



On the stability of J^* -derivations

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ABSTRACT

In this paper, we establish the stability and superstability of J^* -derivations in J^* -algebras for the generalized Jensen-type functional equation

$$rf\left(\frac{x+y}{r}\right) + rf\left(\frac{x-y}{r}\right) = 2f(x).$$

Finally, we investigate the stability of J^* -derivations by using the fixed point alternative.

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1. Introduction

Let \mathcal{H} , \mathcal{K} be two Hilbert spaces and let $\mathcal{B}(\mathcal{H}, \mathcal{K})$ be the space of all bounded operators from \mathcal{H} into \mathcal{K} . By a J^* -algebra we mean a closed subspace \mathcal{A} of $\mathcal{B}(\mathcal{H}, \mathcal{K})$ such that $xx^*x \in \mathcal{A}$ whenever $x \in \mathcal{A}$. Many familiar spaces are J^* -algebras [1]. Of course J^* -algebras are not algebras in the ordinary sense. However from the point of view they may be considered a generalization of C^* -algebras; see [2,1,3]. In particular any Hilbert space may be thought of as a J^* -algebra identified with $\mathcal{L}(\mathcal{H}, \mathbb{C})$. Also any C^* -algebra in $\mathcal{B}(\mathcal{H})$ is a J^* -algebra. Other important examples of J^* -algebras are the so-called Cartan factors of type I, II, III and IV. A J^* -derivation on a J^* -algebra \mathcal{A} is defined to be a \mathbb{C} -linear mapping $d : \mathcal{A} \rightarrow \mathcal{A}$ such that

$$d(aa^*a) = d(a)a^*a + a(d(a))^*a + aa^*d(a)$$

for all $a \in \mathcal{A}$.

In particular, every $*$ -derivation on a C^* -algebra is a J^* -derivation.

The stability of functional equations was first introduced by Ulam [4] in 1940. More precisely, he proposed the following problem: Given a group G_1 , a metric group (G_2, d) and a positive number ϵ , does there exist a $\delta > 0$ such that if a function

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$f : G_1 \rightarrow G_2$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $T : G_1 \rightarrow G_2$ such that $d(f(x), T(x)) < \epsilon$ for all $x \in G_1$? As mentioned above, when this problem has a solution, we say that the homomorphisms from G_1 to G_2 are stable. In 1941, Hyers [5] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1978, Rassias [6] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Rassias [6] is called the Hyers–Ulam–Rassias stability.

Theorem 1.1. Let $f : E \rightarrow E'$ be a mapping from a normed space E into a Banach space E' subject to the inequality

$$\|f(x + y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p) \tag{1.1}$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \rightarrow E'$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2^p} \|x\|^p \tag{1.2}$$

for all $x \in E$. If $p < 0$ then inequality (1.1) holds for all $x, y \neq 0$, and (1.2) for $x \neq 0$. Also, if the function $t \mapsto f(tx)$ from \mathbb{R} into E' is continuous for each fixed $x \in E$, then T is linear.

During the last decades several stability problems of functional equations have been investigated by many mathematicians. A large list of references concerning the stability of functional equations can be found in [7–11].

Recently, Cădariu and Radu applied the fixed point method to the investigation of the functional equations. (see also [12–17]). In [18], Park establish the stability of homomorphisms between C^* -algebras (see also [19,20,15]). In Section 2 of the present paper, we establish the stability and superstability of J^* -derivations in J^* -algebras for the generalized Jensen-type functional equation

$$rf\left(\frac{x+y}{r}\right) + rf\left(\frac{x-y}{r}\right) = 2f(x). \tag{1.3}$$

In Section 3, we will use the fixed point alternative of Cădariu and Radu to prove the stability and superstability of J^* -derivations on J^* -algebras for the generalized Jensen-type functional equation (1.3).

Throughout this paper assume that \mathcal{A} is a J^* -algebra.

2. Stability

We start our work by a theorem in superstability of J^* -derivations.

Theorem 2.1. Let $r, s \in (1, \infty)$, and let $D : \mathcal{A} \rightarrow \mathcal{A}$ be a mapping for which $D(sa) = sD(a)$ for all $a \in \mathcal{A}$. Suppose there exists a function $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$ such that

$$\begin{aligned} \lim_n s^{-n} \phi(s^n a, s^n b, s^n c) &= 0, \\ \left\| r\mu D\left(\frac{a+b}{r}\right) + r\mu D\left(\frac{a-b}{r}\right) - 2D(\mu a) + D(cc^*c) - D(c)(c)^*c - cD(c)^*c - cc^*D(c) \right\| &\leq \phi(a, b, c), \end{aligned} \tag{2.1}$$

for all $\mu \in \mathbb{T}$ and all $a, b, c \in \mathcal{A}$. Then D is a J^* -derivation.

Proof. Put $\mu = a = b = 0$ in (2.1). Then

$$\begin{aligned} \|D(cc^*c) - D(c)c^*c - cD(c)^*c - cc^*D(c)\| &= \frac{1}{s^{3n}} \|D((s^n c)(s^n c^*)(s^n c)) - D(s^n c)(s^n c^*)(s^n c) - (s^n c)D(s^n c^*)(s^n c) \\ &\quad - (s^n c)(s^n c^*)D(s^n c)\| \leq \frac{1}{s^{3n}} \phi(0, 0, s^n c) \leq \frac{1}{s^n} \phi(0, 0, s^n c) \end{aligned}$$

for all $c \in \mathcal{A}$. The right-hand side tends to zero as $n \rightarrow \infty$. So

$$D(cc^*c) = D(c)c^*c + cD(c)^*c + cc^*D(c)$$

for all $c \in \mathcal{A}$. Similarly, put $c = 0$ in (2.1). Then

$$\begin{aligned} \left\| r\mu D\left(\frac{a+b}{r}\right) + r\mu D\left(\frac{a-b}{r}\right) - 2D(\mu a) \right\| &= \frac{1}{s^n} \left\| r\mu D\left(\frac{s^n a + s^n b}{r}\right) + r\mu D\left(\frac{s^n a - s^n b}{r}\right) - 2D(\mu s^n a) \right\| \\ &\leq \frac{1}{s^n} \phi(s^n a, s^n b, 0) \end{aligned}$$

for all $a, b \in \mathcal{A}$. The right-hand side tends to zero as $n \rightarrow \infty$. So,

$$r\mu D\left(\frac{a+b}{r}\right) + r\mu D\left(\frac{a-b}{r}\right) = 2D(\mu a) \quad (2.2)$$

for all $\mu \in \mathbb{T}$ and all $a, b \in \mathcal{A}$. Put $\mu = 1$ in above equation. Then

$$rD\left(\frac{a+b}{r}\right) + rD\left(\frac{a-b}{r}\right) = 2D(a)$$

for all $a, b \in \mathcal{A}$. This means that D satisfies (1.3). It is easy to show that D is additive. Putting $\mu = 1, b = 0$ in (2.2), we get

$$rD\left(\frac{a}{r}\right) = D(a)$$

for all $a \in \mathcal{A}$. Then by (2.2), we obtain that

$$\mu D(a+b) + \mu D(a-b) = 2D(\mu a)$$

for all $\mu \in \mathbb{T}$ and all $a, b \in \mathcal{A}$. Replacing b with a in above equation, then by additivity of D , we obtain that $\mu D(a) = D(\mu a)$ for all $a \in \mathcal{A}$ and all $\mu \in \mathbb{T}$. So it is easy to show that D is \mathbb{C} -linear (see for example Theorem 1 of [21]). \square

Theorem 2.2. Let $r \in (1, \infty)$, and let $f : \mathcal{A} \rightarrow \mathcal{A}$ be a mapping with $f(0) = 0$ for which there exists a function $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$ such that

$$\begin{aligned} \Phi(a, b, c) &:= \sum_{n=0}^{\infty} r^{-n} \phi(r^n a, r^n b, r^n c) < \infty, \\ \left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| &\leq \phi(a, b, c), \end{aligned} \quad (2.3)$$

for all $\mu \in \mathbb{T}$ and all $a, b, c \in \mathcal{A}$. Then there exists a unique J^* -derivation $D : \mathcal{A} \rightarrow \mathcal{A}$ such that

$$\|f(a) - D(a)\| \leq \frac{1}{2} \Phi(a, 0, 0) \quad (2.4)$$

for all $a \in \mathcal{A}$.

Proof. Put $\mu = 1$ and $b = c = 0$ in (2.3). It follows that

$$\|f(a) - r^{-1}f(ra)\| \leq \frac{1}{2} \phi(ra, 0, 0)$$

for all $a \in \mathcal{A}$. By induction, we can show that

$$\|f(a) - r^{-n}f(r^n a)\| \leq \frac{1}{2} \sum_{k=1}^n r^{-k} \phi(r^k a, 0, 0) \quad (2.5)$$

for all $a \in \mathcal{A}$. Replacing a by a^m in (2.5) and then dividing by r^m , we get

$$\|f(a^m) - r^{-n-m}f(r^{n+m}a)\| \leq \frac{1}{2r^m} \sum_m^{m+n} \phi(r^k a, 0, 0)$$

for all $a \in \mathcal{A}$. Hence, $\{r^{-n}f(r^n a)\}$ is a Cauchy sequence. Since \mathcal{A} is complete,

$$D(a) := \lim_n r^{-n}f(r^n a)$$

exists for all $a \in \mathcal{A}$. Now, (2.4) follows from (2.5). By using (2.1) one can show that

$$\begin{aligned} \left\| rD\left(\frac{a+b}{r}\right) + rD\left(\frac{a-b}{r}\right) - 2D(a) \right\| &= \lim_n \frac{1}{r^n} \|rf(r^{n-1}(a+b)) + rf(r^{n-1}(a-b)) - 2f(r^n a)\| \\ &\leq \lim_n \frac{1}{r^n} \phi(r^n a, r^n b, 0) = 0 \end{aligned}$$

for all $a, b \in \mathcal{A}$. So

$$rD\left(\frac{a+b}{r}\right) + rD\left(\frac{a-b}{r}\right) = 2D(a)$$

for all $a, b \in \mathcal{A}$. Putting $U = \frac{a+b}{r}$, $V = \frac{a-b}{r}$ in above equation, we get

$$r(D(U) + D(V)) = 2D\left(\frac{r(U + V)}{2}\right)$$

for all $U, V \in \mathcal{A}$. Hence, D is a Jensen type function. On the other hand, we have

$$\|D(\mu a) - \mu D(a)\| = \lim_n \frac{1}{r^n} \|f(\mu r^n a) - \mu f(r^n a)\| \leq \lim_n \frac{1}{r^n} \phi(r^n a, r^n a, 0) = 0$$

for all $\mu \in \mathbb{T}$, and all $a \in \mathcal{A}$. So it is easy to show that D is \mathbb{C} -linear. It follows from (2.1) that

$$\begin{aligned} \|D(cc^*c) - D(c)c^*c - cD(c^*)c - cc^*D(c)\| &= \lim_n \left\| \frac{1}{r^{3n}} f((r^n c)(r^n c^*)(r^n c)) - \frac{1}{r^n} f(r^n c) \frac{r^n c^*}{r^n} \frac{r^n c}{r^n} \right. \\ &\quad \left. - \frac{r^n c}{r^n} \frac{1}{r^n} f(r^n c^*) \frac{r^n c}{r^n} - \frac{r^n c}{r^n} \frac{r^n c^*}{r^n} \frac{1}{r^n} f(r^n c) \right\| \leq \lim_n \frac{1}{r^{3n}} \phi(0, 0, r^n c) \leq \lim_n \frac{1}{r^n} \phi(0, 0, r^n c) = 0 \end{aligned}$$

for all $c \in \mathcal{A}$. Thus $D : \mathcal{A} \rightarrow \mathcal{A}$ is a J^* -derivation satisfying (2.4), as desired. \square

We prove the following Hyers–Ulam–Rassias stability problem for J^* -derivations on J^* -algebras.

Corollary 2.3. Let $p \in (0, 1)$, $\theta \in [0, \infty)$ and $r \in (1, \infty)$ be real numbers. Suppose $f : A \rightarrow A$ satisfies

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| \leq \theta(\|a\|^p + \|b\|^p + \|c\|^p),$$

for all $\mu \in \mathbb{T}$ and all $a, b, c \in \mathcal{A}$. Then there exists a unique J^* -derivation $D : \mathcal{A} \rightarrow \mathcal{A}$ such that

$$\|f(a) - D(a)\| \leq \frac{2^p \theta}{2^{p-1} - 1} \|a\|^p$$

for all $a \in \mathcal{A}$.

Proof. It follows from Theorem 2.2 by putting $\phi(a, b, c) := \theta(\|a\|^p + \|b\|^p + \|c\|^p)$ for all $a, b, c \in \mathcal{A}$. \square

3. Stability by using alternative fixed point

Before proceeding to the main results of this section, we will state the following theorem.

Theorem 3.1 (The Alternative of Fixed Point [22]). Suppose that we are given a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \rightarrow \Omega$ with Lipschitz constant L . Then for each given $x \in \Omega$, either

$d(T^m x, T^{m+1} x) = \infty$ for all $m \geq 0$, or other exists a natural number m_0 such that

- ★ $d(T^m x, T^{m+1} x) < \infty$ for all $m \geq m_0$;
- ★ the sequence $\{T^m x\}$ is convergent to a fixed point y^* of T ;
- ★ y^* is the unique fixed point of T in the set $\Lambda = \{y \in \Omega : d(T^{m_0} x, y) < \infty\}$;
- ★ $d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$ for all $y \in \Lambda$.

Theorem 3.2. Let $r \in (1, \infty)$ be a real number. Let $f : \mathcal{A} \rightarrow \mathcal{A}$ be a mapping for which there exists a function $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$ such that

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| \leq \phi(a, b, c), \tag{3.1}$$

for all $\mu \in \mathbb{T}$ and all $a, b, c \in \mathcal{A}$. If there exists an $L < 1$ such that

$$\phi(a, b, c) \leq rL\phi\left(\frac{a}{r}, \frac{b}{r}, \frac{c}{r}\right) \tag{3.2}$$

for all $a, b, c \in \mathcal{A}$, then there exists a unique J^* -derivation $D : \mathcal{A} \rightarrow \mathcal{A}$ such that

$$\|f(a) - D(a)\| \leq \frac{L}{1-L} \phi(a, 0, 0) \tag{3.3}$$

for all $a \in \mathcal{A}$.

Proof. Putting $\mu = 1, b = c = 0$ in (3.1), we obtain

$$\left\| 2rf\left(\frac{a}{r}\right) - 2f(a) \right\| \leq \phi(a, 0, 0) \tag{3.4}$$

for all $a \in \mathcal{A}$. Hence,

$$\left\| \frac{1}{r}f(ra) - f(a) \right\| \leq \frac{1}{2r}\phi(ra, 0, 0) \leq L\phi(ra, 0, 0) \tag{3.5}$$

for all $a \in \mathcal{A}$.

Consider the set $X := \{g \mid g : A \rightarrow \mathcal{A}\}$ and introduce the generalized metric on X :

$$d(h, g) := \inf\{C \in \mathbb{R}^+ : \|g(a) - h(a)\| \leq C\phi(a, 0, 0)\forall a \in \mathcal{A}\}.$$

It is easy to show that (X, d) is complete. Now we define the linear mapping $J : X \rightarrow X$ by

$$J(h)(a) = \frac{1}{r}h(ra)$$

for all $a \in \mathcal{A}$. By Theorem 3.1 of [22],

$$d(J(g), J(h)) \leq Ld(g, h)$$

for all $g, h \in X$.

It follows from (3.5) that

$$d(f, J(f)) \leq L.$$

By Theorem 3.1, J has a unique fixed point in the set $X_1 := \{h \in X : d(f, h) < \infty\}$. Let D be the fixed point of J . D is the unique mapping satisfying

$$D(ra) = rD(a)$$

for all $a \in \mathcal{A}$ such that there exists $C \in (0, \infty)$ satisfying

$$\|D(a) - f(a)\| \leq C\phi(a, 0, 0)$$

for all $a \in \mathcal{A}$. On the other hand we have $\lim_n d(J^n(f), D) = 0$. It follows that

$$\lim_n \frac{1}{2^n}f(2^n a) = D(a)$$

for all $a \in \mathcal{A}$. It follows from $d(f, h) \leq \frac{1}{1-L}d(f, J(f))$, that

$$d(f, h) \leq \frac{L}{1-L}.$$

This implies the inequality (3.3).

It follows from (3.2) that

$$\lim_j r^{-j}\phi(r^j a, r^j b, r^j c) = 0$$

for all $a, b, c \in \mathcal{A}$.

By the same reasoning as in the proof of Theorem 2.2, one can show that the mapping $D : \mathcal{A} \rightarrow \mathcal{A}$ is a J^* -derivation satisfying (3.3), as desired. \square

We prove the following Hyers–Ulam–Rassias stability problem for J^* -derivations on J^* -algebras.

Corollary 3.3. Let $p \in (0, 1), \theta \in [0, \infty)$ be real numbers. Suppose $f : A \rightarrow A$ satisfies

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| \leq \theta(\|a\|^p + \|b\|^p + \|c\|^p),$$

for all $\mu \in \mathbb{T}$ and all $a, b, c \in \mathcal{A}$. Then there exists a unique J^* -derivation $D : \mathcal{A} \rightarrow \mathcal{A}$ such that

$$\|f(a) - D(a)\| \leq \frac{2^p\theta}{2-2^p}\|a\|^p$$

for all $a \in \mathcal{A}$.

Proof. Set $\phi(a, b, c) := \theta(\|a\|^p + \|b\|^p + \|c\|^p)$ all $a, b, c \in \mathcal{A}$. Letting $L = 2^{p-1}$, we get the desired result. \square

Now we establish the superstability of J^* -derivations by using the alternative of a fixed point.

Theorem 3.4. Let $s > 1$, and let $f : \mathcal{A} \rightarrow \mathcal{A}$ be a mapping satisfying $f(sx) = sf(x)$ for all $x \in \mathcal{A}$. Let $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$ be a mapping satisfying (3.1). If there exists an $L < 1$ such that

$$\phi(x, y, z) \leq rL\phi\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$$

for all $x, y, z \in \mathcal{A}$, then f is a J^* -derivation.

Proof. It is similar to the proof of Theorem 2.1. \square

Corollary 3.5. Let $r, p \in (0, 1)$, $\theta \in [0, \infty)$ be real numbers. Suppose that $f : A \rightarrow A$ is a function satisfying $f(rx) = rf(x)$ for all $x \in \mathcal{A}$. Let $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$ be a mapping satisfying (3.1). Then f is a J^* -derivation.

Proof. Set $\phi(x, y, z) := \theta(\|x\|^p + \|y\|^p + \|z\|^p)$ all $x, y, z \in \mathcal{A}$. Letting $L = 2^{p-1}$, we get the desired result. \square

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