nx}}=\delta(\widetilde{u_n v_n})_x=\delta\widetilde{v_n v_{nx}}=0$.

The Lagrange multipliers, therefore, can be identified as

$$\lambda_1=\lambda_2=\lambda_3=-1. \tag{28}$$

Substituting Eq. (28) into Eqs. (25)–(27), we obtain

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^t \left\{u_{nt}-\frac{1}{2}u_{nxxx}+3u_n u_{nx}-3(v_n w_n)_x\right\}\mathrm{d}\tau, \tag{29}$$

$$v_{n+1}(x,t)=v_n(x,t)-\int_0^t \{v_{nt}+v_{nxxx}-3u_n v_{nx}\}\,\mathrm{d}\tau, \tag{30}$$

$$w_{n+1}(x,t)=w_n(x,t)-\int_0^t \{w_{nt}+w_{nxxx}-3u_n w_{nx}\}\,\mathrm{d}\tau. \tag{31}$$

With the aid of the initial approximations given by Eqs. (22)–(24) and the iteration formula (29)–(31), we get the other rest of components as follows:

$$u_1(x,t)=-2\frac{\cosh x+2t\sinh x}{\cosh^3 x}, \tag{32}$$

$$v_1(x,t)=\frac{\cosh^2 x+\cosh x+t\sinh x}{\cosh^2 x}, \tag{33}$$

$$w_1(x,t)=2\left(\frac{-\cosh^2 x+\cosh x+t\sinh x}{\cosh^2 x}\right), \tag{34}$$

$$\begin{aligned}u_2(x,t)=\frac{-2}{\cosh^7 x}&(\cosh^5 x-3t^2\cosh^3 x+2t\sinh x\cosh^4 x+2t^3\sinh x\cosh^4 x\\&-20t^3\sinh x\cosh^2 x+2t^2\cosh^5 x+24t^3\sinh x),\end{aligned} \tag{35}$$

$$\begin{aligned}v_2(x,t)=\frac{1}{\cosh^6 x}&(\cosh^5 x+4t^3\cosh^2 x\sinh x-8t^3\sinh x+t\sinh x\cosh^4 x\\&+t^2\cosh^5 x-2t^2\cosh^3 x+2\cosh^6 x),\end{aligned} \tag{36}$$

$$\begin{aligned}w_2(x,t)=\frac{1}{\cosh^6 x}&(2\cosh^5 x+t^2\cosh^5 x-2t^2\cosh^3 x+2t\sinh x\cosh^4 x\\&+8t^3\cosh^2 x\sinh x-2\cosh^6 x-16t^3\sinh x),\end{aligned} \tag{37}$$

and so on, the rest of components of $u_n(x,t)$, $v_n(x,t)$, and $w_n(x,t)$ can be directly evaluated via Eqs. (29)–(31).

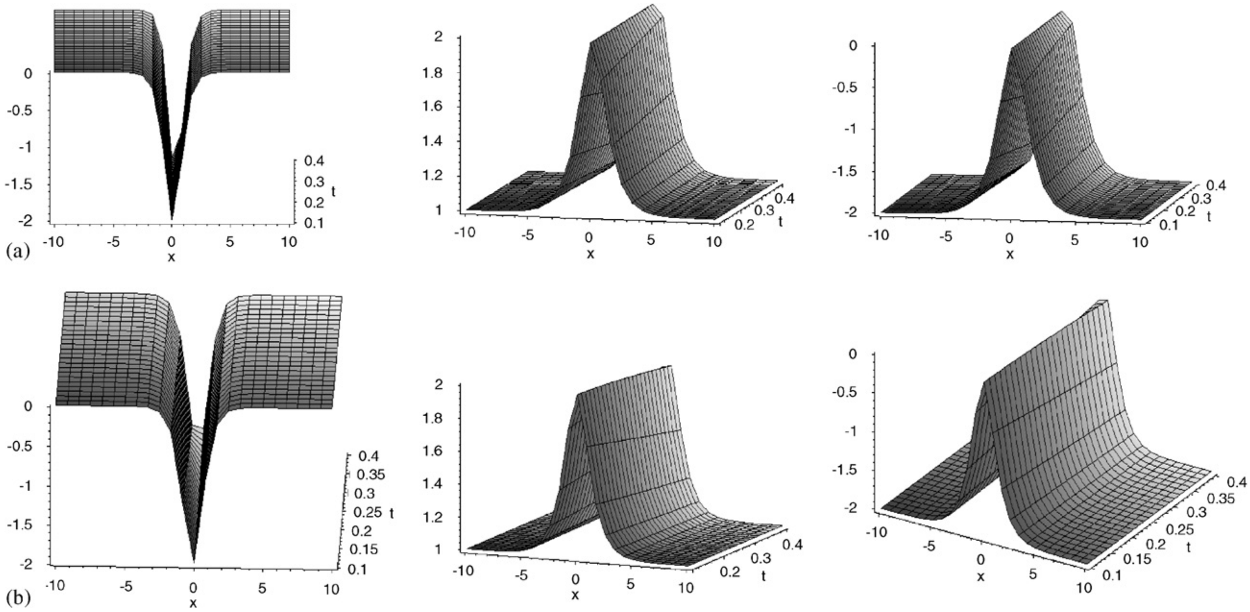

Fig. 2. (a) The numerical solution of $u(x,t)$ obtained by variational iteration method with different values of time with the fixed values of $C_0 = C_1 = k = \beta = 1$. The numerical solution of $v(x,t)$ obtained by variational iteration method with different values of time with the fixed values of $C_0 = C_1 = k = \beta = 1$. The numerical solution of $w(x,t)$ obtained by variational iteration method with different values of time with the fixed values of $C_0 = C_1 = k = \beta = 1$. (b) The exact solution of $u(x,t)$ (Eq. (22)) with different values of time with the fixed values of $C_0 = C_1 = k = \beta = 1$. The exact solution of $v(x,t)$ (Eq. (23)) with different values of time with the fixed values of $C_0 = C_1 = k = \beta = 1$. The exact solution of $w(x,t)$ (Eq. (24)) with different values of time with the fixed values of $C_0 = C_1 = k = \beta = 1$.

It is noted that the exact solutions $u(x,t)$, $v(x,t)$, and $w(x,t)$ are given by [8]

$$u(x,t) = \tfrac{1}{3}(\beta - 2k^2) + 2k^2 \tanh^2(k(x+\beta t)), \tag{38}$$

$$v(x,t) = \frac{-4k^2 C_0(\beta + k^2)}{3C_1^2} + \frac{4k^2(\beta + k^2)}{3C_1} \tanh(k(x+\beta t)), \tag{39}$$

$$w(x,t) = C_0 + C_1 \tanh(k(x+\beta t)). \tag{40}$$

The behavior of the approximate solutions obtained by variational iteration method with the exact solutions (Eqs. (38)–(40)) for different values of times is shown in Figs. 2(a) and (b).

3.3. Example 3: Drinefel'd–Sokolov–Wilson equations

The Drinefel'd–Sokolov–Wilson equations [31] read

$$u_t + pvv_x = 0, \tag{41}$$

$$v_t + qv_{xxx} + ruv_x + su_x v = 0, \tag{42}$$

where p, q, r, and s are arbitrary constants, and the initial conditions of u and v are given by

$$u(x,0) = 2\,\mathrm{sech}^2\, x, \quad v(x,0) = 2\,\mathrm{sech}\, x. \tag{43}$$

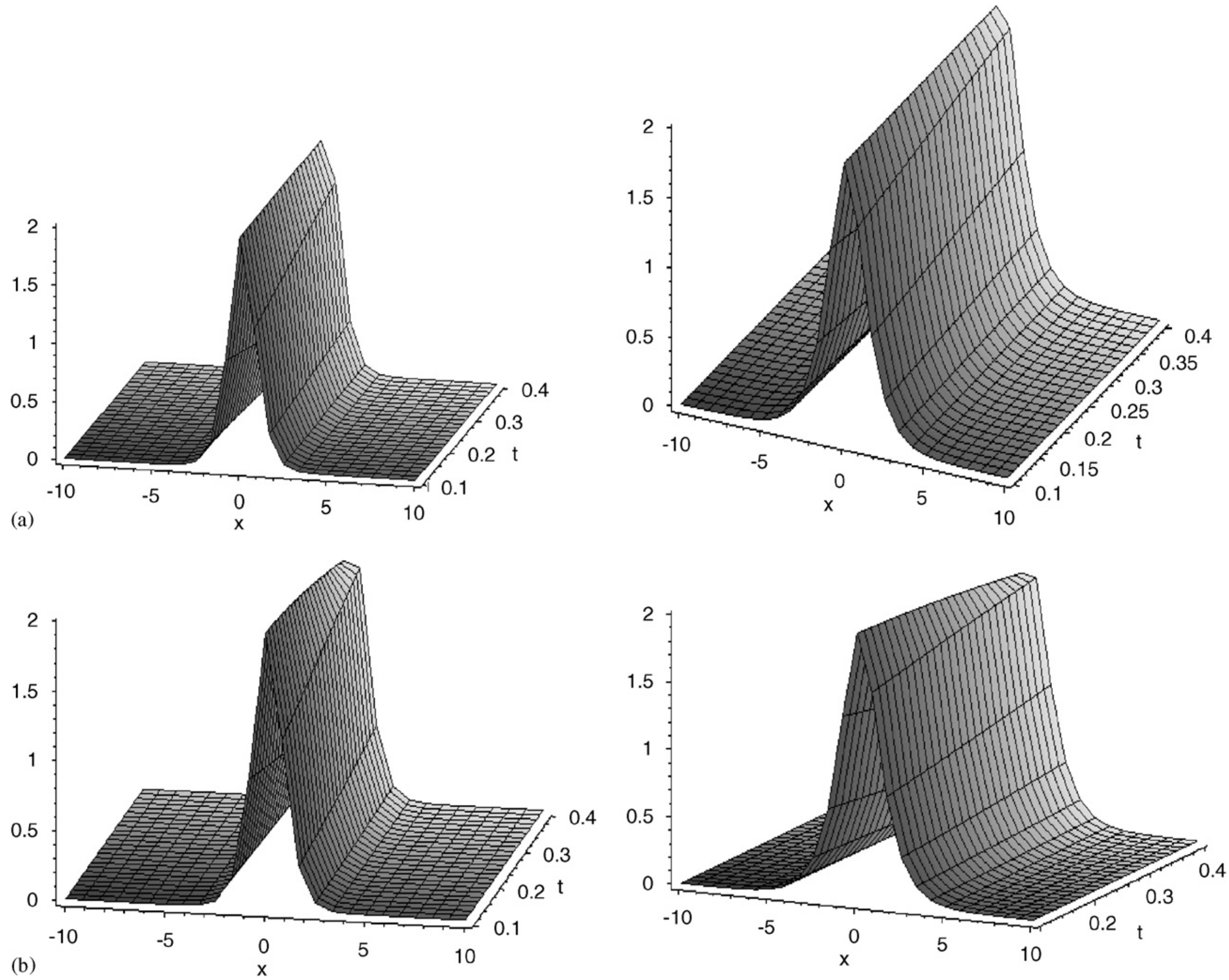

Fig. 3. (a) The numerical solution $u(x,t)$ obtained by variational iteration method with different values of time. The numerical solution of $v(x,t)$ obtained by variational iteration method with different values of time. (b) The exact solution of $u(x,t)$ with different values of time. The exact solution of $v(x,t)$ (Eq. (43)) with different values of time.

Similarly as before, to solve Eqs. (41), (42) using the variational iteration method, we consider

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda_1\{u_{nt} + p\widetilde{v_n v_{nx}}\}\,\mathrm{d}\tau, \tag{44}$$

$$v_{n+1}(x,t) = v_n(x,t) + \int_0^t \lambda_2\{v_{nt} + qv_{nxxx} + r\widetilde{u_n v_{nx}} + s\widetilde{u_{nx} v_n}\}\,\mathrm{d}\tau, \tag{45}$$

where λ_1 and λ_2 are general Lagrange multipliers, and $\widetilde{v_n v_{nx}}$, $\widetilde{u_n v_{nx}}$, and $\widetilde{u_{nx} v_n}$ denotes restricted variations i.e., $\delta\widetilde{v_n v_{nx}} = \delta\widetilde{u_n v_{nx}} = \delta\widetilde{u_{nx} v_n} = 0$.

By means of the above correction functional with fixed values of $(p = q = r = 1)$, we get

$$1 + \lambda_1(\tau)\rfloor_{\tau=t} = 0, \quad 1 + \lambda_2(\tau)\rfloor_{\tau=t} = 0, \tag{46}$$

$$\lambda_1'(\tau) = 0, \quad \lambda_2'(\tau) = 0. \tag{47}$$

In this case, the Lagrange multipliers, therefore, can be identified as

$$\lambda_1 = \lambda_2 = -1. \tag{48}$$

Inserting Eq. (48) into Eqs. (44), (45), we have

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \{u_{nt} + v_n v_{nx}\}\,\mathrm{d}\tau, \tag{49}$$

$$v_{n+1}(x,t) = v_n(x,t) - \int_0^t \{v_{nt} + v_{nxxx} + u_n v_{nx} + u_{nx} v_n\}\,\mathrm{d}\tau. \tag{50}$$

Identifying the zero components of $u(x,0)$ and $v(x,0)$, the remaining components can be determined as follows:

$$u_1(x,t) = 2\frac{\cosh x + 2t\sinh x}{\cosh^3 x}, \tag{51}$$

$$v_1(x,t) = 2\frac{\cosh x + 2t\sinh x}{\cosh^2 x}, \tag{52}$$

$$u_2(x,t) = \frac{2}{3\cosh^5 x}\begin{pmatrix} 3\cosh^3 x + 6t\sinh x\cosh^2 x + 2t^3\sinh x\cosh^2 x \\ -4t^3\sinh x + 6t^2\cosh^3 x - 9t^2\cosh x \end{pmatrix}, \tag{53}$$

$$v_2(x,t) = \frac{1}{3\cosh^6 x}\begin{pmatrix} 6\cosh^5 x + 6t\sinh x\cosh^4 x + 3t^2\cosh^5 x \\ -6t^2\cosh^3 x + 24t^3\sinh x\cosh^2 x - 40t^3\sinh x \end{pmatrix}, \tag{54}$$

the rest of components of $u_n(x,t)$ and $v_n(x,t)$ are obtained with the aid of Eqs. (44), (45).

The exact solutions of $u(x,t)$ and $v(x,t)$ [31] read

$$u(x,t) = 2\,\mathrm{sech}^2(x-t), \quad v(x,t) = 2\,\mathrm{sech}(x-t). \tag{55}$$

The graphical behavior of the approximate solutions obtained by variational iteration method with the exact solutions (Eqs. (55)) for different values of times is shown in Figs. 3(a) and (b).

4. Conclusions

In this paper, the extended variational iteration method is used to find the approximate solutions of the three kinds of equations which are chosen to illustrate this method in mathematical physics. The validity of the method has been successful by applying it for the nonlinear RDEs, Hirota–Satsuma coupled KdV system, and Drinefel'd–Sokolov–Wilson equations. The approximate solutions obtained by the variation iteration method are compared with the exact solutions. The results show that the variational iteration method is a powerful mathematical tool for finding the exact and numerical solutions of nonlinear equations. In our work we use the Maple Package to calculate the series obtained from the variational iteration method.

Finally, it is worthwhile to mention that the method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in mathematical physics. This is our task in the future work.

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