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Geometry of $N = 1$ super Yang–Mills theory in curved superspace

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Abstract

We give a new description of $N = 1$ super Yang–Mills (SYM) theory in curved superspace. It is based on the induced geometry approach to a curved superspace in which it is viewed as a surface embedded into $C^{4|2}$. The complex structure on $C^{4|2}$ supplied with a standard volume element induces a special Cauchy–Riemann (SCR)-structure on the embedded surface. We give an explicit construction of SYM theory in terms of intrinsic geometry of the superspace defined by this SCR-structure and a CR-bundle over the superspace. We write a manifestly SCR-covariant Lagrangian for SYM coupled with matter. We also show that in a special gauge our formulation coincides with the standard one which uses Lorentz connections. Some useful auxiliary results about the integration over surfaces in superspace are obtained.

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1. Introduction

The main purpose of this work is to apply the induced geometry approach to $N = 1$ supergravity to the construction of $N = 1$ Super Yang–Mills (SYM) theory on a curved superspace. This approach was introduced in paper [1]. We refer the reader to that paper for all details about the geometric constructions we use in the present one. However, we give all necessary definitions so that the paper can be read independently. The main benefit of induced geometry approach is that it does not require any additional constraints like

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those one has to impose on a curvature and torsion tensors in the standard formulation of supergravity (see [2] for a good exposition). The induced Special Couchy–Riemann (SCR)-structure on a superspace incorporates all the necessary constraints and seems to be a more natural geometric construction than the Lorentz connections of the conventional approach. The induced geometry approach to $N = 1$ supergravity was further developed in papers [3–5]. In paper [6] the application to the construction of SYM theory over curved superspace was proposed. In the present work we develop another point of view on this problem.

The paper is organized as follows. In Section 2 we give some auxiliary results about the integration over integral surfaces defined by $(0, 2)$ -dimensional distribution. The derivation of these results is postponed until Appendix A. Then we give the description of a curved superspace and Cauchy–Riemann (CR)-structure on it within the framework of induced geometry approach. After that we explain how the results about the integration over integral surfaces can be applied to the construction of a chiral projecting operator. In Section 3 we introduce some more geometric notions and define the gauge fields as sections of CR-bundles over the superspace. Then we formulate the main result and explain all the ingredients. The derivation of the Lagrangian is given in Appendix B. We conclude this section by considering the proper reality conditions imposed on the fields used in the construction. As a result of these restrictions the set of fields reduces to the standard one. This procedure is similar to the one introduced in [7]. In Section 4 we compare our constructions to the conventional ones.

2. Integration over integral surfaces determined by $(0, 2)$ -dimensional distributions

Let \mathcal{M} be a real (m, n) -dimensional supermanifold. Consider a pair of odd vector fields E_α (where $\alpha = 1, 2$) which are closed with respect to the anticommutator, i.e.

$$\{E_\alpha, E_\beta\} = c_{\alpha\beta}{}^\gamma E_\gamma, \quad (1)$$

where $c_{\alpha\beta}{}^\gamma$ are some odd functions on \mathcal{M} . By a super version of the Frobenius theorem, condition (1) means that the vector fields E_α determine an integrable distribution, i.e. for every point in \mathcal{M} there exists a $(0, 2)$ -dimensional surface Σ going through it such that its tangent plane at each point coincides with the one spanned by E_α (equivalently our distribution defines a foliation which has the surfaces Σ as leaves). We are interested in the functions on \mathcal{M} which are stable under the action of the vector fields E_α , i.e. the functions Φ such that $E_\alpha \Phi = 0$. Here and below we use the same symbol for vector fields and for the first order differential operators corresponding to them.

On every surface Σ we have a natural volume element defined by the requirement that the value of the volume form taken on the fields E_1 and E_2 is equal to one. Given an arbitrary function Φ one can get a function with the property we want by integrating Φ over the leaves Σ . We will denote this operation by $\square_E \Phi$ when applied to Φ . The explicit formula for \square_E in terms of given E_α and functions $c_{\alpha\beta}{}^\gamma$ reads as

$$\begin{aligned} \square_E \Phi = & E^\alpha E_\alpha \Phi + c^\alpha{}_\sigma{}^\sigma E_\alpha \Phi \\ & + \Phi \left(\frac{2}{3} E^\alpha c_{\alpha\sigma}{}^\sigma + \frac{1}{3} c^\alpha{}_\sigma{}^\sigma c_{\alpha\beta}{}^\beta + \frac{1}{6} c_{\alpha\beta}{}^\sigma c_\sigma{}^{\alpha\beta} + \frac{1}{12} c_{\alpha\beta\sigma} c^{\alpha\beta\sigma} \right). \end{aligned} \quad (2)$$

Here we are raising indices by means of the spinor metric tensor $\epsilon^{\alpha\beta}$ ($\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$, $\epsilon^{12} = 1$) and lowering by means of the inverse matrix $\epsilon_{\alpha\beta}$. We give a proof of this formula in Appendix A. It is worth noting that the expression in parentheses in formula (2) is just the volume of the leaf evaluated with respect to the volume element specified above.

In order to get a function which is invariant with respect to a given integrable distribution it is not necessary to use the natural volume element related to the chosen pair of vector fields when integrating. One can perform the integration using any volume element as well. But since it differs from the natural volume element by multiplication by some function, all possible freedom is reflected by the following formula for a generic operation \square :

$$\square\Phi = \square_{\rho}\Phi \equiv \square_E(\rho\Phi), \quad (3)$$

where ρ is some function. In short, this freedom is the same as modifying the initial function by multiplying it by some fixed function.

Now let us consider the case of a complex supermanifold \mathcal{M} with a pair of complex vector fields E_{α} on it satisfying the integrability condition (1). In this case we have to modify the consideration above slightly. Formula (2) again defines an operation yielding a E_{α} -invariant function if one assumes that E_{α} is a holomorphic vector field and Φ is a holomorphic function. As before by the Frobenius theorem we have an integral complex surface Σ_C going through every point in \mathcal{M} . Our vector fields define a natural holomorphic volume element on Σ_C . For a generic real submanifold Σ_R , a real basis in a tangent space to Σ_R can be considered as a complex basis in a tangent space to Σ_C . This means that the holomorphic volume form determines a nondegenerate volume element in a tangent space to Σ_R . Given such a generic submanifold $\Sigma_R \subset \Sigma_C$, one can perform an integration of a holomorphic function over it. One can show that when we have a purely odd supermanifold Σ_C , the result of integration does not depend on the particular choice of Σ_R . Therefore we see that in the case of a complex manifold, the operation $\square_{E_{\alpha}}$ (formula (2)) can also be interpreted in terms of integration over the leaves.

We are interested in applications of formula (2) in the framework of the induced geometry approach to the description of curved superspace. In this approach a curved superspace is described as a generic real $(4, 4)$ -dimensional surface Ω embedded into $\mathbb{C}^{4|2}$ (see [1] for details). The complex structure on $\mathbb{C}^{4|2}$ induces a CR-structure on Ω . This means that a complex plane is singled out in every tangent space to Ω . Namely if $T_z(\Omega)$ is the real $(4, 4)$ -dimensional tangent space at a point z and J denotes the linear operator given by the multiplication by i , then $T_z(\Omega) \cap JT_z(\Omega)$ is the maximal complex subspace contained in $T_z(\Omega)$. It is not difficult to figure out that this complex subspace is of dimension $(0, 2)$. One can choose vector fields

$$E_{\alpha}, \bar{E}_{\dot{\alpha}}, E_c \quad (\alpha, \dot{\alpha} = 1, 2; c = 1, \dots, 4) \quad (4)$$

tangent to Ω such that the fields E_{α} form a (complex) basis of the complex subspace at each point, the fields $\bar{E}_{\dot{\alpha}}$ define a basis in the complex conjugate plane and the fields E_c

complete $E_\alpha, \bar{E}_{\dot{\alpha}}$ to a (real) basis of the whole tangent space. The (anti)commutator of two vector fields tangent to Ω is also a vector field tangent to Ω . Thus we have

$$[E_A, E_B] = c_{AB}^D E_D, \quad (5)$$

where c_{AB}^D are some functions and the indices take on the values of the indices $\alpha, \dot{\alpha}, c$. The fact that our CR-structure defined on Ω is induced by a $GL(4, 2|\mathbb{C})$ -structure in the ambient space implies

$$\{E_\alpha, E_\beta\} = c_{\alpha\beta}^\gamma E_\gamma \quad (6)$$

and the corresponding complex conjugate equations. This means that we are dealing with an integrable CR-structure on Ω . We call a function Φ defined on Ω chiral if $\bar{E}_{\dot{\alpha}}\Phi = 0$. A function Φ^+ is called antichiral if $E_\alpha\Phi^+ = 0$. Note that the restriction to Ω of any holomorphic function defined in some neighborhood of Ω in $\mathbb{C}^{4|2}$ is a chiral function (the converse is also true in some sense, see [1]). Formula (6) looks exactly like formula (1). The only difference is that in the case at hand the complex fields E_α are defined on a real manifold Ω . Formula (2) can be used to construct an (anti)chiral function from an arbitrary given one. Moreover one can give an interpretation of formula (2) similar to those we gave for the purely real and complex cases using the complexification of Ω .

3. Formulation of $N = 1$ SYM in curved superspace in terms of induced geometry and CR-bundles

To construct the Lagrangian of $N = 1$ SYM theory in the induced geometry approach, first we need to say more about induced CR-structures and introduce some useful geometric notions.

Note that the basis (4) of tangent vectors defining a CR-structure on Ω is fixed up to linear transformations of the form

$$\begin{aligned} E'_a &= g_a^b E_b + g_a^\beta E_\beta + \bar{g}_a^{\dot{\beta}} \bar{E}_{\dot{\beta}}, \\ E'_\alpha &= g_\alpha^\beta E_\beta, \quad \bar{E}'_{\dot{\alpha}} = \bar{g}_{\dot{\alpha}}^{\dot{\beta}} \bar{E}_{\dot{\beta}}, \end{aligned} \quad (7)$$

where (g_a^b) is a real matrix and $(\bar{g}_{\dot{\alpha}}^{\dot{\beta}})$ is the complex conjugate matrix of $(g_\alpha^\beta E_\beta)$. If $\mathbb{C}^{4|2}$ is equipped with a volume element one can choose E_a, E_α to be a unimodular complex basis in the tangent space to $\mathbb{C}^{4|2}$. This allows one to restrict the transformations (7) by the requirement

$$\det(g_a^b) = \det(g_\alpha^\beta). \quad (8)$$

In this case we say that there is an induced SCR-structure on Ω . From now on we will assume that the basis (4) defines a SCR-structure.

For the functions c_{AB}^D defined by formula (5) in the case of induced SCR-structure we have the following identities:

$$c_{\alpha\beta}{}^{\dot{\gamma}} = c_{\alpha\beta}{}^d = 0, \quad c_{\alpha\dot{\beta}}{}^{\dot{\beta}} = c_{\alpha d}{}^d \quad (9)$$

and the corresponding complex conjugate ones.

We define the Levi matrix of the surface Ω by the expression

$$\Gamma_b^a = i\bar{\sigma}_b^{\alpha\beta} c_{\alpha\beta}^a, \quad (10)$$

where $\bar{\sigma}_b$ are the Pauli matrices for $b = 1, 2, 3$ and the identity matrix for $b = 0$. The matrix Γ_b^a coincides with the matrix of the Levi form defined in the standard way (see [1]).

To construct a Yang–Mills theory on $\Omega \subset \mathbb{C}^{4|2}$, we start with two complex vector bundles \mathcal{F} and \mathcal{F}^+ with structure group G . Denote the Lie algebra corresponding to G by \mathcal{G} . Trivializing our bundles we can represent their sections Φ and Φ^+ locally as vector functions, called fields. They describe matter and charge conjugated matter, respectively. Gauge transformations correspond to the change of trivialization. They have the form

$$\Phi' = e^{i\Lambda} \Phi, \quad (\Phi^+)' = e^{-i\bar{\Lambda}} \Phi^+, \quad (11)$$

where Λ and $\bar{\Lambda}$ are some functions (sections of corresponding homomorphism bundles) with values in the representation of the Lie algebra \mathcal{G} corresponding to the field Φ . We want to stress the fact that for now we consider \mathcal{F} and \mathcal{F}^+ separately, not requiring them to be complex conjugate bundles (the functions Λ , $\bar{\Lambda}$ in (11) are also independent). By SYM fields we understand two pairs of semiconnections

$$\nabla_\alpha \Phi^+ \equiv (E_\alpha + \mathcal{A}_\alpha) \Phi^+, \quad \bar{\nabla}_{\dot{\alpha}} \Phi \equiv (\bar{E}_{\dot{\alpha}} + \bar{\mathcal{A}}_{\dot{\alpha}}) \Phi$$

restricted by the conditions

$$\{\nabla_\alpha, \nabla_\beta\} = c_{\alpha\beta}{}^\gamma \nabla_\gamma, \quad \{\bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\dot{\beta}}\} = c_{\dot{\alpha}\dot{\beta}}{}^{\dot{\gamma}} \bar{\nabla}_{\dot{\gamma}}. \quad (12)$$

These conditions mean that the corresponding semiconnections have vanishing curvature. It can be shown (see for example [5]) that semiconnections $\bar{\nabla}_{\dot{\alpha}}$ satisfying (12) determine a CR-bundle structure on \mathcal{F}^+ , i.e. \mathcal{F}^+ can be pasted together from trivial bundles by chiral gluing functions. One can define the chiral sections as those annihilated by $\bar{\nabla}_{\dot{\alpha}}$. Then condition (12) guarantees that there are sufficiently many of them. Analogously, given ∇_α satisfying (12), one obtains a $\overline{\text{CR}}$ -bundle. By $\overline{\text{CR}}$ -bundle we mean a bundle whose gluing functions are antichiral. Moreover, one can take this property as a definition of CR- and $\overline{\text{CR}}$ -bundles (see [5] for details). Thus the basic geometrical objects we start with are the surface $\Omega \subset \mathbb{C}^{4|2}$, the CR-bundle \mathcal{F}^+ and the $\overline{\text{CR}}$ -bundle \mathcal{F} , both defined over Ω . The solutions to the zero curvature equations (12) can be written locally as

$$\mathcal{A}_\alpha = e^U E_\alpha e^{-U}, \quad \bar{\mathcal{A}}_{\dot{\alpha}} = e^{\tilde{U}} \bar{E}_{\dot{\alpha}} e^{-\tilde{U}}, \quad (13)$$

where U and \tilde{U} are some \mathcal{G} -valued fields. Note that the fields e^{-U} and $e^{\tilde{U}}$ are determined by (13) only up to the left multiplication by arbitrary antichiral and chiral fields, respectively. If we want to write a gauge invariant Lagrange function for chiral fields we will immediately encounter the difficulty in writing the kinetic term, which for the case of free chiral fields is simply $\Phi^+ \Phi$. This difficulty is due to the fact that in the case at hand these fields are sections

of different bundles. Therefore we are forced to identify CR- and $\overline{\text{CR}}$ -bundles choosing a section of the bundle $\mathcal{F}^+ \otimes \mathcal{F}^*$. Here \mathcal{F}^* is the dual to the bundle \mathcal{F} . This section we denote by e^V . Under the gauge transformations the field e^V transforms in the following way:

$$e^{V'} = e^{-i\Lambda'} e^V e^{i\bar{\Lambda}}. \quad (14)$$

Now we can take the gauge invariant combination $\Phi^+ e^V \Phi$ as a kinetic density term.

Our next goal is to describe a gauge invariant theory in terms of the fields Φ_i , Φ_i^+ , e^{-U} , $e^{\tilde{U}}$, e^V defined on our curved superspace Ω . This will be done in a manifestly SCR-covariant way, i.e. independently of the choice of basis vector fields (4) up to local SCR-transformations (7). Instead of the customary Lorentz connections in our construction of the Lagrangian, we use only objects defined by the internal geometry of the superspace Ω , namely the Levi matrix Γ and the functions c_{AB}^D . We postpone the details of this construction until Appendix B. Now we want to formulate the main result. The Lagrangian has the following form:

$$\begin{aligned} \mathcal{S} = & \int dV \left[\frac{1}{k} (\det \Gamma)^{-1} (\square_{\tilde{D}} e^G E^\alpha e^{-G}) (\square_{\tilde{D}} e^G E_\alpha e^{-G}) \right] \\ & + \int dV \left[\square_{\tilde{E}} \left| \frac{1}{4} \det \Gamma \right|^{-1/3} \Phi^+ e^V \Phi \right] \\ & + \int dV \left[a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \right] + \text{h.c.} \end{aligned} \quad (15)$$

Here as in flat space the $N = 1$ SYM Lagrangian contains a Lagrangian of gauge fields, a kinetic term of chiral fields and a term describing the interaction between chiral fields. In (15) we are using the following notations: k is a coupling constant, $e^G = e^{\tilde{U}} e^{-V} e^{-U'}$, a_i , m_{ij} , g_{ijk} are coupling constants which must be chosen in a way that ensures the gauge invariance of matter–matter interaction, $\square_{\tilde{E}}$ is a chiral projector whose general form was described in Section 2, and $\square_{\tilde{D}}$ is a “covariant” chiral projector which is constructed as $\square_{\tilde{E}}$ with derivatives \tilde{E}_α replaced by “covariant” derivatives \tilde{D}_α . More precisely, \tilde{D}_α acts on an arbitrary tensor V carrying the spinor index α in the following way:

$$\tilde{D}_\alpha V_\beta = \tilde{E}_\alpha V_\beta - \check{c}_{\alpha, \dot{\sigma} \beta}^{\dot{\sigma} \sigma} V_\sigma,$$

where $\check{c}_{\alpha, \dot{\sigma} \beta}^{\dot{\sigma} \sigma}$ are some functions which can be expressed in terms of c_{AB}^D (see formulae (B.6)–(B.8) in Appendix B). Note that the integrands in (15) are chiral functions (for the first term this fact follows from its construction, which is explained in detail in Appendix B). The integration in (15) should be understood as a chiral integration. If a chiral function can be extended to a holomorphic function in some domain in $\mathbb{C}^{4|2}$, then it can be integrated with the holomorphic volume element over a real $(4, 2)$ -submanifold contained in Ω .

We have constructed a Lagrangian depending on the field V , the specific combination of the fields e^U , $e^{\tilde{U}}$, e^V which we denote by e^G , and matter fields Φ_i , Φ_i^+ . All these fields are complex. In order to perform a functional integration over the fields U , \tilde{U} , V , Φ_i one has to restrict these fields to a real surface in functional space. Once this surface is chosen it will

restrict our large gauge group to a smaller one. The real surface we choose is given by the equations

$$\bar{\Phi}_i = \Phi_i^+, \quad V = \bar{V}', \quad -\bar{U} = \tilde{U}, \quad (16)$$

where the upper bar denotes complex conjugation. The gauge transformations preserving these reality conditions are of the form (11) and (14) where Λ and $\bar{\Lambda}$ are complex conjugates of each other and $T = \bar{S}^+ \equiv S$. Still we have a rather large gauge group. Let us do a partial gauge fixing by requiring that $e^U = 1$. By reality conditions (16) this implies $e^{-\bar{U}} = 1$ and therefore $e^{-G} = e^V$, which means that our Lagrangian (15) contains only the field V in this partial gauge fixing. The remaining gauge group contains the transformations with $i\Lambda = S^+$, $-i\bar{\Lambda} = S$, i.e. the antichiral transformations of the fields Φ_i and the corresponding complex conjugate chiral transformations of the fields Φ_i^+ .

4. Comparison with the conventional approach

In this section we want to compare our constructions with the conventional (Wess–Zumino) approach to supergravity and SYM theory in curved superspace which is presented in [2] in detail. We start with a comparison of formula (2) with the standard formula for (anti)chiral projection operators (see [2, Chap. 19]). But first let us recall briefly the main constituents of the conventional approach. In this approach we have a connection defined on a tangent bundle over (4, 4)-dimensional real superspace with the Lorentz group as structure group. Another dynamical variable in this approach is a vielbein E_A^M which defines the covariant derivatives \mathcal{D}_M on the tangent bundle and also identifies the Lorentz bundle with the tangent one allowing to transform world indices into Lorentz indices and vice versa. Here we are working with Lorentz indices. Under a certain set of constraints (see [2]) the vector fields $E_\alpha = E_\alpha^M \partial_M$ are closed with respect to the anticommutator. Moreover the functions $c_{\alpha\beta}{}^\gamma$ defining the anticommutation relations satisfy the following condition:

$$c_{\alpha\beta}{}^\gamma = \omega_{\alpha\beta}{}^\gamma + \omega_{\beta\alpha}{}^\gamma, \quad (17)$$

where $\omega_{\alpha\beta}{}^\gamma$ are the connection coefficients for the covariant differentiation of spinor fields, having only Lorentz indices.

The antichiral projector acts on arbitrary function Φ as follows:

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^+) \Phi \quad (18)$$

and gives an antichiral function as a result. Here $R^+ = \frac{1}{24} R_{\alpha\beta}{}^{\alpha\beta}$ denotes the invariant obtained from the curvature tensor

$$R_{\alpha\beta\delta}{}^\gamma = E_\alpha \omega_{\beta\delta}{}^\gamma + E_\beta \omega_{\alpha\delta}{}^\gamma + \omega_{\alpha\sigma}{}^\gamma \omega_{\beta\delta}{}^\sigma + \omega_{\beta\sigma}{}^\gamma \omega_{\alpha\delta}{}^\sigma - c_{\alpha\beta}{}^\sigma \omega_{\sigma\delta}{}^\gamma.$$

After the contraction of indices this gives the following expression for $-8R^+$:

$$-8R^+ = -(E_\alpha \omega_\beta{}^{\alpha\beta} + E_\beta \omega_\alpha{}^{\alpha\beta} + \omega_{\beta\sigma}{}^\beta \omega_\alpha{}^{\alpha\sigma}) + \omega^{\delta\alpha\beta} \omega_{\alpha\beta\delta}. \quad (19)$$

The induced geometry approach to supergravity has been shown to be equivalent to the Wess–Zumino one [1]. Thus one can use formula (2) to obtain (18). We identify the pair of vector fields E_α appearing in the Wess–Zumino approach with those in definition of the $\overline{\text{CR}}$ -structure induced on Ω (i.e. complex conjugate to the corresponding CR-structure). Formulae (18) and (2) must be equivalent at least up to the freedom described by formula (3). Indeed as one can easily check, substituting relation (17) in (2), we will get exactly formula (18) with the term $-8R^+$ expressed as in (19).

If one starts with the induced geometry approach then in order to get the Lorentz gauge group one has to require the following gauge condition:

$$c_{\alpha,\dot{\beta}}{}^a = 2i\sigma_{\alpha\dot{\beta}}^a,$$

where $\sigma_{\alpha\dot{\beta}}^a$ are the Pauli matrices. This condition fixes the SCR-basis (4) up to transformations of the form (7), where $\det(g_a^b) = \det(g_\alpha^\beta) = \det(\bar{g}_{\dot{\alpha}}^{\dot{\beta}}) = 1$, i.e. up to the Lorentz transformations. In this gauge $4(\det \Gamma)^{-1} = 1$, which as it is shown in Appendix B (see formulae (B.9) and (B.10)) implies

$$\check{c}_{\gamma,\sigma\dot{\sigma}}{}^{\sigma\dot{\sigma}} = \check{c}_{\dot{\gamma},\dot{\sigma}\sigma}{}^{\dot{\sigma}\sigma} = 0.$$

Moreover, the quantities $-\check{c}_{\dot{\alpha},\dot{\sigma}\beta}{}^{\dot{\sigma}\sigma}$, $-\check{c}_{\alpha,\sigma\dot{\beta}}{}^{\sigma\dot{\sigma}}$ transform now as coefficients of the Lorentz connection (as can be seen from (B.13) for Lorentz transformations). Thus it seems reasonable to identify $-\check{c}_{\dot{\alpha},\dot{\sigma}\beta}{}^{\dot{\sigma}\sigma}$ and $-\check{c}_{\alpha,\sigma\dot{\beta}}{}^{\sigma\dot{\sigma}}$ with the connection coefficients $\omega_{\dot{\alpha}\sigma}^{\dot{\beta}}$ and $\omega_{\alpha\dot{\sigma}}^{\dot{\beta}}$, respectively, from the conventional approach. The last assumption implies

$$\square_{\overline{\mathcal{D}}} V_\alpha = (\bar{\mathcal{D}}_{\dot{\gamma}} \bar{\mathcal{D}}^{\dot{\gamma}} - 8R) V_\alpha$$

and the corresponding complex conjugate identity. This shows that in the gauge specified above, the Yang–Mills Lagrangian term from (15) reduces to the standard one, which is written in terms of field strengths $\bar{W}_{\dot{\alpha}}$.

Indeed, the identities $-\check{c}_{\dot{\alpha},\dot{\sigma}\beta}{}^{\dot{\sigma}\sigma} = \omega_{\dot{\alpha}\sigma}^{\dot{\beta}}$, $-\check{c}_{\alpha,\sigma\dot{\beta}}{}^{\sigma\dot{\sigma}} = \omega_{\alpha\dot{\sigma}}^{\dot{\beta}}$ are true. One can derive them from the standard set of torsion constraints

$$T_{\underline{\alpha}\underline{\beta}}^{\underline{\gamma}} = 0, \quad T_{\alpha\beta}^a = T_{\dot{\alpha}\dot{\beta}}^a = 0, \quad T_{\alpha\beta}^a = 2i\sigma_{\alpha\dot{\beta}}^a, \quad T_{\underline{\alpha}b}^a = 0, \quad T_{ab}^c = 0, \quad (20)$$

where $\underline{\alpha}$ denotes either α or $\dot{\alpha}$. Finally, the volume element dV we used in (15) is nothing but the chiral volume element of the conventional approach, usually denoted as $\mathcal{E} d^2\Theta$. This completes the derivation of the conventional picture from that of induced geometry.

Appendix A. Explicit formula for the chiral projector

To prove formula (2) let us first consider the case of anticommuting vector fields, i.e. the functions $c_{\alpha\beta}{}^\gamma$ are identically zero.

Let X^M be coordinates on \mathcal{M} ($M = 1, \dots, m+n$), so we have $E_\alpha = E_\alpha^M \partial_M$ where ∂_M is the derivative with respect to X^M and E_α^M are the coordinates of the field E_α . Let the surface Σ be defined in a parametric form $X^M = X^M(\xi)$, where $\xi = (\xi^\alpha)$, ($\alpha = 1, 2$) are odd coordinates on Σ . Then we have the following equation for Σ :

$$\frac{\partial X^M(\xi)}{\partial \xi^\alpha} = E_\alpha^M(X(\xi)). \quad (\text{A.1})$$

Since we assume the fields E_α anticommute we can integrate this equation expanding $X^M(X_0, \xi)$ in ξ (here X_0 is an initial data for the system (A.1)). The solution to (A.1) reads as follows:

$$X^M(X_0, \xi) = X_0^M + \xi^\alpha E_\alpha^M + \frac{1}{4} \xi \xi E^\sigma(E_\sigma^M), \quad (\text{A.2})$$

where E_α^M and $E^\sigma(E_\sigma^M)$ are evaluated at the point $\xi = 0$ and $\xi \xi = \xi^\alpha \xi_\alpha$. The natural volume element on Σ in ξ -coordinates is simply $d\xi d\xi$. Expanding the given function Φ in ξ and performing the integration we get

$$(\square_E \Phi)(X_0) = \int \Phi(X(X_0, \xi)) d\xi d\xi = E^\alpha E_\alpha(\Phi), \quad (\text{A.3})$$

which is obviously annihilated by \tilde{E}_α . As one can easily see formula (A.3) coincides with (2) when $c_{\alpha\beta}^\gamma = 0$. Thus for the case of anticommuting vector fields formula (2) is proved.

Now let us consider two different pairs of vector fields E_α and \tilde{E}_α defining the same integrable distribution. Let $c_{\alpha\beta}^\gamma$ and $\tilde{c}_{\alpha\beta}^\gamma$ be the corresponding functions defining the anticommutation relations for each pair. The fields E_α and \tilde{E}_α are connected by means of some nondegenerate even matrix A :

$$\tilde{E}_\alpha = A_\alpha^\beta E_\beta. \quad (\text{A.4})$$

The transformation law for the functions $c_{\alpha\beta}^\gamma$ reads as follows:

$$\tilde{c}_{\alpha\beta}^\gamma = A_\alpha^\delta A_\beta^\sigma (A^{-1})_\mu^\gamma c_{\delta\sigma}^\mu + (A^{-1})_\delta^\gamma (A_\alpha^\sigma E_\sigma A_\beta^\delta + A_\beta^\sigma E_\sigma A_\alpha^\delta). \quad (\text{A.5})$$

Since the ratio of the natural supervolume element corresponding to the vector fields \tilde{E}_α to the one of the fields E_α equals $\det A$, the operations \square_E and $\square_{\tilde{E}}$ are related by the following formula:

$$\square_{\tilde{E}} \Phi = \square_E (\det A \Phi). \quad (\text{A.6})$$

Note that since our distribution is integrable, given the vector fields E_α we can perform a linear transformation (A.4) such that the new vector fields \tilde{E}_α will anticommute. Thus if we prove that the right-hand side of (2) under the transformations (A.4) satisfies (A.6) then we will have proved formula (2). We are going to show this for the case of infinitesimal transformations $A_\alpha^\beta = \delta_\alpha^\beta + a_\alpha^\beta$, where a_α^β is an infinitesimally small matrix. But this also proves (2) because every finite transformation (A.4) can be obtained as a succession of infinitesimally small ones.

Keeping only the terms of the first order in a , we have the following transformations:

$$\begin{aligned} \tilde{E}_\alpha &= E_\alpha + a_\alpha^\beta E_\beta, & \tilde{E}^\alpha &= E^\alpha - \tilde{a}_\beta^\alpha E^\beta, \\ \tilde{c}_{\alpha\beta}^\gamma &= c_{\alpha\beta}^\gamma + a_\alpha^\sigma c_{\sigma\beta}^\gamma + a_\beta^\sigma c_{\sigma\alpha}^\gamma - a_\sigma^\gamma c_{\alpha\beta}^\sigma + E_\alpha a_\beta^\gamma + E_\beta a_\alpha^\gamma, \end{aligned} \quad (\text{A.7})$$

where $\bar{a}_\alpha^\beta = \epsilon_{\alpha\nu} \epsilon^{\beta\mu} a_\mu^\nu$. Note that $a_\alpha^\beta - \bar{a}_\alpha^\beta = \delta_\alpha^\beta \text{tr}(a)$, where δ_α^β is the Kronecker symbol. Another useful identity is

$$E^\delta E_\sigma a_\delta^\sigma = \frac{1}{2} E^\delta E_\delta (\text{tr}(a)) + \frac{1}{2} c^\delta{}_\sigma{}^\gamma E_\gamma a_\delta^\sigma.$$

For the bilinear combinations in $c_{\alpha\beta}{}^\gamma$ which enter (2) and for the operator $E^\alpha E_\alpha$, up to the first order in a we have

$$\begin{aligned} \bar{c}^\alpha{}_\sigma{}^\gamma \bar{c}_{\alpha\beta}{}^\beta &= (1 + \text{tr}(a)) c^\alpha{}_\sigma{}^\gamma c_{\alpha\beta}{}^\beta + 2c^\alpha{}_\sigma{}^\gamma E_\alpha a_\beta^\beta + 2c^\alpha{}_\sigma{}^\gamma E_\beta a_\alpha^\beta, \\ \bar{c}_{\alpha\beta}{}^\sigma \bar{c}_\sigma{}^{\alpha\beta} &= (1 + \text{tr}(a)) c_{\alpha\beta}{}^\sigma c_\sigma{}^{\alpha\beta} + 2c_{\sigma\alpha}{}^\beta E^\alpha a_\beta^\sigma - 2c_\sigma{}^{\alpha\beta} E_\beta a_\alpha^\sigma, \\ \bar{c}_{\alpha\beta\sigma} \bar{c}^{\alpha\beta\sigma} &= (1 + \text{tr}(a)) c_{\alpha\beta\sigma} c^{\alpha\beta\sigma} + 4c^{\alpha\beta}{}_\sigma E_\alpha a_\beta^\sigma, \\ \bar{E}^\alpha \bar{E}_\alpha &= (1 + \text{tr}(a)) E^\alpha E_\alpha + (E^\alpha a_\alpha^\beta) E_\beta. \end{aligned} \quad (\text{A.8})$$

Note that the appearance of the combination $1 + \text{tr}(a)$ in (A.8) is natural because for the volume preserving transformations (A.4) the contraction of upper and lower indices becomes an invariant operation with respect to the derivative independent part of transformation (A.5). Substituting expressions (A.7) and (A.8) in the right hand side of (2) using the identities mentioned above and keeping terms only up to the first order in a , we obtain the following expression:

$$\begin{aligned} (1 + \text{tr}(a)) E^\alpha E_\alpha \Phi &+ ((1 + \text{tr}(a)) c^\alpha{}_\sigma{}^\sigma + 2(E^\alpha \text{tr}(a))) E_\alpha \Phi \\ &+ \Phi (E^\alpha E_\alpha \text{tr}(a) + c^\delta{}_\sigma{}^\sigma E_\delta \text{tr}(a) + (1 + \text{tr}(a)) \mathcal{V}(c)), \end{aligned} \quad (\text{A.9})$$

where for compactness of notation we denoted the term multiplying Φ in (2) by $\mathcal{V}(c)$. Now after some trivial transformations one can easily single out the factor $(1 + \text{tr}(a))$ in (A.9) and get

$$\begin{aligned} E^\alpha E_\alpha (\Phi (1 + \text{tr}(a))) &+ c^\alpha{}_\sigma{}^\sigma E_\alpha (\Phi (1 + \text{tr}(a))) + \Phi (1 + \text{tr}(a)) \mathcal{V}(c) \\ &= \square_E (\Phi (1 + \text{tr}(a))). \end{aligned} \quad (\text{A.10})$$

The last equation means that the right-hand side of (2) satisfies the infinitesimal version of (A.6), so that by the above considerations it really represents the operation \square_E .

Appendix B. Construction of the Lagrangian

We start with the term describing the interaction between matter chiral fields Φ_i . It has the form

$$\int dV \mathcal{U}(\Phi) + \text{h.c.} = \int dV \left[a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \right] + \text{h.c.}, \quad (\text{B.1})$$

where the function $\mathcal{U}(\Phi)$ is assumed to be gauge invariant. The integration here is as described in Section 3. As we do not have any metric, we have to restrict ourselves to SCR-transformations in order to preserve the volume element dV .

To construct the kinetic term one needs to apply first a chiral projector \square to the gauge invariant quantity $\Phi^+ e^V \Phi$ and then perform the integration described above. The explicit

construction of a chiral projector in terms of internal geometry of the surface Ω was given in Section 2. It was shown there that the action of a generic chiral projector can be written as follows:

$$\square_{\bar{E}} \rho \Phi^+ e^V \Phi,$$

where $\square_{\bar{E}}$ is a chiral projector corresponding to a natural volume element defined by the basis fields \bar{E}_{α} , the explicit construction of which is given by formula (2), and ρ is an arbitrary function. Note that under a change of vector fields \bar{E}_{α} corresponding to (7) the operation $\square_{\bar{E}}$ behaves as follows:

$$\square_{\bar{E}'} = \square_{\bar{E}} \det(\bar{g})$$

(see also formulae (A.4) and (A.6) in Appendix A). Therefore to have SCR covariance one has to choose the transformation law for ρ to be $\rho' = \rho / \det \bar{g}$. It can be easily checked that $\rho = |\det \Gamma|^{-1/3}$ has this property. Here $|\det \Gamma|$ stands for the absolute value of the determinant of the Levi form corresponding to the surface Ω . Therefore the SCR covariant expression of the kinetic term reads as

$$\int dV \left[\square_{\bar{E}} \left| \frac{1}{4} \det \Gamma \right|^{-1/3} \Phi^+ e^V \Phi \right] + \text{h.c.}, \quad (\text{B.2})$$

where $\frac{1}{4}$ is inserted for normalization purposes.

Finally let us turn to the construction of the Lagrangian of gauge fields. This turns out to be the most technically complicated part of the whole construction. First note that due to our large gauge invariance group (11), when Λ and $\tilde{\Lambda}$ are arbitrary functions with values in the gauge Lie algebra, it is impossible to construct the gauge strength fields W_{α} , $W_{\tilde{\alpha}}$ corresponding to each of fields e^V , e^{-U} , $e^{\tilde{U}}$. Consider the combination $e^G = e^{\tilde{U}} e^{-V} e^{-U'}$, which is invariant under gauge transformations. Due to the arbitrariness in the choice of solutions U and \tilde{U} to the zero curvature equations (12) mentioned in Section 3, there is an ambiguity in the definition of e^G described by the transformation

$$e^{G'} = e^T e^G e^{(S^+)'}, \quad (\text{B.3})$$

where T and S^+ are respectively, arbitrary, chiral and antichiral \mathcal{G} -valued functions.

The generic form of chiral field strengths for e^G reads as

$$W_{\alpha} = \square_{\bar{E}} B_{\alpha}^{\beta} e^G E_{\beta} e^{-G}, \quad (\text{B.4})$$

where B_{α}^{β} is a matrix to be defined by the invariance properties of W_{α} . The transformation law for B_{α}^{β} under (7) can be derived in terms of the invariance of construction (B.4) under (7), and has the form

$$(B')_{\alpha}^{\beta} = \frac{1}{\det(\bar{g})} B_{\alpha}^{\gamma} (g^{-1})_{\gamma}^{\beta}. \quad (\text{B.5})$$

Further restrictions on the matrix B come from the covariance requirement under the substitution (B.3). For W_{α} to transform covariantly under (B.3), the operator $\square_{\bar{E}} B_{\alpha}^{\beta} E_{\beta}$ ought

to annihilate chiral functions (the annihilation of antichiral functions is obvious). It is convenient to derive the restriction on the matrix B first in a special basis, namely where $\{\bar{E}_{\dot{\alpha}}, \bar{E}_{\dot{\beta}}\} = 0$ (see Appendix A for details). But first let us introduce some useful notations. One can choose the vectors $E_{\alpha\dot{\beta}} = \{E_{\alpha}, \bar{E}_{\dot{\beta}}\}$ as a basis of the real tangent space to Ω . Then we have the following commutation relations:

$$[\bar{E}_{\dot{\gamma}}, E_{\alpha\dot{\beta}}] = \check{c}_{\dot{\gamma},\alpha\dot{\beta}}^{\sigma\dot{\sigma}} E_{\sigma\dot{\sigma}} + \check{c}_{\dot{\gamma},\alpha\dot{\beta}}^{\sigma} E_{\sigma} + \check{c}_{\dot{\gamma},\alpha\dot{\beta}}^{\dot{\sigma}} \bar{E}_{\dot{\sigma}}, \quad (\text{B.6})$$

where $\check{c}_{\dot{\gamma},\alpha\dot{\beta}}^A$ are some functions. The connection between the two bases introduced in real space is given by the following formulae:

$$\begin{aligned} E_{\alpha\dot{\beta}} &= c_{\alpha\dot{\beta}}^a E_a + c_{\alpha\dot{\beta}}^{\gamma} E_{\gamma} + c_{\alpha\dot{\beta}}^{\dot{\gamma}} \bar{E}_{\dot{\gamma}}, \\ E_a &= \bar{c}_a^{\alpha\dot{\beta}} E_{\alpha\dot{\beta}} - \bar{c}_a^{\gamma} E_{\gamma} - \bar{c}_a^{\dot{\gamma}} \bar{E}_{\dot{\gamma}}, \end{aligned} \quad (\text{B.7})$$

where $\bar{c}_a^A = i(\Gamma^{-1})_a^b \bar{\sigma}_b^{\alpha\dot{\beta}} c_{\alpha\dot{\beta}}^A$. We use the notation $\bar{\sigma}_b^{\alpha\dot{\beta}}$ for the Pauli matrices and Γ^{-1} is the inverse of the Levi form matrix $\Gamma_a^b = i\bar{\sigma}_a^{\alpha\dot{\beta}} c_{\alpha\dot{\beta}}^b$. Substituting the first expression of (B.7) into the commutator on the left-hand side of (B.6), one obtains the following expression for the quantity $\check{c}_{\dot{\gamma},\sigma\dot{\beta}}^{\sigma\dot{\sigma}}$ in terms of the functions c_{AB}^D :

$$\check{c}_{\dot{\gamma},\sigma\dot{\beta}}^{\sigma\dot{\sigma}} = \bar{c}_a^{\sigma\dot{\sigma}} E_{\gamma} c_{\sigma\dot{\beta}}^a + c_{\sigma\dot{\beta}}^a \bar{c}_b^{\sigma\dot{\sigma}} c_{\gamma a}^b - c_{\gamma\dot{\beta}}^{\dot{\sigma}}. \quad (\text{B.8})$$

Contracting the upper index $\dot{\sigma}$ with the lower $\dot{\beta}$ we obtain the following important identity which we will need later:

$$\check{c}_{\dot{\gamma},\sigma\dot{\sigma}}^{\sigma\dot{\sigma}} = \bar{c}_a^{\sigma\dot{\sigma}} E_{\gamma} c_{\sigma\dot{\sigma}}^a + (c_{\gamma a}^a - c_{\gamma\dot{\sigma}}^{\dot{\sigma}}) = (\det \Gamma)^{-1} E_{\gamma} \det \Gamma, \quad (\text{B.9})$$

where in the last step we used the identity (9). The complex conjugate to Eq. (B.9) reads as

$$\check{c}_{\dot{\gamma},\dot{\sigma}\sigma}^{\dot{\sigma}\sigma} = (\det \Gamma)^{-1} \bar{E}_{\dot{\gamma}} \det \Gamma. \quad (\text{B.10})$$

Now we are ready to explore the restrictions on the matrix B in (B.4). For the basis specified above, applying the operator $\square_{\bar{E}} B_{\alpha}^{\beta} E_{\beta}$ to some chiral function Φ we get

$$\begin{aligned} \square_{\bar{E}} B_{\alpha}^{\beta} E_{\beta} \Phi &= \bar{E}^{\dot{\gamma}} \bar{E}_{\dot{\gamma}} B_{\alpha}^{\beta} E_{\beta} \Phi = [\bar{E}^{\dot{\gamma}}, \{ \bar{E}_{\dot{\gamma}}, B_{\alpha}^{\beta} E_{\beta} \}] \Phi \\ &= 2B_{\alpha}^{\delta} \epsilon^{\dot{\sigma}\dot{\delta}} ((B^{-1})_{\delta}^{\gamma} \bar{E}_{\dot{\delta}} B_{\gamma}^{\sigma} + \check{c}_{\delta,\dot{\sigma}\dot{\delta}}^{\dot{\sigma}\sigma}) E_{\sigma\dot{\sigma}} \Phi + f_{\alpha}^{\beta} E_{\beta} \Phi, \end{aligned} \quad (\text{B.11})$$

where by f_{α}^{β} we denote some functions which can be expressed in terms of the matrix B and its derivatives, but are unessential for further analysis. The last remark is due to the fact that if the coefficient of $E_{\sigma\dot{\sigma}}$ vanishes, then the whole operator vanishes on chiral functions. This is because the operator on the left-hand side of (B.11) acts from chiral functions to chiral ones (for the fixed index α) and it can be easily shown that an operator with this property and of the form $f_{\alpha}^{\beta} E_{\beta}$ is identically zero. Therefore we have the following equation for the matrix B :

$$(B^{-1})_{\delta}^{\gamma} \bar{E}_{\dot{\delta}} B_{\gamma}^{\sigma} = -\check{c}_{\delta,\dot{\sigma}\dot{\delta}}^{\dot{\sigma}\sigma}. \quad (\text{B.12})$$

It can be derived directly from definition (B.6) that the transformation law for the coefficients $-\check{c}_{\delta,\dot{\sigma}\delta}^{\dot{\sigma}\sigma}$ under (7) is as follows:

$$-(\check{c}')_{\delta,\dot{\sigma}\delta}^{\dot{\sigma}\sigma} = -\check{c}_{\dot{\nu},\dot{\sigma}\gamma}^{\dot{\sigma}\beta} \bar{g}_{\delta}^{\dot{\nu}} g_{\delta}^{\gamma} (g^{-1})_{\beta}^{\sigma} + \bar{g}_{\delta}^{\dot{\nu}} g_{\delta}^{\gamma} \bar{E}_{\dot{\nu}} (g^{-1})_{\gamma}^{\sigma} - \delta_{\delta}^{\sigma} \bar{g}_{\delta}^{\dot{\nu}} (\det \bar{g})^{-1} \bar{E}_{\dot{\nu}} \det \bar{g}, \quad (\text{B.13})$$

where δ_{δ}^{σ} is the Kronecker symbol. The first two terms in this expression are of the same form as the transformation law for semiconnection coefficients. One can easily check using (B.5) that the transformation rule for the expression on the left-hand side of (B.12) is of the same form, which is of course what one should expect since the operator under consideration is defined invariantly. This invariance implies that it is sufficient to prove the solvability of (B.12) in the special basis which we have already used. In this basis the integrability condition for (B.12) takes the form

$$\bar{E}_{\dot{\alpha}} \check{c}_{\dot{\beta},\dot{\sigma}\delta}^{\dot{\sigma}\sigma} + \bar{E}_{\dot{\beta}} \check{c}_{\dot{\alpha},\dot{\sigma}\delta}^{\dot{\sigma}\sigma} - \check{c}_{\dot{\alpha},\dot{\sigma}\delta}^{\dot{\sigma}\gamma} \check{c}_{\dot{\beta},\dot{\nu}\gamma}^{\dot{\nu}\sigma} - \check{c}_{\dot{\beta},\dot{\sigma}\delta}^{\dot{\sigma}\gamma} \check{c}_{\dot{\alpha},\dot{\nu}\gamma}^{\dot{\nu}\sigma} = 0. \quad (\text{B.14})$$

This formula follows from two identities:

$$\{\bar{E}_{\dot{\alpha}}, [\bar{E}_{\dot{\beta}}, E_{\gamma\dot{\gamma}}]\} = 0, \quad [\bar{E}_{\dot{\beta}}, E_{\gamma\dot{\alpha}}] + [\bar{E}_{\dot{\alpha}}, E_{\gamma\dot{\beta}}] = 0, \quad (\text{B.15})$$

which are true when $\{\bar{E}_{\dot{\alpha}}, \bar{E}_{\dot{\beta}}\} = 0$. The second identity in (B.15) can be equivalently written as

$$\check{c}_{\dot{\alpha},\dot{\beta}\delta}^A + \check{c}_{\dot{\beta},\dot{\alpha}\delta}^A = 0.$$

Writing the first identity in (B.15) in terms of the basis $E_{\alpha}, \bar{E}_{\dot{\alpha}}, E_{\alpha\dot{\alpha}}$, for the coefficient of $E_{\sigma\dot{\sigma}}$ we have

$$-\check{c}_{\dot{\beta},\dot{\gamma}\gamma}^{\dot{\mu}\nu} \check{c}_{\dot{\alpha},\dot{\mu}\nu}^{\dot{\sigma}\sigma} + \bar{E}_{\dot{\alpha}} \check{c}_{\dot{\beta},\dot{\gamma}\gamma}^{\dot{\sigma}\sigma} + \delta_{\dot{\alpha}}^{\dot{\sigma}} \check{c}_{\dot{\beta},\dot{\gamma}\gamma}^{\sigma} = 0.$$

Contracting the indices $\dot{\sigma}$ and $\dot{\gamma}$ in the last expression, symmetrizing it over the indices $\dot{\alpha}, \dot{\beta}$ and using the second identity in (B.15) we get (B.14). To summarize, we showed that Eq. (B.12) is solvable. Note, however, that there is a large arbitrariness in the choice of solutions to (B.12). Namely, one can multiply any solution by a matrix with chiral entries and get another one. We will show how to narrow the choice of solution. But first let us write the Lagrangian $W^{\alpha} W_{\alpha}$ in terms of the construction (B.4) taking into account (B.12). After some trivial transformations we will get

$$W^{\alpha} W_{\alpha} = \det B (\square_{\bar{D}} e^G E^{\alpha} e^{-G}) (\square_{\bar{D}} e^G E_{\alpha} e^{-G}), \quad (\text{B.16})$$

where $\square_{\bar{D}}$ is obtained from the usual chiral projector $\square_{\bar{E}}$ by substitution of the derivatives $\bar{E}_{\dot{\alpha}}$ by covariant derivatives $\bar{D}_{\dot{\alpha}}$ which act on a tensor V carrying a spinor index α in the following way:

$$\bar{D}_{\dot{\alpha}} V_{\beta} = \bar{E}_{\dot{\alpha}} V_{\beta} - \check{c}_{\dot{\alpha},\dot{\sigma}\beta}^{\dot{\sigma}\sigma} V_{\sigma}.$$

The only quantity in (B.16) which depends on the choice of the solution to (B.12) is $\det B$. From (B.12) we have

$$\det B \bar{E}_{\gamma} (\det B)^{-1} = \check{c}_{\dot{\gamma},\dot{\sigma}\sigma}^{\dot{\sigma}\sigma}.$$

As one can see from (B.10), the determinant of the Levi form Γ satisfies exactly the same equation. Therefore one can take $\det \Gamma$ multiplied by a constant factor as a solution to the last equation. We choose $\det B = 4(\det \Gamma)^{-1}$. With this choice of constant factor we will get the standard form for our Lagrangian in a Wess–Zumino gauge for the basis vector fields.

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