



# On the stability of $J^*$ -derivations

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## ABSTRACT

In this paper, we establish the stability and superstability of  $J^*$ -derivations in  $J^*$ -algebras for the generalized Jensen-type functional equation

$$rf\left(\frac{x+y}{r}\right) + rf\left(\frac{x-y}{r}\right) = 2f(x).$$

Finally, we investigate the stability of  $J^*$ -derivations by using the fixed point alternative.

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## 1. Introduction

Let  $\mathcal{H}$ ,  $\mathcal{K}$  be two Hilbert spaces and let  $\mathcal{B}(\mathcal{H}, \mathcal{K})$  be the space of all bounded operators from  $\mathcal{H}$  into  $\mathcal{K}$ . By a  $J^*$ -algebra we mean a closed subspace  $\mathcal{A}$  of  $\mathcal{B}(\mathcal{H}, \mathcal{K})$  such that  $xx^*x \in \mathcal{A}$  whenever  $x \in \mathcal{A}$ . Many familiar spaces are  $J^*$ -algebras [1]. Of course  $J^*$ -algebras are not algebras in the ordinary sense. However from the point of view they may be considered a generalization of  $C^*$ -algebras; see [2,1,3]. In particular any Hilbert space may be thought of as a  $J^*$ -algebra identified with  $\mathcal{L}(\mathcal{H}, \mathbb{C})$ . Also any  $C^*$ -algebra in  $\mathcal{B}(\mathcal{H})$  is a  $J^*$ -algebra. Other important examples of  $J^*$ -algebras are the so-called Cartan factors of type I, II, III and IV. A  $J^*$ -derivation on a  $J^*$ -algebra  $\mathcal{A}$  is defined to be a  $\mathbb{C}$ -linear mapping  $d : \mathcal{A} \rightarrow \mathcal{A}$  such that

$$d(aa^*a) = d(a)a^*a + a(d(a))^*a + aa^*d(a)$$

for all  $a \in \mathcal{A}$ .

In particular, every  $*$ -derivation on a  $C^*$ -algebra is a  $J^*$ -derivation.

The stability of functional equations was first introduced by Ulam [4] in 1940. More precisely, he proposed the following problem: Given a group  $G_1$ , a metric group  $(G_2, d)$  and a positive number  $\epsilon$ , does there exist a  $\delta > 0$  such that if a function

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$f : G_1 \longrightarrow G_2$  satisfies the inequality  $d(f(xy), f(x)f(y)) < \delta$  for all  $x, y \in G_1$ , then there exists a homomorphism  $T : G_1 \rightarrow G_2$  such that  $d(f(x), T(x)) < \epsilon$  for all  $x \in G_1$ ? As mentioned above, when this problem has a solution, we say that the homomorphisms from  $G_1$  to  $G_2$  are stable. In 1941, Hyers [5] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that  $G_1$  and  $G_2$  are Banach spaces. In 1978, Rassias [6] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Rassias [6] is called the Hyers–Ulam–Rassias stability.

**Theorem 1.1.** Let  $f : E \longrightarrow E'$  be a mapping from a normed space  $E$  into a Banach space  $E'$  subject to the inequality

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p) \quad (1.1)$$

for all  $x, y \in E$ , where  $\epsilon$  and  $p$  are constants with  $\epsilon > 0$  and  $p < 1$ . Then there exists a unique additive mapping  $T : E \longrightarrow E'$  such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2-2^p} \|x\|^p \quad (1.2)$$

for all  $x \in E$ . If  $p < 0$  then inequality (1.1) holds for all  $x, y \neq 0$ , and (1.2) for  $x \neq 0$ . Also, if the function  $t \mapsto f(tx)$  from  $\mathbb{R}$  into  $E'$  is continuous for each fixed  $x \in E$ , then  $T$  is linear.

During the last decades several stability problems of functional equations have been investigated by many mathematicians. A large list of references concerning the stability of functional equations can be found in [7–11].

Recently, Cădariu and Radu applied the fixed point method to the investigation of the functional equations. (see also [12–17]). In [18], Park establish the stability of homomorphisms between  $C^*$ -algebras (see also [19,20,15]). In Section 2 of the present paper, we establish the stability and superstability of  $J^*$ -derivations in  $J^*$ -algebras for the generalized Jensen-type functional equation

$$rf\left(\frac{x+y}{r}\right) + rf\left(\frac{x-y}{r}\right) = 2f(x). \quad (1.3)$$

In Section 3, we will use the fixed point alternative of Cădariu and Radu to prove the stability and superstability of  $J^*$ -derivations on  $J^*$ -algebras for the generalized Jensen-type functional equation (1.3).

Throughout this paper assume that  $\mathcal{A}$  is a  $J^*$ -algebra.

## 2. Stability

We start our work by a theorem in superstability of  $J^*$ -derivations.

**Theorem 2.1.** Let  $r, s \in (1, \infty)$ , and let  $D : \mathcal{A} \rightarrow \mathcal{A}$  be a mapping for which  $D(sa) = sD(a)$  for all  $a \in \mathcal{A}$ . Suppose there exists a function  $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$  such that

$$\lim_n s^{-n} \phi(s^n a, s^n b, s^n c) = 0, \\ \left\| r\mu D\left(\frac{a+b}{r}\right) + r\mu D\left(\frac{a-b}{r}\right) - 2D(\mu a) + D(cc^*c) - D(c)(c)^*c - cD(c)^*c - cc^*D(c) \right\| \leq \phi(a, b, c), \quad (2.1)$$

for all  $\mu \in \mathbb{T}$  and all  $a, b, c \in \mathcal{A}$ . Then  $D$  is a  $J^*$ -derivation.

**Proof.** Put  $\mu = a = b = 0$  in (2.1). Then

$$\|D(cc^*c) - D(c)c^*c - cD(c)^*c - cc^*D(c)\| = \frac{1}{s^{3n}} \|D((s^n c)(s^n c^*)(s^n c)) - D(s^n c)(s^n c^*)(s^n c) - (s^n c)D(s^n c^*)(s^n c) \\ - (s^n c)(s^n c^*)D(s^n c)\| \leq \frac{1}{s^{3n}} \phi(0, 0, s^n c) \leq \frac{1}{s^n} \phi(0, 0, s^n c)$$

for all  $c \in \mathcal{A}$ . The right-hand side tends to zero as  $n \rightarrow \infty$ . So

$$D(cc^*c) = D(c)c^*c + cD(c)^*c + cc^*D(c)$$

for all  $c \in \mathcal{A}$ . Similarly, put  $c = 0$  in (2.1). Then

$$\left\| r\mu D\left(\frac{a+b}{r}\right) + r\mu D\left(\frac{a-b}{r}\right) - 2D(\mu a) \right\| = \frac{1}{s^n} \left\| r\mu D\left(\frac{s^n a + s^n b}{r}\right) + r\mu D\left(\frac{s^n a - s^n b}{r}\right) - 2D(\mu s^n a) \right\| \\ \leq \frac{1}{s^n} \phi(s^n a, s^n b, 0)$$

for all  $a, b \in \mathcal{A}$ . The right-hand side tends to zero as  $n \rightarrow \infty$ . So,

$$r\mu D\left(\frac{a+b}{r}\right) + r\mu D\left(\frac{a-b}{r}\right) = 2D(\mu a) \quad (2.2)$$

for all  $\mu \in \mathbb{T}$  and all  $a, b \in \mathcal{A}$ . Put  $\mu = 1$  in above equation. Then

$$rD\left(\frac{a+b}{r}\right) + rD\left(\frac{a-b}{r}\right) = 2D(a)$$

for all  $a, b \in \mathcal{A}$ . This means that  $D$  satisfies (1.3). It is easy to show that  $D$  is additive. Putting  $\mu = 1, b = 0$  in (2.2), we get

$$rD\left(\frac{a}{r}\right) = D(a)$$

for all  $a \in \mathcal{A}$ . Then by (2.2), we obtain that

$$\mu D(a+b) + \mu D(a-b) = 2D(\mu a)$$

for all  $\mu \in \mathbb{T}$  and all  $a, b \in \mathcal{A}$ . Replacing  $b$  with  $a$  in above equation, then by additivity of  $D$ , we obtain that  $\mu D(a) = D(\mu a)$  for all  $a \in \mathcal{A}$  and all  $\mu \in \mathbb{T}$ . So it is easy to show that  $D$  is  $\mathbb{C}$ -linear (see for example Theorem 1 of [21]).  $\square$

**Theorem 2.2.** Let  $r \in (1, \infty)$ , and let  $f : \mathcal{A} \rightarrow \mathcal{A}$  be a mapping with  $f(0) = 0$  for which there exists a function  $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$  such that

$$\begin{aligned} \Phi(a, b, c) &:= \sum_{n=0}^{\infty} r^{-n} \phi(r^n a, r^n b, r^n c) < \infty, \\ \left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| &\leq \phi(a, b, c), \end{aligned} \quad (2.3)$$

for all  $\mu \in \mathbb{T}$  and all  $a, b, c \in \mathcal{A}$ . Then there exists a unique  $J^*$ -derivation  $D : \mathcal{A} \rightarrow \mathcal{A}$  such that

$$\|f(a) - D(a)\| \leq \frac{1}{2} \Phi(a, 0, 0) \quad (2.4)$$

for all  $a \in \mathcal{A}$ .

**Proof.** Put  $\mu = 1$  and  $b = c = 0$  in (2.3). It follows that

$$\|f(a) - r^{-1}f(ra)\| \leq \frac{1}{2} \phi(ra, 0, 0)$$

for all  $a \in \mathcal{A}$ . By induction, we can show that

$$\|f(a) - r^{-n}f(r^n a)\| \leq \frac{1}{2} \sum_{k=1}^n r^{-k} \phi(r^k a, 0, 0) \quad (2.5)$$

for all  $a \in \mathcal{A}$ . Replacing  $a$  by  $a^m$  in (2.5) and then dividing by  $r^m$ , we get

$$\|f(a^m) - r^{-n-m}f(r^{n+m}a)\| \leq \frac{1}{2r^m} \sum_{k=m}^{m+n} \phi(r^k a, 0, 0)$$

for all  $a \in \mathcal{A}$ . Hence,  $\{r^{-n}f(r^n a)\}$  is a Cauchy sequence. Since  $\mathcal{A}$  is complete,

$$D(a) := \lim_n r^{-n}f(r^n a)$$

exists for all  $a \in \mathcal{A}$ . Now, (2.4) follows from (2.5). By using (2.1) one can show that

$$\begin{aligned} \left\| rD\left(\frac{a+b}{r}\right) + rD\left(\frac{a-b}{r}\right) - 2D(a) \right\| &= \lim_n \frac{1}{r^n} \|rf(r^{n-1}(a+b)) + rf(r^{n-1}(a-b)) - 2f(r^n a)\| \\ &\leq \lim_n \frac{1}{r^n} \phi(r^n a, r^n b, 0) = 0 \end{aligned}$$

for all  $a, b \in \mathcal{A}$ . So

$$rD\left(\frac{a+b}{r}\right) + rD\left(\frac{a-b}{r}\right) = 2D(a)$$

for all  $a, b \in \mathcal{A}$ . Putting  $U = \frac{a+b}{r}$ ,  $V = \frac{a-b}{r}$  in above equation, we get

$$r(D(U) + D(V)) = 2D\left(\frac{r(U+V)}{2}\right)$$

for all  $U, V \in \mathcal{A}$ . Hence,  $D$  is a Jensen type function. On the other hand, we have

$$\|D(\mu a) - \mu D(a)\| = \lim_n \frac{1}{r^n} \|f(\mu r^n a) - \mu f(r^n a)\| \leq \lim_n \frac{1}{r^n} \phi(r^n a, r^n a, 0) = 0$$

for all  $\mu \in \mathbb{T}$ , and all  $a \in \mathcal{A}$ . So it is easy to show that  $D$  is  $\mathbb{C}$ -linear. It follows from (2.1) that

$$\begin{aligned} \|D(cc^*c) - D(c)c^*c - cD(c^*)c - cc^*D(c)\| &= \lim_n \left\| \frac{1}{r^{3n}} f((r^n c)(r^n c^*)(r^n c)) - \frac{1}{r^n} f(r^n c) \frac{r^n c^*}{r^n} \frac{r^n c}{r^n} \right. \\ &\quad \left. - \frac{r^n c}{r^n} \frac{1}{r^n} f(r^n c^*) \frac{r^n c}{r^n} - \frac{r^n c}{r^n} \frac{r^n c^*}{r^n} \frac{1}{r^n} f(r^n c) \right\| \leq \lim_n \frac{1}{r^{3n}} \phi(0, 0, r^n c) \leq \lim_n \frac{1}{r^n} \phi(0, 0, r^n c) = 0 \end{aligned}$$

for all  $c \in \mathcal{A}$ . Thus  $D : \mathcal{A} \rightarrow \mathcal{A}$  is a  $J^*$ -derivation satisfying (2.4), as desired.  $\square$

We prove the following Hyers–Ulam–Rassias stability problem for  $J^*$ -derivations on  $J^*$ -algebras.

**Corollary 2.3.** Let  $p \in (0, 1)$ ,  $\theta \in [0, \infty)$  and  $r \in (1, \infty)$  be real numbers. Suppose  $f : A \rightarrow A$  satisfies

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| \leq \theta(\|a\|^p + \|b\|^p + \|c\|^p),$$

for all  $\mu \in \mathbb{T}$  and all  $a, b, c \in \mathcal{A}$ . Then there exists a unique  $J^*$ -derivation  $D : \mathcal{A} \rightarrow \mathcal{A}$  such that

$$\|f(a) - D(a)\| \leq \frac{2^p \theta}{2^{p-1} - 1} \|a\|^p$$

for all  $a \in \mathcal{A}$ .

**Proof.** It follows from Theorem 2.2 by putting  $\phi(a, b, c) := \theta(\|a\|^p + \|b\|^p + \|c\|^p)$  for all  $a, b, c \in \mathcal{A}$ .  $\square$

### 3. Stability by using alternative fixed point

Before proceeding to the main results of this section, we will state the following theorem.

**Theorem 3.1** (The Alternative of Fixed Point [22]). Suppose that we are given a complete generalized metric space  $(\Omega, d)$  and a strictly contractive mapping  $T : \Omega \rightarrow \Omega$  with Lipschitz constant  $L$ . Then for each given  $x \in \Omega$ , either

$d(T^m x, T^{m+1} x) = \infty$  for all  $m \geq 0$ , or other exists a natural number  $m_0$  such that

- ★  $d(T^m x, T^{m+1} x) < \infty$  for all  $m \geq m_0$ ;
- ★ the sequence  $\{T^m x\}$  is convergent to a fixed point  $y^*$  of  $T$ ;
- ★  $y^*$  is the unique fixed point of  $T$  in the set  $\Lambda = \{y \in \Omega : d(T^{m_0} x, y) < \infty\}$ ;
- ★  $d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$  for all  $y \in \Lambda$ .

**Theorem 3.2.** Let  $r \in (1, \infty)$  be a real number. Let  $f : \mathcal{A} \rightarrow \mathcal{A}$  be a mapping for which there exists a function  $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$  such that

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| \leq \phi(a, b, c), \quad (3.1)$$

for all  $\mu \in \mathbb{T}$  and all  $a, b, c \in \mathcal{A}$ . If there exists an  $L < 1$  such that

$$\phi(a, b, c) \leq rL\phi\left(\frac{a}{r}, \frac{b}{r}, \frac{c}{r}\right) \quad (3.2)$$

for all  $a, b, c \in \mathcal{A}$ , then there exists a unique  $J^*$ -derivation  $D : \mathcal{A} \rightarrow \mathcal{A}$  such that

$$\|f(a) - D(a)\| \leq \frac{L}{1-L} \phi(a, 0, 0) \quad (3.3)$$

for all  $a \in \mathcal{A}$ .

**Proof.** Putting  $\mu = 1$ ,  $b = c = 0$  in (3.1), we obtain

$$\left\| 2rf\left(\frac{a}{r}\right) - 2f(a) \right\| \leq \phi(a, 0, 0) \quad (3.4)$$

for all  $a \in \mathcal{A}$ . Hence,

$$\left\| \frac{1}{r}f(ra) - f(a) \right\| \leq \frac{1}{2r}\phi(ra, 0, 0) \leq L\phi(ra, 0, 0) \quad (3.5)$$

for all  $a \in \mathcal{A}$ .

Consider the set  $X := \{g \mid g : A \rightarrow \mathcal{A}\}$  and introduce the generalized metric on  $X$ :

$$d(h, g) := \inf\{C \in \mathbb{R}^+ : \|g(a) - h(a)\| \leq C\phi(a, 0, 0) \forall a \in \mathcal{A}\}.$$

It is easy to show that  $(X, d)$  is complete. Now we define the linear mapping  $J : X \rightarrow X$  by

$$J(h)(a) = \frac{1}{r}h(ra)$$

for all  $a \in \mathcal{A}$ . By Theorem 3.1 of [22],

$$d(J(g), J(h)) \leq Ld(g, h)$$

for all  $g, h \in X$ .

It follows from (3.5) that

$$d(f, J(f)) \leq L.$$

By Theorem 3.1,  $J$  has a unique fixed point in the set  $X_1 := \{h \in X : d(f, h) < \infty\}$ . Let  $D$  be the fixed point of  $J$ .  $D$  is the unique mapping satisfying

$$D(ra) = rD(a)$$

for all  $a \in \mathcal{A}$  such that there exists  $C \in (0, \infty)$  satisfying

$$\|D(a) - f(a)\| \leq C\phi(a, 0, 0)$$

for all  $a \in \mathcal{A}$ . On the other hand we have  $\lim_n d(J^n(f), D) = 0$ . It follows that

$$\lim_n \frac{1}{2^n}f(2^n a) = D(a)$$

for all  $a \in \mathcal{A}$ . It follows from  $d(f, h) \leq \frac{1}{1-L}d(f, J(f))$ , that

$$d(f, h) \leq \frac{L}{1-L}.$$

This implies the inequality (3.3).

It follows from (3.2) that

$$\lim_j r^{-j}\phi(r^j a, r^j b, r^j c) = 0$$

for all  $a, b, c \in \mathcal{A}$ .

By the same reasoning as in the proof of Theorem 2.2, one can show that the mapping  $D : \mathcal{A} \rightarrow \mathcal{A}$  is a  $J^*$ -derivation satisfying (3.3), as desired.  $\square$

We prove the following Hyers–Ulam–Rassias stability problem for  $J^*$ -derivations on  $J^*$ -algebras.

**Corollary 3.3.** Let  $p \in (0, 1)$ ,  $\theta \in [0, \infty)$  be real numbers. Suppose  $f : A \rightarrow A$  satisfies

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) + f(cc^*c) - f(c)(c)^*c - cf(c)^*c - cc^*f(c) \right\| \leq \theta(\|a\|^p + \|b\|^p + \|c\|^p),$$

for all  $\mu \in \mathbb{T}$  and all  $a, b, c \in \mathcal{A}$ . Then there exists a unique  $J^*$ -derivation  $D : \mathcal{A} \rightarrow \mathcal{A}$  such that

$$\|f(a) - D(a)\| \leq \frac{2^p\theta}{2-2^p}\|a\|^p$$

for all  $a \in \mathcal{A}$ .

**Proof.** Set  $\phi(a, b, c) := \theta(\|a\|^p + \|b\|^p + \|c\|^p)$  all  $a, b, c \in \mathcal{A}$ . Letting  $L = 2^{p-1}$ , we get the desired result.  $\square$

Now we establish the superstability of  $J^*$ -derivations by using the alternative of a fixed point.

**Theorem 3.4.** Let  $s > 1$ , and let  $f : \mathcal{A} \rightarrow \mathcal{A}$  be a mapping satisfying  $f(sx) = sf(x)$  for all  $x \in \mathcal{A}$ . Let  $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$  be a mapping satisfying (3.1). If there exists an  $L < 1$  such that

$$\phi(x, y, z) \leq rL\phi\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$$

for all  $x, y, z \in \mathcal{A}$ , then  $f$  is a  $J^*$ -derivation.

**Proof.** It is similar to the proof of Theorem 2.1.  $\square$

**Corollary 3.5.** Let  $r, p \in (0, 1)$ ,  $\theta \in [0, \infty)$  be real numbers. Suppose that  $f : A \rightarrow A$  is a function satisfying  $f(rx) = rf(x)$  for all  $x \in \mathcal{A}$ . Let  $\phi : \mathcal{A}^3 \rightarrow [0, \infty)$  be a mapping satisfying (3.1). Then  $f$  is a  $J^*$ -derivation.

**Proof.** Set  $\phi(x, y, z) := \theta(\|x\|^p + \|y\|^p + \|z\|^p)$  all  $x, y, z \in \mathcal{A}$ . Letting  $L = 2^{p-1}$ , we get the desired result.  $\square$

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