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Tatsuya Seko, Shun Maeta

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CLASSIFICATION OF ALMOST YAMABE SOLITONS IN EUCLIDEAN SPACES

TATSUYA SEKO AND SHUN MAETA

Abstract. In this paper, we completely classify almost Yamabe solitons on hypersurfaces in Euclidean spaces arisen from the position vector field. Some results of almost Yamabe solitons with a concurrent vector field and almost Yamabe solitons on submanifolds in Riemannian manifolds equipped with a concurrent vector field are also presented. Moreover, we classify complete Ricci solitons on minimal submanifolds in non-positively curved space forms. For almost Yamabe solitons, all of the results in this paper can be applied to Yamabe solitons.

1. Introduction

A Riemannian manifold (M, g) is called a Yamabe soliton if it satisfies

$$(R-\rho)g = \frac{1}{2}\mathcal{L}_v g, \tag{1}$$

where v is a vector field on M, $\mathcal{L}_v g$ is the Lie-derivative of g, R is the scalar curvature of M and ρ is a constant. We denote a Yamabe soliton by (M, g, v, ρ). If $\rho > 0$, $\rho = 0$, $\rho < 0$, then a Yamabe soliton (M, g, v, ρ) is called a shrinking, a steady or an expanding Yamabe soliton, respectively. A Yamabe soliton (M, g, v, ρ) is called a gradient Yamabe soliton if v is the gradient of some function f on M. We denote a gradient Yamabe soliton by (M, g, f, ρ). Yamabe solitons are special solutions to the Yamabe flow introduced by Hamilton [13]. Under some conditions, Yamabe solitons have been studied (cf. [5], [8], [12], [14] and [15]). In particular, any compact Yamabe soliton has constant scalar curvature (cf. [5], [8] and [14]).

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E. Barbosa and E. Ribeiro introduced a generalization of Yamabe solitons in [2] as follows.

Definition 1.1 ([2])**.** A Riemannian manifold (M, g, v, ρ) is an almost Yamabe soliton if there exist a complete vector field v and a smooth soliton function ρ on M satisfying

$$(R - \rho)g = \frac{1}{2}\mathcal{L}_v g. \tag{2}$$

An almost Yamabe soliton (M, g, v, ρ) is called a gradient almost Yamabe soliton if v is the gradient of some function f on M. We denote a gradient almost Yamabe soliton by (M, g, f, ρ).

From the definition, if ρ is constant, almost Yamabe solitons are Yamabe solitons. Obviously, Einstein manifolds are almost Yamabe solitons. It is well known that if M is an Einstein manifold with $\mathrm{Ric} = \rho g$, then the function ρ is constant, where Ric is the Ricci tensor on M. However, many examples of almost Yamabe solitons are constructed by E. Barbosa and E. Ribeiro (cf. [2]). In particular, examples of compact almost Yamabe solitons are known. Therefore the "almost" condition is interesting. The simplest example of almost Yamane soliton is $(\mathbb{R}^n,\ g_{\mathrm{can}},\ \frac{1}{2}|x|^2,\ -1)$, where g_{can} is the standard Euclidean metric.

Example 1 ([2])**.** Let v be a fixed vector in $\mathbb{R}^{n+1}$ and v^T stands for its orthogonal projection over the tangent bundle of a standard sphere (S^n, g_{S^n}). Here we notice that v^T is the gradient of the right function h_v. Then $(S^n,\ g_{S^n},\ v^T,\ \rho = \frac{1}{n}(\Delta h_v + R))$ is a compact gradient almost Yamabe soliton.

A non-compact almost Yamabe soliton is also known.

Example 2 ([2])**.** A product manifold $(\mathbb{R}\times_{\cosh t} S^n,\ dt^2+\cosh^2 t g_{S^n},\ f = \sinh t,\ \rho = \sinh t + n)$, where g_{S^n} is the standard metric of S^n, is a non-compact gradient almost Yamabe soliton.

We also construct some examples of gradient almost Yamabe solitons.

Example 3. Let $f = \log(e^{x^1} + e^{x^2} + 1)$ be a function on $\mathbb{R}^2$ with coordinate system $\{x^1, x^2\}$. Assume that $g = \nabla\nabla f$, then g is a Riemannian metric on $\mathbb{R}^2$. By an easy argument, $(\mathbb{R}^2,\ g,\ f,\ \rho = -\frac{2e^{x^1}+2e^{x^2}+3}{2(1+e^{x^1}+e^{x^2})^2} - 1)$ is a gradient almost Yamabe soliton.

Inspired by constructing Hamilton's cigar soliton, we can construct many examples of almost gradient Yamabe solitons.

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Example 4. If a metric takes the form $g = ds^2 + \phi^2(s)d\theta^2$ on $\mathbb{R}^2$, then the scalar curvature is $R(s,\theta) = -2\frac{\phi''}{\phi}$. Therefore $(\mathbb{R}^2, g = ds^2+\phi^2(s)d\theta^2, f = as+b\theta, \rho = -2\frac{\phi''}{\phi})$, $(a, b \in \mathbb{R})$, is a gradient almost Yamabe soliton. By using this, we can construct many examples of gradient almost Yamabe solitons. In particular, $(\mathbb{R}^2, g = ds^2 + e^{4s}d\theta^2, f = as + b\theta, \rho = -4(1 + 2s^2))$ is a gradient almost Yamabe soliton.

In this paper, we classify almost Yamabe solitons on hypersurfaces in Euclidean spaces. We first consider a more general condition, that is, we consider almost Yamabe solitons on submanifolds in Riemannian manifolds with a concurrent vector field. A vector field v on M is called a concurrent vector field if it satisfies

$$\nabla_X v = X, \tag{3}$$

for any vector field X on M, where ∇ is the Levi-Civita connection on M. The notion of the concurrent vector field is a generalization of some basic notions in Riemannian geometry. One of the most important example of Riemannian manifolds with a concurrent vector field is Euclidean spaces. Because, the position vector field on Euclidean spaces satisfies (3).

Example 5 ([3])**.** Let $x_1, \cdots, x_n$ be a standard chart on the real vector space V and (a_{ij}) be a constant positive definite matrix. Set $g(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}) = a_{ij}$, $(i, j = 1, \cdots, n)$. Then g is a Riemannian metric on V. The corresponding connection is the standard one. Then, the radial vector field is concurrent with respect to g.

Riemannian manifolds endowed with a concurrent vector field have been studied (cf. [3], [7], [18] and [19]).

To classify almost Yamabe solitons on hypersurfaces in a Euclidean space $\mathbb{E}^{n+1}$, as the metric g of M is induced from $\mathbb{E}^{n+1}$, it seems natural to take v from $\mathbb{E}^{n+1}$. Since Euclidean spaces have a position vector, we take v as a tangential component of the position vector of the Euclidean space. We denote the tangential and the normal components of the position vector v of Euclidean spaces by v^T and $v^\perp$, respectively.

Theorem 1.2 (Theorem 5.1)**.** *Any almost Yamabe soliton (M, g, v^T, ρ) on a hypersurface in a Euclidean space $\mathbb{E}^{n+1}$ is contained in either a hyperplane or a sphere.*

Remark 1.3. For almost Yamabe solitons, all of the results in this paper can be applied to Yamabe solitons.

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The remaining sections are organized as follows. Section 2 contains some necessary definitions and preliminary geometric results. In section 3, we show that any Yamabe soliton (M, g, v, ρ) with a concurrent vector field v is a gradient expanding Yamabe soliton with $\rho = -1$ and the scalar curvature is zero. In section 4, we consider almost Yamabe solitons on submanifolds in Riemannian manifolds endowed with a concurrent vector field. Section 5 is devoted to the proof of Theorem 1.2. Finally in Appendix, we classify complete gradient Ricci solitons on minimal submanifolds in non-positively curved space forms.

2. Preliminaries

Let $(N, \tilde{g})$ be an m-dimensional Riemannian manifold and (M, g) be an n-dimensional submanifold in $(N, \tilde{g})$. We denote Levi-Civita connections on (M, g) and $(N, \tilde{g})$ by ∇ and $\tilde{\nabla}$ respectively.

For any vector fields X, Y tangent to M and η normal to M, the formula of Gauss is given by

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X, Y),$$

where $\nabla_X Y$ and $h(X, Y)$ are the tangential and the normal components of $\tilde{\nabla}_X Y$. The formula of Weingarten is given by

$$\tilde{\nabla}_X \eta = -A_\eta(X) + D_X \eta,$$

where $-A_\eta(X)$ and $D_X \eta$ are the tangential and the normal components of $\tilde{\nabla}_X \eta$. $A_\eta(X)$ and $h(X, Y)$ are related by

$$g(A_\eta(X), Y) = \tilde{g}(h(X, Y), \eta).$$

The mean curvature vector H of M in N is given by

$$H = \frac{1}{n} \text{ trace } h.$$

For any vector fields X, Y, Z, W tangent to M, the equation of Gauss is given by

$$\begin{aligned} \tilde{g}(\tilde{R}m(X, Y)Z, W) = \; & g(Rm(X, Y)Z, W) \\ & + \tilde{g}(h(X, Z), h(Y, W)) \\ & - \tilde{g}(h(X, W), h(Y, Z)), \end{aligned}$$

where Rm and $\tilde{R}m$ are Riemannian curvature tensors of M and N, respectively. The equation of Codazzi is given by

$$(\tilde{R}m(X, Y)Z)^\perp = (\bar{\nabla}_X h)(Y, Z) - (\bar{\nabla}_Y h)(X, Z),$$

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where $(\tilde{Rm}(X,Y)Z)^{\perp}$ is the normal component of $\tilde{Rm}(X,Y)Z$ and $\bar{\nabla}_X h$ is defined by

$$(\bar{\nabla}_X h)(Y,Z) = D_X h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z).$$

If N is a space of constant curvature, then the equation of Codazzi reduces to

$$0 = (\bar{\nabla}_X h)(Y,Z) - (\bar{\nabla}_Y h)(X,Z)$$

3. Almost Yamabe solitons with a concurrent vector field

Firstly, we show a formula of almost Yamabe solitons which is useful for study of almost Yamabe solitons.

Lemma 3.1.

$$(n-1)\Delta(R-\rho) + \frac{1}{2}\, g(\nabla R, \nabla f) + R(R-\rho) = 0. \tag{4}$$

Proof. Since

$$\Delta \nabla_i f = \nabla_i \Delta f + R_{ij}\nabla_j f,$$
$$\Delta \nabla_i f = \nabla_k \nabla_k \nabla_i f = \nabla_k((R-\rho)g_{ki}) = \nabla_i(R-\rho),$$

and

$$\nabla_i \Delta f = \nabla_i(n(R-\rho)) = n\nabla_i(R-\rho),$$

we have

$$(n-1)\nabla_i(R-\rho) + R_{ij}\nabla_j f = 0, \tag{5}$$

where R_{ij} is the Ricci curvature of M. By applying ∇_l to the both side of (5), we obtain

$$(n-1)\nabla_l\nabla_i(R-\rho) + \nabla_l R_{ij}\cdot\nabla_j f + R_{ij}\nabla_l\nabla_j f = 0. \tag{6}$$

Taking the trace, we obtain (4). □

Proposition 3.2. *If an almost Yamabe soliton (M,g,v,ρ) has a concurrent vector field v, then M is a gradient almost Yamabe soliton with $R = \rho + 1$.*

Proof. Firstly, we show that an almost Yamabe soliton with a concurrent vector field is a gradient almost Yamabe soliton. Set

$$f = \frac{1}{2}\, g(v,v).$$

Then we have

$$g(\nabla f, X) = X(f) = g(v, \nabla_X v) = g(v, X),$$

for any vector field X on M.

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Secondly, we show that $R = \rho + 1$. Since v is a concurrent vector field, we have

$$\mathcal{L}_v g = 2g. \tag{7}$$

Combining (7) with (2), we obtain

$$R = \rho + 1. \tag{8}$$

□

Since, any almost Yamabe soliton with a concurrent vector field is gradient and $R = \rho + 1$, the equation of the almost Yamabe soliton with a concurrent vector field is as follows:

$$g = \nabla\nabla f.$$

Therefore g should be a Hessian metric. Hessian metric is an important notion of Geometry and Physics (cf. [1], [15], [17], and [20]). It is known that S. Y. Cheng and S. T. Yau [2] used Hessian metrics to solve Monge-Ampére equations (see also [16]).

Proposition 3.3. *A Hessian manifold (M, g) with $\rho = R - 1$ is an almost Yamabe soliton.*

Therefore we can get many examples of almost Yamabe solitons. In fact, Example 3 is a Hessian manifold with $\rho = R - 1$.

On the other hand, if M is compact, then there exists no non trivial almost Yamabe solitons with a concurrent vector field.

Corollary 3.4. *There exists no non trivial compact almost Yamabe solitons with a concurrent vector field.*

Proof. By Proposition 3.2 and (2), we have $\Delta f = n$. By applying maximum principle, we get f is constant. From this and (4), we have $R = 0$ and $\rho = -1$. Thus, M is a Yamabe soliton. However, there exists no non trivial Yamabe solitons. □

By applying Proposition 3.2 to Yamabe solitons, we can get the following.

Corollary 3.5. *Any Yamabe soliton with a concurrent vector field is a gradient expanding Yamabe soliton with zero scalar curvature and $\rho = -1$.*

Proof. Since ρ is constant, by Proposition 3.2 and (4), we have $R = 0$ and $\rho = -1$. □

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4. Almost Yamabe solitons on submanifolds

In this section, we assume that $(N, \tilde{g})$ is a Riemannian manifold endowed with a concurrent vector field v and (M, g) is a submanifold in $(N, \tilde{g})$. We denote the tangential and the normal components of v by v^T and $v^\perp$, respectively.

First of all, we construct an example of almost Yamabe soliton hypersurfaces in Euclidean spaces.

Example 6. In general, a Riemannian manifold with $\rho = R$ and a function f satisfying that ∇f is a constant vector field is a gradient almost Yamabe soliton. In particular, we can construct an example of gradient almost Yamabe soliton surfaces in Euclidean spaces. The Helicoid $S = (t\cos s, t\sin s, bs), (b \neq 0)$ is a minimal surface in $\mathbb{E}^3$. The scalar curvature is $R = -\frac{2b^2}{(b^2+t^2)}$. If we take $\rho = -\frac{2b^2}{(b^2+t^2)}$ and $f = at + bs$, $(a, b \in \mathbb{R})$, then it is a gradient almost Yamabe soliton surface in $\mathbb{E}^3$.

To classify almost Yamabe solitons on a submanifold, we show the following lemma which will be used in the proof of Proposition 4.3 and Theorem 5.1.

Lemma 4.1. *Any almost Yamabe soliton (M, g, v^T, ρ) on a submanifold M in N satisfies*

$$(R - \rho - 1)g(X, Y) = g(A_{v^\perp}(X), Y), \tag{9}$$

for any vector fields X, Y on M.

Proof. Since v is a concurrent vector field and by using formulas of Gauss and Weingarten we have

$$\begin{aligned} X = \nabla_X v &= \tilde{\nabla}_X(v^T + v^\perp) \\ &= (\nabla_X v^T + h(X, v^T)) + (-A_{v^\perp}(X) + D_X v^\perp), \end{aligned} \tag{10}$$

for any vector field X on M. By comparing the tangential and the normal components of (10), we obtain

$$\nabla_X v^T = X + A_{v^\perp}(X),\ h(X, v^T) = -D_X v^\perp. \tag{11}$$

From the definition of Lie-derivative and (11), we have

$$(\mathcal{L}_{v^T} g)(X, Y) = 2g(X, Y) + 2g(A_{v^\perp}(X), Y), \tag{12}$$

for any vector fields X, Y on M. Combining (12) with (2), we obtain (3). □

Proposition 4.2. *Any almost Yamabe soliton (M, g, v^T, ρ) on a submanifold M in N is a gradient almost Yamabe soliton.*

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Proof. Set

$$f = \frac{1}{2}\,\tilde{g}(v, v).$$

Then we have

$$g(\nabla f, X) = X(f) = \tilde{g}(v, \tilde{\nabla}_X v) = \tilde{g}(v, X) = g(v^T, X),$$

for any vector field X on M. □

Proposition 4.3. *If an almost Yamabe soliton (M, g, v^T, ρ) on a submanifold M in N is minimal, then $R = \rho + 1$.*

Proof. From Proposition 4.2, we know that any almost Yamabe soliton on a submanifold is a gradient almost Yamabe soliton. Let $\{e_1, \cdots, e_n\}$ be an orthonormal frame on M. From Lemma 4.1, we have

$$(R - \rho - 1)g_{ij} = g(A_{v^\perp}(e_i), e_j).$$

Since M is minimal and taking the trace, we obtain

$$n(R - \rho - 1) = n\langle H, v^\perp\rangle = 0.$$

Therefore we conclude that

$$R = \rho + 1.$$

□

Corollary 4.4. *There exists no non trivial compact almost Yamabe solitons on a minimal submanifold in N.*

Proof. By Proposition 4.3 and (2), we have $\Delta f = n$. By applying maximum principle, we get f is constant. From this and (4), we have $R = 0$ and $\rho = -1$. Thus, M is a Yamabe soliton. However, there exists no non trivial Yamabe solitons. □

Corollary 4.5. *Any Yamabe soliton on a minimal submanifold in N is a gradient expanding Yamabe soliton with zero scalar curvature and $\rho = -1$.*

Proof. Since ρ is constant, by Proposition 4.3 and (4), we have $R = 0$ and $\rho = -1$. □

5. Classification of almost Yamabe solitons in Euclidean spaces

In this section, we give the proof of Theorem 1.2, namely, we completely classify almost Yamabe solitons on hypersurfaces in Euclidean spaces arisen from the position vector field. Let v be the position vector field on Euclidean spaces.

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Theorem 5.1. *Any almost Yamabe soliton (M, g, v^T, ρ) or a hypersurface in a Euclidean space $\mathbb{E}^{n+1}$ is contained in either a hyperplane or a sphere.*

Proof. Let α be a mean curvature and λ be a support function of M, i.e. $H = \alpha N$ and $\lambda = \tilde{g}(N, v)$ with a unit normal vector field N. From Lemma 4.1,

$$(R - \rho - 1)g_{ij} = \tilde{g}(h(e_i, e_j), v^\perp) = \tilde{g}(\kappa_i g_{ij} N, v) = \kappa_i g_{ij}\lambda,$$

where $A_N(e_i) = \kappa_i e_i,\ i = 1, \cdots, n$. So we have

$$R - \rho - 1 = \lambda\kappa_i. \tag{13}$$

Taking the summation, we obtain

$$R - \rho - 1 = \lambda\alpha. \tag{14}$$

Comparing (13) and (14), we have

$$\kappa_i = \alpha.$$

Therefore M is a totally umbilical submanifold with $A_N(e_i) = \alpha e_i$ and h satisfies $h(X, Y) = \alpha g(X, Y)N$. Now we have

$$0 = \tilde{\nabla}_X(\tilde{g}(N, N)) = 2\tilde{g}(\tilde{\nabla}_X N, N) = 2\tilde{g}(D_X N, N).$$

Therefore $D_X N = 0$. So we obtain

$$\begin{aligned}(\bar{\nabla}_X h)(Y, Z) &= D_X h(Y, Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z)\\ &= X(\alpha)g(Y, Z)N,\end{aligned}$$

for any vector fields X, Y, Z on M. From the equation of Codazzi, we have

$$X(\alpha)Y = Y(\alpha)X.$$

Taking X and Y linearly independent, we conclude that α is a constant.
Case 1: $\alpha = 0$. From $\tilde{\nabla}_X N = 0$, N, restricted to M, is a constant in $\mathbb{E}^{n+1}$ and we have

$$\tilde{\nabla}_X(\tilde{g}(v, N)) = \tilde{g}(\tilde{\nabla}_X v, N) + \tilde{g}(v, \tilde{\nabla}_X N) = \tilde{g}(X, N) = 0.$$

This shows that $\tilde{g}(v, N)$ is constant when v and N is restricted to M. Therefore M is contained in the hyperplane normal to N.
Case 2. $\alpha \neq 0$. We have

$$\tilde{\nabla}_X(v + \alpha^{-1}N) = X + \alpha^{-1}\tilde{\nabla}_X N = X + \alpha^{-1}(-A_N(X)) = 0.$$

This shows that the vector field $v+\alpha^{-1}N$, restricted to M, is a constant in $\mathbb{E}^{n+1}$. Therefore M is contained in the sphere. □

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6. Appendix

In this appendix, we classify complete gradient Ricci solitons on minimal submanifolds in Euclidean spaces or hyperbolic spaces. (M, g, f, ρ) is called a gradient Ricci soliton if it satisfies (see for example [8]),

$$\mathrm{Ric} + \nabla\nabla f + \frac{\rho}{2} g = 0. \tag{15}$$

There are many studies of Ricci solitons on submanifolds in Riemannian manifolds (cf. [7], [9], [10] and [11]). Some recent progress on the subject can be found in [4].

Theorem 6.1. *(i) Any complete gradient shrinking or steady Ricci soliton on a minimal submanifold M in a Euclidean space is an affine subspace.*
(ii) There is no complete gradient Ricci soliton on a minimal submanifold M in a hyperbolic space.

Proof. Let c be a sectional curvature of a Euclidean space or a hyperbolic spaces, namely, $c = 0$ or -1. From the equation of Gauss,

$$\mathrm{Ric}(X, Y) = c(n-1)g(X, Y) - \tilde{g}(h(X, e_i), h(e_i, Y)) + \tilde{g}(h(X, Y), nH),$$

for any vector fields X, Y on M. Since M is a minimal, we have

$$\mathrm{Ric}(X, Y) = c(n-1)g(X, Y) - \tilde{g}(h(X, e_i), h(e_i, Y)).$$

Therefore,

$$R = cn(n-1) - |h|^2. \tag{16}$$

In [6], B. L. Chen showed that (a) any complete gradient shrinking or steady Ricci soliton has non-negative scalar curvature $R \geq 0$. (b) any complete gradient expanding Ricci soliton has $R \geq -\frac{n}{2}$.

We will show (i), that is, the case $c = 0$. By (16), $0 \leq R = -|h|^2$. Therefore, M is a totally geodesic submanifold and it is an affine subspace in the Euclidean space.

We will show (ii), that is, the case $c = -1$. (a) M is shrinking or steady. By the same argument, $|h|^2 \leq -n(n-1)$, which can not happen. (b) M is expanding: By the same argument, $|h|^2 \leq -n(n-\frac{3}{2})$, which cannot happen. □

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Department of Mathematics, Shimane University, Nishikawatsu 1060 Matsue, 690-8504, Japan.
E-mail address: seko92903991@gmail.com

Department of Mathematics, Shimane University, Nishikawatsu 1060 Matsue, 690-8504, Japan.
E-mail address: shun.maeta@gmail.com *or* maeta@riko.shimane-u.ac.jp

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