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Global existence and asymptotic behavior for the 3D compressible Navier–Stokes equations without heat conductivity in a bounded domain ☆

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Abstract

In this paper, we investigate the global existence and uniqueness of strong solutions to the initial boundary value problem for the 3D compressible Navier–Stokes equations without heat conductivity in a bounded domain with slip boundary. The global existence and uniqueness of strong solutions are obtained when the initial data is near its equilibrium in $H^2(\Omega)$. Furthermore, the exponential convergence rates of the pressure and velocity are also proved by delicate energy methods.

MSC: 76W05; 35Q35; 35D05

Keywords: Navier–Stokes equations; Global existence; Asymptotic behavior; Bounded domain

1. Introduction

Let $\Omega \subset \mathbb{R}^3$ be a smooth bounded domain, we consider the following well-known compressible Navier–Stokes equations for the motion of compressible viscous fluids:

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$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla p = \mathrm{div} T, \\ (\rho E)_t + \mathrm{div}(\rho E u + p u) = \mathrm{div}(u T) + \kappa \Delta \theta, \end{cases} \tag{1.1}$$

for $(x, t) \in \Omega \times \mathbb{R}^+$. Here ρ, u, p, θ denote the density, velocity, pressure and temperature respectively. The specific total energy $E = \frac{1}{2}|u|^2 + \mathcal{E}$, $\mathcal{E}$ is the specific internal energy. The stress tensor is given by

$$T = \mu(\nabla u + \nabla u^T) + \lambda(\mathrm{div} u) I.$$

μ and λ are the coefficient of viscosity and second coefficient of viscosity, respectively. κ is the coefficient of heat conduction. In this paper, it will be always assumed that

$$\mu > 0, \quad 3\lambda + 2\mu > 0, \quad \kappa = 0. \tag{1.2}$$

We will consider only polytropic fluids, so that the equations of state for the fluid is given by

$$p = R\rho\theta, \quad \mathcal{E} = c_v \theta, \quad p = A e^{\frac{s}{c_v}} \rho^\gamma, \tag{1.3}$$

where $A > 0$ is a constant, $\gamma > 1$ is the adiabatic exponent, s is the entropy, and $c_v = R/(\gamma - 1)$.

To begin with, we note the fact that all thermodynamics variables $\rho, \theta, \mathcal{E}, p$ as well as the entropy s can be represented by functions of any two of them. To overcome the difficulties arising from the non-dissipation on θ, we will rewrite system (1.1). We take the two variables to be p and s. In light of the state equation (1.3), we deduce that

$$\rho = A^{-\frac{c_v}{c_v + R}} p^{\frac{c_v}{c_v + R}} e^{-\frac{s}{c_v + R}}. \tag{1.4}$$

Under the aforementioned assumptions, we can rewrite the system (1.1) in terms of (p, u, s) as follows:

$$\begin{cases} p_t + \gamma p \mathrm{div} u + u \cdot \nabla p = \frac{\Phi[u]}{c_v}, \\ \rho u_t + \rho u \cdot \nabla u + \nabla p = \mu \Delta u + (\mu + \lambda) \nabla \mathrm{div} u, \\ s_t + u \cdot \nabla s = \frac{\Phi[u]}{p}, \end{cases} \tag{1.5}$$

where $\Phi[u]$ is the classical dissipation function:

$$\Phi[u] = \frac{\mu}{2} |\nabla u + \nabla u^T|^2 + \lambda (\mathrm{div} u)^2. \tag{1.6}$$

It should be mentioned that system (1.5) is a hyperbolic–parabolic system, while the dissipation property comes from viscosity. In this paper, we consider the initial boundary value problem for system (1.5), which is supplemented by the following initial and boundary conditions:

$$\begin{cases} (p, u, s)(x, 0) = (p_0, u_0, s_0), & x = (x_1, x_2, x_3) \in \Omega, \\ u|_{\partial\Omega} = 0, & t \geq 0, \\ \int_\Omega p_0^{\frac{1}{\gamma}} dx / |\Omega| = \bar{p}_0^{\frac{1}{\gamma}} > 0. \end{cases} \tag{1.7}$$

As one of the most important systems in continuum mechanics, there is a huge literature on the large-time existence, stability and behavior of solutions to the compressible Navier–Stokes equations. The theory of global well-posedness of solution to the Cauchy problem and initial-boundary-value problem for the system (1.1) has been studied extensively in [1–15,17,18,20–22] and the references therein. The global classical solutions were first obtained by Matsumura–Nishida [20–22] for initial data close to a non-vacuum equilibrium in $H^3(\mathbb{R}^3)$. In particular, the theory requires that the solution has small oscillations from a uniform non-vacuum state so that the density is strictly away from the vacuum and the gradient of the density remains bounded uniformly in time. Later, Hoff [9,10] and Huang et al. [13] studied the problem for discontinuous initial data. For the existence of solutions for arbitrary data, the major breakthrough is due to Lions [17] (see also Feireisl et al. [7]), where he obtained global existence of weak solutions, defined as solutions with finite energy. The main restriction on initial data is that the initial energy is finite, so that the density vanishes at far fields, or even has compact support. When $\kappa = 0$, the one-dimensional system in the Lagrangian coordinates was studied by Liu and Zeng [19], they showed that the elaborate pointwise estimates and large-time behavior of solutions to (1.5) by studying the Green's function and the nonlinear interaction of waves. In the three dimensional case, the global existence solutions of system (1.5) were first announced by Kawashima [16]. Later, Duan et al. [2] and Tan et al. [23] studied the global existence and convergence rates of solutions to system (1.5) when the initial data is near its equilibrium in $H^\ell(\ell = 2, 3)$-framework. The key point in [2,23] is to observe that in the Eulerian's coordinates, the dissipative variables p and u satisfy $(1.5)_1$ and $(1.5)_2$ whose linear parts possess the same structure as ones of the isentropic viscous compressible Navier–Stokes equations, and the non-dissipative variable s satisfies the transport equation $(1.5)_3$ with the nonlinear source term. Then by the standard energy method as in [20,21], the uniform bound of (p, u) under a priori assumption that $\|(p, u, s)(t)\|_\ell$ is sufficiently small. Finally, the uniform bound of s will be obtained by making a priori decay-in-time estimates on (p, u), which is based on the decay property of the linearized equations together with energy estimates of higher order. The boundedness of L^1-norm is needed in the proof of the global existence, which is different from the previous work [20,21] for the case of $\kappa > 0$, where just H^3-norm of the perturbation is supposed for the global existence.

Before we state the main results, let us introduce some notations for the use throughout this paper. C denotes some positive constant. The norms in the Sobolev Spaces $H^m(\Omega)$ and $W^{m,q}(\Omega)$ are denoted respectively by $||\cdot||_m$ and $||\cdot||_{m,q}$ for $m \geq 0$ and $q \geq 1$. In particular, for $m = 0$ we will simply use $||\cdot||$ and $||\cdot||_{L^q}$. Finally,

$$\nabla = (\partial_1, \partial_2, \partial_3), \quad \partial_i = \partial_{x_i}, \ i = 1, 2, 3,$$

and for any integer $\ell \geq 0$, $\nabla^\ell f$ denotes all derivatives of order ℓ of the function f.

For the global existence and large time behavior of strong solutions, we have the following:

Theorem 1.1. *Given two constants $\bar{p}_0 > 0$ and $\bar{s}$, assume that the initial boundary value $(p_0 - \bar{p}_0, u_0, s_0 - \bar{s}) \in H^2(\Omega)$ satisfies the compatibility condition, i.e., $\partial_t^\ell u(x, 0)|_{\partial\Omega} = 0, \ell = 0, 1,$ where*

$$\partial_t u(x, 0) = \frac{\mu \Delta u_0 + (\mu + \lambda)\nabla divu_0 - \rho_0 u_0 \cdot \nabla u_0 - \nabla p_0}{\rho_0}.$$

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Then there exists a constant δ_0 *such that if*

$$\|(p_0-\bar{p}_0, u_0, s_0-\bar{s})\|_2 \le \delta_0, \tag{1.8}$$

the initial boundary value problem (1.5)–(1.7) *admits a unique solution* (p, u, s) *globally in time with* $p > 0$*, which satisfies*

$$\begin{aligned} p-\bar{p}, s-\bar{s} &\in C^0([0,\infty); H^2(\Omega)) \cap C^1([0,\infty); H^1(\Omega)),\\ u &\in C^0([0,\infty); H^2(\Omega)\cap H_0^1(\Omega)) \cap C^1([0,\infty); L^2(\Omega)), \end{aligned}$$

where $\bar{p}$ *satisfies* $\bar{p}^{\frac{1}{\gamma}}(t) = \int_\Omega p^{\frac{1}{\gamma}}(t)dx/|\Omega|$*. Moreover, there exist two positive constants* $C_0 > 0$ *and* $\eta_0 > 0$ *such that for any* $t \ge 0$*, the following estimates hold*

$$\bar{p}(t) \ge \bar{p}_0, \tag{1.9}$$

$$\|(p-\bar{p}, u)(t)\|_2^2 + \int_0^t (\|p(\tau)-\bar{p}(\tau)\|_2^2 + \|u(\tau)\|_3^2)d\tau \le C_0\|(p_0-\bar{p}_0, u_0)\|_2^2, \tag{1.10}$$

$$\|s(t)-\bar{s}\|_2 \le C_0\|(p_0-\bar{p}_0, u_0, s_0-\bar{s})\|_2 \exp\{C_0\|(p_0-\bar{p}_0, u_0)\|_2\}, \tag{1.11}$$

$$\|(p-\bar{p}, u)(t)\|_2 + \|\partial_t(p-\bar{p}, u, s)(t)\| \le C_0\|(p_0-\bar{p}_0, u_0)\|_2 \exp\{-\eta_0 t\}. \tag{1.12}$$

Finally, $\bar{p}(t)$ *is a monotonically increasing function on* t*. Let* $\lim\limits_{t\to\infty}\bar{p}(t) = \tilde{p}$*, then there exists positive constant* c_0 *such that*

$$\tilde{p} - \bar{p}(t) \le C_0\|(p_0-\bar{p}, u_0)\|_2^2 \exp\{-\eta_0 t\} \quad \textit{with} \quad \tilde{p} \ge \bar{p}_0 + c_0. \tag{1.13}$$

Remark 1.1. It is worth noting that the convergence of pressure in (1.12) is somewhat surprising, since the solution relaxes in the maximum norm to the constant background state at a rate of $(1+t)^{-5/4}$ in Cauchy problem case (see [2,23]).

Remark 1.2. Our results of this paper are also right for the two-dimensional case. However, since $\|\nabla(P, u)(t)\|_1$ of the linear solution to system (1.5) decays only as $(1+t)^{-1}$ in Cauchy problem (see [2,23]), which is not integrable, in particular making the strategy of [2,23] difficult to apply, construction of global existence and optimal convergence rates for Cauchy problem of system (1.5) in the two-dimensional case is still an open problem.

Let us now outline the main points for the study and explain some of the main difficulties and techniques in this paper. System (1.5) is a typical example of the quasilinear hyperbolic–parabolic system in [16], for which local well-posedness of initial boundary value problems have been studied with full generality. As usual, the global existence of the strong solutions can be established by combining a priori estimates and the local existence result. As [2] likes to point out, one of main observations is that the dissipative variables p and u satisfy $(1.5)_1$ and $(1.5)_2$ whose linear parts possess the same structure as that of the compressible isentropic Navier–Stokes equations, while the non-dissipative variable s satisfies the homogeneous transport equation $(1.5)_3$. Thus, in order to obtain a priori estimates to (1.5)–(1.7), we can apply the similar energy method as in [22] to the first two equations of (1.5) to obtain the uniform bound of $(p-\bar{p}, u)$ under the assumption that $\|(p-\bar{p}, u, s-\bar{s})(t)\|_2$ is sufficiently small. With these in hand, the norm $\|(p-\bar{p}, u)(t)\|_2$ can be shown to converge exponentially to zero from the Poincaré's and Gronwall's inequality. However, we should point out that we can't use Poincaré's inequality directly to estimate $\|p(t)-\bar{p}(t)\|$ since $\bar{p}(t)$ is not a constant, which make the problem much more difficult and need us to develop some new energy estimates. It is worth mentioning that the crucial part of the proof is to obtain a Lyapunov-type energy inequality. Then, the bound of s will be derived by the exponential decay estimates on $(p-\bar{p}, u)$ and the Gronwall's inequality. Second, comparing to Cauchy problem [2], when establishing a priori estimates by the standard energy method, a new difficulty arises since the spatial derivatives are unknown on the boundary. To overcome this difficulty, we separate the energy estimates for the spatial derivatives into that over the region away from the boundary and near the boundary in spirit of Matsumura and Nishida [22]. In other words, we establish the energy estimates for the spatial derivatives by using cutoff functions and localizations of $\partial\Omega$. Although our proofs are in spirit of those for the isentropic Navier–Stokes equations, we should derive the new estimates due to the different dissipative effect and the special nonlinearity of (1.5). We point out the essential point in the proof of the global existence of small solutions is that under the initial condition (1.8) the initial density is bounded far away from the vacuum. A natural question to ask is whether one can still obtain the strong solutions when there exists vacuum initially and surely small initial data. However, the answer is negative. It is well known in [24,25] that the smooth or strong solutions will blow up in finite time if the initial data has an isolated mass group, no matter how small the initial data are.

The rest of this paper is devoted to prove Theorem 1.1. In Section 2, we give some basic facts that will be used in this paper together with the local existence result. In Section 3, we do some careful a priori estimates for the strong solutions and then the global existence of the strong solutions is established by combining a priori estimates and the local existence result.

2. Local existence and preliminaries

In this section, we will recall some known facts and elementary inequalities that will be used frequently later.

We start with the local existence and uniqueness of the strong solutions of problem (1.5)–(1.7). This does not rely much on the structure of the equations. Recently, Kagei and Kawashima [16] have proved the local H^s-solvability ($s \geq [n/2]+1$ being an integer) of the initial boundary problem for a general class of hyperbolic–parabolic system. In fact, we have the following local well-posedness theorem, which is directly from the classical result in [16].

Proposition 2.1. *(Local existence). Let $(p_0, u_0, s_0) \in H^2(\Omega)$ be such that*

$$\inf_{x\in\bar{\Omega}}\{p_0(x)\} > 0 \quad \textit{and} \quad \partial_t^\ell u_0|_{\partial\Omega} = 0, \ell = 0, 1.$$

Then there exist positive numbers T and C such that problem (1.5)–(1.7) has a unique solution $(p, u, s) \in C([0, T]; H^2(\Omega))$. Moreover, the solution satisfies $\inf_{t\in[0,T],x\in\bar{\Omega}}\{p(t, x)\} > 0$, $p_t, s_t \in C([0, T]; H^1(\Omega))$, $u \in L^2([0, T]; H^3(\Omega))$, $u_t \in C([0, T]; L^2(\Omega))$ and

$$\|(p, u, s)(t)\|_2 \leq C\|(p_0, u_0, s_0)\|_2.$$

For later use we list some inequalities of Sobolev type.

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Lemma 2.1. *Let Ω be any bounded domain in $\mathbb{R}^3$ with smooth boundary. Then*

$$
\begin{aligned}
&(i) \quad \|f\|_{L^\infty} \leq C\|f\|_2,\\
&(ii) \quad \|f\|_{L^p} \leq C\|f\|_1, \quad 2 \leq p \leq 6,
\end{aligned}
$$

for some constant $C > 0$ depending only on Ω.

Finally, we introduce the following lemma on the stationary Stokes equations to get the estimates on the tangential derivatives of both u and p, cf. [22].

Lemma 2.2. *Let Ω be any bounded domain in $\mathbb{R}^3$ with smooth boundary. Consider the problem*

$$
\begin{cases}
-\mu\Delta u + \nabla p = g,\\
divu = f,\\
u|_{\partial\Omega} = 0,
\end{cases}
$$

where $f \in H^{k+1}(\Omega)$ and $g \in H^k(\Omega)$ $(k \geq 0)$. Then the above problem has a solution $(p, u) \in H^{k+1} \times H^{k+2} \cap H_0^1$ which is unique modulo a constant of integration for p. Moreover, this solution satisfies

$$
\|u\|_{k+2}^2 + \|\nabla p\|_k^2 \leq C\{\|f\|_{k+1}^2 + \|g\|_k^2\}. \tag{2.1}
$$

3. The proof of global existence

In this section, we shall prove the global existence of the solution with small initial data (Theorem 1.1). The global existence of smooth solution of problem (1.5)–(1.7) can be established by the local existence theory, the uniformly a priori estimates, and the continuity argument. Thus it suffices for us to establish a priori estimate. Therefore, we assume a priori that

$$
\|(p - \bar{p}, u, s - \bar{s})(t)\|_2 \leq \delta \ll 1, \quad \text{for any } \ t \geq 0. \tag{3.1}
$$

In order to derive both the time-independent low and upper bound for the pressure, we start with the basic energy estimate and the initial layer analysis, and succeed in deriving an estimate on the time-independent low and upper bound for $\bar{p}(t)$.

Lemma 3.1. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant c_1 such that*

$$
\bar{p}_0 < \bar{p}(t) \leq \bar{p}_0 + c_1, \tag{3.2}
$$

for any $t \geq 0$.

Proof. First, we rewrite the system (1.1) in terms of (ρ, u, θ) as follows:

$$
\begin{cases}
\rho_t + \mathrm{div}(\rho u) = 0,\\
\rho u_t + \rho u \cdot \nabla u + \nabla p = \mu\Delta u + (\mu + \lambda)\nabla \mathrm{div} u,\\
c_\nu \rho(\theta_t + u \cdot \nabla\theta) = -p\mathrm{div} u + \Phi[u].
\end{cases} \tag{3.3}
$$

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Adding $(3.3)_2$ multiplied by $\frac{u}{\bar\theta_0}$ to $(3.3)_3$ multiplied by $\frac{1}{\bar\theta_0}-\frac{1}{\theta}$, we obtain after integrating the resulting equality over $(0,t)\times\Omega$ and using $(3.3)_1$ and $u|_{\partial\Omega}=0$ that

$$\begin{aligned}&\int_\Omega\Big(\tfrac{1}{2\bar\theta_0}|u|^2+R(\bar\rho_0+\rho\log\tfrac{\rho}{\bar\rho_0}-\rho)+c_\nu\rho(\tfrac{\theta}{\bar\theta_0}-\log\tfrac{\theta}{\bar\theta_0}-1)\Big)(t)dx\\&+\int_0^t\int_\Omega\tfrac{1}{\theta}\Phi[u](s)dxds\\=&\int_\Omega\Big(\tfrac{1}{2\bar\theta_0}|u_0|^2+R(\bar\rho_0+\rho_0\log\tfrac{\rho_0}{\bar\rho_0}-\rho_0)+c_\nu\rho_0(\tfrac{\theta_0}{\bar\theta_0}-\log\tfrac{\theta_0}{\bar\theta_0}-1)\Big)dx,\end{aligned}\tag{3.4}$$

where $\bar\rho_0,\bar\theta_0$ can be calculated by taking $\bar p_0,\bar s$ into (1.3) and (1.4). We rewrite $(1.5)_1$ in the following form:

$$(p^{\frac{1}{\gamma}})_t+\operatorname{div}(p^{\frac{1}{\gamma}}u)=\frac{\Phi[u]p^{\frac{1}{\gamma}-1}}{R+c_\nu}.\tag{3.5}$$

Integrating the above equality over $(0,t)\times\Omega$ and noticing that $\theta^{-1}=RA^{-\frac{1}{\gamma}}p^{\frac{1}{\gamma}-1}e^{-\frac{s}{\gamma c_\nu}}$, which together with (3.4) gives (3.2) under the assumption (3.1). □

Under the assumption (3.1), (3.2) together with Sobolev's inequality implies in particular that $\|(p-\bar p)(t)\|_2$ is equivalent to $\|(p^{\frac{1}{\gamma}}-\bar p^{\frac{1}{\gamma}})(t)\|_2$, i.e. there exists a constant $C>1$, such that

$$\frac{1}{C}\|(p-\bar p)(t)\|_2\le\|(p^{\frac{1}{\gamma}}-\bar p^{\frac{1}{\gamma}})(t)\|_2\le C\|(p-\bar p)(t)\|_2,\tag{3.6}$$

and

$$\frac{1}{2}\bar p_0\le p(t)\le 2\bar p_0,\quad \frac{1}{C}\le\rho(t)\le C\quad\text{for any }\ t\ge 0.\tag{3.7}$$

This should be kept in mind in the rest of this paper.

In order to deduce a priori estimate, in what follows, we will give some energy estimates in a few lemmas. First of all, the energy estimate of lower order for (p,u) is obtained in the following lemma.

Lemma 3.2. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant C such that*

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_\Omega\rho(t)|u(t)|^2+\frac{(p(t)-\bar p(t))^2}{\gamma\bar p(t)}dx+\mu\int_\Omega|\nabla u(t)|^2dx+(\mu+\lambda)\int_\Omega|divu(t)|^2dx\\&\le C\delta\|(\nabla p,\nabla u)(t)\|^2,\end{aligned}\tag{3.8}$$

for any $t\ge 0$.

Proof. A standard energy estimate for the equation $(3.3)_2$ on u gives

$$\frac{1}{2}\frac{d}{dt}\int_\Omega\rho|u|^2dx+\mu\int_\Omega|\nabla u|^2dx+(\mu+\lambda)\int_\Omega|\operatorname{div}u|^2dx+\int_\Omega u\cdot\nabla p\,dx=0.\tag{3.9}$$

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To get the estimate on p, we shall deduce the equation of $\bar{p}$. By integrating (3.5) over Ω gives

$$\bar{p}_t = \frac{\gamma}{|\Omega|}\bar{p}^{1-\frac{1}{\gamma}}\int_\Omega \frac{\Phi[u]p^{\frac{1}{\gamma}-1}}{R+c_\nu}dx. \tag{3.10}$$

Combining the above equality, (3.1) and (3.2), we obtain

$$\left|\bar{p}_t\right| \le C\|\nabla u\|^2. \tag{3.11}$$

We rewrite $(1.5)_1$ in the following form:

$$\frac{(p-\bar{p})_t}{\gamma\bar{p}} + \mathrm{div}u + \frac{\bar{p}_t + \gamma(p-\bar{p})\mathrm{div}u + u\cdot\nabla p}{\gamma\bar{p}} = \frac{\Phi[u]}{\gamma c_\nu\bar{p}}. \tag{3.12}$$

Multiplying the above equality by $p-\bar{p}$ and integrating over Ω gives

$$\begin{aligned}&\tfrac{1}{2}\tfrac{d}{dt}\int_\Omega \tfrac{(p-\bar{p})^2}{\gamma\bar{p}}dx + \tfrac{1}{2}\int_\Omega \tfrac{(p-\bar{p})^2\bar{p}_t}{\gamma\bar{p}^2}dx + \int_\Omega \mathrm{div}u(p-\bar{p})dx\\ &= \int_\Omega \tfrac{\Phi[u](p-\bar{p})}{\gamma c_\nu\bar{p}}dx - \int_\Omega \tfrac{[\bar{p}_t+\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p](p-\bar{p})}{\gamma\bar{p}}dx.\end{aligned} \tag{3.13}$$

Adding the above equality to (3.9), we finally obtain

$$\begin{aligned}&\tfrac{1}{2}\tfrac{d}{dt}\int_\Omega \rho|u|^2 + \tfrac{(p-\bar{p})^2}{\gamma\bar{p}}dx + \mu\int_\Omega |\nabla u|^2dx + (\mu+\lambda)\int_\Omega|\mathrm{div}u|^2dx\\ &= -\tfrac{1}{2}\int_\Omega \tfrac{(p-\bar{p})^2\bar{p}_t}{\gamma\bar{p}^2}dx + \int_\Omega \tfrac{\Phi[u](p-\bar{p})}{\gamma c_\nu\bar{p}}dx - \int_\Omega \tfrac{[\bar{p}_t+\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p](p-\bar{p})}{\gamma\bar{p}}dx.\end{aligned} \tag{3.14}$$

The right terms of the above equation can be estimated by using (3.1), (3.2), (3.11), Lemma 2.1, Hölder's inequality and Poincaré's inequality. In fact, it holds that

$$\Big|\int_\Omega \frac{(p-\bar{p})^2\bar{p}_t}{\gamma\bar{p}^2}dx\Big| \le C\Big|\frac{\bar{p}_t}{\bar{p}^2}\Big|\|p-\bar{p}\|^2 \le C\|\nabla u\|^2\|p-\bar{p}\|^2 \le C\delta\|\nabla u\|^2, \tag{3.15}$$

$$\Big|\int_\Omega \frac{\Phi[u](p-\bar{p})}{\gamma c_\nu\bar{p}}dx\Big| \le C\Big\|\frac{p-\bar{p}}{\bar{p}}\Big\|_{L^\infty}\|\nabla u\|^2 \le C\delta\|\nabla u\|^2, \tag{3.16}$$

$$\begin{aligned}&\Big|\int_\Omega \tfrac{[\bar{p}_t+\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p](p-\bar{p})}{\gamma\bar{p}}dx\Big|\\ =\ &\Big|\int_\Omega \tfrac{[\bar{p}_t+(1-2\gamma)u\cdot\nabla p](p-\bar{p})}{\gamma\bar{p}}dx\Big|\\ \le\ &C[\big|\tfrac{\bar{p}_t}{\bar{p}}\big|\|p-\bar{p}\| + \|\nabla p\|\|u\|_{L^3}\|p-\bar{p}\|_{L^6}]\\ \le\ &C[\|\nabla u\|^2\|p-\bar{p}\| + \|\nabla p\|^2\|\nabla u\|]\\ \le\ &C\delta(\|\nabla p\|^2 + \|\nabla u\|^2).\end{aligned} \tag{3.17}$$

Taking the above three estimates into (3.14) gives (3.8). The proof of lemma is completed. □

Our next goal is to deal with the energy estimate of the time derivative for (p,u).

Lemma 3.3. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant C such that*

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_\Omega \rho(t)|u_t(t)|^2+\frac{(p_t(t)-\bar{p}_t(t))^2}{\gamma\bar{p}(t)}dx+\mu\int_\Omega|\nabla u_t(t)|^2dx+(\mu+\lambda)\int_\Omega|divu_t(t)|^2dx\\&\le C\delta(\|\nabla u(t)\|_1^2+\|\nabla u_t(t)\|^2),\end{aligned}\tag{3.18}$$

for any $t\ge 0$.

Proof. Differentiating (1.5)$_2$ and (3.12) with respect to t, we have

$$\begin{cases}\rho u_{tt}+\rho_t u_t+(\rho u\cdot\nabla u)_t+\nabla p_t=\mu\Delta u_t+(\mu+\lambda)\nabla\mathrm{div}u_t,\\ \frac{(p-\bar{p})_{tt}}{\gamma\bar{p}}-\frac{(p-\bar{p})_t\bar{p}_t}{\gamma\bar{p}^2}+\mathrm{div}u_t+\{\frac{\bar{p}_t+\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p}{\gamma\bar{p}}\}_t=\{\frac{\Phi[u]}{\gamma c_v\bar{p}}\}_t.\end{cases}\tag{3.19}$$

Multiplying the above equation by u_t, $(p-\bar{p})_t$ respectively, we get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_\Omega\rho|u_t|^2+\frac{(p_t-\bar{p}_t)^2}{\gamma\bar{p}}dx+\mu\int_\Omega|\nabla u_t|^2dx+(\mu+\lambda)\int_\Omega|\mathrm{div}u_t|^2dx\\&=-\frac{1}{2}\int_\Omega\rho_t|u_t|^2-\frac{(p_t-\bar{p}_t)^2\bar{p}_t}{\gamma\bar{p}^2}dx-\int_\Omega(\rho u\cdot\nabla u)_tu_tdx\\&\quad+\int_\Omega\{\frac{\Phi[u]}{\gamma c_v\bar{p}}\}_t(p-\bar{p})_tdx-\int_\Omega\{\frac{\bar{p}_t+\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p}{\gamma\bar{p}}\}_t(p-\bar{p})_tdx.\end{aligned}\tag{3.20}$$

Noticing that $u_t\big|_{\partial\Omega}=0$, it follows from (3.1), (3.2), (3.7), (3.11), Lemma 2.1, Hölder's inequality and Poincaré's inequality that

$$\begin{aligned}&\Big|\int_\Omega\rho_t|u_t|^2-\frac{(p_t-\bar{p}_t)^2\bar{p}_t}{\gamma\bar{p}^2}dx\Big|\\&=\Big|\int_\Omega\mathrm{div}[\rho u]|u_t|^2+\frac{(p_t-\bar{p}_t)^2\bar{p}_t}{\gamma\bar{p}^2}dx\Big|\\&=\Big|\int_\Omega 2\rho u\nabla u_tu_t-\frac{(p_t-\bar{p}_t)^2\bar{p}_t}{\gamma\bar{p}^2}dx\Big|\\&\le C[\|\rho\|_{L^\infty}\|u\|_{L^3}\|\nabla u_t\|_{L^2}\|u_t\|_{L^6}+\Big|\frac{\bar{p}_t}{\bar{p}^2}\Big|\|(p_t-\bar{p}_t)\|^2]\\&\le C\delta(\|\nabla u_t\|^2+\|\nabla u\|^2),\end{aligned}\tag{3.21}$$

where by (3.1) and (3.12), we have used the fact

$$\|p_t-\bar{p}_t\|\le C(\|\nabla u\|+\delta\|\nabla p\|).\tag{3.22}$$

Similarly,

$$\begin{aligned}&\Big|\int_\Omega(\rho u\cdot\nabla u)_tu_tdx\Big|\\&=\Big|\int_\Omega\rho_tu\cdot\nabla uu_t+\rho u_t\cdot\nabla uu_t+\rho u\cdot\nabla u_tu_tdx\Big|\\&=\Big|\int_\Omega-[\nabla\rho u+\rho\mathrm{div}u]u\cdot\nabla uu_t+\rho u_t\cdot\nabla uu_t+\rho u\cdot\nabla u_tu_tdx\Big|\\&\le C[\|\nabla\rho\|_{L^3}\|u\|_{L^\infty}^2\|\nabla u\|_{L^2}\|u_t\|_{L^6}+\|\rho\|_{L^\infty}\|u\|_{L^\infty}\|u_t\|_{L^6}\|\nabla u\|_{L^2}\|\nabla u\|_{L^3}\\&\quad+\|\rho\|_{L^\infty}\|\nabla u\|_{L^3}\|u_t\|_{L^3}^2+\|\rho\|_{L^\infty}\|u\|_{L^3}\|\nabla u_t\|_{L^2}\|u_t\|_{L^6}]\\&\le C\delta(\|\nabla u\|^2+\|\nabla u_t\|^2),\end{aligned}\tag{3.23}$$

where by (1.4) and (3.1), we have used the fact

$$\|\nabla\rho\|_1 \le C(\|\nabla p\|_1 + \|\nabla s\|_1) \le C\delta. \tag{3.24}$$

Next, from (3.1) and (3.12), we have

$$\begin{aligned} \|(p-\bar{p})_t\|_{L^3} &\le C(\|\nabla u\|_{L^3} + \|\nabla u\|^2 + \|u\nabla p\|_{L^3} + \|\nabla u\|_{L^6}^2) \\ &\le C(\|\nabla u\|_{L^3} + \|\nabla u\|^2 + \|u\|_{L^\infty}\|\nabla p\|_{L^3} + \|\nabla u\|_{L^6}^2) \\ &\le C\|\nabla u\|_1. \end{aligned} \tag{3.25}$$

Combining (3.22) and (3.25), the third term on the right hand side of (3.20) can be estimated as follows

$$\begin{aligned} &\Big|\int_\Omega \{\tfrac{\Phi[u]}{\gamma c_v \bar{p}}\}_t (p-\bar{p})_t dx\Big| \\ \le\ & C\Big|\int_\Omega |\nabla u \nabla u_t (p-\bar{p})_t| + |\nabla u|^2 |p_t - \bar{p}_t| dx \\ \le\ & C(\|\nabla u\|_{L^6}\|\nabla u_t\|\|p_t-\bar{p}_t\|_{L^3} + \|\nabla u\|_{L^3}^2\|p_t-\bar{p}_t\|_{L^3} \\ \le\ & C\delta(\|\nabla u\|_1^2 + \|\nabla u_t\|^2). \end{aligned} \tag{3.26}$$

For the last term on the right hand side of (3.20), we first obtain through integration by parts that

$$\Big|\int_\Omega \frac{u\cdot\nabla p_t (p-\bar{p})_t}{\gamma\bar{p}} dx\Big| = \Big|\int_\Omega \frac{\mathrm{divu}(\mathrm{p_t}-\bar{\mathrm{p}}_\mathrm{t})^2}{2\gamma\bar{p}} dx\Big| \le C\delta\|\nabla u\|_1^2, \tag{3.27}$$

Then taking the same idea as in (3.21), (3.23) and (3.26), we finally conclude that

$$\Big|\int_\Omega \{\frac{\bar{p}_t + \gamma(p-\bar{p})\mathrm{div}u + u\cdot\nabla p}{\gamma\bar{p}}\}_t (p-\bar{p})_t dx\Big| \le C\delta(\|\nabla u\|_1^2 + \|\nabla u_t\|^2). \tag{3.28}$$

Taking the above (3.21), (3.23), (3.26) and (3.28) into (3.20) gives (3.18). The proof of lemma is completed. □

To deal with the energy estimate of the spatial derivatives for (p, u), we use the standard technique in [22] that involves separating the estimates of solution into that over the region away from the boundary and near the boundary. Let χ_0 be an arbitrary but fixed function in $C_0^\infty(\Omega)$. Then we have the following as the estimate on the region away from the boundary.

Lemma 3.4. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant C such that*

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\int_\Omega \rho(t)|\nabla u(t)\chi_0|^2 + \frac{|\nabla p(t)\chi_0|^2}{\gamma\bar{p}(t)} dx + \mu\int_\Omega |\nabla^2 u(t)\chi_0|^2 dx \\ &+ (\mu+\lambda)\int_\Omega |\nabla divu(t)\chi_0|^2 dx \\ \le\ & C\delta(\|\nabla u(t)\|_1^2 + \|\nabla u_t(t)\|^2 + \|\nabla p(t)\|_1^2) + C\|\nabla u(t)\|\|(\nabla^2 u(t), \nabla p(t))\|, \end{aligned} \tag{3.29}$$

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\int_\Omega \rho(t)|\nabla^2 u(t)\chi_0|^2 + \frac{|\nabla^2 p(t)\chi_0|^2}{\gamma\bar{p}(t)} dx + \mu\int_\Omega |\nabla^3 u(t)\chi_0|^2 dx \\ &+ (\mu+\lambda)\int_\Omega |\nabla^2 divu(t)\chi_0|^2 dx \\ \le\ & C\delta(\|\nabla u(t)\|_2^2 + \|\nabla u_t(t)\|^2 + \|\nabla p(t)\|_1^2) + C\|\nabla^2 u(t)\|\|(\nabla^3 u(t), \nabla^2 p(t))\|, \end{aligned} \tag{3.30}$$

for any $t \ge 0$.

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Proof. As in Lemma 3.3, here, we only sketch the outline and shall omit the detailed calculations for simplicity. Differentiating $(1.5)_2$ and (3.12) with respect to x_i, multiplying the resulting equations by $u_{x_i}\chi_0^2$, $p_{x_i}\chi_0^2$ respectively, we obtain

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_\Omega \rho|u_{x_i}\chi_0|^2+\frac{|p_{x_i}\chi_0|^2}{\gamma\bar{p}}dx+\mu\int_\Omega|\nabla u_{x_i}(t)\chi_0|^2dx+(\mu+\lambda)\int_\Omega|\mathrm{div}u_{x_i}\chi_0|^2dx\\
=&\ \frac{1}{2}\int_\Omega \rho_t|u_{x_i}\chi_0|^2-\frac{|p_{x_i}\chi_0|^2\bar{p}_t}{\gamma\bar{p}^2}dx-\int_\Omega[\rho_{x_i}u_t+(\rho u\cdot\nabla u)_{x_i}]u_{x_i}\chi_0^2dx\\
&+\int_\Omega\{\frac{\Phi[u]}{\gamma c_v\bar{p}}\}_{x_i}p_{x_i}\chi_0^2dx-\int_\Omega\{\frac{\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p}{\gamma\bar{p}(t)}\}_{x_i}p_{x_i}(t)\chi_0^2dx\\
&-\mu\int_\Omega u_{x_i}(t)\nabla u_{x_i}(t)\nabla\chi_0^2dx-(\mu+\lambda)\int_\Omega \mathrm{div}u_{x_i}(t)u_{x_i}(t)\nabla\chi_0^2dx+\int_\Omega p_{x_i}(t)u_{x_i}(t)\nabla\chi_0^2dx\\
\le&\ C\delta(\|\nabla u(t)\|_1^2+\|\nabla u_t(t)\|^2+\|\nabla p(t)\|_1^2)+C\|\nabla u(t)\|\|(\nabla^2u(t)\chi_0,\nabla p(t)\chi_0)\|,
\end{aligned}$$

which gives (3.29). Repeating the above procedure again for 2nd order spatial derivatives we get the following

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_\Omega \rho|u_{x_ix_j}\chi_0|^2+\frac{|p_{x_ix_j}\chi_0|^2}{\gamma\bar{p}}dx+\mu\int_\Omega|\nabla u_{x_ix_j}\chi_0|^2dx+(\mu+\lambda)\int_\Omega|\mathrm{div}u_{x_ix_j}\chi_0|^2dx\\
=&\ \frac{1}{2}\int_\Omega \rho_t|u_{x_ix_j}\chi_0|^2-\frac{|p_{x_ix_j}\chi_0|^2\bar{p}_t}{\gamma\bar{p}^2}dx+\int_\Omega(\rho u\cdot\nabla u)_{x_ix_j}u_{x_ix_j}\chi_0^2dx\\
&+\int_\Omega\{\frac{\Phi[u]}{\gamma c_v\bar{p}}\}_{x_ix_j}p_{x_ix_j}\chi_0^2dx-\int_\Omega\{\frac{\gamma(p-\bar{p})\mathrm{div}u+u\cdot\nabla p}{\gamma\bar{p}}\}_{x_ix_j}p_{x_ix_j}\chi_0^2dx\\
&-\mu\int_\Omega u_{x_ix_j}\nabla u_{x_ix_j}\nabla\chi_0^2dx-(\mu+\lambda)\int_\Omega \mathrm{div}u_{x_ix_j}u_{x_ix_j}\nabla\chi_0^2dx\\
&+\int_\Omega p_{x_ix_j}u_{x_ix_j}\nabla\chi_0^2dx-\int_\Omega[\rho_{x_ix_j}u_t+\rho_{x_i}u_{x_jt}+\rho_{x_j}u_{x_it}]u_{x_ix_j}\chi_0^2dx\\
\le&\ C\delta(\|\nabla u\|_2^2+\|\nabla u_t\|^2+\|\nabla p\|_1^2)+C\|\nabla^2u\chi_0\|\|(\nabla^3u,\nabla^2p)\|,
\end{aligned}$$

which gives (3.30). The proof of lemma is completed. □

Finally, let us establish the estimates near the boundary. Similar to that in [22], we need a more detailed argument using the trick of estimating the tangential derivatives and the normal derivatives separately. We choose a finite number of bounded open sets $\{\mathcal{O}_j\}_{j=1}^N$ in $\mathbb{R}^3$, such that $\partial\Omega\subset\cup_{j=1}^N\mathcal{O}_j$. In each open set $\mathcal{O}_j$ we choose the local coordinates $y=(y_1,y_2,y_3)$ as follows:

- The surface $\mathcal{O}^j\cap\partial\Omega$ is the image of a smooth vector function $z^j(y_1,y_2)=(z_1^j,z_2^j,z_3^j)(y_1,y_2)$ (e.g., take the local geodesic polar coordinate), satisfying

$$|z_{y_1}^j|=1,\ z_{y_1}^j\cdot z_{y_2}^j=0 \text{ and } |z_{y_2}^j|\ge\delta_1>0, \tag{3.31}$$

  where δ is some positive constant independent of $1\le j\le N$.
- Any $x=(x_1,x_2,x_3)\in\mathcal{O}^j$ is represented by

$$x_i:=\Psi_i(y)=y_3n_i(z^j(y_1,y_2))+z_i^j(y_1,y_2) \text{ for } i=1,\ 2,\ 3, \tag{3.32}$$

  where $n^j(y_1,y_2)=(n_1^j,n_2^j,n_3^j)(z^j(y_1,y_2))$ represents the internal unit normal vector at the point $z^j(y_1,y_2)$ of the surface $\partial\Omega$.

We omit the subscript j in what follows for the simplicity of presentation. For $k=1,2$, we define the unit vectors

$$e_1 = z_{y_1} \text{ and } e_2 = \frac{z_{y_2}}{|z_{y_2}|}.$$

Then Frenet–Serret's formula gives that there exist smooth functions $(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2)$ of (y_1, y_2) satisfying

$$\frac{\partial}{\partial y_1}\begin{pmatrix} e_1 \\ e_2 \\ n \end{pmatrix}^i = \begin{pmatrix} 0 & -\gamma_1 & -\alpha_1 \\ \gamma_1 & 0 & -\beta_1 \\ \alpha_1 & \beta_1 & 0 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ n \end{pmatrix}^i,$$

$$\frac{\partial}{\partial y_2}\begin{pmatrix} e_1 \\ e_2 \\ n \end{pmatrix}^i = \begin{pmatrix} 0 & -\gamma_2 & -\alpha_2 \\ \gamma_2 & 0 & -\beta_2 \\ \alpha_2 & \beta_2 & 0 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ n \end{pmatrix}^i,$$

where e_m^i denotes the i-th component of e_m. An elementary calculation shows that the Jacobian J of the transform (3.32) is

$$J = \Psi_{y_1} \times \Psi_{y_2} \cdot n = |z_{y_2}| + (\alpha_1 |z_{y_2}| + \beta_2) y_3 + (\alpha_1\beta_2 - \beta_1\alpha_2) y_3^2. \tag{3.33}$$

By (3.33), we have the transform (3.32) is regular by choosing y_3 so small that $J \geq \delta/2$. Therefore, the inverse function of $\Psi(y) := (\Psi_1, \Psi_2, \Psi_3)(y)$ exits, and we denote it by $y = \Psi^{-1}(x)$. Moreover $(y_1, y_2, y_3)_{x_i}(x)$ make sense and can be expressed by, using a straightforward calculation,

$$\begin{cases} \partial_{x_i} y_1 = \dfrac{1}{J}(\Psi_{y_2} \times \Psi_{y_3})_i = \dfrac{1}{J}(\mathcal{A} e_i^1 + \mathcal{B} e_i^2) =: a_{1i}, \\ \partial_{x_i} y_2 = \dfrac{1}{J}(\Psi_{y_3} \times \Psi_{y_1})_i = \dfrac{1}{J}(\mathcal{C} e_i^1 + \mathcal{D} e_i^2) =: a_{2i}, \\ \partial_{x_i} y_3 = \dfrac{1}{J}(\Psi_{y_1} \times \Psi_{y_2})_i = n^i =: a_{3i}, \end{cases} \tag{3.34}$$

where $\mathcal{A} = |z_{y_2}| + \beta_2 y_3$, $\mathcal{B} = -y_3\alpha_2$, $\mathcal{C} = -\beta_1 y_3$, $\mathcal{D} = 1 + \alpha_1 y_3$,

$$J = \mathcal{A}\mathcal{D} - \mathcal{B}\mathcal{C} \geq \delta/2. \tag{3.35}$$

Obviously, (3.34) gives

$$\sum_{i=1}^{3} a_{3i}^2 = |n|^2 = 1, \ a_{1i}a_{3i} = a_{2i}a_{3i} = 0, \ J^2 = (\mathcal{A}\mathcal{C} + \mathcal{B}\mathcal{D})^2 - (\mathcal{A}^2 + \mathcal{B}^2)(\mathcal{C}^2 + \mathcal{D}^2)$$

and

$$\partial_{x_i} = a_{ki}\partial_{y_k}, \tag{3.36}$$

where we have used the Einstein convention of summing over repeated indices.

Thus, in each $\mathcal{O}_j$, the first two equations of (1.5) can be rewritten in the local coordinates (y_1, y_2, y_3) as follows:

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$$\begin{aligned}\mathcal{L}^p &:= \frac{dp}{dt} + \frac{\gamma\bar{p}}{J}[(\mathcal{A}e_1+\mathcal{B}e_2)\cdot u_{y_1} + (\mathcal{C}e_1+\mathcal{D}e_2)\cdot u_{y_2} + Jn\cdot u_{y_3}] = f^0,\\ \mathcal{L}^u &:= \rho u_t - \frac{\mu}{J^2}[(\mathcal{A}^2+\mathcal{B}^2)u_{y_1y_1} + 2(\mathcal{AC}+\mathcal{BD})u_{y_1y_2} + (\mathcal{C}^2+\mathcal{D}^2)u_{y_2y_2} + J^2u_{y_3y_3}]\\ &\quad + \text{one order terms of } u + \frac{1}{J}(\mathcal{A}e_1+\mathcal{B}e_2)[\frac{\mu+\lambda}{\gamma\bar{p}}\frac{dp}{dt}+p]_{y_1}\\ &\quad + \frac{1}{J}(\mathcal{C}e_1+\mathcal{D}e_2)[\frac{\mu+\lambda}{\gamma\bar{p}}\frac{dp}{dt}+p]_{y_2} + n[\frac{\mu+\lambda}{\gamma\bar{p}}\frac{dp}{dt}+p]_{y_3} = f,\end{aligned}$$

where

$$\begin{aligned}\frac{d}{dt} &:= \partial_t + u\cdot\nabla \text{ denotes the material derivative,}\\ f^0 &:= \frac{\Phi[u]}{c_v} - \gamma(p-\bar{p})\mathrm{div}u,\\ f &:= \rho u\cdot\nabla u + \frac{\mu+\lambda}{\gamma\bar{p}}\nabla f^0,\\ J^2 &:= (\mathcal{AC}+\mathcal{BD})^2 - (\mathcal{A}^2+\mathcal{B}^2)(\mathcal{C}^2+\mathcal{D}^2).\end{aligned}$$

Let us denote the tangential derivatives by $\partial = (\partial_{y_1}, \partial_{y_2})$ and χ_j be arbitrary but fixed function in $C_0^\infty(\mathcal{O}_j)$. Obviously, $\chi_j\partial^k u = 0$ on $\partial\Omega_j^{-1}$, where $0 \le k \le 2$ and $\Omega_j{}^{-1}(y) := \{y|y = \Psi^{-1}(x), x \in \Omega_j = \mathcal{O}_j \cap \Omega\}$. Estimating the tangential derivatives in a similar manner as in Lemma 3.4, we have

Lemma 3.5. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant C such that*

$$\begin{aligned}&\frac{d}{dt}\int_{\Omega_j^{-1}} \rho(t)|\partial u(t)\chi_j|^2 + \frac{|\partial p(t)\chi_j|^2}{\gamma\bar{p}(t)}dy + \int_{\Omega_j{}^{-1}} |\partial\nabla u(t)\chi_j|^2 dy + \int_{\Omega_j^{-1}} |\partial\frac{dp(t)}{dt}\chi_j|^2 dy\\ &\le C\delta(\|\nabla u(t)\|_1^2 + \|\nabla u_t(t)\|^2 + \|\nabla p(t)\|_1^2) + C\|\nabla u(t)\|(\|(\nabla u(t)\|_1 + \|\nabla p(t)\|),\end{aligned} \tag{3.37}$$

$$\begin{aligned}&\frac{d}{dt}\int_{\Omega_j^{-1}} \rho(t)|\partial^2 u(t)\chi_j|^2 + \frac{|\partial^2 p(t)\chi_j|^2}{\gamma\bar{p}(t)}dy + \int_{\Omega_j^{-1}} |\partial^2\nabla u(t)\chi_j|^2 dy\\ &+ \int_{\Omega_j^{-1}} |\partial^2\frac{dp(t)}{dt}\chi_j|^2 dy\\ &\le C\delta(\|\nabla u(t)\|_2^2 + \|\nabla u_t(t)\|^2 + \|\nabla p(t)\|_1^2) + C\|\nabla^2 u(t)\|(\|\nabla u(t)\|_2 + \|\nabla^2 p(t)\|),\end{aligned} \tag{3.38}$$

for any $t \ge 0$.

Next, we turn to estimates of derivatives in the normal directions.

Lemma 3.6. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant C such that*

$$\begin{aligned}&\frac{d}{dt}\int_{\Omega_j^{-1}} |p_{y_3}(t)\chi_j|^2 dy + \int_{\Omega_j^{-1}} |(\frac{dp(t)}{dt})_{y_3}\chi_j|^2 dy\\ &\le C(\|(\nabla u(t), u_t(t))\|^2 + \delta\|(\nabla p(t), \nabla u(t))\|_1^2 + \int_{\Omega_j{}^{-1}} |\partial\nabla u(t)\chi_j|^2 dy),\end{aligned} \tag{3.39}$$

$$\begin{aligned}&\frac{d}{dt}\int_{\Omega_j^{-1}} |\partial^k\partial_{y_3}^{\ell+1} p(t)\chi_j|^2 dy + \int_{\Omega_j^{-1}} |\partial^k\partial_{y_3}^{\ell+1}(\frac{dp(t)}{dt})\chi_j|^2 dy\\ &\le C(\|(\nabla u(t), u_t(t))\|_1^2 + \delta\|(\nabla p(t), \nabla^2 u(t))\|_1^2 + \int_{\Omega_j{}^{-1}} |\partial^{k+1}\partial_{y_3}^l\nabla u(t)\chi_j|^2 dy),\end{aligned} \tag{3.40}$$

for any $t \ge 0$, $k + \ell = 1$.

Proof. First, we use the equations $\partial_{y_3}(\mathcal{L}^p - f^0) = 0$ and $n(\mathcal{L}^u - f) = 0$, which have the following form:

$$
\begin{aligned}
&(\frac{dp}{dt})_{y_3} + \frac{\gamma\bar{p}}{J}[(\mathcal{A}e_1 + \mathcal{B}e_2)\cdot u_{y_1y_3} + (\mathcal{C}e_1 + \mathcal{D}e_2)\cdot u_{y_2y_3} + Jn\cdot u_{y_3y_3}] \\
&+ \text{one order terms of } u = f^0_{y_3},
\end{aligned} \tag{3.41}
$$

$$
\begin{aligned}
&n\rho u_t - \tfrac{\mu}{J^2}[(\mathcal{A}^2+\mathcal{B}^2)nu_{y_1y_1} + 2(\mathcal{AC}+\mathcal{BD})nu_{y_1y_2} + (\mathcal{C}^2+\mathcal{D}^2)nu_{y_2y_2} + J^2 nu_{y_3y_3}] \\
&+ \text{one order terms of } u + [\tfrac{\mu+\lambda}{\gamma\bar{p}}\tfrac{dp}{dt} + p]_{y_3} = nf,
\end{aligned} \tag{3.42}
$$

To eliminate $nu_{y_3y_3}$ in (3.42), we take the summation $\frac{\mu}{\gamma\bar{p}} \times$ (3.41) + (3.42) which gives

$$
\begin{aligned}
\tfrac{2\mu+\lambda}{\gamma\bar{p}}(\tfrac{dp}{dt})_{y_3} + p_{y_3} = {} & \tfrac{\mu}{J^2}[(\mathcal{A}^2+\mathcal{B}^2)nu_{y_1y_1} + 2(\mathcal{AC}+\mathcal{BD})nu_{y_1y_2} + (\mathcal{C}^2+\mathcal{D}^2)nu_{y_2y_2}] \\
& - n\rho u_t - \tfrac{\mu}{J}[(\mathcal{A}e_1+\mathcal{B}e_2)\cdot u_{y_1y_3} + (\mathcal{C}e_1+\mathcal{D}e_2)\cdot u_{y_2y_3}] \\
& + \text{one order terms of } u + nf + \tfrac{\mu}{\gamma\bar{p}}f^0 = F.
\end{aligned} \tag{3.43}
$$

Multiplying the above equation by $\chi_j^2(\frac{dp}{dt})_{y_3}$ and integrating on Ω_j^{-1}, we can bound the one order derivatives in the normal direction to the boundary as follows:

$$
\begin{aligned}
&\tfrac{1}{2}\tfrac{d}{dt}\int_{\Omega_j^{-1}} |p_{y_3}\chi_j|^2 dy + \tfrac{2\mu+\lambda}{\gamma\bar{p}}\int_{\Omega_j^{-1}} |(\tfrac{dp}{dt})_{y_3}\chi_j|^2 dy \\
&= \int_{\Omega_j^{-1}} -(u\cdot\nabla p)_{y_3} p_{y_3}\chi_j^2 + (\tfrac{dp}{dt})_{y_3} F\chi_j^2 dy.
\end{aligned} \tag{3.44}
$$

It follows from Lemma 2.1, Hölder's inequality and Cauchy's inequality, for the first term in right side of (3.44), it holds that

$$
\begin{aligned}
&\big|\int_{\Omega_j^{-1}} (u\cdot\nabla p)_{y_3} p_{y_3}\chi_j^2 dy\big| \\
&\le \big|\int_{\Omega_j^{-1}} u_{y_3}\cdot\nabla p p_{y_3}\chi_j^2 dy\big| + \tfrac{1}{2}\big|\int_{\Omega_j^{-1}} (p_{y_3})^2 \mathrm{div}(u\chi_j^2) dy\big| \\
&\le C\|\nabla u\|_1\|\nabla p\|_1^2 \le C\delta\|\nabla p\|_1^2.
\end{aligned} \tag{3.45}
$$

For the second term, we have

$$
\begin{aligned}
&\big|\int_{\Omega_j^{-1}} (\tfrac{dp}{dt})_{y_3} F\chi_j^2 dy\big| \\
&\le \tfrac{2\mu+\lambda}{2\gamma\bar{p}}\int_{\Omega_j^{-1}} |(\tfrac{dp}{dt})_{y_3}\chi_j|^2 dy + C\int_{\Omega_j^{-1}} |F\chi_j|^2 dy \\
&\le \tfrac{2\mu+\lambda}{2\gamma\bar{p}}\int_{\Omega_j^{-1}} |(\tfrac{dp}{dt})_{y_3}\chi_j|^2 dy + C(\|(\nabla u, u_t)\|^2 + \delta\|\nabla u\|_1^2 + \int_{\Omega_j{}^{-1}} |\partial\nabla u\chi_j|^2 dy).
\end{aligned} \tag{3.46}
$$

Substituting (3.45) and (3.46) into (3.44), we obtain

$$
\begin{aligned}
&\tfrac{d}{dt}\int_{\Omega_j^{-1}} |p_{y_3}\chi_j|^2 dy + \tfrac{2\mu+\lambda}{\gamma\bar{p}(t)}\int_{\Omega_j^{-1}} |(\tfrac{dp}{dt})_{y_3}\chi_j|^2 dy \\
&\le C(\|(\nabla u, u_t)\|^2 + \delta\|(\nabla p, \nabla u)\|_1^2 + \int_{\Omega_j{}^{-1}} |\partial\nabla u\chi_j|^2 dy),
\end{aligned}
$$

which gives (3.39).

If we apply $\partial^k\partial_{y_3}^\ell (k+\ell=1)$ to (3.43), multiply it by $\chi_j^2\partial^k\partial_{y_3}^{\ell+1}(\frac{dp}{dt})$ and integrate it in the similar way as in (3.39), we get

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$$\begin{aligned}
&\frac{d}{dt}\int_{\Omega_j^{-1}}|\partial^k\partial_{y_3}^{\ell+1}p\chi_j|^2dy+\int_{\Omega_j^{-1}}|\partial^k\partial_{y_3}^{\ell+1}(\tfrac{dp}{dt})\chi_j|^2dy\\
&\le C(\|(\nabla u,u_t)\|_1^2+\delta\|(\nabla p,\nabla^2u)\|_1^2+\int_{\Omega_j{}^{-1}}|\partial^{k+1}\partial_{y_3}^{\ell}\nabla u\chi_j|^2dy),
\end{aligned}$$

which gives (3.40). The proof of lemma is completed. □

Finally, we use Lemma 2.2 to get the estimates on the tangential derivatives of p and u.

Lemma 3.7. *Under the conditions of Theorem 1.1 and (3.1), there exists a positive constant C such that*

$$\|\nabla^2u(t)\|^2+\|\nabla p(t)\|^2\le C(\|\frac{dp(t)}{dt}\|_1^2+\|u_t(t)\|^2+\|\nabla u(t)\|_1^2\|\nabla^2u(t)\|^2) \tag{3.47}$$

$$\begin{aligned}
&\int_{\Omega_j^{-1}}|\partial\nabla^2u(t)\chi_j|^2dy+\int_{\Omega_j^{-1}}|\partial\nabla p(t)\chi_j|^2dy\\
&\le C(\|(\nabla u(t),u_t(t))\|_1^2+\|\nabla p(t)\|^2+\int_{\Omega_j^{-1}}|\partial\nabla\tfrac{dp(t)}{dt}\chi_j|^2dy\\
&\quad+\|\nabla p(t)\|\|\nabla\tfrac{dp(t)}{dt}\|+\|\nabla u(t)\|_1^2\|\nabla^3u(t)\|^2),
\end{aligned}\tag{3.48}$$

for any $t\ge 0$.

Proof. We rewrite equations $(1.5)_{1,2}$ as the Stokes problem:

$$\begin{cases}
\mathrm{div}u=\frac{1}{\gamma p}[-\frac{dp}{dt}+\frac{\Phi[u]}{c_v}],\\
-\mu\Delta u+\nabla p=(\mu+\lambda)\nabla\mathrm{div}u-(\rho u_t+\rho u\cdot\nabla u),\\
u|_{\partial\Omega}=0.
\end{cases}\tag{3.49}$$

Thus applying Lemma 2.2 to (3.49) gives (3.47).

Next operating $\chi_j\partial$ to Stokes equation $(3.49)_2$, then together with $(3.49)_{1,3}$, implies that the following Stokes problem:

$$\begin{cases}
\mathrm{div}(\chi_j\partial u)=\chi_j\partial\{\frac{1}{\gamma p}[-\frac{dp}{dt}+\frac{\Phi[u]}{c_v}]\}+\nabla\chi_j\partial u,\\
-\mu\Delta(\chi_j\partial u)+\nabla(\chi_j\partial p)=-2\mu\nabla\chi_j\nabla(\partial u)-\Delta\chi_j\partial u+\nabla\chi_j\partial p\\
\qquad\qquad+(\mu+\lambda)\chi_j\nabla\partial\{\frac{1}{\gamma p}[-\frac{dp}{dt}+\frac{\Phi[u]}{c_v}]\}-\chi_j\partial(\rho u_t+\rho u\cdot\nabla u),\\
\chi_j\partial u|_{\partial\Omega_j^{-1}}=0.
\end{cases}\tag{3.50}$$

Thus applying Lemma 2.2 to (3.50) gives (3.48). The proof of lemma is completed. □

Now we are in a position to prove Theorem 1.1.

Proof of Theorem 1.1. We do it by four steps.

Step 1: We first estimate the lower order derivatives for (p,u). Let D be a fixed but large positive constant. By summing up

$$D^2\times((3.8)+(3.18))+D\times((3.29)+(3.37))+(3.39),$$

there exists a function $H_1(p,u)$ which is equivalent to $\|(u,p-\bar p,u_t,p_t-\bar p_t,\nabla p)\|^2$ and satisfies

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$$
\begin{aligned}
&\frac{d}{dt}\Big\{H_1(t)+\int_\Omega \rho(t)|\nabla u(t)\chi_0|^2dx+\sum_{j=1}^N\int_{\Omega_j^{-1}}\rho(t)|\partial u(t)\chi_0|^2dy\Big\}+D\|(\nabla u(t),\nabla u_t(t))\|^2\\
&+\|\nabla\tfrac{dp(t)}{dt}\|^2+\int_\Omega|\nabla^2u(t)\chi_0|^2dx+\sum_{j=1}^N\int_{\Omega_j^{-1}}|\partial\nabla u(t)\chi_j|^2dy\\
&\le \frac{1}{D^{1/3}}\|(\nabla p(t),\nabla^2u(t))\|^2+C\delta\|\nabla^2p(t)\|^2.
\end{aligned}
\tag{3.51}
$$

Taking (3.47) into the above inequality, and using the fact $\frac{dp}{dt}=-\gamma p\mathrm{div}u+\frac{\Phi[u]}{c_v}$, we obtain

$$
\begin{aligned}
&\frac{d}{dt}\Big\{H_1(t)+\int_\Omega \rho(t)|\nabla u(t)\chi_0|^2dx+\sum_{j=1}^N\int_{\Omega_j^{-1}}\rho(t)|\partial u(t)\chi_0|^2dy\Big\}+\|\nabla u(t)\|_1^2\\
&+\|(\nabla p(t),\nabla u_t(t))\|^2+\|\nabla\tfrac{dp(t)}{dt}\|^2\le C\delta\|\nabla^2p(t)\|^2,
\end{aligned}
\tag{3.52}
$$

since D is large and δ is small.

Step 2: Next, we estimate the higher order derivatives for (p,u). Taking $\ell=0$ in (3.40) and summing up $D\times[2\times(3.30)+(3.38)]+(3.40)$, we have

$$
\begin{aligned}
&\frac{d}{dt}\{D\int_\Omega \rho(t)|\nabla^2u(t)\chi_0|^2+\frac{|\nabla^2p(t)\chi_0|^2}{\gamma\bar p(t)}dx+D\sum_{j=1}^N\int_{\Omega_j^{-1}}\rho(t)|\partial^2u(t)\chi_j|^2+\frac{|\partial^2p(t)\chi_j|^2}{\gamma\bar p(t)}dy\\
&+\int_{\Omega_j^{-1}}|\partial\partial_{y_3}p(t)\chi_j|^2dy\}+\int_\Omega|\nabla^3u(t)\chi_0|^2dx+\int_\Omega|\nabla^2\tfrac{dp(t)}{dt}\chi_0|^2dx\\
&+\sum_{j=1}^N\int_{\Omega_j^{-1}}|\partial^2\nabla u(t)\chi_j|^2+|\partial\nabla\tfrac{dp(t)}{dt}\chi_j|^2dy\\
&\le CD(\|\nabla u(t)\|_1^2+\|\nabla u_t(t)\|^2)+CD\delta\|(\nabla p(t),\nabla^2u(t))\|_1^2\\
&\quad+CD\|\nabla^2u(t)\|(\|\nabla u(t)\|_2+\|\nabla^2p(t)\|).
\end{aligned}
\tag{3.53}
$$

Then taking $\ell=1$ in (3.40) and substituting (3.48) into (3.40), we have

$$
\begin{aligned}
&\frac{d}{dt}\int_{\Omega_j^{-1}}|\partial_{y_3}^2p(t)\chi_j|^2dy+\int_{\Omega_j^{-1}}|\partial_{y_3}^2(\tfrac{dp(t)}{dt})\chi_j|^2dy\\
&\le C(\|(\nabla u(t),u_t(t))\|_1^2+\|\nabla p(t)\|\|(\nabla p(t),\nabla\tfrac{dp(t)}{dt})\|+\delta\|(\nabla p(t),\nabla^2u(t))\|^2).
\end{aligned}
\tag{3.54}
$$

Adding $D\times$ (3.53) to (3.54), there exists $H_2(t)$ which is equivalent to $\|\nabla^2p(t)\|^2$, such that

$$
\begin{aligned}
&\frac{d}{dt}\{D^2\int_\Omega\rho(t)|\nabla^2u(t)\chi_0|^2dx+D^2\sum_{j=1}^N\int_{\Omega_j^{-1}}\rho(t)|\partial^2u(t)\chi_j|^2dy+H_2(t)\}\\
&+\int_\Omega|\nabla^3u(t)\chi_0|^2dx+\sum_{j=1}^N\int_{\Omega_j^{-1}}|\partial^2\nabla u(t)\chi_j|^2dy+\int_\Omega|\nabla^2\tfrac{dp(t)}{dt}|^2dx\\
&\le CD^2(\|\nabla u(t)\|_1^2+\|(\nabla u_t(t),\nabla p(t))\|^2)+CD^2\delta\|(\nabla p(t),\nabla^2u(t))\|_1^2\\
&\quad+CD^2\|\nabla^2u(t)\|(\|\nabla u(t)\|_2+\|\nabla^2p(t)\|).
\end{aligned}
\tag{3.55}
$$

Applying Lemma 2.2 to (3.49) as (3.48), we obtain

$$
\begin{aligned}
\|(\nabla^3u(t),\nabla^2p(t))\|^2\le{}& C(\|(\nabla u(t),u_t(t))\|_1^2+\|\nabla p(t)\|^2+\|\nabla\tfrac{dp(t)}{dt}\|_1^2\\
&+\|\nabla u(t)\|_1^2\|\nabla^3u(t)\|^2).
\end{aligned}
\tag{3.56}
$$

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Step 3: Now, we are able to establish the energy estimate of Gronwall-type. An application of the L^p-estimate of elliptic system to (1.5)$_2$ gives

$$\|\nabla^2 u(t)\|^2 \le C\|(u_t(t), \nabla p(t), \nabla u(t))\|^2. \tag{3.57}$$

Thus by summing up $D^4 \times$ (3.52) $+ D \times$ (3.55) $+$ (3.56), there exists a function $H_3(p, u)$ which is equivalent to $\|(p(t), u(t))\|_2^2 + \|(p_t(t), u_t(t))\|^2$ such that

$$\frac{dH_3(p(t), u(t))}{dt} + CH_3(p(t), u(t)) + C\|\nabla^3 u(t)\|^2 \le 0, \tag{3.58}$$

where we use the Poincaré's inequality $\|p(t) - \bar{p}(t)\| \le C\|\nabla p(t)\|$. Integrating the above inequality over $[0, t]$ gives (1.9).

By Gronwall's inequality, (3.58) leads to

$$H_3(p(t), u(t)) \le CH_3(p(0), u(0))e^{-Ct}.$$

Taking the above estimate into the homogeneous transport equation (1.5)$_3$ we arrive at (1.11).

Step 4: By symmetry and some tedious but straightforward calculation, we have the energy estimates on the entropy as following:

$$\frac{d}{dt}\|s(t) - \bar{s}\|_2^2 \le C\|u(t)\|_2\|s(t) - \bar{s}\|_2^2 + C\delta^{\frac{1}{2}}\|u(t)\|_3^2.$$

Adding the above inequality to (3.58), and by Gronwall's inequality, we have

$$\|s(t) - \bar{s}\|_2^2 \le C\|(p_0 - \bar{p}_0, u_0, s_0 - \bar{s})\|_2 \exp\{C\int_0^t \|u(\tau)\|_2 d\tau\},$$

taking (1.11) into the above inequality, we arrive at (1.10).

Finally, from (3.5) we have

$$\tilde{p} - \bar{p}(t) = \frac{1}{|\Omega|}\int_t^\infty \int_\Omega \frac{\Phi[u]p^{\frac{1}{\gamma}-1}}{R + c_v} dx d\tau,$$

then taking (1.11) into the above equality we prove (1.12) and this completes the proof of Theorem 1.1. □

References

[1] K. Deckelnick, Decay estimates for the compressible Navier–Stokes equations in unbounded domains, Math. Z. 209 (1992) 115–130.

[2] R.J. Duan, H.F. Ma, Global existence and convergence rates for the 3-D compressible Navier–Stokes equations without heat conductivity, Indiana Univ. Math. J. 5 (2008) 2299–2319.

[3] R.J. Duan, H.X. Liu, S. Ukai, T. Yang, Optimal $L^p - L^q$ convergence rates for the compressible Navier–Stokes equations with potential force, J. Differential Equations 238 (2007) 220–233.

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[4] R.J. Duan, S. Ukai, T. Yang, H.J. Zhao, Optimal convergence rates for the compressible Navier–Stokes equations with potential forces, Math. Models Methods Appl. Sci. 17 (2007) 737–758.
[5] Jishan Fan, Song Jiang, Yaobin Ou, A blow-up criterion for compressible viscous heat-conductive ows, Ann. Inst. H. Poincaré Anal. Non Linéaire 27 (1) (2010) 337–350.
[6] E. Feireisl, H. Petzeltová, Large-time behavior of solutions to the Navier–Stokes equations of compressible flow, Arch. Ration. Mech. Anal. 150 (1999) 77–96.
[7] E. Feireisl, A. Novotný, H. Petzeltová, On the existence of globally defined weak solutions to the Navier–Stokes equations, J. Math. Fluid Mech. 3 (2001) 358–392.
[8] Y. Guo, Y.J. Wang, Decay of dissipative equations and negative Sobolev spaces, Comm. Partial Differential Equations 37 (2012) 2165–2208.
[9] D. Hoff, Global solutions of the Navier–Stokes equations for multidimensional compressible flow with discontinuous initial data, J. Differential Equations 120 (1995) 215–254.
[10] D. Hoff, Strong convergence to global solutions for multidimensional flows of compressible, viscous fluids with polytropic equations of state and discontinuous initial data, Arch. Ration. Mech. Anal. 132 (1995) 1–14.
[11] D. Hoff, K. Zumbrun, Multi-dimensional diffusion waves for the Navier–Stokes equations of compressible flow, Indiana Univ. Math. J. 44 (1995) 604–676.
[12] D. Hoff, K. Zumbrun, Pointwise decay estimates for multidimensional Navier–Stokes diffusion waves, Z. Angew. Math. Phys. 48 (1997) 597–614.
[13] X. Huang, J. Li, Z.P. Xin, Global well-posedness of classical solutions with large oscillations and vacuum to the three-dimensional isentropic compressible Navier–Stokes equations, Comm. Pure Appl. Math. 65 (4) (2012) 549–585.
[14] F. Jiang, S. Jiang, J. Yin, Global weak solutions to the two-dimensional Navier–Stokes equations of compressible heat-conducting flows with symmetric data and forces, Discrete Contin. Dyn. Syst. Ser. A 34 (2) (2014) 567–587.
[15] F. Jiang, Nonlinear thermal instability in compressible viscous flows without heat conductivity, preprinted.
[16] Y. Kagei, S. Kawashima, Local solvability of initial boundary value problem for a quasilinear hyperbolic–parabolic system, J. Hyperbolic Differ. Equ. 3 (2006) 195–232.
[17] P.L. Lions, Mathematical Topics in Fluid Mechanics, vol. 2. Compressible Models, Oxford University Press, New York, 1998.
[18] T.P. Liu, W.K. Wang, The pointwise estimates of diffusion waves for the Navier–Stokes equations in odd multidimensions, Comm. Math. Phys. 196 (1998) 145–173.
[19] T.P. Liu, Y. Zeng, Compressible Navier–Stokes equations with zero heat conductivity, J. Differential Equations 153 (1999) 225–291.
[20] A. Matsumura, T. Nishida, The initial value problem for the equations of motion of compressible viscous and heat conductive fluids, Proc. Japan Acad. Ser. A Math. Sci. 55 (1979) 337–342.
[21] A. Matsumura, T. Nishida, The initial value problem for the equations of motion of viscous and heat-conductive gases, J. Math. Kyoto Univ. 20 (1) (1980) 67–104.
[22] A. Matsumura, T. Nishida, Initial boundary value problems for the equations of motion of compressible viscous and heat-conductive fluids, Comm. Math. Phys. 89 (1983) 445–464.
[23] Z. Tan, H. Wang, Global existence and optimal decay rate for the strong solutions in H^2 to the 3-D compressible Navier–Stokes equations without heat conductivity, J. Math. Anal. Appl. 394 (2) (2012) 571–580.
[24] Z. Xin, W. Yan, On blowup of classical solutions to the compressible Navier–Stokes equations, Comm. Math. Phys. 321 (2) (2013) 529–541.
[25] Z. Xin, Blowup of smooth solutions to the compressible Navier–Stokes equation with compact density, Comm. Pure Appl. Math. 51 (1998) 229–240.

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