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# Positive rotationally symmetric solutions for a Dirichlet problem involving the higher mean curvature operator in Minkowski space

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In this paper we investigate the global behavior of positive rotationally symmetric solutions for Dirichlet problem involving the $k$-th mean curvature operator in Minkowski space $\mathbb{L}^{n+1}$, which is not elliptic for $k > 1$. The proofs of our main results are based upon bifurcation techniques.



## 1. Introduction

Let $\mathbb{L}^{n+1}$ be the $(n+1)$-dimensional Lorentzian spacetime endowed with its standard Lorentzian metric

$$\langle\ ,\ \rangle = -dx_1^2 + \sum_{j=2}^{n+1} dx_j^2$$

and with the time orientation defined by $\partial/\partial x_1$. For a spacelike hypersurface in $\mathbb{L}^{n+1}$, the $k$-th mean curvatures are geometric invariants which encode the geometry of the hypersurface.

Let us consider a smooth immersion $\varphi : \Sigma \to \mathbb{L}^{n+1}$ of an $n$-dimensional manifold $\Sigma$ in $\mathbb{L}^{n+1}$, which is spacelike (i.e., the induced metric via $\varphi$ is Riemannian). Assume $N$ is a unit normal vector field along $\varphi$, which we choose pointing to the future. The shape operator of $\Sigma$ relative to $N$, is defined by O'Neill [20, Chap. 4] and de la Fuente et al. [11],

$$A(X) = -\nabla_X N,$$

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where $X \in T_p\Sigma$, $p \in \Sigma$, and $\nabla$ denotes the Levi-Civita connection of $\mathbb{L}^{n+1}$ which is given by

$$\nabla_X N = (X(N_1), \cdots, X(N_{n+1})),$$

where $N = (N_1, \cdots, N_{n+1})$, contemplated as a map from $\Sigma$ to $\mathbb{L}^{n+1}$.

The linear operator $A$ of $T_p\Sigma$, $p \in \Sigma$, is self-adjoint with respect to the induced metric. Its eigenvalues $\kappa_1(p), \kappa_2(p), \cdots, \kappa_n(p)$ are called the principal curvatures of the hypersurface. Consider the characteristic polynomial of $A$,

$$\det(tI - A) = \sum_{k=0}^{n} c_k t^{n-k} = \prod_{i=1}^{n} (t - \kappa_i), \tag{1.1}$$

where we put $c_0 = 1$. It is not difficult to see that

$$\begin{aligned} c_1 &= -\sum_{i=1}^{n} \kappa_i, \\ c_k &= (-1)^k \sum_{i_1 < \cdots < i_k} \kappa_{i_1} \cdots \kappa_{i_k}, \quad 2 \le k \le n. \end{aligned} \tag{1.2}$$

The *k-th mean curvature* $S_k$ of $\Sigma$ is defined as follows,

$$S_k = \frac{1}{C_n^k} c_k, \tag{1.3}$$

where $C_n^k = \frac{n!}{k!(n-k)!}$. For instance, when $k = 1$, we get $S_1 = \frac{c_1}{n} = -\frac{1}{n}\text{trace}(A)$, the usual mean curvature of $\Sigma$. Moreover, $S_2$ is, up to a constant, the scalar curvature of $\Sigma$ and, when $k = n$, we recover the Gauss–Kronecker curvature $S_n = (-1)^n \det(A)$ of $\Sigma$. It is interesting to note that $k$-th mean curvatures are in fact intrinsic geometric invariants of the hypersurfaces when $k$ is even. Precisely, the parity of $k$ plays an important role in the treatment of the equations, as it will be shown in next sections.

The prescribed $k$-th mean curvature problem in $\mathbb{L}^{n+1}$ consists in finding, for a given prescription function $H_k$, a spacelike hypersurface $\Sigma$ in $\mathbb{L}^{n+1}$ which satisfies

$$S_k(p) = H_k(p) \quad \text{for all } p \in \Sigma. \tag{1.4}$$

We will focus here the problem as follows. Consider a line $\gamma$ in $\mathbb{L}^{n+1}$ and put $\Pi$ the hyperplane through $p = \gamma(0)$ and orthogonal to $\gamma$ in $\mathbb{L}^{n+1}$. We will look for $\Sigma$ as a spacelike graph for a suitable function $v$ defined on $\Pi$, i.e., $\Sigma = \{(v(x), x) : x \in \Pi\} \subset \mathbb{R} \times \mathbb{R}^n$. If the prescription function $H_k$ was assumed rotationally symmetric with respect to $\gamma$, then it would be natural to assume $v$ also has the same symmetry, i.e., $v(x) = v(r)$ where $r = r(x)$ is the distance in $\Pi$ from $x$ to $\gamma(0)$.

The differential operators $\mathcal{M}_k$, $1 \le k \le n$, associated to the $k$-curvature of rotationally symmetric graphs in $\mathbb{L}^{n+1}$, can be written as follows,

$$\mathcal{M}_k : \ \{v \in C^2(\mathbb{R}^+) : \ v'(0) = 0, \ |v'| < 1\} \to \mathbb{R}, \tag{1.5}$$

$$\mathcal{M}_k(v)(r) = \begin{cases} \dfrac{1}{nr^{n-1}} \left(r^{n-k} \phi^k(v')\right)' & \text{in } (0, \infty), \\ 0 & \text{in } r = 0, \end{cases} \tag{1.6}$$

where $\phi(s) := \frac{s}{\sqrt{1-s^2}}$. Note that, in general, the differential operators $\mathcal{M}_k$ $(2 \le k \le n)$ are not elliptic. To investigate the existence of rotationally symmetric entire spacelike graph with prescribed $k$-th mean curvature function $H_k$ in Minkowski spacetime $\mathbb{L}^{n+1}$, it is enough to find the solutions of the equations

$$\mathcal{M}_k(v)(r) = H_k(v(r), r) \qquad r \in \mathbb{R}^+, \tag{1.7}$$

for a given prescription function $H_k : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$.

From a physical perspective, the $k$-th mean curvatures have a relevant role in General Relativity. A spacelike hypersurface is a suitable subset of the spacetime where the initial value problems for the different field equations are naturally stated.

In the Minkowski context, the pioneer works was given by Cheng and Yau [7], they proved the Bernstein's property for entire solutions of the maximal (i.e., zero mean curvature) hypersurface equation and, later, Treibergs [22] classified the entire solutions of the constant mean curvature spacelike hypersurface equation. An important universal existence result was proved by Bartnik and Simon [3], and Bartnik proved the existence of prescribed mean curvature spacelike hypersurfaces under certain asymptotic assumptions [2]. More recently, there are more contributions (see for instance, [1]) and the interest is many times focused on the existence of positive solutions, by using a combination of variational techniques, critical point theory, sub–supersolutions and topological degree (see for instance [5,6,8–10,19] and the references therein). Bayard proved the existence of prescribed scalar entire spacelike hypersurfaces in Minkowski spacetime [4], by using other previous works on the Dirichlet problem and Gerhardt [14] obtained important results on the case of more general ambient spacetimes. Finally, the Gauss–Kronecker curvature has been also quite well studied. In Minkowski spacetime, we highlight the work of Li [16] on constant Gauss curvature and Delanoè [13], in which the existence of entire spacelike hypersurfaces asymptotic to a lightcone with prescribed Gauss–Kronecker curvature function is proved.

Up to the last decade, little attention has been paid to hypersurfaces with prescribed $k$-th mean curvature when $3 \le k < n$, see [11,15].

Associated to equation (1.7), de la Fuente, Romero and Torres [11] used the polar coordinates to transfer (1.7) into the following boundary value problem of ordinary differential equation

$$\begin{aligned} &(r^{n-k}\phi^k(v'))' = nr^{n-1}H_k(v(r), r), && r \in (0, R),\\ &|v'| < 1, && r \in (0, R),\\ &v'(0) = 0 = v(R), \end{aligned} \tag{1.8}$$

and then, they applied Schauder fixed point theorem to get the existence and multiplicity results of rotationally symmetric solutions. Their main results are selected in following two lemmas, see [11, Proposition 3.1–3.3] and their proofs for the detail.

**Lemma A.** *Let $k$ be odd. Then for every continuous function $H_k : \mathbb{R} \times [0, R] \to \mathbb{R}$, (1.8) has at least one solution.*

**Lemma B.** *Let $k$ be even. Assume*

$$\int_0^r s^{n-1} H_k(v(s), s)ds \ge 0 \quad \textit{for all } r \in [0, R], \textit{ and } v \in C^1, |v'| < 1. \tag{1.9}$$

*Then, problem (1.8) has at least two different solutions. Moreover, one of them is increasing and negative in $(0, R)$ and the other one is decreasing and positive in $(0, R)$.*

It is easy to see that Lemma A contains no any information about the sign of the solutions of (1.8), and Lemma B provides no any *Geometric interpretation* about the occurrence of two solutions.

If $H_k(0, r) \equiv 0$ for $r \in [0, R]$, then $u = 0$ is a solution of (1.8). Of course the natural question is whether or not (1.8) has positive solutions or negative solutions?

It is the purpose of this paper to study the global structure of positive (or negative) solutions of the following problem

$$\begin{aligned}
&(r^{n-k}\phi^k(v'))' = \lambda n r^{n-1} H_k(v(r), r), \qquad && r \in (0, R),\\
&|v'| < 1, && r \in (0, R), \qquad (1.10)\\
&v'(0) = 0 = v(R),
\end{aligned}$$

where $\lambda > 0$ is a parameter, and $\phi(s) := \frac{s}{\sqrt{1-s^2}}$. The main tools we used are the global bifurcation theorem, see Zeidler [24, Corollary 15.12] and the property of the superior limit of a sequence of connected components due to Ma and An [17, Lemma 2.4] and [18, Lemma 2.2].

Let $\mathbb{R}^+ = [0, \infty)$. Let $C$ be the Banach space of the real continuous functions in $[0, R]$, with the maximum norm $||\cdot||_C$, and $C^1$ the space of continuously differentiable functions with its usual norm $||v||_{C^1} = \max\{||v||_C, ||v'||_C\}$. For $h \in C$, we write $h \succ 0$ if $h(t) \geq 0$ for $t \in [0, R]$ and it is positive in a set of positive measure. For $w \in C^1$, we write $w \gg 0$ if $w(r) > 0$ for $[0, R)$ and $w'(R) < 0$. And $w \ll 0$ if $-w \gg 0$.

The main results of this paper are the following

**Theorem 1.1.** *Let $k \in \mathbb{N}$ with $1 \leq k \leq n$. Assume that*

(A1) $(-1)^k H_k(t, r) \geq 0$ *for* $(t, r) \in \mathbb{R} \times [0, R]$;
(A2) $H_k(0, r) = 0$ *for* $r \in [0, R]$;
(A3) *there exists a function* $(H_k)_0 \in C$ *with* $(H_k)_0 \succ 0$, *such that*

$$\lim_{t \to 0} \frac{(-1)^k H_k(t, r)}{t} = (H_k)_0. \qquad (1.11)$$

*Then the set*

$$\mathcal{D}^+ := \overline{\{(\lambda, u) : \lambda > 0,\ u \succ 0, (\lambda, u) \text{ satisfies } (1.10)\}}^{\mathbb{R}^+ \times C^1}$$

*contains a connected component $\xi$ satisfying*

(1) $(\lambda, u) \in \xi \setminus \{0\} \ \Rightarrow\ u \gg 0$ *and* $u'(r) < 0$ *in* $(0, R]$;
(2) $\xi$ *joins* $(0, 0)$ *to infinity in the* $\lambda$*-direction.*

**Remark 1.1.** It is an immediate consequence that under the assumptions of Theorem 1.1, (1.8) has at least one positive solution. However, Lemma A gives no any information about the sign of solutions.

**Theorem 1.2.** *Let $k \in \mathbb{N}$ be even and $1 \leq k \leq n$. Assume that (A1)–(A3) hold. Then the set*

$$\mathcal{D}^- := \overline{\{(\lambda, u) : \lambda > 0,\ u \ll 0, (\lambda, u) \text{ satisfies } (1.10)\}}^{\mathbb{R}^+ \times C^1}$$

*contains a connected component $\zeta$ satisfying*

(1) $(\lambda, u) \in \zeta \setminus \{0\} \ \Rightarrow\ u \ll 0$ *and* $u'(r) > 0$ *in* $(0, R]$;
(2) $\zeta$ *joins* $(0, 0)$ *to infinity in the* $\lambda$*-direction.*

**Remark 1.2.** It is easy to see that $u = 0$ is a solution of (1.8). Let $\lambda = 1$ in (1.10). Then Theorem 1.2 guarantees the existence of one positive solution and one negative solution for (1.8) besides the trivial solution $u = 0$. In other words, we get three solutions for (1.8).

**Remark 1.3.** Clearly, Lemma A and B gives no information on the interesting problem as to what happens to the norms of positive solutions of (1.10) as $\lambda$ varies in $(0,\infty)$. However, the connected components in Theorem 1.1 and 1.2 are very useful for computing the numerical solutions of (1.8) as they can be used to guide the numerical work. For example, they can be used to estimate the $u$-interval in advance in applying the finite difference method, and they together with the fact

$$|u'(t)| < 1, \quad 0 \le u(t) \le R$$

can be used to restrict the range of initial values we need to consider in applying the shooting method.

The rest of the paper is arranged as follows. In Section 2, we rewrite (1.10) into an equivalent bifurcation problem and we investigate the principal eigenvalue and its eigenfunction for the associated positive linear operator via the well-known Krein–Rutman theorem. Section 3 is devoted to show our main results by using the global bifurcation theorem and the property of the superior limit of a sequence of connected components. Finally in Section 4, we prove the existence of entire spacelike graphs in Minkowski space with prescribed higher mean curvature.

## 2. An equivalent operator equation

In this section, we shall rewrite (1.10) into an equivalent operator equation. Let us define

$$K:\ C^1 \to C^1,$$
$$K(v)(r) = \int_r^R v(t)dt,$$
$$S:\ C \to C^1,$$
$$S(v)(r) = \frac{n}{r^{n-k}} \int_0^r t^{n-1} v(t)dt, \quad r \in (0,R), \quad S(v)(0) = 0.$$

Let

$$B_{R,1} := \{u \in C^1 :\ ||u||_\infty < R,\ ||u'||_\infty < 1\}.$$

Let us consider the Nemytskii operator associated to $H_k$,

$$N_{H_k}:\ B_{R,1} \subset C^1 \to C, \quad N_{H_k}(v)(r) = H_k(v(r), r). \tag{2.1}$$

Obviously, $N_{H_k}$ is continuous and $N_{H_k}(B_{R,1})$ is a bounded subset of $C$. Finally, we define the operator

$$A:\ \bar{B}_{R,1} \subset C^1 \to C^1, \quad A = K \circ (\phi^{-1})^{1/k} \circ S \circ (\lambda N_{H_k}), \tag{2.2}$$

where $(\phi^{-1})^{1/k}:\ \mathbb{R}^+ \to [0,1)$ with $(\phi^{-1})^{1/k}(s) = \phi^{-1}(s^{1/k})$. More explicitly, operator $A$ can be written as

$$A(v)(r) = \int_r^R \phi^{-1}\Big[\Big(\frac{n(-1)^k}{s^{n-k}} \int_0^s \tau^{n-1}\ \lambda\ H_k(v(\tau),\tau)d\tau\Big)^{1/k}\Big]ds. \tag{2.3}$$

Note that $A$ is well-defined thanks to condition (A1). Note that $A$ is a composition of continuous operators, hence it is continuous. Moreover, from the compactness of $K$, $A$ is a compact and continuous operator. Note

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that the image of the operator $A$ is contained in $C^2[0,R]$, so the fixed points (solutions of the equation (1.10)) will be of class $C^2[0,R]$.

A straightforward checking shows that if a function $v \in C^1$ is a fixed point of the nonlinear compact operator (2.2), then $v$ is a solution of equation (1.10).

To apply global bifurcation theorem, we firstly find the Fréchet derivative of $A$.

For $h \in C^1$, it follows from $\phi^{-1} \in C^1(\mathbb{R})$ and the fact

$$(y_0+\Delta y)^{1/k}-(y_0)^{1/k}=\frac{1}{k}y_0^{\frac{1}{k}-1}\ \Delta y+\circ(|\Delta y|)$$

that

$$\begin{aligned}
&A(v_0+h)(r)-A(v_0)(r)\\
&=\int_r^R \phi^{-1}\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau)+h(\tau))d\tau\Big)^{1/k}\Big]ds\\
&\quad-\int_r^R \phi^{-1}\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big]ds\\
&=\int_r^R \{\phi^{-1}\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau)+h(\tau))d\tau\Big)^{1/k}\Big]\\
&\quad-\phi^{-1}\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big]\}ds\\
&\to\int_r^R \{(\phi^{-1})'\Big(\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big)\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau)+h(\tau))d\tau\Big)^{1/k}\\
&\quad-\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big]\}ds\\
&\to\int_r^R \{(\phi^{-1})'\Big(\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big)\frac{1}{k}\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{\frac{1}{k}-1}\\
&\quad\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau)+h(\tau))d\tau\Big)-\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)\Big]\}ds\\
&=\int_r^R \{(\phi^{-1})'\Big(\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big)\frac{1}{k}\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{\frac{1}{k}-1}\\
&\quad\Big[\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ \big(H_k(v_0(\tau)+h(\tau))-H(v_0(\tau))\big)d\tau\Big]\}ds\\
&\to\int_r^R \{(\phi^{-1})'\Big(\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big)\frac{1}{k}\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{\frac{1}{k}-1}
\end{aligned}$$

$$\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (H_k)'(v_0)h(\tau)d\tau\Big\}ds \tag{2.4}$$

as $||h||_C \to 0$.

Let

$$\begin{aligned}A'(\lambda, v_0)h(r) := \int_r^R \Big\{(\phi^{-1})'\Big(\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{1/k}\Big)\\ \frac{1}{k}\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v_0(\tau))d\tau\Big)^{\frac{1}{k}-1}\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (H_k)'(v_0)h(\tau)d\tau\Big\}ds.\end{aligned} \tag{2.5}$$

Then $A'(\lambda, 0)$ is not well-defined since

$$\lim_{\epsilon\to 0^+}\frac{1}{k}|\epsilon|^{\frac{1}{k}-1} = +\infty. \tag{2.6}$$

So, we consider a family of auxiliary problems

$$\begin{cases}(r^{n-k}\phi^k(v'))' = \lambda n r^{n-1}[H_k(v(r), r)) + \dfrac{(-1)^k}{m}], & r \in (0, R),\\ |v'| < 1, & r \in (0, R),\\ v'(0) = 0 = v(R),\end{cases} \tag*{$(2.7)_m$}$$

where $m \in \mathbb{N}$. It is easy to verify that $(2.7)_m$ is equivalent to the operator equation

$$v = A_m(\lambda, v)(r), \tag*{$(2.8)_m$}$$

where

$$A_m(v)(r) := \int_r^R \phi^{-1}\Big[\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ [H_k(v(\tau), \tau) + \frac{(-1)^k}{m}]d\tau\Big)^{1/k}\Big]ds. \tag*{$(2.9)_m$}$$

Define $\mathcal{T}_m : C^1 \to C^1$ by

$$\begin{aligned}\mathcal{T}_m(\lambda, v_0)h(r) := \int_r^R \Big\{(\phi^{-1})'\Big(\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (H_k(v_0(\tau)) + \frac{(-1)^k}{m})d\tau\Big)^{1/k}\Big)\\ \frac{1}{k}\Big(\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (H_k(v_0(\tau)) + \frac{(-1)^k}{m})d\tau\Big)^{\frac{1}{k}-1}\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (H_k)'(v_0)h(\tau)d\tau\Big\}ds.\end{aligned} \tag{2.10}$$

Then

$$\begin{aligned}\mathcal{T}_m(\lambda, 0)h(r) := \int_r^R \Big\{(\phi^{-1})'\Big(\Big(\frac{n}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (\frac{1}{m})d\tau\Big)^{1/k}\Big)\frac{1}{k}\Big(\frac{n}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (\frac{1}{m})d\tau\Big)^{\frac{1}{k}-1}\\ \frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ (H_k)_0 h(\tau)d\tau\Big\}ds.\end{aligned} \tag{2.11}$$

Let

$$\mathcal{B}_m(\lambda,0)h(r) := \int_r^R \{(\phi^{-1})'\Big(s(\frac{\lambda}{m})^{1/k}\Big)\frac{1}{k}\Big(s^k\frac{1}{m}\Big)^{\frac{1}{k}-1}\ \frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ (H_k)_0 h(\tau)d\tau\}ds. \tag{2.12}$$

Then

$$\mathcal{B}_m(0,0)h(r) := \int_r^R \{(\phi^{-1})'(0)\frac{1}{k}\big(s^k\frac{1}{m}\big)^{\frac{1}{k}-1}\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ (H_k)_0 h(\tau)d\tau\}ds, \tag{2.13}$$

$$\mathcal{F}_m := \mathcal{B}_m(0,0). \tag{2.14}$$

**Lemma 2.1.** *Let*

$$P = \{u \in C^1 : u'(0) = u(R) = 0,\ u(r) \geq ||u||_C\frac{R-r}{R}\ \ \textit{for } r\in[0,R]\}.$$

*Let $h \in C$ with $h \succ 0$. Then for given $m \in \mathbb{N}$,*

$$\mathcal{F}_m h \in \mathit{int}\,P.$$

**Proof.** Let

$$\beta(s) = (\phi^{-1})'(0)\frac{1}{k}\Big(s^k\frac{1}{m}\Big)^{\frac{1}{k}-1}\frac{n(-1)^k}{s^{n-k}}\int_0^s \tau^{n-1}\ (H_k)_0 h(\tau)d\tau.$$

From (A3), it follows

$$\beta(r) \geq 0, \qquad \beta(r) \not\equiv 0 \qquad r\in(0,R). \tag{2.15}$$

This together with the fact that $\beta$ is strictly increasing in $(0,R)$ implies that

$$\beta'(s) \geq 0, \qquad s\in(0,R).$$

Thus

$$\big(\mathcal{F}_m h\big)''(r) = -\beta'(r) \leq 0,$$

and accordingly, the function $\big(\mathcal{F}_m h\big)(\cdot)$ is concave down in $(0,R)$. Combining this with (2.15) and using the fact $\big(\mathcal{F}_m\beta(r)\big)' \leq 0$ in $(0,R]$, it is easy to deduce that

$$\big(\mathcal{F}_m h\big)(r) \geq \big(\mathcal{F}_m h\big)(0)\frac{R-r}{R} = ||\mathcal{F}_m h||_C\frac{R-r}{R}. \qquad \square$$

Now, it follows from Lemma 2.1 and the well-known Krein–Rutman theorem Deimling [12, Theorem 19.2], we obtain the following

**Lemma 2.2.** *The operator $\mathcal{F}_m$ has a positive eigenvalue $\mu_1^{[\mathcal{F}_m]}$ and the eigenspace corresponding to $\mu_1^{[\mathcal{F}_m]}$ is spanned by an eigenfunction $\psi_1^{[\mathcal{F}_m]}$ with $\psi_1^{[\mathcal{F}_m]} \gg 0$. Moreover, the algebraic multiplicity of $\mu_1^{[\mathcal{F}_m]}$ as a characteristic value of $\mathcal{F}_m$ is 1.*

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From $\mathcal{F}_m$ is strongly positive and

$$||\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0) - \mathcal{F}_m||_{\mathcal{L}[C^1,C^1]} \to 0 \quad \text{as } m \to \infty,$$

it follows that $\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)$ is strongly positive. By the similar argument to prove Lemma 2.2, we may deduce the following

**Lemma 2.3.** *The operator $\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)$ has a positive eigenvalue $\mu_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]},0)]}$ and the eigenspace corresponding to $\mu_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]},0)]}$ is spanned by an eigenfunction $\psi_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]},0)]}$ with $\psi_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]},0)]} \gg 0$. Moreover, the algebraic multiplicity of $\mu_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]},0)]}$ as a characteristic value of $\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)]$ is 1.*

**Lemma 2.4.**

$$\lim_{m\to\infty} \mu_1^{[\mathcal{F}_m]} = 0.$$

**Proof.** It is an immediate consequence of (2.13), (2.14) and the fact

$$\lim_{m\to\infty} \Big(\frac{1}{m}\Big)^{\frac{1}{k}-1} = \infty. \qquad \square$$

**Lemma 2.5.**

$$\lim_{m\to\infty} \mu_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]},0)]} = \lim_{m\to\infty} \mu_1^{[\mathcal{F}_m]}. \tag{2.16}$$

**Proof.** Since $(\phi^{-1})''(0) = 0$ and

$$(\phi^{-1})'\Big(\frac{\mu_1^{[\mathcal{F}_m]}}{m^{1/k}}s\Big) - (\phi^{-1})'\Big(0\Big) = (\phi^{-1})''(0)\Big(\frac{\mu_1^{[\mathcal{F}_m]}}{m^{1/k}}s - 0\Big) + \circ(\Big|\frac{1}{m}\Big|^{\frac{1}{k}}) = \circ(\Big|\frac{1}{m}\Big|^{\frac{1}{k}}), \quad \text{as } m \to \infty \tag{2.17}$$

uniformly for $\lambda$ in any compact subinterval of $(0, \infty)$, it is easy to check that

$$||\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0) - \mathcal{F}_m||_{\mathcal{L}[C^1,C^1]} \to 0, \qquad \text{as } m \to \infty. \tag{2.18}$$

From (2.18) and the well known Gelfand's formula,

$$\begin{aligned} \lim_{m\to\infty} r(\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)) &= \lim_{m\to\infty} ||\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)||^{1/m} \\ &= \lim_{m\to\infty} ||\mathcal{F}_m||^{1/m} = \lim_{m\to\infty} r(\mathcal{F}_m), \end{aligned} \tag{2.19}$$

where $r(T)$ is the spectrum radius of a linear completely continuous operator $T$. Obviously, (2.19) implies (2.16). $\square$

Now, we rewrite $(2.8)_m$ into the form

$$v = \mu\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0))v + \mathcal{E}_m(\mu, v), \tag{2.20}$$

where

$$\mathcal{E}_m(\mu, v) = A_m(\mu^k, v) - \mu\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0))v = \circ(||v||_C). \tag{2.21}$$

## 3. Proof of the main results

In this section, we shall prove Theorem 1.1 and 1.2.

**Proof of Theorem 1.1.** Let us divide the proof into three steps.

*Step 1.* Global structure of positive solutions for the approximation problem $(2.7)_m$.

Recall that $(2.7)_m$ is equivalent to the operator equation

$$v = \mu\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)v + \mathcal{E}_m(\mu, v), \tag{3.1}$$

where $\mu = \lambda^{\frac{1}{k}}$, $\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0) : C^1 \to C^1$ is completely continuous, and

$$\mathcal{E}_m(\mu, v) := A_m(\mu^k, v) - \mu\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)v, \tag{3.2}$$
$$\lim_{||v||_C \to 0} \frac{\mathcal{E}_m(\mu, v)}{||v||_C} = 0.$$

Notice that we only decompose each nonlinear functions into its linear principal part adds a infinitesimal of higher order (as $||v||_C \to 0$) in the process to define the linear operator $\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)$, so we may get the limit in (3.2).

Applying the well-known global bifurcation theorem [24, Zeidle, Corollary 15.12] and using the standard method, it follows that for each $m \in \mathbb{N}$, there exists a connected component $C^{[m]}$ which joint $(\mu_1^{[\mathcal{B}_m(\mu_1^{[\mathcal{F}_m]}, 0)]}, 0)$ to infinity in $\mu$-direction.

*Step 2.* Finding a connected component in $\limsup\limits_{m\to\infty} C^{[m]}$.

Using the fact

$$\sup\{||u||_{C^1} : (\lambda, u) \in \cup_{j=1}^{\infty} C^{(j)}\} \le \max\{R, 1\} < \infty$$

and [17, Lemma 2.4] and [18, Lemma 2.2], it follows that the set $\limsup C^{[m]}$ contains an unbounded connected component $\mathcal{C}^*$:

$$(0, 0) \in \mathcal{C}^* \subset \limsup_{m\to\infty} C^{[m]}$$

which joins $(0, 0)$ to infinity in $\mu$-direction.

Put

$$\mathcal{C}^\diamond := \{(\lambda, u)|\ \lambda = \mu^k \text{ and } (\mu, u) \in C^*\}.$$

Then $\mathcal{C}^\diamond$ joins $(0, 0)$ to infinity in $\lambda$-direction.

*Step 3.* We show that

$$\mathcal{C}^\diamond \cap \big\{(0, \infty) \times \{0\}\big\} = \emptyset. \tag{3.3}$$

Assumption on the contrary that there exists a sequence $\{(\eta_j, w_j)\} \subset \mathcal{C}^\diamond$, such that

$$\eta_j \to \hat{\lambda} > 0, \qquad ||w_j||_{C^1} \to 0 \qquad \text{as } j \to \infty. \tag{3.4}$$

By the definition of $\limsup C^{[j]}$, see Whyburn [23] and Ma [17,18], for each $j \in \mathbb{N}$, after taking a subsequence if necessary, there exists $(\beta_j, u_j) \in C^{[j]}$ with

$$|\beta_j - \eta_j| \leq \frac{1}{j}, \qquad ||u_j - w_j||_{C^1} < \frac{1}{j}. \tag{3.5}$$

Thus

$$u_j = \beta_j \mathcal{B}_j(\mu_1^{[\mathcal{F}_m]}, 0)u_j + \mathcal{E}_j(\beta_j, u_j) \tag{3.6}$$

with

$$\lim_{||u_j||_C \to 0} \frac{\mathcal{E}_j(\beta_j, u_j)}{||u_j||_C} = \lim_{||u_j||_C \to 0} \frac{\mathcal{E}_j(\beta_j, u_j)}{u_j} \frac{u_j}{||u_j||_C} = 0, \tag{3.7}$$

which implies that

$$\lim_{||u_j||_{C^1} \to 0} \frac{\mathcal{E}_j(\beta_j, u_j)}{||u_j||_{C^1}} = 0$$

since

$$||u_j||_{C^1} \to 0 \;\Rightarrow\; ||u_j||_C \to 0.$$

Let $v_j := \frac{u_j}{||u_j||_C}$. Then (3.7) yields

$$\lim_{||u_j||_C \to 0} \frac{\mathcal{E}_j(\beta_j, u_j)}{u_j} v_j = 0. \tag{3.8}$$

Let $[\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)]^*$ be the adjoint operator of $\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)$. Then it follows from Deimling [12, Theorem 19.5] that

$$[\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)]^* \psi_j^* = \mu_1^{[\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)]} \psi_j^* \tag{3.9}$$

for some strictly positive $\psi^* \in P^*$, where $P^*$ is the dual cone of $P$.

It is easy to from the fact $u_n(r) \geq ||u_n||_C \frac{R-r}{R}$ that

$$v_j(r) \geq \frac{R-r}{R}, \qquad r \in [0, R]. \tag{3.10}$$

Combining (3.6), (3.8), (3.9) and (3.10) and using the fact $\psi_j^*$ is strictly positive, it follows that

$$\begin{aligned}
0 &< \langle \psi_j^*, v_j \rangle \\
&= \beta_j \langle \psi_j^*, \mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)v_j \rangle + \langle \psi_j^*, \frac{\mathcal{E}_j(\beta_j, u_j)}{||u_j||_C} \rangle \\
&= \beta_j \langle \psi_j^*, \mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)v_j \rangle + \langle \psi_j^*, \frac{\mathcal{E}_j(\beta_j, u_j)}{u_j} \frac{u_j}{||u_j||_C} \rangle \\
&= \beta_j \langle \big[\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)\big]^* \psi_j^*, v_j \rangle + \langle \psi_j^*, \frac{\mathcal{E}_j(\beta_j, u_j)}{u_j} \frac{u_j}{||u_j||_C} \rangle \\
&= \beta_j \mu_1^{[\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)]} \langle \psi_j^*, v_j \rangle + \langle \psi_j^*, \frac{\mathcal{E}_j(\beta_j, u_j)}{u_j} v_j \rangle.
\end{aligned} \tag{3.11}$$

However, this is impossible since $\lim_{j \to \infty} \mu_1^{[\mathcal{B}_j(\mu_1^{[\mathcal{F}_j]}, 0)]} = 0$.

Therefore, (3.3) is valid.

Finally, (2.3) and (A1) yield

$$v'(r) = -\phi^{-1}\Big[\Big(\frac{n(-1)^k}{r^{n-k}}\int_0^r \tau^{n-1}\ \lambda\ H_k(v(\tau),\tau)d\tau\Big)^{1/k}\Big] < 0 \quad \text{for } r \in (0,R]. \qquad \square$$

**Proof of Theorem 1.2.** To get the connected component of negative solutions of (1.10) in the case $k$ is even, we may consider

$$[\phi(v')]^k(r) = \frac{n}{r^{n-k}}\int_0^r \tau^{n-1}\ \lambda\ H_k(v(\tau),\tau)d\tau, \tag{3.12}$$

and accordingly,

$$\phi(v')(r) = \big[\frac{n}{r^{n-k}}\int_0^r \tau^{n-1}\ \lambda\ H_k(v(\tau),\tau)d\tau\big]^{1/k}.$$

Define

$$\tilde{A}(v)(r) = -\int_r^R \phi^{-1}\Big[\Big(\frac{n}{s^{n-k}}\int_0^s \tau^{n-1}\ \lambda\ H_k(v(\tau),\tau)d\tau\Big)^{1/k}\Big]ds. \tag{3.13}$$

By the same argument used in the proof of Theorem 1.1, with obvious changes, we may get a connected component $\zeta \in \mathcal{D}_-$ with

$$\mathcal{D}^- := \overline{\{(\lambda,u) : \lambda > 0,\ u \ll 0, (\lambda,u) \text{ satisfies (1.10)}\}}^{\mathbb{R}^+\times C^1}$$

which satisfying

(1) $(\lambda,u) \in \zeta \setminus \{0\} \ \Rightarrow\ u \ll 0$ and $u'(r) > 0$ in $(0,R]$;
(2) $\zeta$ joins $(0,0)$ to infinity in the $\lambda$-direction. $\square$

## 4. Existence of entire spacelike graphs

In this section, we provide conditions to guarantee that every solution $u$, given by Theorem 1.1 and Theorem 1.2, once $R$ and $\lambda$ are fixed, can be continued until $\infty$ as a solution of equations

$$\mathcal{M}_k(u)(r) = \lambda H_k(u(r),r), \qquad r \in \mathbb{R}^+. \tag{4.1}$$

We shall make the following assumptions:

(C1) $(-1)^k H_k(t,r) \geq 0$ for $(t,r) \in \mathbb{R}\times\mathbb{R}^+$;
(C2) $H_k(0,r) = 0$ for $r \in \mathbb{R}^+$.

**Theorem 4.1.** *Let $H_k : \mathbb{L}^{n+1} \to \mathbb{R}$, with $k$ an odd positive integer, be a continuous function which is rotationally symmetric with respect to an inertial observer $\gamma$ of $\mathbb{L}^{n+1}$. Let $R$ and $\lambda$ be two positive constants. Assume that (C1), (C2) and (A3) hold. Then there exists at least an entire spacelike graph, rotationally* *symmetric respect to $\gamma$, whose $k$-th mean curvature equals to $\lambda H_k$ and such that it intersects the hyperplane orthogonal to $\gamma$ at $\gamma(0)$ in an $(n-1)$-sphere with radius $R$ centered at $\gamma(0)$.*

**Theorem 4.2.** *Let $H_k : \mathbb{L}^{n+1} \to \mathbb{R}$, with $k$ an even positive integer, be a continuous function which is rotationally symmetric with respect to an inertial observer $\gamma$ of $\mathbb{L}^{n+1}$. Let $R$ and $\lambda$ be two positive constants. Assume that (C1), (C2) and (A3) hold. Then there exists at least two different entire spacelike graphs and rotationally symmetric whose $k$-th mean curvature equals to $\lambda H_k$ and such that it intersects the hyperplane orthogonal to $\gamma$ at $\gamma(0)$ in an $(n-1)$-sphere with radius $R$ centered in $\gamma(0)$. Moreover, the radial profile curve of one of them is increasing and the other one is decreasing.*

We need the following lemma.

**Lemma 4.1.** *Let $\lambda, \rho \in (0, \infty)$ be fixed. Assume that $(\lambda, u)$ is a solution of*

$$\begin{aligned} (r^{n-k}\phi^k(u'))' &= \lambda n r^{n-1} H_k(u(r), r), && r \in (0, \rho), \\ |u'| &< 1, && r \in (0, \rho), \\ u'(0) &= 0 = u(\rho). \end{aligned} \tag{4.2}$$

*Then there exists a constant $\sigma \in (0, 1)$ such that*

$$|u'(r)| < 1 - \sigma, \qquad r \in [0, \rho]. \tag{4.3}$$

**Proof.** (1.10) is equivalent to

$$|u'(r)| = \Big|\, \phi^{-1}\Big[\Big(\frac{n(-1)^k}{r^{n-k}} \int_0^r \tau^{n-1}\ \lambda\ H_k(u(\tau), \tau) d\tau\Big)^{1/k}\Big]\, \Big| \quad \text{for } r \in (0, \rho].$$

This together with the definition of $\phi$ and the fact

$$\max\Big\{\Big(\frac{n(-1)^k}{r^{n-k}} \int_0^r \tau^{n-1}\ \lambda\ H_k(u(\tau), \tau) d\tau\Big)^{1/k}\ :\ r \in (0, \rho]\Big\} < \infty$$

imply that there exists $\sigma \in (0, 1)$ such that (4.3) is valid. $\square$

**Proof of Theorem 4.1.** Assume that $k$ is odd. Let $u$ be a solution of equation (4.1), and let $[0, b)$ be the maximal interval of definition of $u$. Suppose on the contrary that $b < \infty$. We can rewrite equation (4.1) as a system of two ordinary differential equations of first order

$$\begin{aligned} u'(r) &= \phi^{-1}\big[\big(\frac{z(r)}{r^{n-k}}\big)^{1/k}\big], \\ z'(r) &= \lambda n r^{n-1} H_k(u(r), r)), \end{aligned}$$

which we can abbreviate

$$\begin{pmatrix} u' \\ z' \end{pmatrix} = \mathcal{F}(r, u, z),$$

where $\mathcal{F} : \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^2$.

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By the standard prolongability theorem of ordinary differential equations (see for instance [21, Section 2.5]), we have that the graph $\{(r, u(r), z(r)) : r \in [R/2, b)\}$ goes out of any compact subset of $\mathbb{R}^+ \times \mathbb{R} \times \mathbb{R}$.

However, by Lemma 4.1, $|u'(r)| < 1 - \sigma$ for $r \in [0, b)$, then

$$|u(r)| < b(1 - \sigma), \qquad r \in [0, b).$$

Therefore, the graph can not go out of the compact subset $[R/2, b] \times [-b(1-\sigma), b(1-\sigma)] \times [-b^{n-k}\phi^k(1-\sigma), b^{n-k}\phi^k(1-\sigma)]$ contained in the domain of $\mathcal{F}$. This is a contradiction. Therefore, $b = \infty$. □

**Proof of Theorem 4.2.** Proof of Theorem 4.2 is similar to that of Theorem 4.1, so we omit it. □

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