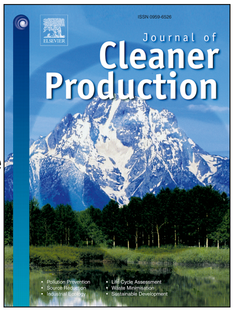
Accepted Manuscript



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PII: S0959-6526(17)32455-1

DOI: 10.1016/j.jclepro.2017.10.222

Reference: JCLP 11014

To appear in: *Journal of Cleaner Production*

Received Date: 13 May 2017

Revised Date: 6 October 2017

Accepted Date: 13 October 2017

Sustainable Economic Production Quantity Models for Inventory Systems with Shortage

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Abstract

Recently, new economic order/production quantity models have shifted away from focusing only on economic issues and towards combined economic-environmental concerns because of sustainable development goals. Despite this shift, only a few works have addressed sustainable Economic Production Quantity (EPQ). The theoretical sustainable EOQ and EPQ models are basic models that ignore many real-life conditions such as the possibility of stock-out in inventory systems. In this paper, we develop four new sustainable economic production quantity models that consider different shortage situations. To find optimal values of inventory system variables, we solve four independent profit maximization problems for four different situations. These proposed models include a *basic model* in which shortages are not allowed, and when shortages are allowed, the *lost sale*, *full backordering* and *partial backordering* models can be selected by operations managers depending on the manufacturer's motivation to improve service levels. We have also proposed an algorithm for determining optimum values of the decision variables for these sustainable economic production quantity models. Finally, the formulated models are explained with some different examples and the obtained results have been analyzed and discussed. These results show that the *sustainable economic production quantity with partial backordering* model is a general and more realistic model that can be used in many real cases with a reasonable profit amount, compared with the three other proposed models.

Keywords: Sustainable Economic Production Quantity Models, Shortage, Backordering, Inventory Management, Sustainable Development, Environmental Considerations

1. Introduction

For more than a century, the act of determining order quantity (or lot sizing) for a firm's requirements has been a primary consideration. As early as 1913 Harris developed a simple model for determining order quantity based on basic economic considerations (including holding and ordering costs) that was called an Economic Order Quantity (EOQ) model. Two years later, Harris (1915) presented a similar model that determines Economic Production Quantity (EPQ) and, in 1918, Taft proposed a similar formula for EPQ. Over the years, many models have been developed based on Harris' masterworks, but most of them merely

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customized the initial EOQ model by adding other economic considerations and did not address non-economic considerations.

Now, because of the importance of environmental issues and comprehensive relations between industrial development and environmental management, sustainable development and the concept of sustainability receives much more attention (Kannan, 2017). The current rise in global warming has motivated both consumers and producers to be more concerned about controlling emissions and protecting the environment. Supply chains are focusing on their environmental performance in addition to competing on cost and service (Khan et al., 2012).

As will be seen in the literature review section (Section 2), some works seek to determine sustainable EOQ and fewer works attempt to determine sustainable EPQ. No work considers shortage issues, so we address this research gap in this paper. Our research question is: how can we formulate a sustainable EPQ problem considering shortage issues? To answer this question in this work, we model and solve four sustainable EPQ models for different shortage situations while considering environmental parameters.

This article consists of 6 other sections. In Section 2 (literature review), we review previous related researches and studies. In Section 3 we provide the motivation of our study and describe the problem. Section 4 introduces the notations used throughout the paper, describes the developed inventory models, and proposes the optimum solution for each model. Numerical examples are presented in Section 5 to illustrate the theoretical results. Results from these examples are analyzed and interpreted in the results and discussion sections (Section 6 and 7). We describe the managerial implications of our research in the Implication section (Section 8). Finally, we present our conclusions and further research in Section 9.

2. Literature Review

In recent years, a few works have examined sustainability issues in EOQ models. Inman (2002) presented a number of primary propositions to set research guidelines in the field of environmentally conscious operations management. Barbosa-Póvoa (2009) performed an overview of sustainability at the supply chain level as an emerging area that needs to be studied in a systematic way. Turkay (2008) developed a lot-sizing model by considering business' carbon footprint in the model and analyzing five different approaches: Carbon Tax, Carbon Offsets, Direct Accounting, Cap & Trade, and Direct Cap. Direct accounting is a careful approach because sustainability issues are translated into costs and modeled as a part of the total cost function, but other approaches focus more on governmental policies (Bouchery et al., 2010). Bonney and Jaber (2011) provide a short list of some of the environmental costs and they propose a responsible EOQ model. Wahab et al. (2011) focused especially on transportation emission costs; they incorporated environmental issues in order to establish an optimal strategy by calculating fixed and variable carbon emission costs. Bouchery et al. (2010) prepared a basic sustainable lot-sizing model. They also presented a multi-objective EOQ model that minimizes the cost and environmental damages (Bouchery et al., 2012). Absi et al. (2012) presented a model for single-item multi-sourcing lot-sizing problems with fixed and variable carbon emissions so that each sourcing mode includes source location and transportation models. At the same time, Heuvel et al. (2012) modeled a bi-objective sustainable lot-sizing problem for minimizing costs of lot-sizing and to minimize emissions from the steps of setup, production, and inventory. Abdallah et al. (2012) prepared a closed-loop model that considers waste disposal cost, and they proposed a framework for product recovery using carbon credit allocations and trading. Csutora et al. (2012) analyzed the effects of introducing carbon emissions in the model as an endogenous variable by employing a comparative static analysis. They suggested that carbon costs may significantly modify the EOQ ordering policy. Glock et al. (2012) presented a supply chain including a single supplier and single manufacturer and studied tradeoffs between demand, sustainability, costs, and profit. Chen et al. (2013) studied the effects of parameters of carbon emission in lot sizing models in supply chain management (SCM) and showed the effect of carbon emissions in their work. Oslo (2013) analyzed a retailer's joint decision on inventory replenishment and investment for carbon emission reduction. He used the EOQ model considering Cap & Trade, carbon cap, and carbon tax approaches. Digiesi et al. (2013) prepared an EOQ model considering environmental aspects with demand uncertainty. Andriolo et al. (2013) discussed a “Sustainable Inventory Management Framework” that identifies associated sub-problems, decision variables, and the sources of sustainable achievement. They also explained that material transportation and waste have a major role in environmental sustainability. Jawad et al (2014) proposed a new sustainable EOQ model with an extended energy analysis (EEA) approach that considers capital, environment, and labor as the factors of sustainability. Gurtu et al. (2014) studied the effect of emissions and changes in fuel price in a two-echelon supply chain, and they analyzed how these costs can affect inventory policies of the chain members.

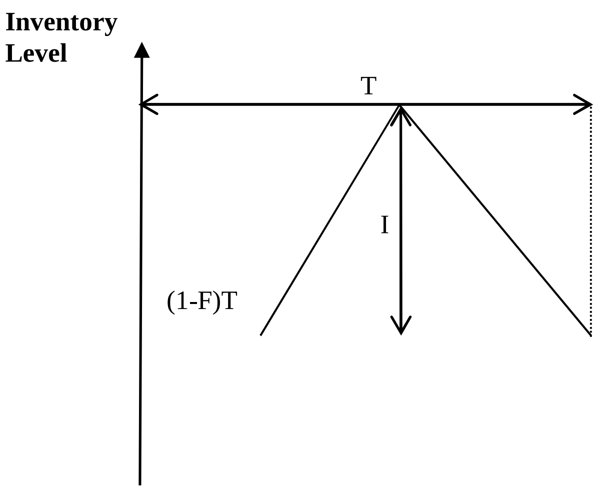
In the recent years, Battini et al. (2014) proposed a new model with the direct accounting approach to calculate a sustainable economic order quantity (called S-EOQ) which considered on one hand, the ordering, holding of inventory, and obsolescence costs, and on the other hand, emissions of obsolescence, transportation, and holding of inventory costs. Digiesi et al. (2015) examined sustainability issues and effects in spare parts logistics. They incorporated repair/replacement costs, such as scrapping cost, in their sustainable EOQ model. Hovelaque and Bironneau (2015) proposed a new sustainable EOQ model with a variable demand that depends upon the price of the product. Andriolo et al. (2014) prepared a comprehensive survey on EOQ literature and predicted that future important challenges in lot sizing problems are expected for both sustainable inventory and manufacturing models. Hammami et al. (2015) integrated carbon emissions to other production-inventory costs in a multi-echelon system with fixed due dates. Their model considers production, transportation, and holding of inventory emission costs with carbon taxes and direct cap approaches. Massaro et al. (2015) prepared a cost-benefit evaluation model which considers economic, environmental, and social aspects using a life cycle analysis (LCA) approach. Kazemi et al. (2016) developed an EOQ model for a retailer considering environmental issues and imperfect quality of products. Scheel (2016) proposed a framework called Sustainable Wealth based on Innovation and Technology (SWIT) that explores sustainable value sharing in a community. He argues that it is possible to move beyond the sustainability mandate and create sustainable wealth.

An important issue that is neglected in previous studies of sustainable inventory models is shortage. Depending on the shortage choices of a particular company – whether it is allowed or not allowed – it can be determined which sustainable inventory model should be applied. No previous studies on sustainable inventory models has considered the inventory shortage issue. To bridge the major research gap in this area, in the current paper we concentrate on developing sustainable EPQ models with different shortage choices. Considering shortage issues in a sustainable EPQ problem makes the model more realistic and more applicable to real world conditions. Any company that faces different shortage situations must have a suitable plan in place. Consequences accompany lost sales, full backordering or partial backordering, for companies with both sustainable or non-sustainable inventory systems. In the next section, to cover this research gap, we describe the problem and explain the motivation of this study.

3. Motivation of study and Problem Description

As reviewed above, no considerable work has been accomplished on Sustainable Economic Production Quantity (SEPQ) models, especially within the context of shortage. Gunasekaran et al.'s (2014) research shows that a few previous studies, which integrate both economic and environmental aspects with a direct accounting approach, have translated these concerns to a tangible indicator such as profitability. In this paper, we apply a direct accounting approach that translates environmental issues of an inventory system into economic parameters which can be used in developing new sustainable EPQ models. We consider ordering, holding of inventory, obsolescence, emission of inventory obsolescence, and holding costs, just as in the "sustainable EOQ model" presented by Battini et al. (2014). However, we have considered "emission of production" instead of their "emission of transportation" approach, and we have added new cost functions such as lost sales and backordering costs to the models. In this paper, we propose four new models based on different shortage situations. These models are *Basic SEPQ*, *SEPQ with lost sales*, *SEPQ with full backordering*, and finally *SEPQ with partial backordering*. We used the approaches of San José et al. (2009) and Pentico et al. (2009) for modeling the SEPQ problem with partial backordering.

Figure (1) shows the graph of shortage and backordering in an EPQ problem with "first in/first out" (FIFO) backorder filling; it assumes that at first backorders, and then new orders are addressed.



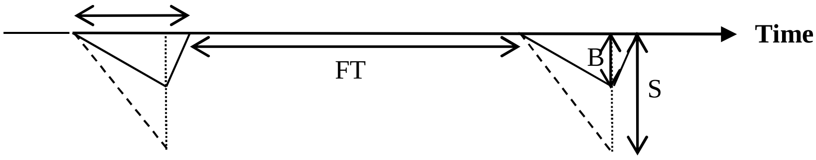


Figure 1. Graph of Backorder Case for EPQ with FIFO backorder filling (*D W Pentico, M J Drake, C Toews. The deterministic EPQ with partial backordering: A new approach, Omega, 2009, 37: 624-636.*)

4. Formulation of the sustainable EPQ models

For formulating our new models, the following notations are used.

Parameters

D: Annual demand rate (unit/year)
P: Maximum annual rate of production (unit/year)
s: Price of a product unit ($/unit)
s': Scrap price per unit ($/unit)
C_p: Unit production cost ($/unit)
C_s: Setup cost ($/setup)
C_i: Cost of holding a unit of inventory in a time unit ($/unit)
C_b: Backordering cost of a product unit in a time unit ($/unit)
C_g: Goodwill loss of an unsatisfied demand ($/unit)
C_l: Lost sale cost per unit ($C_l = (s - C_p) + C_g$) ($/unit)
β: Backordered portion of stock-outs (percent)
α: Obsolescence rate of inventory (percent)
b: Required space for each unit of product (cubic meters per unit)
a: The weight of an obsolete inventory (ton per unit)
C_{ei}: The average emission cost of carbon for inventory holding ($/m^3)
C_{eo}: Average disposal, waste collection, and emission cost of carbon for inventory obsolescence ($/ton)
C_{ep}: The emission cost of carbon for manufacturing each unit ($/unit)

Decision Variables

T: The inventory cycle or time between two consecutive orders (time)
F: The fraction of period length with positive inventory level (percent)

Dependent Variables

Q: Production quantity (unit/year)
I: The highest quantity of inventory (unit/year)
$\bar{I}$: The annual average level of inventory (unit/year)
S: The highest quantity of shortage (unit/year)
B: The highest quantity of backordered (unit/year)
$\bar{B}$ The annual average quantity of backordered ($B = \beta S$) (unit/year)
$\Pi(T,F)$: Total profit function (denoted by $\Pi_{SEPQ-Basic}(T)$ for the basic SEPQ model, $\Pi_{\mathrm{B}}(T,F)$ for the SEPQ model with full backordering, $\Pi_{\mathrm{LS}}(T,F)$ for the lost sale SEPQ model, and by $\Pi_{\mathrm{PBO}}(T,F)$ for the SEPQ-PBO model) ($/year)

TS: Function of total sales ($/year)
TC: Function of total cost ($/year)
CF_p: Cost function of production ($/year)
CF_{ep}: Cost function of "emission of production" ($/year)
CF_s: Set up cost function ($/year)
CF_i: Inventory holding cost function ($/year)
CF_{ei}: Cost function of "emission of inventory holding" ($/year)
CF_{obs}: The function of obsolescence cost of inventory ($/year)
CF_{eo}: Cost function of "emission of inventory obsolescence" ($/year)
CF_b Backordering cost function ($/year)
CF_g: Goodwill loss cost function ($/year)

The main assumptions in the models development process are:

(1) Single product: the company produces only one type of product.
(2) Single period: all of the periods are similar and thus, we only need to model the problem in one period to find the optimal values of decision variables.
(3) Single transportation mode: All products transport to the customers by only one type of transportation mode.
(4) Deterministic demand: The demand rate is deterministic.
(5) Finite production rate: The production rate is finite and the total production capacity is given.

4.1. Modeling of the *Basic SEPQ*

At first, for modeling of the basic SEPQ model without shortage, we define a total profit ($\Pi_{SEPQ-Basic}$) function as below:

$$\begin{aligned}\Pi_{SEPQ-Basic}(T) &= TS - CF_P - CF_{ep} - CF_s - CF_i - CF_{ei} - CF_{obs} - CF_{eo} \\ &= sD - C_pD - C_{ep}\,D - \tfrac{C_s}{T} - C_i\bar{I} - C_{ei}b\bar{I} - \alpha(s - s')\bar{I} - \alpha a\bar{I}C_{eo}\end{aligned} \tag{1}$$

Where, from Pentico et al. (2009),

$$\bar{I} = \frac{DT}{2}\left(1 - \frac{D}{P}\right) \tag{2}$$

In this work, we use three environmental parameters to determine cost functions of emission of inventory holding (CF_{ei}), emission of inventory obsolescence (CF_{eo}) and emission of production (CF_{ep}). These three parameters are the average emission cost of carbon for inventory holding (C_{ei}), average disposal, waste collection, and emission cost of carbon for inventory obsolescence (C_{eo}), and the emission cost of carbon for manufacturing each product unit (C_{ep}). The optimal inventory cycle of the basic SEPQ model can be derived while maximizing the following annual profit function:

$$\Pi_{SEPQ-Basic}(T) = sD - C_pD - C_{ep}\,D - \tfrac{C_s}{T} - C_iDT/2\left(1 - \tfrac{D}{P}\right) - C_{ei}bDT/2\left(1 - \tfrac{D}{P}\right) - \tag{3}$$

$$\alpha(s-s')DT/2\left(1-\frac{D}{P}\right)-\alpha a C_{eo}DT/2\left(1-\frac{D}{P}\right)$$

To simplify the notation, we define

$$C_i' = C_i(1-\frac{D}{P}) \tag{4}$$

$$C_{ei}' = C_{ei}(1-\frac{D}{P}) \tag{5}$$

$$s'' = (s-s')(1-\frac{D}{P}) \tag{6}$$

$$C_{eo}' = C_{eo}(1-\frac{D}{P}) \tag{7}$$

So the profit function shown in Equation (3) changes as below:

$$\Pi_{SEPQ-Basic} = sD - C_pD - C_{ep}\,D - \frac{C_s}{T} - \frac{C_i'DT}{2} - \frac{C_{ei}'bDT}{2} - \alpha s''\frac{DT}{2} - \alpha a C_{eo}'\frac{DT}{2} \tag{8}$$

To find $T^*_{SEPQ-Basic}$, we must first prove the concavity of the profit function.

Theorem 1. The profit function shown in Equation (8) is concave.

Proof. Taking the first and the second derivative of $\Pi_{SEPQ-Basic}$ with respect to T yields:

$$\frac{d\Pi}{dT} = \frac{C_s}{T^2} - \frac{D}{2}[C_i' + bC_{ei}' + \alpha s'' + \alpha a C_{eo}'] \tag{9}$$
$$\frac{d^2\Pi}{d^2T} = \frac{-2C_o}{T^3} \leq 0$$

Since the second order derivative is always negative, the profit function is strictly concave.

Since the profit function is concave, setting the first derivative equal to zero gives the optimal value of period length as below.

$$T^*_{SEPQ-Basic} = \sqrt{\frac{2C_s}{D\omega}} \tag{10}$$

where,

$$\omega = C_i' + bC_{ei}' + \alpha s'' + \alpha a C_{eo}' \tag{11}$$

Maximizing profit function, presented in Equation (8), is equivalent to minimizing the following cost function. It should be noted that in the following equation, production and emission cost are not included because both are independent from the period length.

$$TC_{SEPQ-Basic} = \frac{C_s}{T} + C_i'\,DT/2 + C_{ei}'bDT/2 + \alpha s''DT/2 + \frac{\alpha a C_{eo}'DT}{2} \tag{12}$$

Substituting Equation (10) into Equation (12), after some algebra, we have

$$TC_{SEPQ-Basic} = \sqrt{2DC_s(C_i' + bC_{ei}' + \alpha s'' + \alpha a C_{eo}')} = \sqrt{2DC_s\omega} \tag{13}$$

Therefore, the maximum profit is

$$\Pi_{SEPQ-Basic} = \left(s - C_p - C_{ep}\right)D - \sqrt{2DC_s\omega} \tag{14}$$

4.2. Modeling of the SEPQ with lost sales

In this section we analyze the SEPQ model where shortages are fully lost sales. In this condition, the profit function is as below:

$$\begin{aligned}\Pi_{LS}(T,F) &= TS - CF_P - CF_{ep} - CF_s - CF_i - CF_{ei} - CF_{obs} - CF_{eo} - CF_g \\ &= sD - C_pD - C_{ep}D - \frac{C_s}{T} - C_i\bar{I} - C_{ei}b\bar{I} - \alpha(s-s')\bar{I} - \alpha a\bar{I}C_{eo} - C_gD(1-F)\end{aligned} \tag{15}$$

Where, from Pentico et al. (2009),

$$\bar{I} = \frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \tag{16}$$

Substituting $\bar{I}$ into the profit function (Equation (15)) we have:

$$\begin{aligned}\Pi_{LS}(T,F) = {} & sD - C_pD - C_{ep}D - \frac{C_s}{T} - C_i\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) - C_{ei}b\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \\ & - \alpha(s-s')\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) - \alpha a\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right)C_{eo} - C_gD(1-F)\end{aligned} \tag{17}$$

Substituting Equations (4) to (7) and then Equation (11) into Equation (17) yields:

$$\Pi_{LS}(T,F) = \left(s - C_p - C_{ep}\right)DF - \frac{C_s}{T} - \frac{\omega DTF^2}{2} - C_gD(1-F) \tag{18}$$

To find $\Pi_{LS}(T,F)$, we must first prove the concavity of the profit function shown in Equation (18).

Theorem 2. The profit function shown in Equation (18) is concave.

Proof. See Appendix A.

Because of concavity of the profit function shown in Equation (18), to find the optimal policy of this system, taking the partial derivative of profit function with respect to period length gives:

$$\frac{\partial\Pi}{\partial T} = \frac{C_s}{T^2} - \frac{\omega DF^2}{2} \tag{19}$$

Setting this derivative to zero, we have

$$T = \frac{1}{F}\sqrt{\frac{2C_s}{\omega D}} \tag{20}$$

Substituting *T* into the profit function (Equation (18)) we obtain

$$\begin{aligned} \Pi_{LS}(F) &= \Pi_{LS}(T(F), F) = \left(s - C_p - C_{ep}\right)DF - \frac{C_s F}{\sqrt{\frac{2C_s}{\omega D}}} - \frac{\omega DF}{2}\sqrt{\frac{2C_s}{\omega D}} - C_g D(1-F) \\ &= \left[\left(C_l - C_{ep}\right)D - \sqrt{2C_s\omega D}\right]F - C_g D \end{aligned} \tag{21}$$

Now, $\Pi_{LS}(F)$ is a linear function with respect to the variable F. The maximum profit is determined taking into account the slope of the function $\Pi_{LS}(F)$. Thus, we have:

(i) If $\left(C_l - C_{ep}\right)D \geq \sqrt{2C_s\omega D}$, then the maximum profit is obtained when *F**=1. This profit is given by

$$\Pi^* = \Pi_{LS}(F^*) = \left[\left(C_l - C_{ep}\right)D - \sqrt{2C_s\omega D}\right] - C_g D \tag{22}$$

In this case, the optimal inventory cycle is

$$T^* = \sqrt{\frac{2C_s}{\omega D}} \tag{23}$$

(ii) If $\left(C_l - C_{ep}\right)D < \sqrt{2C_s\omega D}$, then the maximum profit is obtained when *F**=0 and the optimal inventory cycle is T* = ∞. It means no inventories are carried and there are always lost sales.

4.3. Modeling of the *SEPQ* with full backordering

In this case, the shown profit function in Equation (1) changes to the following equation.

$$\Pi_B(T, F) = TS - CF_P - CF_{ep} - CF_s - CF_i - CF_{ei} - CF_{obs} - CF_{eo} - CF_b \tag{25}$$

Calculating the costs included in the profit function (25), we have that $\Pi_B(T, F)$ is a function of *T* and *F*. Thus, we have:

$$\Pi_B(T, F) = sD - C_pD - C_{ep}\,D - \frac{C_s}{T} - C_i\bar{I} - C_{ei}b\bar{I} - \alpha(s - s')\bar{I} - \alpha a\bar{I}C_{eo} - C_b\bar{B} \tag{26}$$

Where, from Pentico et al. (2009),

$$\bar{I} = \frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \tag{27}$$

$$\bar{B} = \frac{DT(1-F)^2}{2}(1 - D/P) \tag{28}$$

Substituting Equations (27) and (28) into Equation (26), Equation (29) can be written as below:

$$\Pi_B(T,F) = \left(s - C_p - C_{ep}\right)D - \frac{C_s}{T} - \frac{C_i DTF^2}{2}\left(1-\frac{D}{P}\right) - \frac{C_{ei} bDTF^2}{2}\left(1-\frac{D}{P}\right) - \alpha(s - s')\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - \alpha a C_{eo}\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - \frac{C_b DT(1-F)^2}{2}\left(1-\frac{D}{P}\right) \tag{29}$$

According to Equation (11) we finally have:

$$\Pi_B(T,F) = \left(s - C_p - C_{ep}\right)D - \frac{C_s}{T} - \frac{\omega DTF^2}{2} - \frac{C_b DT(1-F)^2}{2}\left(1-\frac{D}{P}\right) \tag{30}$$

Maximizing the objective function presented in Equation (30) is similar to minimizing the following function.

$$\pi_1(T,F) = \frac{\lambda_1}{T} + \left(\lambda_2 F^2 - 2\lambda_3 F + \lambda_3\right)T \tag{31}$$

Where the new parameters λ_1, λ_2, and λ_3 are:

$$\lambda_1 = C_o > 0 \tag{32}$$

$$\lambda_2 = \frac{\omega D}{2} + \left(1-\frac{D}{P}\right)C_b D/2 > 0 \tag{33}$$

$$\lambda_3 = \left(1-\frac{D}{P}\right)C_b D/2 > 0 \tag{34}$$

Theorem 3. The function of cost shown in Equation (31) is convex.

Proof. The function of cost shown in Equation (31) is exactly as same as the functions proposed by Taleizadeh (2014a, 2014b) and, with some changes in the notations, the convexity is not affected. So, according to these works, one can easily prove that the objective function shown in Equation (31) is convex.

After convexity proof of the cost function, setting the first partial derivatives of $\pi_1(T,F)$ with respect to F and T equal to zero, we obtain the optimal values of decision variables. So we have:

$$\frac{\partial \pi_1(T,F)}{F} = \left(2\lambda_2 F - 2\lambda_3\right)T = 0 \tag{35}$$

Then

$$F = \frac{\lambda_3}{\lambda_2} = \frac{\left(1-\frac{D}{P}\right)C_b}{\omega + \left(1-\frac{D}{P}\right)C_b} \tag{36}$$

Also we have:

$$\frac{\partial \pi_1(T,F)}{T} = -\frac{\lambda_1}{T^2} + \left(\lambda_2 F^2 - 2\lambda_3 F + \lambda_3\right) \tag{37}$$

So the optimum length of period is:

$$T = \sqrt{\frac{\lambda_1}{\lambda_2 F^2 - 2\lambda_3 F + \lambda_3}} = \sqrt{\frac{C_s}{\left(\frac{\omega D}{2} + \frac{\left(1-\frac{D}{P}\right) C_b D}{2}\right) F^2 - \left(1-\frac{D}{P}\right) C_b D F + \left(1-\frac{D}{P}\right) C_b D/2}} \tag{38}$$

Substituting Equation (36) to this relation, we get, after some algebra:

$$T = \sqrt{\frac{2C_s(\omega + C_b\left(1-\frac{D}{P}\right))}{\omega\left(1-\frac{D}{P}\right) C_b D}} \tag{39}$$

Finally, the maximum profit $\Pi_B(T,F)$ is calculated by substituting Equations (36) and (39) in Equation (30).

4.4. Modeling of the *SEPQ* with partial backlogging

The function of profit of this case is as below.

$$\begin{aligned} \Pi_{PBO}(T,F) &= TS - CF_P - CF_{ep} - CF_s - CF_i - CF_{ei} - CF_{obs} - CF_{eo} - CF_b - CF_g \\ &= sD[(1-F)\beta + F] - DC_p[(1-F)\beta + F] - C_{ep}\, D[(1-F)\beta + F] - \frac{C_s}{T} \\ &- C_i\bar{I} - C_{ei} b\bar{I} - \alpha(s - s')\bar{I} - \alpha a\bar{I}C_{eo} - C_b\bar{B} - C_g D(1-\beta)(1-F) \end{aligned} \tag{40}$$

Where, from Pentico et al. (2009),

$$\bar{I} = \frac{DTF^2}{2}\left(1-\frac{D}{P}\right) \tag{41}$$

$$\bar{B} = \frac{\beta DT(1-F)^2}{2}(1-\beta D/P) \tag{42}$$

Substituting Equations (41) and (42) into Equation (40) yields:

$$\begin{aligned} \Pi_{PBO}(T,F) &= D\left(s - C_p - C_{ep}\right)[(1-F)\beta + F] - \frac{C_s}{T} - \frac{C_i DTF^2}{2}\left(1-\frac{D}{P}\right) - \\ &\frac{C_{ei} bDTF^2}{2}\left(1-\frac{D}{P}\right) - \alpha(s-s')\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - \alpha a C_{eo}\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - \\ &\frac{C_b \beta DT(1-F)^2}{2}\left(1-\frac{\beta D}{P}\right) - C_g D(1-\beta)(1-F) \end{aligned} \tag{43}$$

For more simplification, let:

$$C_b' = C_b(1-\frac{\beta D}{P}) \tag{44}$$

Substituting Equation (44) and Equations (4) to (7) into Equation (43) we get:

$$\Pi_{PBO}(T,F) = \left(s - C_p - C_{ep}\right)D[F + \beta(1-F)] - \frac{C_s}{T} - \frac{(C_i' + C_{ei}'b + \alpha s'' + \alpha a C_{eo}')}{2}DTF^2 - \frac{C_b'\beta DT(1-F)^2}{2} - C_g D(1-\beta)(1-F) \tag{45}$$

Then, according to Equation (11), we have:

$$\Pi_{PBO}(T,F) = \left(s - C_p - C_{ep}\right)D[F + \beta(1-F)] - \frac{C_s}{T} - \frac{\omega DTF^2}{2} - \frac{C_b'\beta DT(1-F)^2}{2} - C_g D(1-\beta)(1-F) \tag{46}$$

In the next step, we must find the optimal values for T and F by maximizing function $\Pi_{\mathrm{PBO}}(T,F)$. For determining the optimum values of decision variables, first we should prove the concavity of the profit function.

Theorem 4. The profit function shown in Equation (46) is concave.

Proof. The profit function shown in Equation (46) is exactly as same as the function proposed by Pentico et al. (2009); some changes in the notations do not affect the convexity. Thus, according to this work, one can easily prove that the objective function shown in Equation (46) is concave.

Because of the concavity of Equation (46), its partial derivatives with respect to the decision variables can be derived and used to determine the optimal values. So we have:

$$\frac{\partial \Pi}{\partial F} = D\left(s - C_p + C_g - C_{ep}\right)(1-\beta) - \omega DTF + D\beta C_b'T(1-F) \tag{47}$$

Also, we know:

$$C_l = \left(s - C_p\right) + C_g \tag{48}$$

Finally, setting Equation (47) equal to zero yields:

$$\underset{47,48}{\Longrightarrow} F = \frac{(1-\beta)\left(C_l - C_{ep}\right) + \beta C_b'T}{T(\omega + \beta C_b')} \tag{49}$$

Also we have:

$$\frac{\partial \Pi}{\partial T} = \frac{C_s}{T^2} - \frac{\omega DF^2}{2} - \frac{C_b'\beta D(1-F)^2}{2} = 0 \tag{50}$$

Equation (50) is reduced to:

$$\frac{2C_s}{T^2} = DF^2\omega + C_b'\beta D(1-F)^2 = 0 \tag{51}$$

Finally, we have:

$$T = \sqrt{\frac{2C_s}{\omega D F^2 + C_b' \beta D (1-F)^2}} \tag{52}$$

Substituting Equation (49) into Equation (52), we obtain, after some algebra:

$$T^* = \sqrt{\frac{2C_s(\omega + C_b'\beta)}{C_b'\beta\omega D} - \frac{(1-\beta)^2\left(C_l - C_{ep}\right)^2}{C_b'\omega\beta}} \tag{53}$$

F^* value could be calculated by substituting Equation (53) into Equation (49). The maximum profit $\Pi_{PBO}(T, F)$ can be determined by substituting results of Equations (49) and (53) in Equation (46).

4.5. A solution algorithm for optimality, partial backordering case

The profit function shown in Equation (31) is exactly the same as the profit function of San José et al. (2009) with some changes in coefficients of decision variables. They developed a procedure to obtain the optimal inventory policy which we adapted for our partial backordering model (proposed in section 3.4). Thus, based on San José et al. (2009), the following solution algorithm can be used to determine the independent and dependent decision variables:

Step 1. Calculate the values $\Delta = (1-\beta)^2(C_l - C_{ep})^2 D^2 - 2\omega CsD$ and $\xi = C_b'\beta$.

- If $\Delta > 0$, the optimal policy is $F^*=1$, $T^* = \sqrt{\frac{2C_s}{\omega D}}$ and the maximum profit is $\Pi^* = \Pi_{PBO}(T^*, F^*) = \left(s - C_p - C_{ep}\right)D - \sqrt{2C_s\omega D}$. Go to Step 4.

- If $\Delta = 0$, go to Step 2.

- If $\Delta < 0$, go to Step 3.

Step 2.

- If $\xi > 0$, the optimal policy is $F^*=1$, $T^* = \sqrt{\frac{2C_s}{\omega D}}$ and the maximum profit is $\Pi^* = \Pi_{PBO}(T^*, F^*) = \left(s - C_p - C_{ep}\right)D - \sqrt{2C_s\omega D}$. Go to Step 4.

- If $\xi = 0$, then $\beta = 0$ and the maximum profit is achieved at any point of the inventory cycle, with value $\Pi^* = \Pi_{PBO}(T^*, F^*) = -C_g D$. Go to Step 4.

Step 3.

- If $\xi > 0$, the optimal policy (T^*, F^*) is obtained from Equations (49) and (53) and the maximum profit $\Pi^* = \Pi_{PBO}(T^*,F^*)$ is calculated from Equation (46). Go to Step 4.

- If $\xi = 0$, then the optimal policy is $F^* = 0$ and $T^* = \infty$ and $\Pi^* = -C_g D$. Note that in this case, no inventory is carried and there are always lost sales. Go to Step 4.

Step 4.

Finally, determine total demand per cycle and maximum inventory level using DT^* and $I^* = F^*DT^*(1\text{-}D/P)$, respectively. Moreover, the maximum levels of stock-out and backordered can be determined using $S^* = (1 - F^*)DT^*(1\text{-}\beta D/P)$ and $B^* = \beta S^*$, respectively. Finally, determine the production quantity using $Q^* = DT^*[(1\text{-} F^*)\beta + F^*]$.

5. Numerical examples

For better illustration of applying the optimal policies for the SEPQ inventory models (with shortage) and to see how the solution procedure proposed for this system works, we will present some numerical examples. The objective of this section is to show how we can solve various SEPQ problems (with shortage) by models that are presented in this paper. To design and define these numerical examples more accurately and to ensure that the example demonstrates applicability, we will pursue an Iranian petrochemical company as a case study. The production and inventory systems of this company are similar to our developed models that have considered different shortage situations. However, in this section the numerical examples do not utilize real input data because of security purposes and the lack of appropriate data for some parameters. The lack of appropriate data is a limitation of our study that we will reveal later in the conclusion section. Each of these examples can help readers to understand how to select and how to use any of our proposed models.

Example 1. A production system with these parameters is given:

Demand and production rates are 40 units/year and 100 units/year, and the price of new and scrapped products are 10 \$/unit and 5 \$/unit, respectively. Production cost of a product unit and obsolescence rate are 7 \$/unit and α = 10%. Fixed setup cost, inventory holding, backlogging, and goodwill costs are respectively 20 \$/order, 2.5 \$/unit, 3 \$/unit, 1 \$/unit. The required space for each item and weight of each item are 1.7 m^3/unit and 2 ton/unit. The average emission cost of carbon for hold inventory, average disposal, and waste collection, for inventory obsolescence, and for manufacturing each unit and partial backordering rate are 0.55 \$/m^3, 13 \$/ton, and 0.3 \$/unit, 0.45 respectively. So $C_l = (s - C_p) + Cg = 4$ \$/unit. First, we must calculate values of C'_i, C'_{ei}, s'' and C'_{eo} from Equations (4) to (7) as below:

$C'_i = 2.5(1\text{-}40/100)=1.5$

$C'_{ei} = 0.55(1\text{-}40/100)=0.33$

$C'_{eo} = 13(1\text{-}40/100)=7.8$

$s''= (10\text{-}5)(1\text{-}40/100)=3$

Then, from Equation (11) we get $\omega = 1.5 + 1.7 * 0.33 + 0.1 (3) + 0.1 * 2 * 7.8 = 3.921$ \$/unit and from Equation (44) we get $C'_b = 3(1\text{-}0.45{*}40/100)=2.46$ \$/unit. Next, we apply our proposed solution algorithm for determining the optimum solution.

In the first step, we calculate the values of the parameters Δ and ξ, which are:

Δ = (1-β)^2($C_l - C_{ep}$)^2D^2 - 2ωCsD = (0.55)^2(3.7)^2(40)^2-2*3.921*20*40 = 6625.96-6273.6 = 352.36 and $\xi = C_b'\beta$ = 2.46*0.45 = 1.107. As Δ > 0, then the optimum solution (T*, F*) is F*=1 and $T^* = \sqrt{\frac{2C_o}{\omega D}} = \sqrt{\frac{2*20}{3.921*40}} = 0.50$. The maximum profit is $\Pi^* = \Pi_{PBO}(T^*, F^*) = (s - C_p - C_{ep})D - \sqrt{2C_s\omega D} = (2.7) * 40 - \sqrt{2 * 20 * 3.921 * 40} = 108 - 79.206 = 28.794\ \$/year$.

Next, the values of the dependent variables are:

- *DT^*=20.2 units,*
- *$I^* = F^*DT^*(1\text{-}D/P)$=12.12 units,*
- *$(1 - F^*)DT^*(1\text{-}\beta D/P)$ =0,*
- *$B^* = \beta S^*$=0,*
- *$Q^* = [F^* + \beta(1\text{-} F^*)]DT^*$=20.2 units*

Example 2. Now, we assume that all parameters values are similar to *Example 1* parameters but we modify the parameter β and choose a new β = 0.50. Applying the procedure again, first from Equation (34) we get C_b'= *3(1-0. 5*40/100)=2.4* \$/unit.

Step 1. We obtain the values Δ = (1-β)^2($C_l - C_{ep}$)^2D^2 - 2ωCsD = (0.5)^2(3.7)^2(40)^2-2*3.921*20*40 = 5476-6273.6 = - 797.6 and $\xi = C_b'\beta$ = 2.4*0.5 = 1.2. As Δ<0, we go to Step 2.

Step 2. As ξ>0, the optimal policy (*T**, *F**) is obtained from Equations (49) and (53), and the maximum profit $\Pi^* = \Pi_{PBO}$ (*T**, *F**) is calculated from Equation (46). Thus, we have

$$T^* = \sqrt{\frac{2 * 20(3.921 + 2.4 * 0.5)}{2.4 * 0.5 * 40 * 3.921} - \frac{(1 - 0.5)^2(4 - 0.3)^2}{2.4 * 0.5 * 3.921}} = 0.601$$

and

$$F^* = \frac{(1 - 0.5)(4 - 0.3) + 0.5 * 2.4 * 0.601}{0.601(3.921 + 0.5 * 2.4)} = 0.836$$

In addition, from Equation (46) we get the total profit $\Pi_{SEPQ-PBO}$ = \$ 29.259 per year.
Finally, we have:

- Demand of a cycle = 0.601*40 = 24.33 units,
- I* = 40*0.836*0.601*(1-40/100) = 12.049 units,
- S* = (1 – 0.836)*40*0.601*(1- 0.5*40/100) = 3.161 units,
- B* = 0.5*3.161 = 1.580 units,
- Q* = 0.601*40(0.836+0.164*0.5) = 22.057 units.

Example 3. We assume that all parameters values in this example are similar to *Example 1* parameters, but now β = 1. In this case, we are in the full backordering SEPQ model. The optimal policy is given by Equations (36) and (39). Thus, we have

$$F^* = \frac{\left(1-\frac{D}{P}\right)C_b}{\omega + \left(1-\frac{D}{P}\right)C_b} = \frac{0.6 * 3}{3.921 + 0.6 * 3} = 0.315$$

and

$$T^* = \sqrt{\frac{2C_s(\omega + C_b\left(1-\frac{D}{P}\right))}{\omega\left(1-\frac{D}{P}\right)C_b D}} = \sqrt{\frac{2 * 20(3.921 + 3 * 0.6)}{3.921 * 0.6 * 3 * 40}} = \sqrt{\frac{40 * 5.721}{40 * 7.058}} = 0.9\ year.$$

From Equation (30), the maximum profit is $\Pi^*_B = 63.572\ \$/year$.

Example 4. Finally, we assume that all parameters values are similar to *Example 1*, but suppose that β = 0. In this case, we are in the lost sale SEPQ model. The optimal policy depends on the values $(C_l - C_{ep})D = (3.7) * 40 = 148$, and $\sqrt{2C_s\omega D} = \sqrt{2(20)(3.921)40} = 79.206$. As $(C_l - C_{ep})D > \sqrt{2C_s\omega D}$, then the optimal policy is given by $F^* = 1$, and $T^* = \sqrt{\frac{2C_s}{\omega D}} = \sqrt{\frac{2*20}{3.921*40}} = 0.505$. Also, from Equation (22), the maximum profit is

$$\Pi^*_{LS} = \left[(C_l - C_{ep})D - \sqrt{2C_s\omega D}\right] - C_g D = 148 - 79.206 - 40 = 28.794\ \$/year.$$

6. Results

Table 1 briefly shows the results of these examples. In this research, Total profit function is the target function of optimizing all four proposed models. For this reason we concentrate on analyzing the total profit amount of each model in the various situations that were stated in our examples.

Table 1. Summary of examples results

Example	***Related Model***	***β***	***T****	***F****	***Total Profit ($/year)***
1	*SEPQ-PBO* (finally used "*Basic SEPQ* model")	0.45	0.5	1	28.794
2	*SEPQ-PBO*	0.5	0.601	0.836	29.259
3	*SEPQ-full backordering*	1	0.315	0.9	63.572
4	*SEPQ-lost sale*	0	0.505	1	28.794

The optimum values of β, T^* and F^* in models that presented in these examples are shown in Figure 2 and the total profit of different models are compared with each other in Figure 3.

As mentioned before, the maximum profit for the SEPQ-Basic model is

$$\Pi_{SEOQ-Basic} = (s - C_p - C_{ep})D - \sqrt{2DC_s\omega} = (10 - 7 - 0.3) * 40 - \sqrt{2 * 40 * 20 * 3.921} = 108 - 79.206 = \$\, 28.794\ per\ year.$$

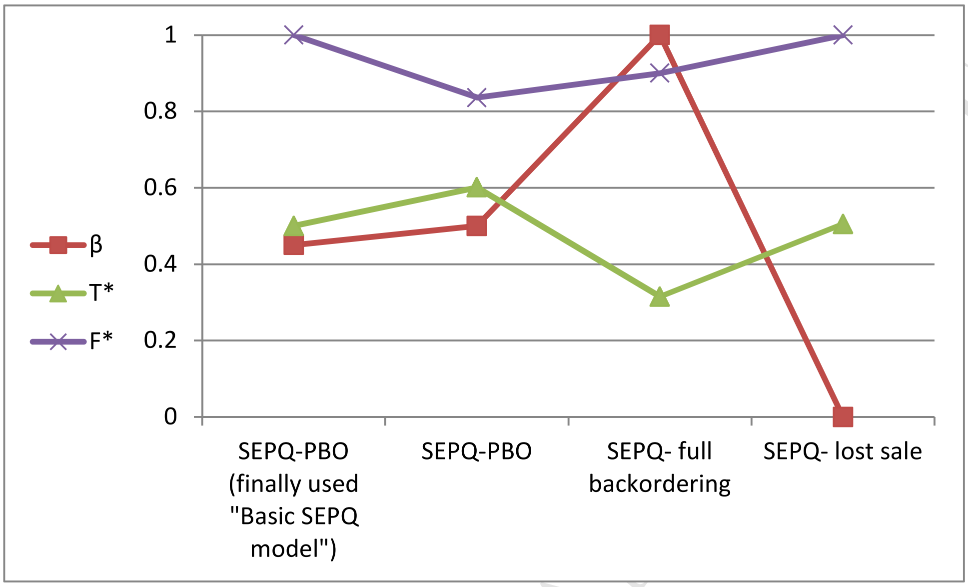


Figure 2. Optimum values of β, T*and F* in different models

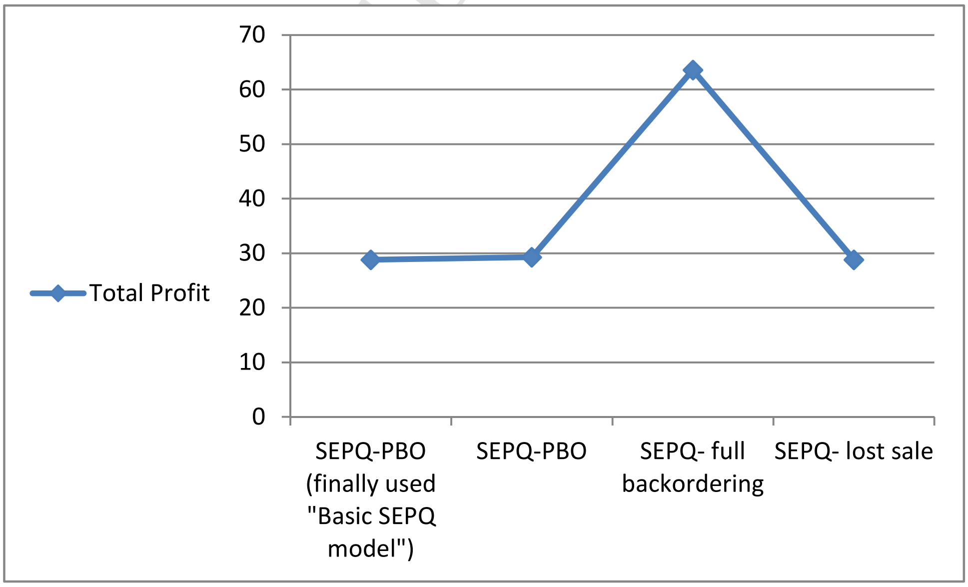


Figure 3. Total profit of different models

7. Discussions

In this section, we discuss the results obtained from numerical examples. As can be seen in Example 1, if β = 0.45, the optimal policy coincides for both *Basic SEPQ* and *SEPQ-PBO* models, and the total profit (Π*) is the same (28.794 $/*year*). However, as shown in Example 2, if β = 0.50, the *SEPQ-PBO* model has a better result, because its profit $\Pi^* = \Pi_{PBO}(T^*,F^*)$ = 29.259 $/*year* is higher than one of the Basic SEPQ models. Also, as can be seen in Example 4, the total profit of the *SEPQ-lost sales* model and the *Basic SEPQ* model is equal (both are 28.794 $/*year*). But the total profit of the *SEPQ-lost sales* model is lower than the *SEPQ-PBO* model with 29.259 $ annual profit, because with β = 0, there will be no backorders and all of the orders will be lost.

In Example 3, as β = 1, all orders will be saved and backordered. For this reason we used the *SEPQ-full backordering* model to determine decision variables (*T* and *F*). Finally, as Figure 3 shows, the amount of this model's total profit is calculated as 63.572 $/year. It is obvious that when we can save all orders faced with shortage (in *SEPQ-full backordering* model), we can obtain more profit than the partial backordering case (with 29.259 $/*year* total profit). But in the real world, because all companies participate in a competitive context (many customers are not loyal enough to restrict their business to only one company), the *SEPQ-partial backordering* model (*SEPQ-PBO*) is a realistic model that considers both economic and environmental aspects.

As Figure 3 shows, the total profit amount of *SEPQ-full backordering* model is more than the *SEPQ-PBO* model and, in the next level, the *Basic SEPQ* and *SEPQ-lost sale* models. In reality, full backordering is not possible in many conditions. Thus, the *SEPQ-PBO* model, based on the premise that "only a portion of orders will be backlogged," may be used when backordering is possible to gain more profit. Figure 2 shows that the inventory cycle (T) value in *SEPQ-PBO* model is greater than other models. Based on an illustration of the inventory level in the partial backordering case in Figure 1 (and illustrated in Figure 2), the time between two consecutive orders (*T*) in the *SEPQ-PBO* model is longer than other models. However, the fraction of period length with positive inventory level (*F*) in *SEPQ-PBO* model is less than other models. In the other words, *SEPQ-PBO* model is a sustainable EPQ model considering shortage issues; it gains reasonable total profit amount with the highest *T* value and the lowest *F* value than any of our proposed models based on the given parameters values of this case.

8. Implications

This research has several practical and managerial implications. Many previous related works focus on sustainable EOQ problem, but in this paper we model a sustainable EPQ problem applicable for researchers and practitioners who work in manufacturing and production contexts to use. Another important feature of our proposed models is sustainability. We consider environmental parameters, such as emission of production, emission of inventory obsolescence, and emission of inventory holding, in our model’s formulation. Actually, a sustainable EPQ model is a more realistic and responsible inventory model than other models that ignore sustainability issues because of its direct or indirect effects on the firm’s longterm profitability. An additional managerial implication of our research is adding inventory shortage possibility as a real and practical dimension to the sustainable EPQ problem. In the real world, any firm can face four different situations in relation with inventory shortage issue: no shortage (basic model), lost sale, full backordering, and partial backordering. In this paper, we cover all possible shortage situations by developing four different SEPQ models. These models can be useful for operations managers who are interested in determining levels of suitable economic production quantity with regard to different shortage situations. In other words, our developed models may be applicable for operations managers and researchers who are interested in modeling and solving EPQ problem considering *sustainability* and inventory *shortage* issues as two of main dimensions that can be noted in inventory models development based on real world conditions.

9. Conclusions

In this paper, we have developed four sustainable EPQ models that consider different inventory shortage situations in a production system. Our proposed models are the *basic SEPQ* model, the *lost sale SEPQ,* the *SEPQ with full backordering,* and the *SEPQ with partial backordering*. The direct accounting approach is applied, so sustainability issues are included by considering inventory emissions costs such as cost of inventory obsolescence emission, cost of inventory holding emission, and cost of production emission, in addition to more common costs of inventory systems under the partial backordering case.

These new models may be useful for companies seeking environmentally conscious production systems because of their applicable and straightforward computational procedures. Our proposed sustainable EPQ (SEPQ) models cover all of main shortage situations with regard to both economic and environmental considerations. These four models are tested, explained, and compared with four examples. We demonstrate the SEPQ-partial backordering model has a good generality with reasonable profit amount with the highest T value and the lowest F value compared with the three other proposed models. The main limitation of our study is sustainable cost estimation. In this paper, because we employ the direct accounting approach, a cost estimation of environmental parameters of sustainability is a critical task to run the proposed models. Unfortunately, one of the major barriers of this research expansion in many countries and companies is the lack of appropriate data with which to determine environmental parameters (such as $C_{ei},$ C_{eo} and C_{ep}). However, other practitioners or researchers may provide these parameters. Also, these models can be improved in several ways. Other sustainability approaches, such as Cap & Trade or Carbon Offsets, can be used in the model. In this work we develop four models in a deterministic environment, but with further research, these models may be extended by considering stochastic demand. Moreover, many classical inventory control models can be developed by adding sustainability issues and parameters.

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Appendix A. Proofing the profit function of lost sale case

According to Equation (18) we know:

$$\Pi_{LS}(T,F) = \left(s - C_p - C_{ep}\right)DF - \frac{C_s}{T} - \frac{\omega DTF^2}{2} - C_g D(1-F)$$

So,

$$\frac{\partial \pi_{LS}}{\partial T} = \frac{C_s}{T^2} - \frac{\omega DF^2}{2}\ ,\ \frac{\partial^2 \pi_{LS}}{\partial^2 T} = -\frac{2C_s}{T^3}$$

$$\frac{\partial \pi_{LS}}{\partial F} = (s - C_p - C_{ep})D - \omega DFT + C_g D$$

$$\frac{\partial^2 \pi_{LS}}{\partial^2 F} = -\omega DT$$

$$\frac{\partial^2 \pi_{LS}}{\partial T \partial F} = \frac{\partial^2 \pi_{LS}}{\partial F \partial T} = -\omega DF$$

In order to show the concavity of the proposed profit function we should show that $[T, F].H.\begin{bmatrix} T \\ F \end{bmatrix} \leq 0$ where, $H = \begin{bmatrix} \frac{\partial^2 \pi_{LS}}{\partial^2 T} & \frac{\partial^2 \pi_{LS}}{\partial T \partial F} \\ \frac{\partial^2 \pi_{LS}}{\partial F \partial T} & \frac{\partial^2 \pi_{LS}}{\partial^2 F} \end{bmatrix}$. So we have:

$$[T, F]\begin{bmatrix} -\frac{2C_s}{T^3} & -\omega DF \\ -\omega DF & -\omega DT \end{bmatrix}\begin{bmatrix} T \\ F \end{bmatrix} = \begin{bmatrix} -\frac{2C_s}{T^2} - \omega DF^2 & -2\omega DFT \end{bmatrix}\begin{bmatrix} T \\ F \end{bmatrix} = -\frac{2C_s}{T} - 4\omega DTF^2 \leq 0$$