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Existence of solution for a nonvariational elliptic system with exponential growth in dimension two

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Abstract

We prove existence of solution for an elliptic system on a bounded domain in dimension two. We use the Galerkin scheme in the product of Hilbert spaces. The nonlinearities may have subcritical or critical exponential growth.

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Keywords: Elliptic system; Existence of solution; Subcritical and critical exponential growth; Galerkin scheme

1. Introduction

We prove existence of solution of the system

$$\begin{cases} -\Delta v = \lambda v^{q_1} + f(u) & \text{in} \quad \Omega \\ -\Delta u = \sigma u^{q_2} + g(v) & \text{in} \quad \Omega \\ v, u > 0 & \text{in} \quad \Omega \\ v = u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

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where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary, $\lambda, \sigma > 0$ are parameters, $0 < q_1, q_2 < 1$, $f, g : \mathbb{R} \to \mathbb{R}$ is a continuous functions and

$$0 \le f(s)s \le C|s|^{p_1} \exp(\alpha s^2), \tag{2}$$

$$0 \le g(s)s \le C|s|^{p_2} \exp(\beta s^2) \tag{3}$$

where $2 < p_1, p_2 < \infty$, $\alpha, \beta > 0$ and $C > 0$ are constants.

Remark 1.1. The results of this paper also work for a system with $u_1, ..., u_m$ variables and m equations $-\Delta u_i = \lambda_i u_i^{q_i} + f_i(u_j)$ where $j = \sigma(i)$, $\sigma : \{1, 2, ..., m\} \to \{1, 2, ..., m\}$ is a permutation such that $\sigma^k(i) \neq i$ for $k = 1, 2, ..., m-1$ and $\sigma^m(i) = i$, the index k stands for composition of functions, see [1] for the concept of m-coupled elliptic systems. Conditions (2)–(3) should be changed accordingly for each f_i.

We state our main result.

Theorem 1.1. *Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are continuous functions satisfying (2) and (3) respectively. Then there exist $\lambda^*, \sigma^* > 0$ such that for every $\lambda \in (0, \lambda^*)$ and $\sigma \in (0, \sigma^*)$ the problem (1) has positive weak solutions $v, u \in H_0^1(\Omega) \cap H^2(\Omega)$.*

A function h has subcritical growth at ∞ if for every $\gamma > 0$

$$\lim_{s \to \infty} \frac{|h(x,s)|}{e^{\gamma s^2}} = 0$$

The critical growth of h at ∞ means that there is $\gamma_0 > 0$ such that

$$\lim_{s \to \infty} \frac{|h(x,s)|}{e^{\gamma s^2}} = 0 \quad \forall \gamma > \gamma_0 \quad \text{and} \quad \lim_{s \to \infty} \frac{|h(x,s)|}{e^{\gamma s^2}} = \infty \quad \forall \gamma < \gamma_0.$$

Equations like $-\Delta u = h(x, u)$ with h having critical or subcritical growth have been studied in [2–7]. The Trudinger–Moser inequality [8–10] has a crucial role, since it indicates the space of functions one has to work to seek for solutions. The inequality reads as follows. Given $u \in H_0^1(\Omega)$, then

$$e^{\zeta |u|^2} \in L^1(\Omega) \;\text{ for every }\; \zeta > 0, \tag{4}$$

and there exists a positive constant L such that

$$\sup_{\|u\|_{H_0^1(\Omega)} \le 1} \int_\Omega e^{\xi |u|^2} dx \le L \;\text{ for every }\; \xi \le 4\pi. \tag{5}$$

Elliptic systems of type

$$\begin{cases} -\Delta v = f(x, v, u) & \text{in} \quad \Omega \\ -\Delta u = g(x, v, u) & \text{in} \quad \Omega \\ v = u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{6}$$

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in $\Omega \subset \mathbb{R}^2$ where $f(x,v,u)$, $g(x,v,u)$ are continuous functions with f behaving like $exp(\alpha|u|^2)$ as $|u| \to \infty$ and g behaving like $exp(\beta|v|^2)$ as $|v| \to \infty$ have been studied in [11–13], see also the references therein. Other applications of elliptic systems with exponential growth are [14,15]. When finding a solution for (6) by means of topological methods, monotonicity assumptions on f and g are required as well as conditions on the behavior of f and g for u and v near zero are imposed. The domain should be convex or the gradients $\nabla_x g$ and $\nabla_x f$ should satisfy some condition. To deal with variational methods, another class of assumptions on f and g is asked in order to find critical points of a functional corresponding to (6). In [16] the authors study a Hamiltonian system of Schrödinger equations with exponential growth at infinity. They have found finite Morse index solutions which correspond to critical points of linking type on Lorentz–Sobolev spaces. The finite dimensional part of the solution is obtained by means of a Galerkin approximation scheme. A similar idea is pursued in [17] in Sobolev spaces. The study is further developed in [18], where the authors prove the existence of semiclassical ground state solutions of a singularly perturbed system. Here we do not assume hypotheses leading to variational techniques, we only require (2)–(3).

2. Approximation of the system

In the course of the proof of Theorem 1.1 we approximate the system (1) by using Lipschitz continuous functions $f_k, g_k : \mathbb{R} \to \mathbb{R}$ which tend uniformly to f, g as $k \to \infty$, respectively. Let

$$f_k(s) = \begin{cases} -k[F(-k-\frac{1}{k}) - F(-k)], & \text{if} \quad s \le -k \\ -k[F(s-\frac{1}{k}) - F(s)], & \text{if} \quad -k \le s \le -\frac{1}{k} \\ k^2 s[F(-\frac{2}{k}) - F(-\frac{1}{k})], & \text{if} \quad -\frac{1}{k} \le s \le 0 \\ k^2 s[F(\frac{2}{k}) - F(\frac{1}{k})], & \text{if} \quad 0 \le s \le \frac{1}{k}. \\ k[F(s+\frac{1}{k}) - F(s)], & \text{if} \quad \frac{1}{k} \le s \le k \\ k[F(k+\frac{1}{k}) - F(k)], & \text{if} \quad s \ge k, \end{cases} \tag{7}$$

where $F(s) = \int_0^s f(\xi)d\xi$. The definition of g_k is analogous, we only replace F by $G(s) = \int_0^s g(\xi)d\xi$.

The proof of the following convergence result was done in [19] and uses explicit expression of the sequence (7). An analogous statement concerning g and g_k also holds.

Lemma 2.1. *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $sf(s) \ge 0$ for every $s \in \mathbb{R}$. Then there exists a sequence $f_k : \mathbb{R} \to \mathbb{R}$ of continuous functions satisfying*

(i) $sf_k(s) \ge 0 \quad \forall s \in \mathbb{R}$;

(ii) $\forall k \in \mathbb{N}\ \exists c_k > 0$ such that $|f_k(\xi) - f_k(\eta)| \le c_k|\xi - \eta| \quad \forall \xi, \eta \in \mathbb{R}$;

(iii) f_k converges uniformly to f on the bounded subsets of $\mathbb{R}$.

The sequences f_k and g_k of the previous lemma enjoy additional growth properties. The following lemmas was proved in [4].

Lemma 2.2. *Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous functions satisfying (2) and (3) for every $s \in \mathbb{R}$. Then the sequence f_k of Lemma 2.1 satisfies*

(i) $\forall k \in \mathbb{N}$, $0 \le sf_k(s) \le C_1|s|^{p_1}\exp(4\alpha s^2) \quad \forall |s| \ge \frac{1}{k}$;

(ii) $\forall k \in \mathbb{N}$, $0 \le sf_k(s) \le C_2|s|^2\exp(4\alpha s^2) \quad \forall |s| \le \frac{1}{k}$,

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where $C_1, C_2 > 0$ are constants independent of k. Analogous statements to (i) and (ii) are true if one replaces f, f_k, p_1 and α by g, g_k, p_2 and β, respectively.

Let f_n and g_n be the sequences given by Lemma 2.1 and Lemma 2.2. Before proving Theorem 1.1 we first obtain a sequence of solutions (v_n, u_n) of the system below

$$\begin{cases} -\Delta v = \lambda v^{q_1} + f_n(u) + \frac{1}{n}, & \text{in} \quad \Omega \\ -\Delta u = \sigma u^{q_2} + g_n(v) + \frac{1}{n}, & \text{in} \quad \Omega \\ v, u > 0 & \text{in} \quad \Omega \\ v = u = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{8}$$

We will use the Galerkin method together with the following fixed point theorem, see [11], [20] and [19].

Lemma 2.3. *Let $\Phi : \mathbb{R}^d \to \mathbb{R}^d$ be a continuous function such that $\langle \Phi(\xi), \xi\rangle \geq 0$ for every $\xi \in \mathbb{R}^d$ with $|\xi| = r$ for some $r > 0$. Then, there exists z_0 in the closed ball $\overline{B}_r(0)$ such that $\Phi(z_0) = 0$.*

The main result in this section is the following.

Lemma 2.4. *There exists $\lambda^*, \sigma^* > 0$ and $n^* \in \mathbb{N}$ such that (8) has a weak solution (v_n, u_n) with $v_n > 0$ and $u_n > 0$ in Ω for every $\lambda \in (0, \lambda^*)$, $\sigma \in (0, \sigma^*)$ and $n \geq n^*$.*

2.1. Proof of Lemma 2.4

Let $\mathcal{B} = \{w_1, w_2, \dots, w_m, \dots\}$ be an orthonormal basis of $H_0^1(\Omega)$ and define

$$W_m = [w_1, w_2, \dots, w_m],$$

to be the space generated by $\{w_1, w_2, \dots, w_m\}$. Define the function $\Phi : \mathbb{R}^{2m} \to \mathbb{R}^{2m}$ such that

$$\Phi(\eta, \xi) = (F_1(\eta, \xi), F_2(\eta, \xi), \dots, F_m(\eta, \xi), G_1(\eta, \xi), G_2(\eta, \xi), \dots, G_m(\eta, \xi))$$

where $(\eta, \xi) = (\eta_1, \eta_2, ..., \eta_m, \xi_1, \xi_2, ..., \xi_m) \in \mathbb{R}^{2m}$,

$$F_j(\eta, \xi) = \int_\Omega \nabla v \nabla w_j - \lambda \int_\Omega (v_+)^{q_1} w_j - \int_\Omega f_n(u_+) w_j - \frac{1}{n} \int_\Omega w_j, \quad j = 1, 2, \dots, m,$$

$$G_j(\eta, \xi) = \int_\Omega \nabla u \nabla w_j - \sigma \int_\Omega (u_+)^{q_2} w_j - \int_\Omega g_n(v_+) w_j - \frac{1}{n} \int_\Omega w_j, \quad j = 1, 2, \dots, m,$$

$$v = \sum_{i=1}^m \xi_i w_i \quad \in W_m$$

and

$$u = \sum_{i=1}^m \eta_i w_i \quad \in W_m.$$

Therefore

$$(\Phi(\eta,\xi),(\eta,\xi)) = \int_\Omega |\nabla v|^2 - \lambda \int_\Omega (v_+)^{q_1+1} - \int_\Omega f_n(u_+)v - \frac{1}{n}\int_\Omega v + \int_\Omega |\nabla u|^2 - \sigma \int_\Omega (u_+)^{q_2+1} - \int_\Omega g_n(v_+)u - \frac{1}{n}\int_\Omega u, \tag{9}$$

where $v_+ = \max\{v, 0\}$ and $v_- = v_+ - v$, $u_+ = \max\{u, 0\}$ and $u_- = u_+ - u$.

Given $v, u \in W_m$ we define

$$\Omega_n^1 = \{x \in \Omega : |u(x)| \geq \frac{1}{n}\}$$

and

$$\Omega_n^2 = \{x \in \Omega : |v(x)| \geq \frac{1}{n}\}.$$

Hence we define

$$(\Phi(\eta,\xi),(\eta,\xi)) = (\Phi(\eta,\xi),(\eta,\xi))_+ + (\Phi(\eta,\xi),(\eta,\xi))_-, \tag{10}$$

where

$$(\Phi(\eta,\xi),(\eta,\xi))_+ = \int_{\Omega_n^1\cap\Omega_n^2} |\nabla v|^2 - \lambda \int_{\Omega_n^1\cap\Omega_n^2} (v_+)^{q_1+1} - \int_{\Omega_n^1\cap\Omega_n^2} f_n(u_+)v - \frac{1}{n}\int_{\Omega_n^1\cap\Omega_n^2} v + \int_{\Omega_n^1\cap\Omega_n^2} |\nabla u|^2 - \sigma \int_{\Omega_n^1\cap\Omega_n^2} (u_+)^{q_2+1} - \int_{\Omega_n^1\cap\Omega_n^2} g_n(v_+)u - \frac{1}{n}\int_{\Omega_n^1\cap\Omega_n^2} u,$$

and

$$(\Phi(\eta,\xi),(\eta,\xi))_- = \int_{(\Omega_n^1\cap\Omega_n^2)^c} |\nabla v|^2 - \lambda \int_{(\Omega_n^1\cap\Omega_n^2)^c} (v_+)^{q_1+1} - \int_{(\Omega_n^1\cap\Omega_n^2)^c} f_n(u_+)v - \frac{1}{n}\int_{(\Omega_n^1\cap\Omega_n^2)^c} v + \int_{(\Omega_n^1\cap\Omega_n^2)^c} |\nabla u|^2 - \sigma \int_{(\Omega_n^1\cap\Omega_n^2)^c} (u_+)^{q_2+1} - \int_{(\Omega_n^1\cap\Omega_n^2)^c} g_n(v_+)u - \frac{1}{n}\int_{(\Omega_n^1\cap\Omega_n^2)^c} u,$$

where $(\Omega_n^1 \cap \Omega_n^2)^c = \{x \in \Omega : x \notin \Omega_n^1 \cap \Omega_n^2\}$.

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Step 1. Estimate in $(\Omega_n^1 \cap \Omega_n^2)$.
Since $0 < q_1, q_2 < 1$, then

$$\int_{\Omega_n^1 \cap \Omega_n^2} (v_+)^{q_1+1} \leq \int_{\Omega} (v_+)^{q_1+1} = \|v\|_{L^{q_1+1}}^{q_1+1} \leq C_1 \|v\|_{H_0^1(\Omega)}^{q_1+1}$$

and

$$\int_{\Omega_n^1 \cap \Omega_n^2} (u_+)^{q_2+1} \leq \int_{\Omega} (u_+)^{q_2+1} = \|u\|_{L^{q_2+1}}^{q_2+1} \leq C_2 \|u\|_{H_0^1(\Omega)}^{q_2+1}.$$

By Lemma 2.2 item (i), we obtain

$$\begin{aligned} &\int_{\Omega_n^1 \cap \Omega_n^2} f_n(u_+) v \\ &\leq C \int_{\Omega} |u_+|^{p_1-1} |v| \exp(4\alpha |u_+|^2) dx \\ &\leq C \int_{\Omega} \left(\frac{p_1-1}{p_1} |u_+|^{p_1} + \frac{1}{p_1} |v|^{p_1}\right) \exp(4\alpha |u_+|^2) dx \\ &\leq C \left(\int_{\Omega} |u_+|^{p_1+1} \right)^{\frac{p_1}{p_1+1}} \left(\int_{\Omega} \exp(4\alpha (p_1+1) |u_+|^2) dx \right)^{\frac{1}{p_1+1}} \\ &+ C \left(\int_{\Omega} |v|^{p_1+1} \right)^{\frac{p_1}{p_1+1}} \left(\int_{\Omega} \exp(4\alpha (p+1) |u_+|^2) dx \right)^{\frac{1}{p_1+1}} \\ &\leq C (\|u\|_{L^{p_1+1}(\Omega)}^{p_1} + \|v\|_{L^{p_1+1}(\Omega)}^{p_1}) \left(\int_{\Omega} \exp(4\alpha (p_1+1) |u|^2) dx \right)^{\frac{1}{p_1+1}}. \end{aligned} \tag{11}$$

Also,

$$\begin{aligned} &\int_{\Omega_n^1 \cap \Omega_n^2} g_n(v_+) u \\ &\leq C (\|u\|_{L^{p_2+1}(\Omega)}^{p_2} + \|v\|_{L^{p_2+1}(\Omega)}^{p_2}) \left(\int_{\Omega} \exp(4\beta (p_2+1) |v|^2) dx \right)^{\frac{1}{p_2+1}}. \end{aligned} \tag{12}$$

From (9) and (11) we conclude that

$$
\begin{aligned}
&(\Phi(\eta,\xi),(\eta,\xi))_+ \\
&\geq \int_{\Omega_n^1\cap\Omega_n^2} |\nabla v|^2 - \lambda C\|v\|_{H_0^1(\Omega)}^{q_1+1} \\
&-C_2(\|u\|_{L^{p_1+1}(\Omega)}^{p_1} + \|v\|_{L^{p_1+1}(\Omega)}^{p_1})\left(\int_\Omega \exp(4\alpha(p_1+1)|u|^2)dx\right)^{\frac{1}{p_1+1}} \\
&-\frac{C_3}{n}\|v\|_{H_0^1(\Omega)} + \int_{\Omega_n^1\cap\Omega_n^2} |\nabla u|^2 - \sigma C\|u\|_{H_0^1(\Omega)}^{q_2+1} \\
&-C_4(\|u\|_{L^{p_2+1}(\Omega)}^{p_2} + \|v\|_{L^{p_2+1}(\Omega)}^{p_2})\left(\int_\Omega \exp(4\beta(p_2+1)|v|^2)dx\right)^{\frac{1}{p_2+1}} \\
&-\frac{C_5}{n}\|u\|_{H_0^1(\Omega)}.
\end{aligned}
\tag{13}
$$

Step 2. Estimate in $(\Omega_n^1\cap\Omega_n^2)^c = \Omega - (\Omega_n^1\cap\Omega_n^2)$.
If $x \in (\Omega_n^1\cap\Omega_n^2)^c$ we have $|u(x)| < \frac{1}{n}$ and $|v(x)| < \frac{1}{n}$, then

$$
\int_{(\Omega_n^1\cap\Omega_n^2)^c} (v_+)^{q_1+1} \leq \int_{(\Omega_n^1\cap\Omega_n^2)^c} |v|^{q_1+1} \leq |\Omega|\frac{1}{n^{q_1+1}} \tag{14}
$$

and

$$
\int_{(\Omega_n^1\cap\Omega_n^2)^c} (u_+)^{q_2+1} \leq \int_{(\Omega_n^1\cap\Omega_n^2)^c} |u|^{q_2+1} \leq |\Omega|\frac{1}{n^{q_2+1}}. \tag{15}
$$

By Lemma 2.2 item (ii) we have

$$
\int_{(\Omega_n^1\cap\Omega_n^2)^c} f_n(u_+)v \leq C \int_{(\Omega_n^1\cap\Omega_n^2)^c} |u_+||v|\exp(4\alpha|v_+|^2)dx \leq |\Omega|\exp(4\alpha)\frac{1}{n^2} \tag{16}
$$

and

$$
\int_{(\Omega_n^1\cap\Omega_n^2)^c} g_n(v_+)u \leq C \int_{(\Omega_n^1\cap\Omega_n^2)^c} |v_+||u|\exp(4\beta|v_+|^2)dx \leq |\Omega|\exp(4\beta)\frac{1}{n^2} \tag{17}
$$

Hence from (14)–(17) we obtain

$$
\begin{aligned}
(\Phi(\eta,\xi),(\eta,\xi))_- \geq\ & \int_{(\Omega_n^1\cap\Omega_n^2)^c} |\nabla v|^2 - \lambda|\Omega|\frac{1}{n^{q_1+1}} - |\Omega|\exp(4\alpha)\frac{1}{n^2} - \frac{|\Omega|}{n^2} \\
&+ \int_{(\Omega_n^1\cap\Omega_n^2)^c} |\nabla u|^2 - \sigma|\Omega|\frac{1}{n^{q_2+1}} - |\Omega|\exp(4\beta)\frac{1}{n^2} - \frac{|\Omega|}{n^2}.
\end{aligned}
\tag{18}
$$

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Thus (10), (13) and (18) imply

$$
\begin{aligned}
(\Phi(\eta,\xi),(\eta,\xi)) \ \geq\ & \|v\|^2_{H_0^1(\Omega)} + \|u\|^2_{H_0^1(\Omega)} - \lambda C\|v\|^{q_1+1}_{H_0^1(\Omega)} - \sigma C\|u\|^{q_2+1}_{H_0^1(\Omega)} \\
& - C_2(\|u\|^{p_1}_{H_0^1(\Omega)} + \|v\|^{p_1}_{H_0^1(\Omega)}) \left(\int_\Omega \exp(4\alpha(p_1+1)|u|^2)dx \right)^{\frac{1}{p_1+1}} \\
& - C_4(\|u\|^{p_2}_{H_0^1(\Omega)} + \|v\|^{p_2}_{H_0^1(\Omega)}) \left(\int_\Omega \exp(4\beta(p_2+1)|v|^2)dx \right)^{\frac{1}{p_2+1}} \\
& - \frac{C_3}{n}\|v\|_{H_0^1(\Omega)} - \frac{C_5}{n}\|u\|_{H_0^1(\Omega)} - \lambda|\Omega|\frac{1}{n^{q_1+1}} - |\Omega|\exp(4\alpha)\frac{1}{n^2} \\
& - \frac{|\Omega|}{n^2} - \sigma|\Omega|\frac{1}{n^{q_2+1}} - |\Omega|\exp(4\beta)\frac{1}{n^2} - \frac{|\Omega|}{n^2}.
\end{aligned} \tag{19}
$$

Step 3. Verifying the hypothesis of Lemma 2.3.
Denoting by

$$
\|(u,v)\|^2 = \|u\|^2_{H_0^1(\Omega)} + \|v\|^2_{H_0^1(\Omega)}
$$

we obtain

$$
\begin{aligned}
(\Phi(\eta,\xi),(\eta,\xi)) \ \geq\ & \|(u,v)\|^2 - \lambda C\|(u,v)\|^{q_1+1} - \sigma C\|(u,v)\|^{q_2+1} \\
& - C_6\|(u,v)\|^{p_1} \left(\int_\Omega \exp(4\alpha(p_1+1)|u|^2)dx \right)^{\frac{1}{p_1+1}} \\
& - C_7\|(u,v)\|^{p_2} \left(\int_\Omega \exp(4\beta(p_2+1)|v|^2)dx \right)^{\frac{1}{p_2+1}} \\
& - \frac{C_8}{n}\|(u,v)\| - \lambda|\Omega|\frac{1}{n^{q_1+1}} - |\Omega|\exp(4\alpha)\frac{1}{n^2} - \frac{|\Omega|}{n^2} \\
& + -\sigma|\Omega|\frac{1}{n^{q_2+1}} - |\Omega|\exp(4\beta)\frac{1}{n^2} - \frac{|\Omega|}{n^2}.
\end{aligned}
$$

Assume that $\|(u,v)\| = r$ for some $r > 0$ to be chose later. We deduce that

$$
\int_\Omega \exp(4\alpha(p_1+1)|u|^2)dx \leq \int_\Omega \exp\left(4\alpha(p_1+1)r^2\left(\frac{u}{\|u\|}\right)^2\right)dx \tag{20}
$$

and

$$
\int_\Omega \exp(4\beta(p_2+1)|v|^2)dx \leq \int_\Omega \exp\left(4\beta(p_2+1)r^2\left(\frac{v}{\|v\|}\right)^2\right)dx. \tag{21}
$$

Observe that in order to apply the Trudinger–Moser inequality (5) we impose

$$4\alpha(p_1+1)r^2 < 4\pi \quad \text{and} \quad 4\beta(p_2+1)r^2 < 4\pi.$$

Therefore,

$$r < \left(\frac{\pi}{\alpha(p_1+1)}\right)^{1/2} \text{ and } r < \left(\frac{\pi}{\beta(p_2+1)}\right)^{1/2}.$$

Then

$$\sup_{\|u\|\leq 1} \int_\Omega \exp\left(4\alpha(p_1+1)r^2\left(\frac{u}{\|u\|}\right)^2\right) dx \leq L_1$$

and

$$\sup_{\|v\|\leq 1} \int_\Omega \exp\left(4\beta(p_2+1)r^2\left(\frac{v}{\|v\|}\right)^2\right) dx \leq L_2.$$

Hence,

$$\begin{aligned}(\Phi(\eta,\xi),(\eta,\xi)) \geq\ & r^2 - \lambda C r^{q_1+1} - \sigma C r^{q_2+1} - C_6 L_1^{\frac{1}{p_1+1}} r^{p_1} - C_7 L_2^{\frac{1}{p_2+1}} r^{p_2} \\ & - \tfrac{C_8}{n} r - \lambda|\Omega|\tfrac{1}{n^{q_1+1}} - |\Omega|\exp(4\alpha)\frac{1}{n^2} - \frac{|\Omega|}{n^2} - \sigma|\Omega|\frac{1}{n^{q_2+1}} \\ & - |\Omega|\exp(4\beta)\frac{1}{n^2} - \frac{|\Omega|}{n^2}.\end{aligned}$$

Choose r such that

$$\frac{r^2}{2} - C_6 L_1^{\frac{1}{p_1+1}} r^{p_1} \geq \frac{r^2}{4}$$

and

$$\frac{r^2}{2} - C_7 L_2^{\frac{1}{p_2+1}} r^{p_2} \geq \frac{r^2}{4}.$$

Thus,

$$r \leq \frac{1}{(4C_6 L_1^{\frac{1}{p_1+1}})^{\frac{1}{p_1-2}}}$$

and

$$r \leq \frac{1}{(4C_7 L_2^{\frac{1}{p_2+1}})^{\frac{1}{p_2-2}}}.$$

The following choice of r is exactly what we need. Let

$$r = \min\left\{ \frac{1}{2(4C_6 L_1^{\frac{1}{p_1+1}})^{\frac{1}{p_1-2}}}, \frac{1}{2(4C_7 L_2^{\frac{1}{p_2+1}})^{\frac{1}{p_2-2}}}, \frac{1}{2}\left(\frac{\pi}{\alpha(p_1+1)}\right)^{\frac{1}{2}}, \frac{1}{2}\left(\frac{\pi}{\beta(p_2+1)}\right)^{\frac{1}{2}} \right\}.$$

Hence

$$\begin{aligned}(\Phi(\eta,\xi),(\eta,\xi)) \geq\ & \tfrac{r^2}{2} - \lambda C r^{q_1+1} - \sigma C r^{q_2+1} - \tfrac{C_8}{n} r - \lambda|\Omega|\tfrac{1}{n^{q_1+1}} \\ & - |\Omega|\exp(4\alpha)\frac{1}{n^2} - \frac{|\Omega|}{n^2} - \sigma|\Omega|\frac{1}{n^{q_2+1}} - |\Omega|\exp(4\beta)\frac{1}{n^2} - \frac{|\Omega|}{n^2}.\end{aligned}$$

Define $\rho_1 = \frac{r^2}{4} - \lambda C r^{q_1+1}$ and $\rho_2 = \frac{r^2}{4} - \sigma C r^{q_2+1}$. We choose $\lambda^* > 0$ such that $\rho_1 > 0$ for $\lambda < \lambda^*$ and $\sigma^* > 0$ such that $\rho_2 > 0$ for $\sigma < \sigma^*$. Therefore, we take $\lambda^* = \frac{r^{1-q_1}}{8C}$ and $\sigma^* = \frac{r^{1-q_2}}{8C}$. We thus take $n^* \in \mathbb{N}$ such that

$$\begin{aligned}\frac{C_8}{n} r + \lambda|\Omega|\frac{1}{n^{q_1+1}} + |\Omega|\exp(4\alpha)\frac{1}{n^2} + \frac{|\Omega|}{n^2} + \sigma|\Omega|\frac{1}{n^{q_2+1}} + |\Omega|\exp(4\beta)\frac{1}{n^2} + \frac{|\Omega|}{n^2} \\ < \frac{\rho_1+\rho_2}{2},\end{aligned}$$

for every $n \geq n^*$. Let $\xi, \eta \in \mathbb{R}^m$ be such that $|(\eta,\xi)| = |\eta| + |\xi| = r$, then for $\sigma < \sigma^*$, $\lambda < \lambda^*$ and $n \geq n^*$ we obtain

$$(\Phi(\eta,\xi),(\eta,\xi)) \geq \frac{\rho_1}{2} + \frac{\rho_2}{2} > 0. \tag{22}$$

Step 4. Existence of a solution pair (v_n, u_n).

Notice that for every $n \in \mathbb{N}$, f_n and g_n are Lipschitz functions, then by Lemma 2.3 for every $m \in \mathbb{N}$ there exists $(y,z) \in \mathbb{R}^{2m}$ with $|(y,z)| \leq r$ such that $\Phi(y,z) = 0$, that is, there exist $u_m, v_m \in W_m$ satisfying

$$\|(u_m, v_m)\| = \|u_m\|_{H_0^1(\Omega)} + \|v_m\|_{H_0^1(\Omega)} \leq r \text{ for every } m \in \mathbb{N}$$

and such that

$$\int_\Omega \nabla v_m \nabla w = \lambda \int_\Omega (v_{m+})^{q_1} w + \int_\Omega f_n(u_{m+}) w + \frac{1}{n}\int_\Omega w \ \ \forall w \in W_m \tag{23}$$

and

$$\int_\Omega \nabla u_m \nabla w = \sigma \int_\Omega (u_{m+})^{q_2} w + \int_\Omega g_n(v_{m+}) w + \frac{1}{n}\int_\Omega w \ \ \forall w \in W_m. \tag{24}$$

Since $W_m \subset H_0^1(\Omega)$ $\forall m \in \mathbb{N}$ and r does not depend on m, then the sequences (v_m) and (u_m) are bounded in $H_0^1(\Omega)$. For a subsequence, there exist $v_n, u_n \in H_0^1(\Omega)$ such that

$$v_m \rightharpoonup v_n \text{ weakly in } H_0^1(\Omega) \tag{25}$$

and

$$u_m \rightharpoonup u_n \text{ weakly in } H_0^1(\Omega). \tag{26}$$

Until the end of this proof we denote $v = v_n$ and $u = u_n$.

Take $k \in \mathbb{N}$, then for every $m \geq k$ one has

$$\int_\Omega \nabla v_m \nabla w_k = \lambda \int_\Omega (u_{m+})^{q_1} w_k + \int_\Omega f_n(u_{m+}) w_k + \frac{1}{n} \int_\Omega w_k \;\; \forall w_k \in W_k \tag{27}$$

and

$$\int_\Omega \nabla u_m \nabla w_k = \sigma \int_\Omega (u_{m+})^{q_2} w_k + \int_\Omega g_n(v_{m+}) w_k + \frac{1}{n} \int_\Omega w_k \;\; \forall w_k \in W_k. \tag{28}$$

Thus from (25) and (26)

$$\int_\Omega \nabla v_m \nabla w_k \to \int_\Omega \nabla v \nabla w_k \tag{29}$$

and

$$\int_\Omega \nabla u_m \nabla w_k \to \int_\Omega \nabla u \nabla w_k. \tag{30}$$

We now use (25), (26) and Sobolev compact imbedding. Letting $m \to \infty$ we obtain

$$\lambda \int_\Omega (v_{m+})^{q_1} w_k + \int_\Omega f_n(u_{m+}) w_k + \frac{1}{n} \int_\Omega w_k \to \lambda \int_\Omega (v_+)^{q_1} w_k + \int_\Omega f_n(u_+) w_k + \frac{1}{n} \int_\Omega w_k \tag{31}$$

and

$$\sigma \int_\Omega (u_{m+})^{q_2} w_k + \int_\Omega g_n(v_{m+}) w_k + \frac{1}{n} \int_\Omega w_k \to \sigma \int_\Omega (u_+)^{q_2} w_k + \int_\Omega g_n(v_+) w_k + \frac{1}{n} \int_\Omega w_k. \tag{32}$$

By (27), (28), (29), (30), (31) and (32)

$$\int_\Omega \nabla v \nabla w_k = \lambda \int_\Omega (v_+)^{q_1} w_k + \int_\Omega f_n(u_+) w_k + \frac{1}{n} \int_\Omega w_k \;\; \forall w_k \in W_k \tag{33}$$

and

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$$\int_\Omega \nabla u \nabla w_k = \sigma \int_\Omega (u_+)^{q_2} w_k + \int_\Omega g_n(v_+) w_k + \frac{1}{n}\int_\Omega w_k \ \ \forall w_k \in W_k. \tag{34}$$

But $[W_k]_{k\in\mathbb{N}}$ is dense in $H_0^1(\Omega)$, hence by linearity we get

$$\int_\Omega \nabla v \nabla w = \lambda \int_\Omega (v_+)^{q_1} w + \int_\Omega f_n(u_+) w + \frac{1}{n}\int_\Omega w \ \ \forall w \in H_0^1(\Omega) \tag{35}$$

and

$$\int_\Omega \nabla u \nabla w = \sigma \int_\Omega (u_+)^{q_2} w + \int_\Omega g_n(v_+) w + \frac{1}{n}\int_\Omega w \ \ \forall w \in H_0^1(\Omega). \tag{36}$$

Step 5. Maximum principle, that is, $v_n, u_n > 0$ in Ω.

We continue to denote $v = v_n$ and $u = u_n$. We are going to prove that $v, u > 0$ in Ω. Notice that $v, u \geq 0$ in Ω. Indeed, inserting $v_-, u_- \in H_0^1(\Omega)$, then from (35) and (36) we obtain

$$\int_\Omega \nabla v \nabla v_- = \lambda \int_\Omega (v_+)^q v_- + \int_\Omega f_n(u_+) v_- + \frac{1}{n}\int_\Omega v_-$$

and

$$\int_\Omega \nabla u \nabla u_- = \lambda \int_\Omega (u_+)^q u_- + \int_\Omega g_n(v_+) u_- + \frac{1}{n}\int_\Omega u_-.$$

Implying

$$-\|v_-\|^2_{H_0^1(\Omega)} = \int_\Omega \nabla v \nabla v_- = \int_\Omega f_n(u_+) v_- + \frac{1}{n}\int_\Omega v_- \geq 0$$

and

$$-\|u_-\|^2_{H_0^1(\Omega)} = \int_\Omega \nabla u \nabla u_- = \int_\Omega g_n(v_+) u_- + \frac{1}{n}\int_\Omega u_- \geq 0,$$

thus $v_- \equiv 0$ and $u_- \equiv 0$ a.e. in Ω. By the strong maximum principle $v, u > 0$ in Ω.

3. Proof of the theorem

In this section we prove Theorem 1.1. We state a lemma already established in [21]. We will use the unique solutions $\widetilde{w}_1$ and $\widetilde{w}_2$ of the problems (37) and (38) to bound from below the solutions v_n and u_n of (8), respectively.

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Lemma 3.1. *Let $0 < q_1, q_2 < 1$. There exist unique smooth solutions $\widetilde{w}_1$ and $\widetilde{w}_2$ of problems*

$$\begin{cases} -\Delta \widetilde{w}_1 = \widetilde{w}_1^{q_1} & in \quad \Omega \\ \widetilde{w}_1 > 0 & in \quad \Omega \\ \widetilde{w}_1 = 0 & on \quad \partial\Omega \end{cases} \tag{37}$$

and

$$\begin{cases} -\Delta \widetilde{w}_2 = \widetilde{w}_2^{q_2} & in \quad \Omega \\ \widetilde{w}_2 > 0 & in \quad \Omega \\ \widetilde{w}_2 = 0 & on \quad \partial\Omega. \end{cases} \tag{38}$$

The following lemma of [19] is used to show that (v_n, u_n) converges to a solution (v, u) of (1).

Lemma 3.2. *Let Ω be a bounded open set in $\mathbb{R}^N$, $h_k : \Omega \to \mathbb{R}$ be a sequence functions and $\varphi_k : \Omega \to \mathbb{R}$ be a sequence of functions such that $\varphi_k(h_k)$ are measurable in Ω for every $k \in \mathbb{N}$. Assume that $\varphi_k(h_k) \to \gamma$ a.e. in Ω, $\int_\Omega |\varphi_k(h_k)u_k|dx < C$ for a constant C independent of k and $s \to \infty$ when $\varphi_k(s) \to \infty$. Then $\gamma \in L^1(\Omega)$ and $\varphi_k(h_k) \to \gamma$ in $L^1(\Omega)$.*

3.1. Proof of Theorem 1.1

By Lemma 2.4 the system (8) has a weak solution (v_n, u_n) with $v_n, u_n \in H_0^1(\Omega)$ and $v_n > 0, u_n > 0$ for each $n \in \mathbb{N}$. Since $0 < q_1, q_2 < 1$ and by virtue of the fact that f_n and g_n are Lipschitz continuous, then $\lambda v_n^{q_1} + f_n(u_n) + \frac{1}{n} \in L^p(\Omega)$ and $\sigma u_n^{q_2} + g_n(v_n) + \frac{1}{n} \in L^p(\Omega)$ for $p > N$. Hence $v_n \in C^{1,\alpha_1}(\overline{\Omega})$ and $u_n \in C^{1,\alpha_2}(\overline{\Omega})$ with $0 < \alpha_1, \alpha_2 < 1$, see [22]. Therefore, $v_n \in H_0^1(\Omega) \cap C^{1,\alpha_1}(\overline{\Omega})$ and $u_n \in H_0^1(\Omega) \cap C^{1,\alpha_2}(\overline{\Omega})$.

Using (25) and (26), for each $n \in \mathbb{N}$, $n > n^*$ we get

$$v_m \rightharpoonup v_n \text{ weakly in } H_0^1(\Omega) \text{ as } m \to \infty \tag{39}$$

and

$$u_m \rightharpoonup u_n \text{ weakly in } H_0^1(\Omega) \text{ as } m \to \infty. \tag{40}$$

Therefore for each $n > n*$,

$$\|v_n\|_{H_0^1(\Omega)} \le \liminf \|v_m\|_{H_0^1(\Omega)} \le r$$

and

$$\|u_n\|_{H_0^1(\Omega)} \le \liminf \|u_m\|_{H_0^1(\Omega)} \le r,$$

where r does not depend on n. Thus there exist $v, u \in H_0^1(\Omega)$ such that

$$v_n \rightharpoonup v \text{ weakly in } H_0^1(\Omega) \text{ as } n \to \infty \tag{41}$$

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and

$$u_n \rightharpoonup u \text{ weakly in } H_0^1(\Omega) \text{ as } n \to \infty. \tag{42}$$

The Sobolev compact imbedding for $1 \le s < \infty$ implies

$$v_n \to v \text{ in } L^s(\Omega) \text{ and a.e. in } \Omega$$

and

$$u_n \to u \text{ in } L^s(\Omega) \text{ and a.e. in } \Omega.$$

Notice that

$$\begin{cases} -\Delta v_n \ge \lambda v_n^{q_1}, & \text{in} \quad \Omega \\ v_n > 0 & \text{in} \quad \Omega \\ v_n = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{43}$$

and

$$\begin{cases} -\Delta u_n \ge \sigma u_n^{q_2}, & \text{in} \quad \Omega \\ u_n > 0 & \text{in} \quad \Omega \\ u_n = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{44}$$

Rescaling as $w_n = \lambda^{\frac{1}{q_1-1}} v_n$ and $z_n = \sigma^{\frac{1}{q_2-1}} u_n$ we obtain

$$-\Delta\left(\frac{w_n}{\lambda^{\frac{1}{q_1-1}}}\right) \ge \lambda \left(\frac{w_n}{\lambda^{\frac{1}{q_1-1}}}\right)^{q_1}$$

and

$$-\Delta\left(\frac{z_n}{\sigma^{\frac{1}{q_2-1}}}\right) \ge \sigma \left(\frac{z_n}{\sigma^{\frac{1}{q_2-1}}}\right)^{q_2}$$

hence

$$-\Delta\, w_n \ge w_n^{q_1} \text{ and } -\Delta\, z_n \ge z_n^{q_2}. \tag{45}$$

The comparison result of [23, Lemma 3.3], implies $w_n \ge \widetilde{w}_1$ and $z_n \ge \widetilde{w}_2$ $\forall n \in \mathbb{N}$, that is,

$$v_n \ge \lambda^{\frac{1}{1-q_1}} \widetilde{w}_1 \text{ a.e. in } \Omega\ \forall n \in \mathbb{N} \tag{46}$$

and

$$u_n \ge \sigma^{\frac{1}{1-q_2}} \widetilde{w}_2 \text{ a.e. in } \Omega\ \forall n \in \mathbb{N}. \tag{47}$$

Letting $n \to \infty$ in (46) and (47) we obtain

$$v \geq \lambda^{\frac{1}{1-q_1}} \widetilde{w}_1 \text{ and } u \geq \sigma^{\frac{1}{1-q_2}} \widetilde{w}_2 \text{ a.e. in } \Omega$$

then $v, u > 0$ in Ω.

We accomplish by showing that v and u are true solutions of (1). Since

$$v_n \to v \text{ a.e. in } \Omega$$

and

$$u_n \to u \text{ a.e. in } \Omega,$$

we obtain

$$f(u_n) \to f(u) \text{ a.e. in } \Omega$$

and

$$g(v_n) \to g(v) \text{ a.e. in } \Omega.$$

Moreover,

$$f_n(u_n) \to f(u_n) \text{ a.e. in } \Omega$$

and

$$g_n(v_n) \to g(v_n) \text{ a.e. in } \Omega.$$

Therefore

$$f_n(u_n(x)) \to f(u(x)) \text{ a.e. in } \Omega \tag{48}$$

and

$$g_n(v_n(x)) \to g(v(x)) \text{ a.e. in } \Omega. \tag{49}$$

Recall from (35) that

$$\int_\Omega \nabla v_n \nabla w = \lambda \int_\Omega (v_n)^{q_1} w + \int_\Omega f_n(u_n) w + \frac{1}{n} \int_\Omega w \ \ \forall w \in H_0^1(\Omega) \tag{50}$$

and

$$\int_\Omega \nabla u_n \nabla w = \sigma \int_\Omega (u_n)^{q_2} w + \int_\Omega g_n(v_n) w + \frac{1}{n} \int_\Omega w \ \ \forall w \in H_0^1(\Omega). \tag{51}$$

Taking $w = u_n$ in (50) and $w = v_n$ in (51), using (39) and (40), Hölder and Young's inequalities we obtain

$$\int_\Omega f_n(u_n)u_n dx \leq C \tag{52}$$

and

$$\int_\Omega g_n(v_n)v_n dx \leq C, \tag{53}$$

for every $n \in \mathbb{N}$, where $C > 0$ is a constant independent on n.

Lemma 3.2 implies

$$f_n(u_n) \to f(u) \text{ in } L^1(\Omega) \tag{54}$$

and

$$g_n(v_n) \to g(v) \text{ in } L^1(\Omega). \tag{55}$$

It follows from (4) that $e^{v^2}, e^{u^2} \in L^1(\Omega)$, and in view of (2) and (3) and Hölder inequality we conclude that $f(u), g(v) \in L^2(\Omega)$.

By (50) and (51) we get

$$\int_\Omega \nabla v \nabla w = \lambda \int_\Omega v^{q_1} w + \int_\Omega f(u)w \;\; \forall w \;\text{ in } H_0^1(\Omega) \tag{56}$$

and

$$\int_\Omega \nabla u \nabla w = \sigma \int_\Omega u^{q_2} w + \int_\Omega g(v)w \;\; \forall w \;\text{ in } H_0^1(\Omega). \tag{57}$$

By the fact that $f(u), g(v) \in L^2(\Omega)$ and $\lambda v^{q_1}, \sigma u^{q_2} \in L^2(\Omega)$, expressions (56) and (57) yield $v, u \in H^2(\Omega)$. Thus

$$-\Delta v = \lambda v^{q_1} + f(u)$$

and

$$-\Delta u = \sigma u^{q_2} + g(v).$$

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