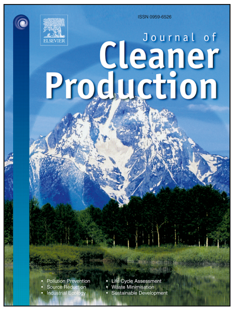
Accepted Manuscript



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PII: S0959-6526(17)33078-0

DOI: 10.1016/j.jclepro.2017.12.119

Reference: JCLP 11514

To appear in: *Journal of Cleaner Production*

Received Date: 11 May 2017

Revised Date: 13 December 2017

Accepted Date: 13 December 2017

Please cite this article as: Pomponi F, Lenzen M, Hybrid life cycle assessment (LCA) will likely yield more accurate results than process-based LCA, *Journal of Cleaner Production* (2018), doi: 10.1016/j.jclepro.2017.12.119.

Hybrid life cycle assessment (LCA) will likely yield more accurate results than process-based LCA

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Abstract

We analyse the comparative investigation into truncation vs. aggregation errors of process-based and hybrid LCA by Yang et al. 2017. We analyse the validity of their findings when the hypothetical five-sector economy is altered to account for realistic technological and sectoral interdependencies. We show that in such cases the truncation error of process-based LCA outweighs the aggregation error of hybrid LCA. To this end, we compare the dominant eigenvalue of our alternative economy with that of real economies, showing good agreement. The same validity check does not hold for the system used by Yang and colleagues. Additionally, we demonstrate that even simple process systems can have higher dominant eigenvalues, provided they are based on realistic data.

Keywords: hybrid; life cycle assessment; input-output; process; aggregation; truncation error; eigenvalues.

1. Introduction

Yang et al. (2017) (YHB in the following) analyse the magnitude of truncation errors in process-based Life Cycle Assessment (LCA) versus that of aggregation errors in hybrid LCA. They conclude that hybrid LCA may not necessarily produce more accurate results than process-based LCA, as aggregation can cause a larger error than truncation. While their proof of concept stands from both a mathematical and a philosophy-of-science perspective, it is worth establishing to what extent it is representative of real systems and economies.

For the remainder of this article we make the following qualification: When we refer to input-output analysis as a tool to complement process-based LCA to form a hybrid LCA approach, we understand its function as a static model, and not as an analysis of economic equilibria. In other words, we understand "accurate" (as for example used in our title) in a static sense, and not in the sense of being capable of capturing a consequential future. After all, capturing the future remains a very challenging—and often unsuccessful—task, as Queen Elizabeth's famous question to academics at the London School of Economics should remind us all (Giles, 2008).

This qualification also relates to an article by Yang (2017a), stating that *"[an economy wide system boundary] assumes all processes would expand (shrink) to some extent as a result of an increase (decrease) in the output of the reference process being studied*". Forecasting future changes in total output as a result of changes in final demand is not an exercise that is intended to be carried out using input-output tables (Gretton, 2013). Leontief's quantity model in input-output analysis usually operates *ex-post*, that is, it describes interdependencies in the past. In addition, input coefficients are average and not marginal coefficients, and as such may not be applied to marginal final demand changes. Outputs may not decrease or increase linearly with inputs, because of slack capacity, availability of stocks, etc. The restrictions that the proportionality assumption imposes on IO tables have been known for a long time (Bullard et al., 1976; Treloar, 1997).

Subject to these qualifications, in this work, we show that in order to construct a case where input-output aggregation errors are larger than process analysis truncation errors, YHB have resorted to quite extreme examples, because they compare a process system with an unusually low truncation error with an input-output system with an unusually high aggregation error. More specifically, we show that:

- YHB assume a weakly coupled process system, i.e. a system with unusually low feedback and little interconnectedness of entities, featuring rapid convergence towards system completeness, and thus leading to a low truncation error;
- YHB's process system does not hold to the scrutiny of generality when compared to comparably simple process systems based on realistic data (for example ecoinvent v3); and
- YHB assume an input-output system with features unlike those of realistic IO systems, i.e. characterised by an aggregation across sectors with substantially different physical attributes, leading to a high aggregation error.

The latter point is particularly important because while it might seem that YHB's article focuses on hybrid LCA and process-based LCA it must be noted that the two are linked precisely by the IO system used. More specifically, YHB start their analysis with a very specific process system that does not reflect well the sectoral interdependencies observed in the real world. This is then used as the input to develop an equally specific IO system that does not resemble a real economy. As such, to prove or disprove the merit of hybrid LCA both an appropriate process system and a realistic IO system must be used.

This article will unfold as follows: In Section 2 we present an example for a strongly coupled process system with higher feedback, and show that truncation errors in this system significantly increase. In Section 3 we present an associated strongly coupled input-output system with higher feedback, and show that aggregation errors do not significantly increase. In Section 4 we show the truncation error in realistic systems. In Section 5 we present an input-output system where aggregation occurs across sectors with less different physical attributes, and show that aggregation errors significantly decrease. In Section 6 we conclude.

2. An alternative, more realistic technology matrix

In a first step, we construct an alternative hypothetical example, and modify YHB's technology matrix $\mathbf{A}_0$ as follows[1]:

$$\mathbf{A}_{0\mathrm{sc}} = \begin{bmatrix} 1 & 0 & -0.5 & 0 & -0.5 \\ 0 & 1 & -0.02 & 0 & 0 \\ -0.4 & -0.2 & 1 & -0.6 & -0.7 \\ -0.25 & -0.02 & -0.04 & 1 & -0.03 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (1)$$

In essence, we introduce additional interdependencies and couplings between the activities. First, we assume that corn and wheat cultivation as well as machinery manufacturing requires twice as much energy (red shadings). Second, we assume that corn cultivation is highly mechanised, and increase machinery use from 0.01 to 0.25 (green). Third, we assume that the energy system is largely run on corn-based biofuels, and set a technology coefficient of 0.5 (yellow). This system involves a wider range of necessary inputs, and represents sufficient information for calculating the cradle-to-gate emissions of popcorn making.

Using the same CO_2 coefficients vector $\mathbf{B}_0 = [2 \quad 7 \quad 1.5 \quad 3 \quad 1]\,\mathrm{kgCO_2}$ and the same functional unit vector $\mathbf{f}_0 = [0\ 0\ 0\ 0\ 1]'$ as YHB, the new cradle-to-gate CO_2 emissions (kg) are $\mathbf{m}_{0\mathrm{sc}} = \mathbf{B}_0 \mathbf{A}_{0\mathrm{sc}}{}^{-1}\mathbf{f}_0 = 8.78$. This value will be used as a benchmark to evaluate the truncation error of process-based LCA and the aggregation error of hybrid LCA as in YHB. If the system is now truncated by the same curtailment as in YHB, that is to[2]

[1] We adopt YHB's notation, and distinguish all variables for the alternative, strongly coupled system that we propose by a "sc" subscript. A table with the correspondence between matrices and vectors as they appear in the original paper and in the present work is given in the appendix.

[2] We adopt YHB's notation, and distinguish all variables for the truncated system by a "1" subscript. A strongly coupled *and* truncated system is denoted by a "1sc" subscript.

$$\mathbf{A}_{1sc} = \begin{bmatrix} 1 & -0.02 & 0 & 0 \\ -0.2 & 1 & -0.6 & -0.7 \\ -0.02 & -0.04 & 1 & -0.03 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

with $\mathbf{B}_1 = [7 \quad 1.5 \quad 3 \quad 1]$ $kgCO_2$ and $\mathbf{f}_1 = [0\ 0\ 0\ 1]'$, the cradle-to-gate CO_2 emissions are $\mathbf{B}_1\mathbf{A}_{1sc}{}^{-1}\mathbf{f}_1 = 2.35$, resulting in a relative truncation error of 70.8% for the process system, which is higher than the relative aggregation errors of YHB's input-output systems.

3. Aggregation error in an input-output system with strong feedback

In the following, we show briefly that for our example in Section 2, aggregation errors are not significantly influenced by variations in feedback. We now need to translate the strongly coupled process system $\mathbf{A}_{0sc}$ into the input-output formulation[3], and then aggregate using YHB's aggregator matrix $\mathbf{j}$. In order to calculate cradle-to-gate CO_2 emissions in the input-output system, variables need to be converted from physical into monetary units (starred variables), with $\mathbf{A}^*_{0sc} = \widehat{\mathbf{p}_0}(\mathbf{I} - \mathbf{A}_{0sc})\widehat{\mathbf{p}_0}^{-1}$ being the input coefficients matrix, $\mathbf{B}^*_0 = \mathbf{B}_0\widehat{\mathbf{p}_0}^{-1}$ the emissions vector, and $\mathbf{f}^*_0 = \widehat{\mathbf{p}_0}\mathbf{f}_0$ the functional unit, with $\mathbf{p}_0$ YHB's vector of unit prices (3).

$$\mathbf{p}_0 = \begin{bmatrix} 2 \\ 1.4 \\ 1 \\ 10 \\ 4 \end{bmatrix} \begin{array}{c} \text{corn (\$/kg)} \\ \text{wheat (\$/kg)} \\ \text{energy (\$/MJ)} \\ \text{machine (\$/unit)} \\ \text{popcorn (\$/kg)} \end{array} \quad (3)$$

Then, cradle-to-gate CO_2 emissions are calculated as $\mathbf{B}^*_0(\mathbf{I} - \mathbf{A}^*_{0sc})^{-1}\mathbf{f}^*_0$.[4] For the disaggregated case, we find $\mathbf{B}^*_0(\mathbf{I} - \mathbf{A}^*_{0sc})^{-1}\mathbf{f}^*_0 = 8.78$, in agreement with the disaggregated process-analytical result $\mathbf{B}_0\mathbf{A}_{0sc}{}^{-1}\mathbf{f}_0$.

In order to aggregate, we first need to calculate the transactions matrix $\mathbf{Z}_{0sc} = \mathbf{A}^*_{0sc}\mathbf{x}_{0sc}$, with $\mathbf{x}_{0sc} = (\mathbf{I} - \mathbf{A}^*_{0sc})^{-1}(\mathbf{I} - \mathbf{A}^*_0)\,\mathbf{x}_0$, and then aggregate as $\mathbf{Z}_{2sc} = \mathbf{j}'\mathbf{Z}_{0sc}\mathbf{j}$, $\mathbf{x}_{2sc} = \mathbf{j}'\mathbf{x}_{0sc}$, $\mathbf{B}_2 = \mathbf{B}_0\widehat{\mathbf{x}_{0sc}}\mathbf{j}\widehat{\mathbf{x}_{2sc}}^{-1}$, and $\mathbf{f}_2 = \mathbf{j}'\mathbf{f}^*_0$. Finally, we can calculate the aggregated coefficients matrix $\mathbf{A}_{2sc} = \mathbf{Z}_{2sc}\widehat{\mathbf{x}_{2sc}}^{-1}$(compare with equations 16-22 in Yang et al. (2017)), and then determine post-aggregation cradle-to-gate CO_2 emissions as $\mathbf{B}_2(\mathbf{I} - \mathbf{A}_{2sc})^{-1}\mathbf{f}_2$. Details for $\mathbf{B}_2$and $\mathbf{A}_{2sc}$ are given in (4) and (5) respectively.

$$\mathbf{B}_2 = [3.33 \quad 1.5 \quad 0.3 \quad 0.25]\ kgCO_2 \quad (4)$$

[3] We adopt YHB's notation, and distinguish all unaggregated input-output variables by a "*" superscript, and all aggregated input-output variables by a "2" subscript. A strongly coupled input-output system is denoted by an additional "sc" subscript.

[4] The differences in notation between the LCA formulation in Heijungs and Suh (2002) and the standard input-output formalism can be confusing, since in LCA, the $\mathbf{A}_0$ matrix is inverted, but in input-output analysis, $\mathbf{I} - \mathbf{A}_0$ is inverted. The same applies to the calculation of eigenvalues.

$$\mathbf{A}_{2\mathrm{sc}} = \begin{bmatrix} 0 & 0.028 & 0 & 0.25 \\ 0.0833 & 0 & 0.03 & 0.175 \\ 0.1042 & 0.4 & 0 & 0.075 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (5)$$

We find $\mathbf{B}_2(\mathbf{I} - \mathbf{A}_{2\mathrm{sc}})^{-1}\mathbf{f}_2 = 15.93$. The relative aggregation error is 70.0%, only slightly above YHB's value, and below the truncation error for the strongly coupled system.

4. Truncation error in realistic systems

The increase of the truncation error can be understood in terms of system convergence. This means that process analysis often omits the pollutant emissions and resource requirements of upstream layers of the supply chain. To show this, we unravel cradle-to-gate CO_2 emissions into an infinite series of production layers (for further reading see Suh and Heijungs, 2007). For example, for the strongly coupled system, this takes the following form:

$$\mathbf{B}_2(\mathbf{I} - \mathbf{A}_{2\mathrm{sc}})^{-1}\mathbf{f}_2 = \mathbf{B}_2\left[\mathbf{I} + \mathbf{A}_{2\mathrm{sc}} + \mathbf{A}_{2\mathrm{sc}}{}^2 + \mathbf{A}_{2\mathrm{sc}}{}^3 + \cdots\right]\mathbf{f}_2 = \mathbf{B}_2 \sum_{n=0}^{\infty} \mathbf{A}_{2\mathrm{sc}}{}^n \mathbf{f}_2, \qquad (6)$$

where $\mathbf{I}$ is an identity matrix with the same dimensions as $\mathbf{A}_{2\mathrm{sc}}$and n is the order of production layer. In equation 3, the term $\mathbf{B}_2\mathbf{I}\mathbf{f}_2 = \mathbf{B}_2\mathbf{f}_2$ covers emissions of popcorn making (direct emissions), $\mathbf{B}_2\mathbf{A}_{2\mathrm{sc}}\mathbf{f}_2$ covers emissions associated with immediate inputs into popcorn making (1st-order emissions), $\mathbf{B}_2\mathbf{A}_{2\mathrm{sc}}{}^2\mathbf{f}_2$ emissions from inputs into immediate inputs into popcorn making (2nd order), and so on. The same is done to YHB's input-output system, which is characterised by the matrix $\mathbf{A}_2$ in the original publication by the authors.

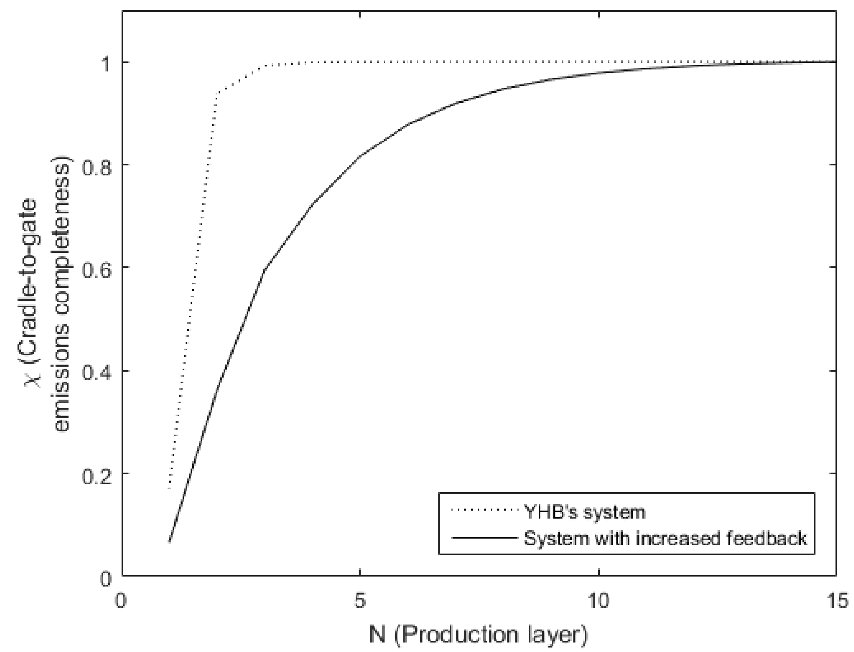

Fig. 1: Cumulative cradle-to-gate CO_2 emissions as a function of upstream production layer coverage.

In Fig. 1, cradle-to-gate emissions completeness is measured as $\chi(N) = \mathbf{B}_2 \sum_{n=0}^{N} {\mathbf{A}_2}^n \mathbf{f}_2 \oslash \mathbf{B}_2(\mathbf{I} - \mathbf{A}_2)^{-1}\mathbf{f}_2$ (dotted curve), and $\chi_{\mathrm{sc}}(N) = \mathbf{B}_2 \sum_{n=0}^{N} {\mathbf{A}_{2\mathrm{sc}}}^n \mathbf{f}_2 \oslash \mathbf{B}_2(\mathbf{I} - \mathbf{A}_{2\mathrm{sc}})^{-1}\mathbf{f}_2$ (solid curve), respectively**.** The '$\oslash$' sign denotes element-wise division.

Indeed, the convergence of the more realistic input-output system $\mathbf{A}_{2\mathrm{sc}}$ towards completeness is much slower than that of YHB's system $\mathbf{A}_2$ (Fig. 1). Hence, any truncation of the system will have a more profound effect on total cradle-to-gate emissions. For example, omitting all 3rd and higher-order production layers will lead to a truncation error of less than 1% in YHB's system, but of about 41% in the process system with increased feedback.

System feedback and convergence of process systems can be characterised by the real part of the dominant eigenvalue $\lambda_{\max}$ of the coefficients matrix $\mathbf{A}$ of an input-output system (the "dominant eigenvalue" or "eigenvalue" hereafter), or for the $(\mathbf{I} - \mathbf{A})$ matrix of a process system (Lenzen, 2000; Lorenzen, 1981; Peters, 2006).[4,5] The higher the dominant eigenvalue, the more feedback is in the system, the slower the system converges toward completeness, and the more vulnerable it is to truncation errors. For YHB's example and the system specified in equation 1, we find $\lambda_{\max}(\mathbf{A}_2) = 0.12$ and $\lambda_{\max}(\mathbf{A}_{2\mathrm{sc}}) = 0.67$, respectively.[4,6]

Eigenvalues larger than 0.5 are very common in modern, interconnected production systems. For example, in a study of the Australian economy between 1975 and 1999, Wood and Lenzen (2009) report dominant eigenvalues of around 0.9. Further examples are provided in Table 1. The technology matrix of the IO system that YHB used in their analysis is interconnected three times more weakly than Rwanda's economy, and as such unlikely to be representative of any real economy.

Tab. 1: Dominant eigenvalues of some input coefficient matrices $\mathbf{A}$ in the case of input-output systems, and of $\mathbf{I} - \mathbf{A}$ in the case of process systems

Country	Year	Dimension	$\lambda_{\max}$	Reference	Comments
Italy	2011	34	0.82	(OECD, 2017)	
Belgium	2010	59	0.79	(Eurostat, 2017)	

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[5] The eigenvalues λ_j of matrices with real entries (such as LCA process matrices or IO input coefficients matrices) may be complex, however these then always appear in complex conjugate pairs. The elements in any vector of powers λ_j^k of eigenvalues also appear in complex conjugate pairs. Given that the system convergence factor $\lim_{k\to\infty} \kappa(k) = \lim_{k\to\infty} \sum_j \xi_j \lambda_j^k / \sum_j \xi_j \lambda_j^{k-1}$ (see Eq. 16 in Lenzen, 2000) involves weighted sums of powers λ_j^k of eigenvalues λ_j , the imaginary parts of the eigenvalues partially cancel out, and the real part dominates in determining system convergence.

[6] Note that the eigenvalues of coefficients matrices such as YHB's $\mathbf{I} - \mathbf{A}_0$, $\mathbf{A}_0^*$ and $\mathbf{A}_2$ (see YHB's equations 12 and 20), as well as our modified versions are invariant to scaling the corresponding flow matrices and total output, that is $\mathbf{Z}_0$ and $\mathbf{x}_0$ as in YHB's equations 10 and 11, and $\mathbf{Z}_2$ and $\mathbf{x}_2$ as in YHB's equations 16 and 19. Note that coefficients matrices such as YHB's $\mathbf{I} - \mathbf{A}_0$, $\mathbf{A}_0^*$ and $\mathbf{A}_2$ may not be scaled since their units are not homogeneous across any one row. In this sense, YHB's equations 11 and 16 show correct units, but YHB's equations 1, 5 and 24 do not, since the matrices in these equations have mixed units across each row. See also our equation 7a below.

Swaziland	2007	22	0.79	(IFPRI, 2014b)	Social accounting matrix
France	2011	34	0.77	(OECD, 2017)	
$\mathbf{I}-\mathbf{P}$ (simplified process system)	/	6	0.72		Based on ecoinvent v3
USA	2011	34	0.71	(OECD, 2017)	
UK	2011	34	0.70	(OECD, 2017)	
Vietnam	2007	15	0.69	(ADB, 2017)	
Singapore	2006	15	0.68	(ADB, 2017)	
China	2005	15	0.67	(ADB, 2017)	
$\mathbf{A}_{2sc}$(this paper)	/	5	0.67		
Australia	1999	344	0.66	(Lenzen and Treloar, 2003)	Includes sparse imports and capital flow, hence $\lambda_{\max}$is lower than in (Wood and Lenzen, 2009).
Malaysia	2005	15	0.66	(ADB, 2017)	
Australia	2015	400	0.65	(Lenzen et al., 2017)	Multi-region input-output table
Taiwan	2006	15	0.64	(ADB, 2017)	
Thailand	2007	15	0.61	(ADB, 2017)	
India	2006	15	0.52	(ADB, 2017)	
Tanzania	2009	58	0.47	(IFPRI, 2014c)	Social accounting matrix
Scotland	2013	98	0.46	(Scottish Government, 2016)	
Egypt	2010	49	0.43	(IFPRI, 2016)	Social accounting matrix
Bangladesh	2006	15	0.42	(ADB, 2017)	
Sri Lanka	2006	15	0.42	(ADB, 2017)	
Rwanda	2011	54	0.39	(IFPRI, 2014a)	Social accounting matrix
$\mathbf{A}_2$ (YHB's paper)	/	5	0.12		

On the other hand, dominant eigenvalues of 0.2 or below can indicate either a very simplified closed system or, more likely, an open system where some inputs are excluded. The New Zealand dairy manufacturing process system compiled by Lenzen and Lundie (2012), for example, is expressed in purely mass flow units, and excludes all non-material inputs for example services such as insurance, accounting, transport, electricity etc. Services have been shown to be quite important in terms of energy, materials and emissions, once supply-chain contributions are taken into account; therefore they may be easily overlooked in process databases. Suh (2006), for example, showed that whilst services contributed only 5% to the US' emissions inventory directly, they were responsible for 38% of the life-cycle emissions of US final consumption. Similarly, Nansai et al. (2009) highlighted the increasing importance of energy and materials required in the supply chains of services consumed in Japan. This means that omitting services and their supply chains from life-cycle databases and inventories is likely to cause significant systematic errors. Whether this situation applies can be indicated by the dominant eigenvalue: in Lenzen and Lundie's process system this is 0.197, above that of YHB's system.

However, even in simple yet realistic process systems, dominant eigenvalues are sensibly higher than that of YHB's system. We show a matrix $\mathbf{P}$ of a fictitious bicycle production system. The numerical values in the table are not randomly or purposely chosen though; they are the results obtained from modelling the system in ecoinvent v3, and therefore they are representative of what a real-world process analysis would be like. The diagonal elements equalling 1 also reflects that "*there is no self-consumption as is in many LCA systems*" (Suh and Heijungs, 2007, p.385). We found the dominant eigenvalue to be $\lambda_{\max}(\mathbf{I}-\mathbf{P}) = 0.72$, which is in line with those of real economies in Table 1, and much higher than that of YHB's weakly coupled system. Adding more components and interconnections would increase the eigenvalue further however even in this simple example, we have shown that the base case underlying YHB's argument is not in line with process systems based on real data. Therefore, YHB's example does not seem to hold to the scrutiny of generality.

$$\begin{array}{cc}
 & \begin{array}{cccccc} \text{Steel production} & \text{Energy generation} & \text{Wheel production} & \text{Bike production} & \text{Rubber production} & \text{Transportation} \end{array} \\
\mathbf{P} = & \begin{bmatrix} 1 & -2.77 & -0.1103 & -0.8 & -0.095 & -0.014 \\ -0.0147 & 1 & -0.011 & -0.17 & -0.0662 & -0.1 \\ 0 & 0 & 1 & -2 & 0 & -0.0001 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -0.0002 & 0 & -0.0539 & -0.44 & 1 & -0.0003 \\ -0.4602 & -2.91237 & -0.1814 & -3.42 & -0.85 & 1 \end{bmatrix} , (7) \begin{array}{l} \text{Steel} \\ \text{Energy} \\ \text{Wheel} \\ \text{Bike} \\ \text{Rubber} \\ \text{Transportation} \end{array}
\end{array}$$

The input coefficients matrix $\mathbf{I} - \mathbf{P}$ that determines convergence is then

$$\begin{bmatrix} 0 & 2.77\text{ kg/GJ} & 0.11\text{ kg/unit} & 0.8\text{ kg/unit} & 0.095\text{ kg/kg} & 0.014\text{ kg/tkm} \\ 0.015\text{ GJ/kg} & 0 & 0.011\text{ GJ/unit} & 0.17\text{ GJ/unit} & 0.066\text{ GJ/kg} & 0.1\text{ GJ/tkm} \\ 0 & 0 & 0 & 2\text{ units/unit} & 0 & 0.0001\text{ units/tkm} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0002\text{ kg/kg} & 0 & 0.054\text{ kg/unit} & 0.44\text{ kg/unit} & 0 & 0.0003\text{ kg/tkm} \\ 0.46\text{ tkm/kg} & 2.91\text{ tkm/GJ} & 0.18\text{ tkm/unit} & 3.42\text{ tkm/unit} & 0.85\text{ tkm/kg} & 0 \end{bmatrix} \quad (7a)$$

Note that the eigenvalues of the disaggregated monetary input coefficients matrices are the same as for their process-analytical equivalents, that is $\lambda_{\max}(\mathbf{A}_0^*) = 0.12$ and $\lambda_{\max}(\mathbf{A}_{0sc}^*) = 0.70$, respectively. Aggregation does not change this significantly, with $\lambda_{\max}(\mathbf{A}_2) = 0.12$ and $\lambda_{\max}(\mathbf{A}_{2sc}) = 0.67$, respectively. Once again, we emphasise that real-world production systems are characterised by eigenvalues in the order of or larger than 0.5.

Overall, the low dominant eigenvalue of YHB's systems $\mathbf{A}_0$ and $\mathbf{A}_0^*$ casts some doubt on their suitability for representing a reference, or "true" system. In reality, popcorn making would require a host of additional, often non-material inputs such as business services, printing, office equipment, insurance, packing and cleaning services, marketing services, advertising signs, building and machinery repairs, road transport, freight handling, and many more. These inputs would in turn depend on other inputs such energy generation, thus introducing feedback loops into the system. It is hence likely that any modern production system would be heavily interconnected with the rest of the economy, thus exhibiting a large eigenvalue and slow convergence, and therefore potentially large truncation errors.

5. Aggregation in input-output systems

The finding from Section 2 is not surprising, because an important determinant of aggregation errors in input-output systems is not feedback, but sector heterogeneity (Lenzen, 2011). In YHB's case, the main heterogeneity lies in the difference between the first two entries in $\mathbf{B}_0^*$, 1 and 5, causing the high aggregation error of 64.3%. When these entries are changed to 2 and 4, respectively, the aggregation error decreases to 25.1%, even though wheat cultivation is still twice as emissions-intensive as corn cultivation.

Sector aggregation is a well-known problem in input-output analysis, and in order to avoid very high aggregation errors very dissimilar sectors should not be combined. Disaggregation is possible and advantageous even if complete information for disaggregating an input-output system does not exist (Lenzen, 2011). During the development of the Australian Industrial Ecology Lab (Lenzen et al., 2014) for example, the input-output sector classification available from the Australian Bureau of Statistics was significantly expanded (from 106 to 1284 sectors) on the basis of partial information. In particular, rice growing was split from other grains because of its water use intensity, aluminium was separated from other non-ferrous metals because of large energy inputs into aluminium smelting, beef cattle were distinguished from dairy cattle because of widespread land clearing and greenhouse gas emissions associated with beef production.

Overall, hybrid LCA is not a panacea, nor is it a blanket solution for all policy-related and environmental decision-making. However, it is important to reflect on (and even criticise) the tool based on its real features. A five-sector, weakly coupled, and strongly aggregated economy might simply not be a sufficient example to truly identify and discuss the limitations of, and further the debate on, hybrid LCA. This is particularly true when considering coupling existing multi-regional input-output (MRIO) databases to a process-based LCA, because the former feature typically between 50-500 sectors for any given country, resulting in thousands of different region-sector pairs (Tukker and Dietzenbacher, 2013) to map product flows and relationships between economies.

6. Conclusions

Hybrid LCA is often under scrutiny for the claims that it produces higher accuracy in its results than process-based LCA (Treloar, 1997; Treloar, 1998). Such debate is certainly positive in advancing the field (for instance Gibon and Schaubroeck, 2017; Schaubroeck and Gibon, 2017; Yang, 2017a, b).

In their recent work, YHB analysed the magnitude of truncation and aggregation errors in — respectively — process-based and hybrid LCA, and concluded that hybrid LCA does not necessarily produce more accurate results than process-based LCA. However, we have shown that the hypothetical five-sector economy they have used to prove their point is an extreme example, which is characterised by a process system with an unusually low truncation error, and by an input-output system with an unusually high aggregation error.

We have demonstrated that their argument does not hold when their case is modified to be similar to a real economy with technological and sectoral interdependencies. Specifically, introducing more feedback and coupling between the sectors of the economy slows down convergence of the cradle-to-gate inventory, and increases the truncation error of the process-based LCA whilst introducing more feedback and coupling does not increase the aggregation error of hybrid LCA as much.

To ensure that our counterargument would not just be a mathematical exercise we have introduced sectoral and technological interdependencies that are realistic and representative of those of a real economy. This is further supported by the calculation of the dominant eigenvalue of the technology matrix which – in our case – is in line with those of real economies whereas YHB's is unusually low. Additionally, we have also developed a simple process system based on ecoinvent data, which further indicates the unrepresentativeness of YHB's technology matrix.

This is important at a time where LCA (in its several forms) is ever growing and being used for ever more important policy decisions. We are still far from perfecting LCA theories and methodologies, and therefore every contribution is useful to foster the debate and promote new insights. Yet, we should try to challenge the *status quo* bearing in mind that LCA will be used as a tool for real-life application and not in a mathematical exercise.

Acknowledgements: This work was financially supported by the UK Engineering and Physical Sciences Research Council (EP/R01468X/1) and the Australian Research Council through its Discovery Projects DP0985522 and DP130101293, and by the National eResearch Collaboration Tools and Resources project (NeCTAR) through its Industrial Ecology Virtual Laboratory. NeCTAR are Australian Government projects conducted as part of the Super Science initiative and financed by the Education Investment Fund. The authors thank Sebastian Juraszek for expertly managing our advanced computation requirements, and Charlotte Jarabak for assistance with collecting data. Grateful thanks to Filippo D'Alimonte for his help with the process system based on ecoinvent data.

References

ADB, 2017. Asian Development Bank – Statistics for Dynamic Policy Making. Input-Output Tables of Selected Economies in Asia and the Pacific – Available at: https://sdbs.adb.org/sdbs/jsp/ICP/IOTDownload.jsp [Last Accessed: 8 May 2017]. .

Bullard, C.W., Penner, P.S., Pilati, D.A., 1976. Net Energy Analysis: Handbook for Combining Process and Input-Output Analysis., Center for Advanced Computation. University of Illinois.

Eurostat, 2017. Supply, Use and Input-Output tables. Available at: http://ec.europa.eu/eurostat/web/esa-supply-use-input-tables/ [Last Accessed: 8th May].

Gibon, T., Schaubroeck, T., 2017. Lifting the fog on characteristics and limitations of hybrid LCA—a reply to "Does hybrid LCA with a complete system boundary yield adequate results for product promotion?" by Yi Yang (Int J Life Cycle Assess 22(3):456–406, doi:10.1007/s11367-016-1256-9. The International Journal of Life Cycle Assessment, 1-4.

Giles, C., 2008. The economic forecasters' failing vision. Available at: https://www.ft.com/content/50007754-ca35-11dd-93e5-000077b07658, Financial Times.

Gretton, P., 2013. On input-output tables: uses and abuses. Staff Research Note. Australian Government Productivity Commission, Canberra.

Heijungs, R., Suh, S., 2002. The computational structure of life cycle assessment. Springer Science & Business Media.

IFPRI, 2014a. International Food Policy Research Institute (IFPRI). 2014. Rwanda Social Accounting Matrix (SAM), 2011. Washington, DC: [dataset].http://dx.doi.org/10.7910/DVN/28532.

IFPRI, 2014b. International Food Policy Research Institute (IFPRI). 2014. Swaziland Social Accounting Matrix (SAM), 2007. Washington, DC: [dataset].http://dx.doi.org/10.7910/DVN/28538

IFPRI, 2014c. International Food Policy Research Institute (IFPRI). 2014. Tanzania Social Accounting Matrix (SAM), 2009. Washington, DC: [dataset].http://dx.doi.org/10.7910/DVN/28540.

IFPRI, 2016. International Food Policy Research Institute (IFPRI); Central Agency for Public Mobilization and Statistics (CAPMAS). 2016. Egypt Disaggregated Social Accounting Matrix, 2010/112. Washington, DC: International Food Policy Research Institute (IFPRI) [dataset]. http://dx.doi.org/10.7910/DVN/DH37H9

Lenzen, M., 2000. Errors in conventional and input-output-based life-cycle inventories. Journal of Industrial Ecology 4, 127-148.

Lenzen, M., 2011. Aggregation versus disaggregation in input-output analysis of the environment. Economic Systems Research 23, 73 – 89.

Lenzen, M., Geschke, A., Malik, A., Fry, J., Lane, J., Wiedmann, T., Kenway, S., Hoang, K., Cadogan-Cowper, A., 2017. New multi-regional input-output databases for Australia – enabling timely and flexible regional analysis. Economic Systems Research 29, in press.

Lenzen, M., Geschke, A., Wiedmann, T., Lane, J., Anderson, N., Baynes, T., Boland, J., Daniels, P., Hadjikakou, M., Kenway, S., Moran, D., Murray, J., Nettleton, S., Poruschi, L., Reynolds, C., Rowley, H., Ugon, J., Webb, D., West, J., 2014. Compiling and using input-output frameworks through collaborative virtual laboratories. Science of the Total Environment 485–486, 241–251.

Lenzen, M., Lundie, S., 2012. Constructing enterprise input-output tables - a case study of New Zealand dairy products. Journal of Economic Structures 1, 6.

Lenzen, M., Treloar, G., 2003. Differential convergence of life-cycle inventories towards upstream production layers. Journal of Industrial Ecology 6, 137-160.
Lorenzen, G., 1981. Zur Charakterisierung der industriellen Verflechtung und der Endnachfragestruktur einer Volkswirtschaft. Jahrbücher für Nationalökonomie und Statistik 196, 503-510.
Nansai, K., Kagawa, S., Suh, S., Fujii, M., Inaba, R., Hashimoto, S., 2009. Material and energy dependence of services and its implications for climate change. ACS Publications.
OECD, 2017. Organisation for Economic Co-operation and Development OECD.Stat - Input-Output Tables. Avaialble at: http://stats.oecd.org/Index.aspx?DataSetCode=IOTS [Last Accessed 10th May].
Peters, G.P., 2006. Efficient algorithms for Life Cycle Assessment, Input-Output Analysis, and Monte-Carlo Analysis. The International Journal of Life Cycle Assessment 12, 373.
Schaubroeck, T., Gibon, T., 2017. Outlining reasons to apply hybrid LCA—a reply to "rethinking system boundary in LCA" by Yi Yang (2017). The International Journal of Life Cycle Assessment, 1-2.
Scottish Government, 2016. Scottish Supply Use and Analytical Input-Output Tables, 1998-2013, published 20th July 2016. Available at: http://www.gov.scot/Topics/Statistics/Browse/Economy/Input-Output/Downloads/IO1998-2013L1 [Last Accessed May 9th].
Suh, S., 2006. Are services better for climate change? ACS Publications.
Suh, S., Heijungs, R., 2007. Power series expansion and structural analysis for life cycle assessment. The International Journal of Life Cycle Assessment 12, 381.
Treloar, G.J., 1997. Extracting embodied energy paths from input–output tables: towards an input–output-based hybrid energy analysis method. Economic Systems Research 9, 375-391.
Treloar, G.J., 1998. A comprehensive embodied energy analysis framework. Faculty of Science and Technology, Deakin University.
Tukker, A., Dietzenbacher, E., 2013. Global multiregional input–output frameworks: an introduction and outlook. Economic Systems Research 25, 1-19.
Wood, R., Lenzen, M., 2009. Aggregate measures of complex economic structure and evolution – a review and case study. Journal of Industrial Ecology 13, 264-283.
Yang, Y., 2017a. Does hybrid LCA with a complete system boundary yield adequate results for product promotion? The International Journal of Life Cycle Assessment 22, 456-460.
Yang, Y., 2017b. Rethinking system boundary in LCA—reply to "Lifting the fog on the characteristics and limitations of hybrid LCA" by Thomas Gibon and Thomas Schaubroeck (2017). The International Journal of Life Cycle Assessment, 1-3.
Yang, Y., Heijungs, R., Brandão, M., 2017. Hybrid life cycle assessment (LCA) does not necessarily yield more accurate results than process-based LCA. Journal of Cleaner Production 150, 237-242.

Appendix

For further clarity, the following table show the equivalence between the original notation for matrices and vectors used in YHB and the modification occurred for the strongly coupled variations in this paper.

	YHB Notation	Strongly-coupled variations in our paper
Complete process based system	A_0	A_{0sc}
	B_0	/
	f_0	/
	m_0	m_{0sc}
Incomplete process based system	A_1	A_{1sc}
	B_1	/
	f_1	/
	m_1	/
Unaggragated IO-based system	q_0	/
	p_0	/
	x_0	x_{0sc}
	Z_0	Z_{0sc}
	A_0*	A_{0sc}*
	B_0*	/
	f_0*	/
	m_0*	/
Aggragated IO-based system	Z_2	Z_{2sc}
	x_2	x_{2sc}
	A_2	A_{2sc}
	B_2	/
	f_2	/
	m_2	m_{2sc}
Hybrid system	A_3	/
	B_3	/
	f_3	/
	m_3	/