

Deflection of concentrically supported and eccentrically loaded thin circular plate

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Abstract

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Within the restrictions of the classical small bending theory of thin plates, the complex variable method is used to obtain exact expressions in series form for the deflection at any point of a thin isotropic circular plate simply supported along two concentric circles and eccentrically loaded. Numerical results and graphs illustrating the variation of the deflection along radii of the plate for two different load positions are presented. Special and limiting cases reduce the solutions derived above to those obtained by other authors.

Keywords: Elasticity, deflection, thin circular plate, complex analysis, biharmonic equation, free boundary, simple support, singular load, Poisson's ratio.

1. Introduction

The transverse flexure of thin isotropic circular plates subject to various distributions of normal loadings has been considered by several authors when the boundary of the plate is rigidly clamped, simply supported or free. Bassali and Nassif [12] and Bassali [7] used the method of complex potentials to deal with the elastically restrained circular plate under uniform and hydrostatic loadings over the area of a concentric ellipse. Exact solutions in closed forms were also obtained by Bassali [5,6] for a thin circular plate having a free boundary and supported at several points, the plate being subject to two types of normal loadings over an eccentric circle which include the uniform and uniformly varying loads as special cases. Bassali and Barsoum [9,10] and Barsoum [4] obtained the deflection at any point of a thin circular plate having a free boundary supported along a concentric circle and subject to three different load distributions over an eccentric circular patch lying within the circle of support. Attia and Nassif [3] dealt with the transverse bending of a thin circular plate supported along a concentric circle and subject to a concentrated load at any point of the plate. Recently Bassali and Anwar [1,8] obtained exact expressions in infinite series for the small deflection, moments and shears in a thin isotropic

circular plate loaded over a concentric ellipse (i) when the plate is subject to normal paraboloidal loading and symmetrically supported at the four corners of a concentric rectangle whose sides are parallel to the axes of the ellipse; (ii) when the plate is uniformly loaded and supported at the four vertices of a concentric rhombus whose diagonals coincide with the axes of the ellipse. In a recent paper the authors [2] dealt with the deflection of a thin isotropic circular plate supported along two concentric circles and acted upon by a point load lying between the two circles of support.

The problem considered in the present paper is a continuation of the previous problem but with the load lying either inside the inner circle of support or outside the outer one. Hence the mathematical manipulations and analysis encountered in the present work vary from those used by the authors [2]. Several previously known solutions appear as limiting cases of the results given here. Numerical results and curves are presented for the two different cases.

2. Basic equations

It is known that the transverse deflection w , measured positively upwards, at any point $z = x + iy = re^{i\theta}$ of the midplane of a normally loaded thin elastic plate of flexural rigidity D satisfies the biharmonic equation

$$D\nabla^4 w = -L, \quad (2.1)$$

where L is the normal load intensity at the point z of the plate. The general solution of (2.1) may be expressed in terms of two regular functions $\Omega(z)$ and $\omega(z)$, called the complex potentials, and a particular integral $W(z, \bar{z})$ in the form

$$w = \bar{z}\Omega(z) + z\bar{\Omega}(\bar{z}) + \omega(z) + \bar{\omega}(\bar{z}) + W(z, \bar{z}), \quad (2.2a)$$

where bars denote conjugate complex quantities.

In an unloaded region $L = 0$ and hence (2.2a) becomes

$$w = \bar{z}\Omega(z) + z\bar{\Omega}(\bar{z}) + \omega(z) + \bar{\omega}(\bar{z}). \quad (2.2b)$$

We suppose that the plate is divided by a closed contour Γ into the two regions 1 and 2 lying totally inside the plate. The regions are assumed to be unloaded and the only load on the plate is a lateral line loading extending along Γ . It has been shown by Bassali and Nassif [11] that the transition conditions across Γ are

$$[w]_1^2 = \left[\frac{\partial w}{\partial z} \right]_1^2 = \left[\frac{\partial^2 w}{\partial z \partial \bar{z}} \right]_1^2 = 0, \quad (2.3a)$$

$$\left[\frac{\partial^3 w}{\partial z^2 \partial \bar{z}} \right]_1^2 = \frac{-p}{8D} e^{-i\alpha}, \quad (2.3b)$$

where p is the normal intensity of the line loading at a point of Γ at which the outward normal to Γ makes an angle α with the x -axis.

When the boundary C of the plate is free, it was shown by Lechnitzky [13] that, apart from constant terms in $\Omega(z)$ and $\omega(z)$, the two complex potentials must satisfy the condition

$$k\Omega(z) + z\bar{\Omega}'(\bar{z}) + \bar{\omega}'(\bar{z}) = 0 \quad \text{on } C, \quad (2.4)$$

where $k = (3 + \nu)/(\nu - 1)$, ν being Poisson's ratio for the material of the plate.

3. Circular plate supported along two concentric circles and singularly loaded

Consider a circular plate of centre O , radius c and circumference C and supported along two concentric circles Γ_1 and Γ_2 of respective radii a and b , where $0 < a < b < c$. Suppose that the plate is acted upon by a concentrated load m at a point S distant f from O , see Fig. 1.

Take O as origin of coordinates and OS as x -axis and let $z = x + iy = re^{i\theta}$ be the complex variable at any point of the plate. Let $p(\theta)$ and $q(\theta)$ be the line intensities of the lateral thrust of the support at any points T_1 of Γ_1 and T_2 of Γ_2 , respectively, where the straight line OT_1T_2 makes an angle θ with the x -axis.

Suppose that due to the symmetry of the system both $p(\theta)$ and $q(\theta)$ are expressible in cosine series in the forms

$$p(\theta) = \sum_{n=0}^{\infty} a_n \cos n\theta \quad (3.1)$$

and

$$q(\theta) = \sum_{n=0}^{\infty} b_n \cos n\theta. \quad (3.2)$$

Thus it can be ascertained that the equilibrium conditions of the plate lead to

$$a_0 a + b_0 b = \frac{m}{2\pi}, \quad (3.3)$$

$$a_1 a^2 + b_1 b^2 = \frac{mf}{\pi}. \quad (3.4)$$

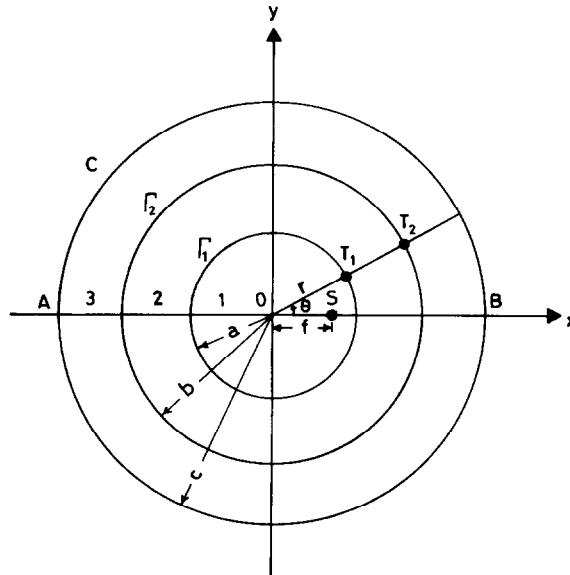


Fig. 1.

Noting that $z = ae^{i\theta}$ on Γ_1 and $z = be^{i\theta}$ on Γ_2 , (3.1) and (3.2) may be rewritten in the forms

$$p(\theta) = \frac{1}{2} \sum_{n=0}^{\infty} a_n \left(\frac{z^n}{a^n} + \frac{a^n}{z^n} \right), \quad (3.5)$$

$$q(\theta) = \frac{1}{2} \sum_{n=0}^{\infty} b_n \left(\frac{z^n}{b^n} + \frac{b^n}{z^n} \right). \quad (3.6)$$

Let 1, 2 and 3 refer to the three regions inside Γ_1 , between Γ_1 and Γ_2 and between Γ_2 and C , respectively.

If $\Omega_j(z)$ and $\omega_j(z)$ are the complex potential functions for the deflection w_j in region j , $j = 1, 2, 3$, then it can be verified [2] that the transition conditions (2.3) in view of (3.5) and (3.6) and the free boundary condition (2.4) lead to

$$\begin{aligned} \Omega_1(z) = \frac{1}{16D} & \left[\alpha_0 + \left(\alpha_1 + \frac{m}{\pi} \right) z + \left\{ \alpha_2 - \frac{1}{2}(a_1 + b_1) \right\} z^2 \right. \\ & + \frac{m}{\pi} (z - f) \left\{ \frac{1}{k} \log \left(1 - \frac{fz}{c^2} \right) - \log(z - f) \right\} \\ & - \sum_{n=2}^{\infty} \left(\frac{a_n}{a^{n-1}} + \frac{b_n}{b^{n-1}} \right) \frac{z^{n+1}}{n(n+1)} \\ & \left. + \frac{1}{k} \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+1} + b_n b^{n+1}}{nc^{n-1}} - \frac{a_n a^{n+3} + b_n b^{n+3}}{(n+1)c^{n+1}} \right\} \frac{z^{n+1}}{c^{n+1}} \right], \end{aligned} \quad (3.7a)$$

$$\begin{aligned} \omega_1(z) = \frac{1}{16D} & \left[\beta_0 - 2(a_0 a^3 + b_0 b^3) + \beta_1 z + \frac{mf}{\pi} (z - f) \log(z - f) \right. \\ & + \frac{mc^2}{\pi} \left(\frac{1}{k} - k \right) I(z) + \frac{m}{\pi} \left\{ k(c^2 - fz) - \frac{c^2 - f^2}{k} \right\} \log \left(1 - \frac{fz}{c^2} \right) \\ & + \sum_{n=2}^{\infty} \left(\frac{a_n}{a^{n-3}} + \frac{b_n}{b^{n-3}} \right) \frac{z^n}{n(n-1)} \\ & \left. + \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+3} + b_n b^{n+3}}{knc^{2n}} - \frac{a_n a^{n+1} + b_n b^{n+1}}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right\} z^n \right], \end{aligned} \quad (3.7b)$$

$$\begin{aligned} \Omega_2(z) = \frac{1}{16D} & \left[\alpha_0 + (\alpha_1 + 2b_0 b) z + \left(\alpha_2 - \frac{1}{2} b_1 \right) z^2 + a(2a_0 z - a_1 a) \log \frac{z}{a} \right. \\ & + \frac{m}{\pi} (z - f) \left\{ \frac{1}{k} \log \left(1 - \frac{fz}{c^2} \right) - \log(z - f) \right\} \\ & + \sum_{n=2}^{\infty} \frac{1}{n} \left\{ \frac{a_n a^{n+1}}{(n-1)z^{n-1}} - \frac{b_n z^{n+1}}{(n+1)b^{n-1}} \right\} \\ & \left. + \frac{1}{k} \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+1} + b_n b^{n+1}}{nc^{n-1}} - \frac{a_n a^{n+3} + b_n b^{n+3}}{(n+1)c^{n+1}} \right\} \frac{z^{n+1}}{c^{n+1}} \right], \end{aligned} \quad (3.8a)$$

$$\begin{aligned}
\omega_2(z) = \frac{1}{16D} & \left[-\frac{a_1 a^4}{2z} + \beta_0 - 2b_0 b^3 + \beta_1 z - a^2(a_1 z - 2a_0 a) \log \frac{z}{a} \right. \\
& + \frac{mc^2}{\pi} \left(\frac{1}{k} - k \right) I(z) + \frac{mf}{\pi} (z-f) \log(z-f) \\
& + \frac{m}{\pi} \left\{ k(c^2 - fz) - \frac{c^2 - f^2}{k} \right\} \log \left(1 - \frac{fz}{c^2} \right) \\
& + \sum_{n=2}^{\infty} \frac{1}{n} \left\{ \frac{b_n z^n}{(n-1)b^{n-3}} - \frac{a_n a^{n+3}}{(n+1)z^n} \right\} \\
& + \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+3} + b_n b^{n+3}}{knc^{2n}} - \frac{a_n a^{n+1} + b_n b^{n+1}}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right\} z^n \Bigg], \quad (3.8b)
\end{aligned}$$

$$\begin{aligned}
\Omega_3(z) = \frac{1}{16D} & \left[\alpha_0 + a_1 a^2 \log a + b_1 b^2 \log b + \{ \alpha_1 - 2(\alpha_0 a \log a + b_0 b \log b) \} z \right. \\
& + \alpha_2 z^2 + \frac{m}{\pi} (z-f) \left\{ \frac{1}{k} \log \left(1 - \frac{fz}{c^2} \right) - \log \left(1 - \frac{f}{z} \right) \right\} \\
& + \sum_{n=2}^{\infty} \frac{a_n a^{n+1} + b_n b^{n+1}}{n(n-1)z^{n-1}} \\
& + \frac{1}{k} \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+1} + b_n b^{n+1}}{nc^{n-1}} - \frac{a_n a^{n+3} + b_n b^{n+3}}{(n+1)c^{n+1}} \right\} \frac{z^{n+1}}{c^{n+1}} \Bigg], \quad (3.9a)
\end{aligned}$$

$$\begin{aligned}
\omega_3(z) = \frac{1}{16D} & \left[-\frac{1}{2z} (a_1 a^4 + b_1 b^4) + \beta_0 - 2(a_0 a^3 \log a + b_0 b^3 \log b) \right. \\
& + (\beta_1 + a_1 a^2 \log a + b_1 b^2 \log b) z \\
& + \frac{mc^2}{\pi} \left(\frac{1}{k} - k \right) I(z) + \frac{mf}{\pi} (z-f) \log(z-f) \\
& - \frac{mf}{\pi} z \log z + 2(a_0 a^3 + b_0 b^3) \log z \\
& + \frac{m}{\pi} \left\{ k(c^2 - fz) - \frac{c^2 - f^2}{k} \right\} \log \left(1 - \frac{fz}{c^2} \right) \\
& - \sum_{n=2}^{\infty} \frac{a_n a^{n+3} + b_n b^{n+3}}{n(n+1)z^n} \\
& + \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+3} + b_n b^{n+3}}{knc^{2n}} - \frac{a_n a^{n+1} + b_n b^{n+1}}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right\} z^n \Bigg], \quad (3.9b)
\end{aligned}$$

where

$$\left. \begin{aligned} \alpha_1 &= 2(a_0 a \log a + b_0 b \log b) - \frac{2(a_0 a^3 + b_0 b^3)}{(k+1)c^2} - \frac{mf^2}{k(k+1)\pi c^2}, \\ \alpha_2 &= \frac{mf}{k\pi c^2} - \frac{1}{2kc^4}(a_1 a^4 + b_1 b^4), \\ \beta_1 &= \frac{mkf}{\pi} - \frac{2mf}{k\pi} + \frac{1}{kc^2}(a_1 a^4 + b_1 b^4) - (k+1)(a_1 a^2 \log a + b_1 b^2 \log b). \end{aligned} \right\} \quad (3.10)$$

4. Case one: the point load is lying inside the inner circle of support

In this section we apply the condition that the deflection vanishes along the circles of support Γ_1 and Γ_2 to evaluate the coefficients $(a_n)_0^\infty$ and $(b_n)_0^\infty$. The region j we start with is chosen in such a way that the expansion of the functions involved in the appropriate deflection w_j will be valid.

Since the load lies in region 1, we have $f < a < b < c$ and for all values of z in region 2 we have $f < |z|$. Thus we start by considering the deflection w_2 and the functions involved in its expression will be expanded in powers of f/z and z/f' , where $ff' = c^2$. Introducing (3.8) in (2.2b) and carrying out the involved expansion we obtain, in view of (3.10), the following expression:

$$\begin{aligned} w_2 &= \frac{1}{8D} \left[\beta_0 - 2b_0 b^3 + 2b_0 b r^2 \left(1 + \log \frac{b}{r} \right) + 2a_0 a^3 \log \frac{r}{a} \right. \\ &\quad - \frac{2(a_0 a^3 + b_0 b^3)}{(k+1)c^2} r^2 + \frac{m}{\pi} \left\{ \frac{f^2 r^2}{(k+1)c^2} - f^2 (1 + \log r) \right\} + \gamma_2 r \sin \theta \\ &\quad + \left\{ \gamma_1 + (a_1 a^4 + b_1 b^4) \left(\frac{1}{kc^2} - \frac{r^2}{2kc^4} \right) - \frac{a_1 a^4}{2r^2} - \frac{b_1 r^2}{2} \right. \\ &\quad \left. - (k+1)(a_1 a^2 \log a + b_1 b^2 \log b) - 2a_1 a^2 \log \frac{r}{a} \right\} r \cos \theta \\ &\quad + \frac{mf}{\pi} \left\{ 1 - k + f^2 \left(\frac{r^2}{2kc^4} - \frac{1}{kc^2} + \frac{1}{2r^2} \right) + 2 \log r \right\} r \cos \theta \\ &\quad \left. + P_n + Q_n + S_1 + S_2 \right], \end{aligned} \quad (4.1)$$

where

$$\operatorname{Re} \alpha_0 \bar{z} = \gamma_1 r \cos \theta + \gamma_2 r \sin \theta, \quad (4.2)$$

$$P_n = \frac{m}{\pi} \sum_{n=2}^{\infty} \left\{ \frac{f^2 r^2}{(n+1)kc^2} - \frac{f^2 + r^2}{nk} + \frac{c^2}{n^2} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right\} \frac{f^n r^n}{c^{2n}} \cos n\theta, \quad (4.3)$$

$$Q_n = \sum_{n=2}^{\infty} \left\{ \frac{a_n a^{n+1} + b_n b^{n+1}}{n c^{2n-2}} \left(\frac{r^2}{k c^2} - \frac{n+1}{n k} - \frac{k}{n(n-1)} \right) + \frac{a_n a^{n+3} + b_n b^{n+3}}{k c^{2n}} \left(\frac{1}{n} - \frac{r^2}{(n+1)c^2} \right) \right\} r^n \cos n\theta, \quad (4.4)$$

$$S_1 = \frac{m}{\pi} \sum_{n=2}^{\infty} \left\{ \frac{f^2}{n+1} - \frac{r^2}{n-1} \right\} \frac{f^n}{n r^n} \cos n\theta, \quad (4.5)$$

$$S_2 = \sum_{n=2}^{\infty} \frac{1}{n} \left\{ \frac{a_n a^{n+1}}{r^n} \left(\frac{r^2}{n-1} - \frac{a^2}{n+1} \right) + \frac{b_n r^n}{b^{n-1}} \left(\frac{b^2}{n-1} - \frac{r^2}{n+1} \right) \right\} \cos n\theta. \quad (4.6)$$

From symmetry we have

$$\gamma_2 = 0, \quad (4.7)$$

and the constants β_0 and γ_1 are to be determined together with the coefficients a_n , b_n , $n \geq 0$, from the condition that the deflection vanishes along the circles of support. Thus the condition $w_2(a, \theta) = 0$ and $w_2(b, \theta) = 0$ for all values of θ lead to the following results:

$$\beta_0 = 2a_0 a \left\{ \frac{b^2(a^2 - b^2)}{(k+1)c^2} - a^2 \log \frac{b}{a} \right\} + \frac{m}{\pi} \left\{ \frac{b^2(b^2 - f^2)}{(k+1)c^2} + f^2(1 + \log b) \right\}, \quad (4.8)$$

$$\gamma_1 = a_1 a^2 (b^2 - a^2) \left\{ \frac{1}{2b^2} + \frac{1}{k c^2} - \frac{a^2}{2k c^4} - \frac{k+1}{b^2 - a^2} \log \frac{b}{a} \right\} + \frac{m f}{\pi} \left\{ \frac{a^2}{2b^2} - \frac{f^2}{2a^2} - \frac{b^2 - f^2}{k c^2} \left(1 - \frac{a^2}{2c^2} \right) + k - 1 + (k+1) \log b - 2 \log a \right\}, \quad (4.9)$$

where

$$2a_0 a = \frac{m}{\pi} \cdot \frac{1 - \frac{b^2 - f^2}{(k+1)c^2} - \frac{a^2 + f^2}{b^2 - a^2} \log \frac{b}{a}}{1 - \frac{b^2 - a^2}{(k+1)c^2} - \frac{2a^2}{b^2 - a^2} \log \frac{b}{a}}, \quad (4.10)$$

$$a_1 a^2 = \frac{m f}{\pi} \cdot \frac{\frac{a^2 + f^2}{2a^2 b^2} + \frac{b^2 - f^2}{2k c^4} - \frac{2}{b^2 - a^2} \log \frac{b}{a}}{\frac{1}{b^2} + \frac{b^2 - a^2}{2k c^4} - \frac{2}{b^2 - a^2} \log \frac{b}{a}}. \quad (4.11)$$

Inserting (4.10) and (4.11) in (3.3) and (3.4) respectively, we obtain

$$2b_0 b = \frac{m}{\pi} (a^2 - f^2) \cdot \frac{\frac{1}{(k+1)c^2} - \frac{1}{b^2 - a^2} \log \frac{b}{a}}{1 - \frac{b^2 - a^2}{(k+1)c^2} - \frac{2a^2}{b^2 - a^2} \log \frac{b}{a}}, \quad (4.12)$$

$$b_1 b^2 = \frac{m f}{\pi} (a^2 - f^2) \cdot \frac{\frac{1}{2a^2 b^2} - \frac{1}{2k c^4}}{\frac{1}{b^2} + \frac{b^2 - a^2}{2k c^4} - \frac{2}{b^2 - a^2} \log \frac{b}{a}}. \quad (4.13)$$

Finally the coefficients a_n and b_n , $n \geq 2$, can be obtained by solving the following two equations:

$$\begin{aligned}
 & a_n a^{n+1} \left\{ \frac{a^{n+2}}{kc^{2n}} \left(\frac{2}{n} - \frac{a^2}{(n+1)c^2} \right) - \frac{a^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) + \frac{2}{n(n^2-1)a^{n-2}} \right\} \\
 & + b_n b^{n+1} \left\{ \frac{a^n}{kc^{2n}} \left(\frac{a^2+b^2}{n} - \frac{a^2 b^2}{(n+1)c^2} \right) - \frac{a^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right. \\
 & \quad \left. - \frac{a^n}{nb^{2n}} \left(\frac{b^2}{n-1} - \frac{a^2}{n+1} \right) \right\} \\
 & = \frac{m}{\pi} \left\{ \frac{f^n}{na^n} \left(\frac{a^2}{n-1} - \frac{f^2}{n+1} \right) \right. \\
 & \quad \left. + \frac{a^n f^n}{c^{2n}} \left(\frac{a^2+f^2}{nk} - \frac{a^2 f^2}{(n+1)kc^2} - \frac{kc^2}{n^2(n-1)} - \frac{n+1}{n^2 k} c^2 \right) \right\}, \quad (4.14a)
 \end{aligned}$$

$$\begin{aligned}
 & a_n a^{n+1} \left\{ \frac{b^n}{kc^{2n}} \left(\frac{a^2+b^2}{n} - \frac{a^2 b^2}{(n+1)c^2} \right) - \frac{b^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right. \\
 & \quad \left. + \frac{1}{nb^n} \left(\frac{b^2}{n-1} - \frac{a^2}{n+1} \right) \right\} \\
 & + b_n b^{n+1} \left\{ \frac{b^{n+2}}{kc^{2n}} \left(\frac{2}{n} - \frac{b^2}{(n+1)c^2} \right) - \frac{b^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) + \frac{2}{n(n^2-1)b^{n-2}} \right\} \\
 & = \frac{m}{\pi} \left\{ \frac{f^n}{nb^n} \left(\frac{b^2}{n-1} - \frac{f^2}{n+1} \right) \right. \\
 & \quad \left. + \frac{b^n f^n}{c^{2n}} \left(\frac{b^2+f^2}{nk} - \frac{b^2 f^2}{(n+1)kc^2} - \frac{kc^2}{n^2(n-1)} - \frac{n+1}{n^2 k} c^2 \right) \right\}. \quad (4.14b)
 \end{aligned}$$

Introducing (4.8) and (4.9) in (4.1) we obtain in view of (3.3) and (3.4) the following expression for the deflection w_2 :

$$\begin{aligned}
 w_2 = \frac{1}{8D} & \left[2a_0 a \left\{ b^2 - r^2 + \frac{(r^2 - b^2)(b^2 - a^2)}{(k+1)c^2} - (r^2 + a^2) \log \frac{b}{r} \right\} \right. \\
 & + \frac{m}{\pi} \left\{ r^2 - b^2 + \frac{(b^2 - r^2)(b^2 - f^2)}{(k+1)c^2} + (r^2 + f^2) \log \frac{b}{r} \right\} \\
 & + a_1 a^2 \left\{ \frac{1}{2}(r^2 - a^2) \left(\frac{r^2 + b^2}{r^2 b^2} + \frac{b^2 - a^2}{kc^4} \right) - 2 \log \frac{r}{a} \right\} r \cos \theta \\
 & - \frac{mf}{\pi} \left\{ \frac{1}{2}(r^2 - a^2) \left(\frac{f^2}{a^2 r^2} + \frac{1}{b^2} + \frac{b^2 - f^2}{kc^4} \right) - 2 \log \frac{r}{a} \right\} r \cos \theta \\
 & \left. + P_n + Q_n + S_1 + S_2 \right], \quad (4.15)
 \end{aligned}$$

where P_n , Q_n , S_1 and S_2 are as given by (4.3)–(4.6), respectively.

Also applying (3.7) in (2.2b) and introducing (4.8) and (4.9), we obtain in view of (3.3), (3.4) and (3.10) the deflection w_1 in the form

$$\begin{aligned}
 w_1 = \frac{1}{8D} & \left[2a_0a \left\{ (b^2 - a^2) \left(1 + \frac{r^2 - b^2}{(k+1)c^2} \right) - (r^2 + a^2) \log \frac{b}{a} \right\} \right. \\
 & - \frac{m}{\pi} \left\{ b^2 + \frac{(r^2 - b^2)(b^2 - f^2)}{(k+1)c^2} - (r^2 + f^2)(1 + \log b) \right\} \\
 & + \frac{1}{2}a_1a^2(r^2 - a^2)(b^2 - a^2) \left(\frac{1}{kc^4} - \frac{1}{a^2b^2} \right) r \cos \theta - \frac{m}{\pi} R^2 \log R \\
 & - \frac{mf}{\pi} \left\{ 1 + \frac{f^2}{2a^2} + \frac{1}{2}(r^2 - a^2) \left(\frac{1}{b^2} + \frac{b^2 - f^2}{kc^4} \right) + 2 \log a \right\} r \cos \theta \\
 & \left. + P_n + Q_n + S_3 \right], \tag{4.16}
 \end{aligned}$$

where

$$R = |z - f|, \tag{4.17}$$

$$S_3 = \sum_{n=2}^{\infty} \frac{1}{n} \left\{ \frac{a_n}{a^{n-1}} \left(\frac{a^2}{n-1} - \frac{r^2}{n+1} \right) + \frac{b_n}{b^{n-1}} \left(\frac{b^2}{n-1} - \frac{r^2}{n+1} \right) \right\} r^n \cos n\theta. \tag{4.18}$$

Now the term $R^2 \log R$ can be expanded in two series forms depending on whether $r \leq f$ or $r \geq f$ as follows:

$$R^2 \log R = r^2 + (r^2 + f^2) \log f - \left\{ 1 + \frac{r^2}{2f^2} + 2 \log f \right\} fr \cos \theta - \frac{\pi}{m} S_4, \quad r \leq f, \tag{4.19}$$

$$R^2 \log R = f^2 + (r^2 + f^2) \log r - \left\{ 1 + \frac{f^2}{2r^2} + 2 \log r \right\} fr \cos \theta - \frac{\pi}{m} S_1, \quad r \geq f, \tag{4.20}$$

where

$$S_4 = \frac{m}{\pi} \sum_{n=2}^{\infty} \left\{ \frac{r^2}{n+1} - \frac{f^2}{n-1} \right\} \frac{r^n}{nf^n} \cos n\theta. \tag{4.21}$$

Hence we get the following two expressions for the deflection w_1 :

$$\begin{aligned}
 w_1 = \frac{1}{8D} & \left[2a_0a \left\{ (b^2 - a^2) \left(1 + \frac{r^2 - b^2}{(k+1)c^2} \right) - (r^2 + a^2) \log \frac{b}{a} \right\} \right. \\
 & + \frac{m}{\pi} \left\{ (f^2 - b^2) \left(1 + \frac{r^2 - b^2}{(k+1)c^2} \right) + (r^2 + f^2) \log \frac{b}{f} \right\} \\
 & + \frac{1}{2}a_1a^2(r^2 - a^2)(b^2 - a^2) \left(\frac{1}{kc^4} - \frac{1}{a^2b^2} \right) r \cos \theta \\
 & - \frac{mf}{\pi} \left\{ \frac{f^2}{2a^2} - \frac{r^2}{2f^2} + \frac{1}{2}(r^2 - a^2) \left(\frac{1}{b^2} + \frac{b^2 - f^2}{kc^4} \right) + 2 \log \frac{a}{f} \right\} r \cos \theta \\
 & \left. + P_n + Q_n + S_3 \right].
 \end{aligned}$$

$$\begin{aligned}
w_1 = \frac{1}{8D} & \left[2a_0a \left\{ (b^2 - a^2) \left(1 + \frac{r^2 - b^2}{(k+1)c^2} \right) - (r^2 + a^2) \log \frac{b}{a} \right\} \right. \\
& + \frac{m}{\pi} \left\{ (r^2 - b^2) \left(1 - \frac{b^2 - f^2}{(k+1)c^2} \right) + (r^2 + f^2) \log \frac{b}{r} \right\} \\
& + \frac{1}{2} a_1 a^2 (r^2 - a^2) (b^2 - a^2) \left(\frac{1}{kc^4} - \frac{1}{a^2 b^2} \right) r \cos \theta \\
& - \frac{mf}{\pi} \left\{ \frac{1}{2} (r^2 - a^2) \left(\frac{1}{b^2} + \frac{f^2}{a^2 r^2} + \frac{b^2 - f^2}{kc^4} \right) + 2 \log \frac{a}{r} \right\} r \cos \theta \\
& \left. + P_n + Q_n + S_3 + S_1 \right], \quad r \geq f.
\end{aligned} \tag{4.22b}$$

To obtain the deflection at the centre of the plate we substitute $r = 0$ in (4.22a). Thus we get

$$\begin{aligned}
w_0 = \frac{1}{8D} & \left[2a_0a \left\{ (b^2 - a^2) \left(1 - \frac{b^2}{(k+1)c^2} \right) - a^2 \log \frac{b}{a} \right\} \right. \\
& \left. + \frac{m}{\pi} \left\{ (f^2 - b^2) \left(1 - \frac{b^2}{(k+1)c^2} \right) + f^2 \log \frac{b}{f} \right\} \right].
\end{aligned} \tag{4.23}$$

Finally we evaluate w_3 from (3.9) to obtain the following expression:

$$\begin{aligned}
w_3 = \frac{1}{8D} & \left[2a_0a(b^2 - a^2) \left\{ \frac{r^2 - b^2}{(k+1)c^2} + \log \frac{b}{r} \right\} \right. \\
& - \frac{m}{\pi} (b^2 - f^2) \left\{ \frac{r^2 - b^2}{(k+1)c^2} + \log \frac{b}{r} \right\} \\
& + a_1 a^2 \left\{ \frac{1}{2} (b^2 - a^2) \left(\frac{r^2 + b^2}{r^2 b^2} + \frac{r^2 - a^2}{kc^4} \right) - 2 \log \frac{b}{a} \right\} r \cos \theta \\
& + \frac{mf}{\pi} \left\{ \frac{a^2}{2b^2} - \frac{f^2}{2a^2} - \frac{1}{2} (b^2 - f^2) \left(\frac{1}{r^2} + \frac{r^2 - a^2}{kc^4} \right) + 2 \log \frac{b}{a} \right\} r \cos \theta \\
& \left. + P_n + Q_n + S_1 + S_5 \right],
\end{aligned} \tag{4.24}$$

where

$$S_5 = \sum_{n=2}^{\infty} \left\{ a_n a^{n+1} \left(\frac{r^2}{n-1} - \frac{a^2}{n+1} \right) + b_n b^{n+1} \left(\frac{r^2}{n-1} - \frac{b^2}{n+1} \right) \right\} \frac{\cos n\theta}{nr^n}. \tag{4.25}$$

Inserting the expressions for a_0 , a_1 and a_n , $n \geq 2$, given by (4.10), (4.11) and (4.14), respectively, in (3.1) we get the thrust of support at any point T_1 of the circle Γ_1 (see Fig. 1). Similarly the thrust of support at any point T_2 of Γ_2 may be obtained by inserting the expressions for b_0 , b_1 and b_n , $n \geq 2$, given by (4.12)–(4.14), respectively, in (3.2).

5. Limiting cases

We now consider the following limiting cases.

(i) The deflection of an infinitely large plate supported along two concentric circles and eccentrically loaded at a point inside the inner circle of support can be investigated by letting c tend to ∞ in the foregoing results.

(ii) Letting $b \rightarrow a$ we obtain expressions for the deflection of a circular plate supported along one circle of support and eccentrically loaded at a point inside the circle of support. It is found that the results coincide with [3, (3.8) and (3.9)].

6. Case two: the point load is lying outside the outer circle of support

In this case $a < b < f < c$, therefore for all values of z in region 2, we see that $|z| < f$. Hence the functions involved are expanded in powers of z/f and z/f' . Therefore the modified form of w_2 as given by (4.1), will be as follows:

$$\begin{aligned}
 w_2 = \frac{1}{8D} & \left[\beta_0 + 2b_0b(r^2 - b^2) + 2r^2(a_0a \log a + b_0b \log b) \right. \\
 & - \frac{2(a_0a^3 + b_0b^3)r^2}{(k+1)c^2} + 2a_0a(r^2 + a^2) \log \frac{r}{a} \\
 & + \frac{m}{\pi} \left\{ \frac{f^2r^2}{(k+1)c^2} - r^2 - (r^2 + f^2) \log f \right\} \\
 & + \left\{ \gamma_1 - \frac{a_1a^4}{2r^2} - \frac{1}{2}b_1r^2 + \frac{a_1a^4 + b_1b^4}{kc^2} \left(1 - \frac{r^2}{2c^2} \right) \right. \\
 & \quad \left. - (k+1)(a_1a^2 \log a + b_1b^2 \log b) - 2a_1a^2 \log \frac{r}{a} \right\} r \cos \theta \\
 & + \frac{mf}{\pi} \left\{ 1 - k + \frac{r^2}{2f^2} - \frac{f^2}{kc^2} + \frac{f^2r^2}{2kc^4} + 2 \log f \right\} r \cos \theta \\
 & \left. + P_n + Q_n + S_2 + S_4 \right], \tag{6.1}
 \end{aligned}$$

where P_n , Q_n , S_2 and S_4 are as given by (4.3), (4.4), (4.6) and (4.21), respectively.

Now using the condition that $w_2(a, \theta) = 0$ and $w_2(b, \theta) = 0$ for all θ we get

$$\begin{aligned}
 \beta_0 = 2a_0a & \left\{ \frac{b^2(a^2 - b^2)}{(k+1)c^2} - a^2 \log \frac{b}{a} \right\} \\
 & + \frac{m}{\pi} \left\{ \frac{b^2(b^2 - f^2)}{(k+1)c^2} + b^2(1 - \log b) + (b^2 + f^2) \log f \right\}, \tag{6.2}
 \end{aligned}$$

$$\gamma_1 = a_1 a^2 \left\{ \frac{b^2 - a^2}{kc^2} \left(1 - \frac{b^2}{2c^2} \right) - \frac{1}{2} \left(1 - \frac{b^2}{a^2} \right) - (k-1) \log \frac{b}{a} \right\} \\ + \frac{mf}{\pi} \left\{ (f^2 - b^2) \left(\frac{1}{kc^2} - \frac{b^2}{2kc^4} \right) - \frac{1}{2} \left(1 + \frac{b^2}{f^2} \right) + k + (k+1) \log b - 2 \log f \right\}, \quad (6.3)$$

where

$$2a_0 a = \frac{m}{\pi} \cdot \frac{\frac{f^2 - b^2}{(k+1)c^2} - \log \frac{f}{b}}{1 - \frac{b^2 - a^2}{(k+1)c^2} - \frac{2a^2}{b^2 - a^2} \log \frac{b}{a}}, \quad (6.4)$$

$$a_1 a^2 = \frac{mf}{\pi} \cdot \frac{\frac{f^2 - b^2}{2f^2 b^2} - \frac{f^2 - b^2}{2kc^4}}{\frac{1}{b^2} + \frac{b^2 - a^2}{2kc^4} - \frac{2}{b^2 - a^2} \log \frac{b}{a}}, \quad (6.5)$$

so that (3.3) and (3.4) give

$$2b_0 b = \frac{m}{\pi} \cdot \frac{1 - \frac{f^2 - a^2}{(k+1)c^2} - \frac{2a^2}{b^2 - a^2} \log \frac{b}{a} + \log \frac{f}{b}}{1 - \frac{b^2 - a^2}{(k+1)c^2} - \frac{2a^2}{b^2 - a^2} \log \frac{b}{a}}, \quad (6.6)$$

$$b_1 b^2 = \frac{mf}{\pi} \cdot \frac{\frac{f^2 + b^2}{2f^2 b^2} + \frac{f^2 - a^2}{2kc^4} - \frac{2}{b^2 - a^2} \log \frac{b}{a}}{\frac{1}{b^2} + \frac{b^2 - a^2}{2kc^4} - \frac{2}{b^2 - a^2} \log \frac{b}{a}}. \quad (6.7)$$

Finally the solution of the following two equations gives the coefficients $(a_n)_2^\infty$ and $(b_n)_2^\infty$:

$$a_n a^{n+1} \left\{ \frac{a^{n+2}}{kc^{2n}} \left(\frac{2}{n} - \frac{a^2}{(n+1)c^2} \right) - \frac{a^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) + \frac{2}{n(n^2-1)a^{n-2}} \right\} \\ + b_n b^{n+1} \left\{ \frac{a^n}{kc^{2n}} \left(\frac{a^2 + b^2}{n} - \frac{a^2 b^2}{(n+1)c^2} \right) - \frac{a^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right. \\ \left. + \frac{a^n}{nb^{2n}} \left(\frac{b^2}{n-1} - \frac{a^2}{n+1} \right) \right\} \\ = \frac{m}{\pi} \left\{ \frac{a^n}{nf^n} \left(\frac{f^2}{n-1} - \frac{a^2}{n+1} \right) \right. \\ \left. + \frac{a^n f^n}{c^{2n}} \left(\frac{a^2 + f^2}{nk} - \frac{a^2 f^2}{(n+1)kc^2} - \frac{kc^2}{n^2(n-1)} - \frac{n+1}{n^2 k} c^2 \right) \right\}, \quad (6.8a)$$

$$\begin{aligned}
& a_n a^{n+1} \left\{ \frac{b^n}{kc^{2n}} \left(\frac{a^2 + b^2}{n} - \frac{a^2 b^2}{(n+1)c^2} \right) - \frac{b^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} + \frac{k}{n-1} \right) \right. \\
& \quad \left. + \frac{1}{nb^n} \left(\frac{b^2}{n-1} - \frac{a^2}{n+1} \right) \right\} \\
& + b_n b^{n+1} \left\{ \frac{b^{n+2}}{kc^{2n}} \left(\frac{2}{n} - \frac{b^2}{(k+1)c^2} \right) - \frac{b^n}{n^2 c^{2n-2}} \left(\frac{n+1}{k} - \frac{k}{n-1} \right) + \frac{2}{n(n^2-1)b^{n-2}} \right\} \\
& = \frac{m}{\pi} \left\{ \frac{b^n}{nf^n} \left(\frac{f^2}{n-1} - \frac{b^2}{n+1} \right) \right. \\
& \quad \left. + \frac{b^n f^n}{c^{2n}} \left(\frac{b^2 + f^2}{nk} - \frac{b^2 f^2}{(n+1)kc^2} - \frac{kc^2}{n^2(n-1)} - \frac{n+1}{n^2 k} c^2 \right) \right\}. \quad (6.8b)
\end{aligned}$$

Hence the expression (6.1) for w_2 now becomes

$$\begin{aligned}
w_2 = \frac{1}{8D} & \left[2a_0 a \left\{ (b^2 - r^2) \left(1 - \frac{b^2 - a^2}{(k+1)c^2} \right) - (r^2 + a^2) \log \frac{b}{r} \right\} \right. \\
& + \frac{m}{\pi} (r^2 - b^2) \left\{ \frac{f^2 - b^2}{(k+1)c^2} - \log \frac{f}{b} \right\} \\
& + a_1 a^2 \left\{ \frac{1}{2} (r^2 - b^2) \left(\frac{r^2 + a^2}{r^2 b^2} + \frac{b^2 - a^2}{kc^4} \right) + 2 \log \frac{b}{r} \right\} r \cos \theta \\
& - \frac{mf}{2\pi} (r^2 - b^2) (f^2 - b^2) \left(\frac{1}{f^2 b^2} - \frac{1}{kc^4} \right) r \cos \theta \\
& \left. + P_n + Q_n + S_2 + S_4 \right]. \quad (6.9)
\end{aligned}$$

Next the expression for the deflection w_1 of the present case is formed from (2.2b) and (3.7) by using (3.10), (4.8) and (4.9) to obtain

$$\begin{aligned}
w_1 = \frac{1}{8D} & \left[2a_0 a \left\{ (b^2 - a^2) \left(1 + \frac{r^2 - b^2}{(k+1)c^2} \right) - (r^2 + a^2) \log \frac{b}{a} \right\} \right. \\
& + \frac{m}{\pi} (b^2 - r^2) \left\{ \frac{b^2 - f^2}{(k+1)c^2} - \log \frac{b}{f} \right\} \\
& + a_1 a^2 \left\{ \frac{1}{2} (b^2 - a^2) \left(\frac{r^2 - b^2}{kc^4} - \frac{r^2 + a^2}{a^2 b^2} \right) + 2 \log \frac{b}{a} \right\} r \cos \theta \\
& + \frac{mf}{2\pi} (b^2 - f^2) (r^2 - b^2) \left(\frac{1}{f^2 b^2} - \frac{1}{kc^4} \right) r \cos \theta \\
& \left. + P_n + Q_n + S_3 + S_4 \right]. \quad (6.10)
\end{aligned}$$

The deflection at the centre of the plate is then obtained by putting $r = 0$ in (6.10) to get

$$w_0 = \frac{1}{8D} \left[2a_0 a \left\{ (b^2 - a^2) \left(1 - \frac{b^2}{(k+1)c^2} \right) - a^2 \log \frac{b}{a} \right\} + \frac{mb^2}{\pi} \left\{ \frac{b^2 - f^2}{(k+1)c^2} - \log \frac{b}{f} \right\} \right]. \quad (6.11)$$

Finally the deflection w_3 is obtained from (3.9) and we get the following expression:

$$w_3 = \frac{1}{8D} \left[2a_0 a (b^2 - a^2) \left\{ \frac{r^2 - b^2}{(k+1)c^2} + \log \frac{b}{r} \right\} + \frac{m}{\pi} \left\{ b^2 + \frac{(r^2 - b^2)(f^2 - b^2)}{(k+1)c^2} + f^2 \log f + r^2 \log r + b^2 \log \frac{fr}{b^2} \right\} - \frac{m}{\pi} R^2 \log R + \frac{1}{2} a_1 a^2 (r^2 - b^2)(b^2 - a^2) \left(\frac{1}{kc^4} - \frac{1}{r^2 b^2} \right) r \cos \theta + \frac{mf}{\pi} \left\{ \frac{(r^2 - b^2)(f^2 - b^2)}{2kc^4} - \frac{b^2(r^2 + f^2)}{2f^2 r^2} - \frac{1}{2} + 2 \log \frac{b}{fr} \right\} r \cos \theta + P_n + Q_n + S_5 \right]. \quad (6.12)$$

Using the expansions (4.19) and (4.20) for the term $R^2 \log R$ we get the following two expressions for w_3 :

$$w_3 = \frac{1}{8D} \left[2a_0 a (b^2 - a^2) \left\{ \frac{r^2 - b^2}{(k+1)c^2} + \log \frac{b}{r} \right\} + \frac{m}{\pi} \left\{ (b^2 - r^2) \left(1 - \frac{f^2 - b^2}{(k+1)c^2} \right) + r^2 \log \frac{r}{f} + b^2 \log \frac{fr}{b^2} \right\} + \frac{1}{2} a_1 a^2 (r^2 - b^2)(b^2 - a^2) \left(\frac{1}{kc^4} - \frac{1}{r^2 b^2} \right) r \cos \theta + \frac{mf}{\pi} \left\{ \frac{1}{2} (r^2 - b^2) \left(\frac{r^2 + f^2}{f^2 r^2} + \frac{f^2 - b^2}{kc^4} \right) + 2 \log \frac{b}{r} \right\} r \cos \theta + P_n + Q_n + S_4 + S_5 \right], \quad r \leq f, \quad (6.13a)$$

$$\begin{aligned}
w_3 = \frac{1}{8D} & \left[2a_0a(b^2 - a^2) \left\{ \frac{r^2 - b^2}{(k+1)c^2} + \log \frac{b}{r} \right\} \right. \\
& + \frac{m}{\pi} \left\{ (b^2 - f^2) \left(1 - \frac{r^2 - b^2}{(k+1)c^2} \right) + f^2 \log \frac{f}{r} + b^2 \log \frac{fr}{b^2} \right\} \\
& + \frac{1}{2}a_1a^2(r^2 - b^2)(b^2 - a^2) \left(\frac{1}{kc^4} - \frac{1}{r^2b^2} \right) r \cos \theta \\
& + \frac{mf}{\pi} \left\{ \frac{1}{2}(f^2 - b^2) \left(\frac{r^2 + f^2}{f^2r^2} + \frac{r^2 - b^2}{kc^4} \right) + 2 \log \frac{b}{f} \right\} r \cos \theta \\
& \left. + P_n + Q_n + S_1 + S_5 \right], \quad r \geq f.
\end{aligned} \tag{6.13b}$$

Now to obtain the thrusts of support p and q at any point T_1 of Γ_1 and T_2 of Γ_2 (Fig. 1) respectively, we insert the values of a_0 , a_1 and a_n , $n \geq 2$, given by (6.4), (6.5) and (6.8), in (3.1) and those of b_0 , b_1 and b_n , $n \geq 2$, given by (6.6), (6.7) and (6.8), in (3.2).

7. Limiting cases

For the above load position we now consider the following limiting cases.

(i) Allowing c to tend to ∞ yields the appropriate solution for an infinitely large plate supported along two concentric circles and eccentrically loaded at a point outside the outer circle of support.

(ii) The case in which b tends to a leads to the problem of a circular plate supported along one circle of support and eccentrically loaded at a point outside the circle of support. The expressions obtained coincide with [3, (4.5) and (4.6)].

(iii) In the limiting case in which a tends to 0 the plate will be loaded with a concentrated load m at the point S , distant f from the centre O and supported along the circle Γ of radius b , where $b < f$. The plate is also singularly supported at its centre O .

8. Numerical results

This section gives the numerical and graphical representation of the deflections at various points along radii of the circular plate for the two previously considered load positions. We take $a/c = 0.3$ and $b/c = 0.7$. For case one we take $f/c = 0.15$ while for case two we put $f/c = 0.85$.

The numerical evaluation of the infinite series denoted by P_n , Q_n , S_i , $i = 1, 2, 3, 4, 5$, and given by expressions (4.3)–(4.6), (4.18), (4.21), (4.25), has been performed in the following manner. A computer program was developed that sums these series for a large number of terms; the partial sums were consecutively computed and tested for convergence. When convergence was detected within a preselected small tolerance (of the order of 10^{-5}), the sum was computed and used in the corresponding expressions.

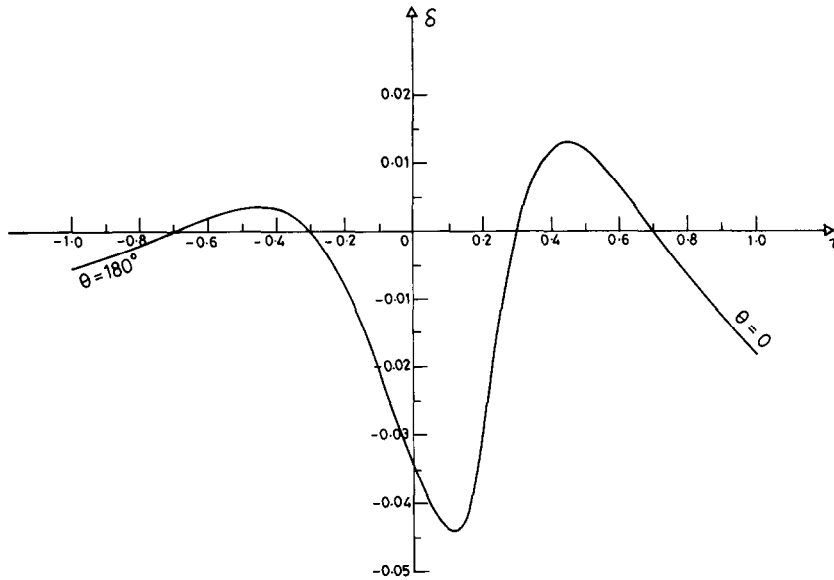


Fig. 2. Variation of the deflection factor along the diameter AB , $f/c = 0.15$.

It should be noticed that the series are indeed convergent in the following sense.

- Series P_n is expanded in powers of r/f' and thus is valid at any point in the plate since we always have $|r| < f'$.
- Series S_1 involves powers of f/r and appears in the expression (4.15) for the deflection w_2 of case one and the expression (6.13b) for the deflection w_3 of case two where we have $|r| > f$.
- Series S_4 contains powers of r/f and occurs in the expression (4.22a) for the deflection w_1 of case one and in the expression (6.9) for the deflection w_2 of case two where we have $|r| < f$.
- Series Q_n , S_2 , S_3 , S_5 have been obtained due to the Fourier representation of the thrust of

Table 1
Values of $10^4\delta$ ($\nu = 0.333$)

r	$\theta = 0$		$\theta = \frac{1}{4}\pi$		$\theta = \frac{1}{2}\pi$		$\theta = \frac{3}{4}\pi$		$\theta = \pi$	
	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
0.0	-345	-72	-345	-72	-345	-72	-345	-72	-345	-72
0.1	-440	-94	-363	-81	-271	-60	-222	-47	-207	-43
0.2	-294	-86	-198	-59	-123	-30	-91	-20	-83	-17
0.3	0	0	0	0	0	0	0	0	0	0
0.4	120	201	86	71	52	15	37	8	33	7
0.5	121	381	90	107	54	17	39	8	35	7
0.6	69	382	52	81	32	10	23	5	20	4
0.7	0	0	0	0	0	0	0	0	0	0
0.8	65	-948	51	111	31	10	22	5	20	4
0.9	126	-1973	98	235	61	20	43	9	38	83
1.0	184	-2904	144	352	89	29	63	13	56	12

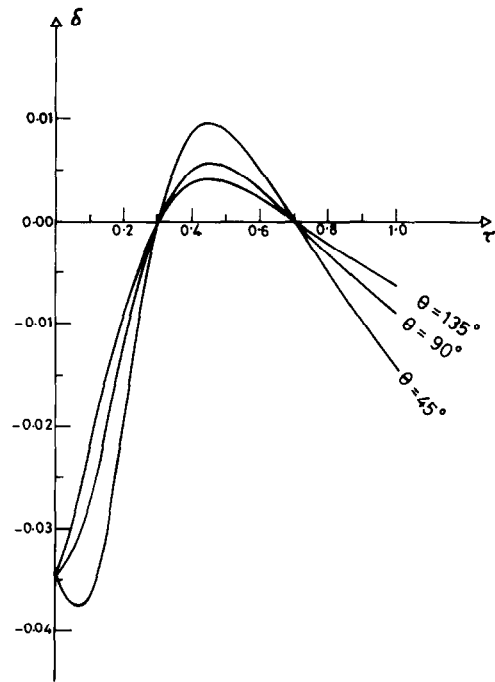


Fig. 3. Variation of the deflection factor along radii, $f/c = 0.15$.

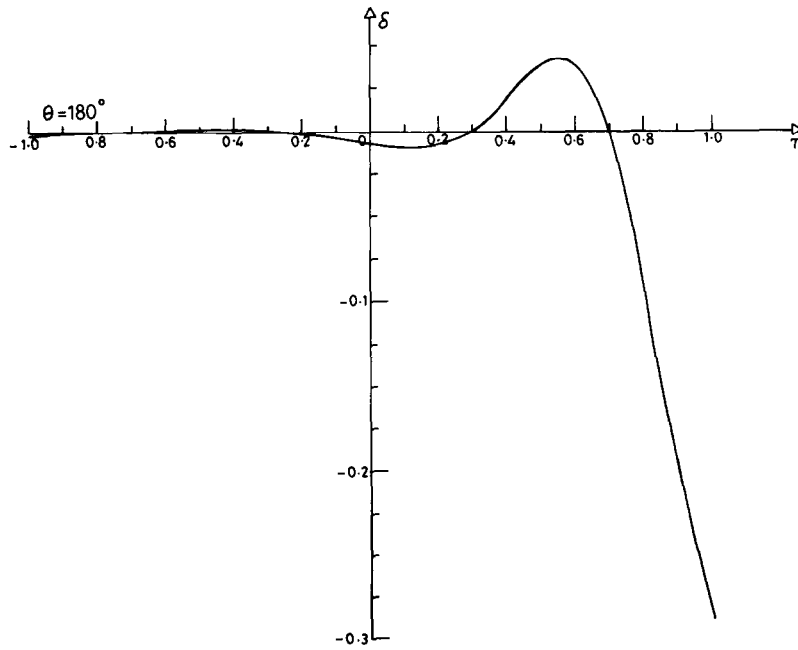


Fig. 4. Variation of the deflection factor along the diameter AB , $f/c = 0.85$.

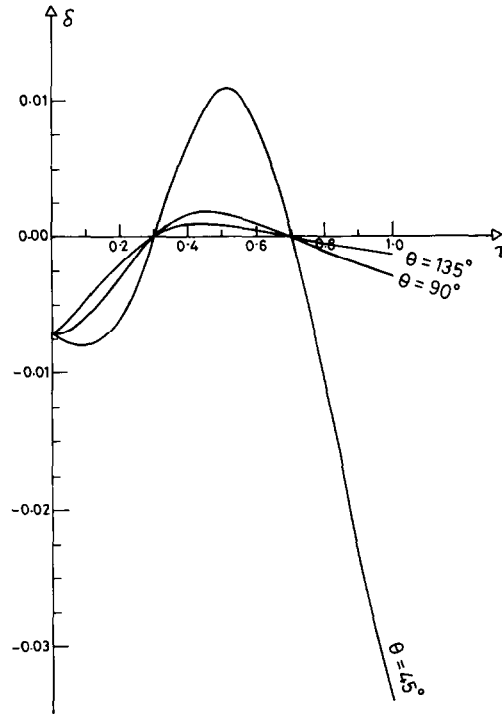


Fig. 5. Variation of the deflection factor along radii, $f/c = 0.85$.

supports and the application of the transition conditions (2.3) and the concept of complex potentials. Thus based on the Fourier theory the convergence of these series is achieved.

Due to the symmetry about the x -axis of the system under investigation, the nondimensional deflection factor $\delta = (8\pi D/mc^2)w$ is plotted against r for the radii $\theta = 0, \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi, \pi$. Figures 2 and 3 represent the results of case one while Figs. 4 and 5 represent those of case two. Table 1 summarises the numerical values obtained for various angles θ and some values of r in the upper half of the plate.

From the given table and figures one can see that for case one as well as for case two, the deflection is downwards inside the inner support circle. Between the two support circles the deflection is upwards, whereas outside the outer support circle, the deflection again reverses its sign.

The absolute value of the deflection reaches its maximum value at the load position for case one and at the edge of the plate for case two. The results obtained reveal that the expected deflection at the plate edge is greatly controlled by the position of the load with respect to the support circles.

It is also noticed that for constant r , the absolute value of the deflection factor decreases with increasing value of θ , so that the deflection of the plate is more considerable in the first and fourth quadrant of the plate than in the second and third ones.

As expected the deflection vanishes along the support circles, which indicates the convergence of the involved series and the validity of the computational procedure.

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