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Submanifolds of Cartan–Hartogs domains and complex Euclidean spaces

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ABSTRACT

We study the non-existence of common submanifolds of a complex Euclidean space and a Cartan–Hartogs domain equipped with their canonical metrics.

1. Introduction

The problem of holomorphic isometric embedding is a classical problem studied by many mathematicians. In a celebrated paper by Calabi [1], many deep results of a holomorphic isometry from a complex manifold into a complex space form have been obtained by using his diastasis functions. In particular, given two complex space forms with different curvature signs, Calabi proved that there does not exist local holomorphic isometric embedding between them with respect to the canonical Kähler metrics. Di Scala and Loi later generalized Calabi's non-embeddability result to Hermitian symmetric spaces of different types in [3].

Following Calabi's idea of diastasis, Umehara [9] studied the existence of common submanifolds of two Kähler manifolds and proved that two complex space forms with different curvature signs cannot have a common Kähler submanifold with the induced metrics. In [4], Di Scala and Loi called two Kähler manifolds are relatives when they share a common Kähler submanifold. They also proved that a bounded domain with its Bergman metric cannot be a relative to a projective algebraic manifold with the induced Fubini–Study

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metric. In fact, the result of Umehara [9] implies that the complex Euclidean space and a projective algebraic manifold with the induced Fubini–Study metric cannot be relatives. More recently, Huang and Yuan [6] proved that a complex Euclidean space and a Hermitian symmetric space of noncompact type cannot be relatives by using different argument. For related problems, the interested reader may refer to [2] and [12].

Cartan–Hartogs domains, introduced by Yin and Roos, are fiber bundles over classical domains in complex Euclidean spaces. They are natural generalizations of bounded symmetric domains and ellipsoids, but in general they are not homogeneous and their Bergman kernel functions are not rational. The Bergman kernels/metrics, proper holomorphic maps of Cartan–Hartogs domains have received considerable attentions thanks to the works by Loi–Zedda [7], Tu–Wang [8], Yin [10], etc. In [11], it was asked whether a complex Euclidean space and a Cartan–Hartogs domain can be relatives. We try to answer this question in this note.

Let the bounded symmetric domain Ω be the Harish-Chandra realization of an irreducible Hermitian symmetric space of noncompact type and let $N_\Omega(z,w)$ be its generic norm. The Cartan–Hartogs domain is defined as $M_\Omega(\mu) = \{(z,w) \in \Omega \times \mathbb{C}^N || w|^{2\mu} < N_\Omega(z,z)\}$, where N is a positive integer, μ is a positive real number. In particular, when Ω is one of the classic domains of Type I, II, III, IV, Yin [10] obtained the Bergman kernels $K_{M_\Omega(\mu)}$ as follows:

- When Ω is of Type I,

$$K_{M_\Omega(\mu)} = K_I(z,w,\bar{z},\bar{w}) = \mu^{-pq}\pi^{-(pq+N)}C(Y)det(I - z\bar{z}^T)^{-(p+q+\frac{N}{\mu})},$$

  where $z \in \Omega$ is a $m \times n$ matrix, $C(Y) = \sum_{i=0}^{pq+1} b_{1i}\Gamma(N+i)Y^{N+i}$, $b_{1i} \in \mathbb{R}$, $Y = (1-X)^{-1}$, $X = |w|^2(det(I - z\bar{z}^T))^{\frac{-1}{\mu}}$;
- When Ω is of Type II,

$$K_{M_\Omega(\mu)} = K_{II}(z,w,\bar{z},\bar{w}) = \mu^{\frac{-(p+1)}{2}}\pi^{-(\frac{p(p+1)}{2}+N)}C(Y)det(I - z\bar{z}^T)^{-(p+1+\frac{N}{\mu})},$$

  where $z \in \Omega$ is a $p \times p$ skew-symmetric matrix, $C(Y) = \sum_{i=0}^{\frac{p(p+1)}{2}+1} b_{2i}\Gamma(N+i)Y^{N+i}$, $b_{2i} \in \mathbb{R}$, Y and X are the same as above;
- When Ω is of Type III,

$$K_{M_\Omega(\mu)} = K_{III}(z,w,\bar{z},\bar{w}) = \mu^{\frac{-q(q-1)}{2}}\pi^{-(\frac{q(q-1)}{2}+N)}C(Y)det(I - z\bar{z}^T)^{-(q-1+\frac{N}{\mu})},$$

  where $z \in \Omega$ is a $q \times q$ skew-symmetric matrix, $C(Y) = \sum_{i=0}^{\frac{q(q-1)}{2}+1} b_{3i}\Gamma(N+i)Y^{N+i}$, $b_{3i} \in \mathbb{R}$, Y and X are the same as above;
- When Ω is of Type IV,

$$K_{M_\Omega(\mu)} = K_{IV}(z,w,\bar{z},\bar{w}) = \mu^{-n}\pi^{-(n+N)}C(Y)(1 + |zz^T|^2 - 2z\bar{z}^T)^{-(n+\frac{N}{\mu})},$$

  where $z \in \Omega \subset \mathbb{C}^n$, $C(Y) = \sum_{i=0}^{n+1} b_{4i}\Gamma(N+i)Y^{N+i}$, $b_{4i} \in \mathbb{R}$, Y is the same as above, $X = |w|^2(1 + |zz^T|^2 - 2z\bar{z}^T)^{\frac{-1}{\mu}}$.

For convenience, we consider $M_\Omega(\mu)$ as a subset in $\mathbb{C}^{pq+N}$ in the following sense, where $pq = p^2$ when Ω is of type II; $pq = q^2$ when Ω is of type III; $pq = n$ when Ω is of type IV. The Bergman metric on $M_\Omega(\mu)$ is given by $\omega_{M_\Omega(\mu)} = \sqrt{-1}\partial\bar{\partial}\log K_{M_\Omega(\mu)}$ up to a positive normalization constant. Assume that $D \subset \mathbb{C}^\kappa$ is a connected open set and ω_D is a Kähler metric on D which is not necessarily complete. We assume, without loss of generality, 0 is contained in D. The question raised in [11] asks whether there simultaneously exist holomorphic isometric immersions $F : (D, \omega_D) \to (\mathbb{C}^n, \omega_{\mathbb{C}^n})$ and $L : (D, \omega_D) \to (M_\Omega(\mu), \omega_{M_\Omega(\mu)})$. In this note, we show that there do not exist such immersions mapping 0 to 0.

Theorem 1.1. *Let $D \subset \mathbb{C}$ be a connected open subset. Suppose that $F : D \to \mathbb{C}^n$ and $L = (G, H) = (g_1, \cdots, g_{pq}, h_1, \cdots, h_N) : D \to M_\Omega(\mu)$ are holomorphic mappings with $L(0) = 0$ such that*

$$F^*\omega_{\mathbb{C}^n} = L^*\omega_{M_\Omega(\mu)} \quad on \quad D, \tag{1.1}$$

where $\omega_{\mathbb{C}^n}$ is the Euclidean metric on $\mathbb{C}^n$. Then F must be a constant map.

Corollary 1.2. *There does not exist a Kähler submanifold of a Cartan–Hartogs domain passing through zero that is also the submanifold of the complex Euclidean space.*

2. Proof of Theorem 1.1

We use the idea in [5,6] to prove Theorem 1.1. Let $F = (f_1, \cdots, f_n) : D \to \mathbb{C}^n$, $L = (G, H) = (g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z)) : D \to M_\Omega(\mu)$ be holomorphic maps satisfying the equation (1.1). Without loss of generality, assume that $0 \in D$ and $F(0) = 0$ by translation. We argue by contradiction by assuming that F is not constant.

By equation (1.1), we have

$$\partial\bar{\partial}(\sum_{i=1}^{n} |f_i(z)|^2) = \partial\bar{\partial}(\log K_{M_\Omega(\mu)}(G(z), H(z), \overline{G(z)}, \overline{H(z)})) \quad \text{for} \quad z \in D, \tag{2.1}$$

where $K_{M_\Omega(\mu)}(\xi, \overline{\eta}) = \sum_l h_l(\xi)\overline{h_l(\eta)}$ is the Bergman kernel on $M_\Omega(\mu)$ and $\{h_l(\xi)\}$ is an orthonormal basis of L^2 integrable holomorphic functions over M_Ω. Since $M_\Omega(\mu)$ is a complete circular domain, we know that $K_{M_\Omega(\mu)}(\xi, \overline{\eta})$ does not contain any nonconstant pure holomorphic terms in ξ, and any nonconstant pure anti-holomorphic terms in η. Hence $K_{M_\Omega(\mu)}(\xi, \overline{\xi})$ does not contain nonconstant pluriharmonic terms in ξ. After normalization, we can assume that $K_{M_\Omega(\mu)}(\xi, 0) = 1$. By the standard argument, one can get rid of $\partial\bar{\partial}$ in (2.1) to obtain the following functional identity by comparing the pure holomorphic and anti-holomorphic terms in z:

$$\sum_{i=1}^{n} |f_i(z)|^2 = \log K_{M_\Omega(\mu)}(G(z), H(z), \overline{G(z)}, \overline{H(z)}). \tag{2.2}$$

After polarization, (2.2) is equivalent to

$$\sum_{i=1}^{n} f_i(z)\bar{f}_i(w) = \log K_{M_\Omega(\mu)}(G(z), H(z), \overline{G}(w), \overline{H}(w))) \quad \text{for} \quad (z, w) \in D \times \text{conj}(D), \tag{2.3}$$

where $\bar{f}_i(w) = \overline{f_i(\overline{w})}$, $\bar{G}(w) = \overline{G(\overline{w})}$, $\bar{H}(w) = \overline{H(\overline{w})}$ and $\text{conj}(D) = \{z \in \mathbb{C} | \bar{z} \in D\}$.

We proceed in three steps to prove Theorem 1.1.

Step 1. We claim that for any $1 \le i \le n$, $f_i(z)$ can be written as a holomorphic polynomials of $L(z) = (G(z), H(z))$, shrinking D toward the origin if needed. Namely, there exist holomorphic polynomials $P_i(z, X)$, $i = 1, 2, \cdots, n$ such that $f_i(z) = P_i(z, L(z))$.

The proof is similar to the algebraic lemma in Proposition 3.1 of [5]. For the sake of completeness, we will give the details. In the following, we will denote the Bergman Kernels by $K_J(z, w, \overline{z}, \overline{w})$ $(J = I, II, III, IV)$ for the four types of Cartan–Hartogs domains. Applying the differentiation $\frac{\partial}{\partial w}$ to equation (2.3), for w near 0, we get the equations:

$$\sum_{i=1}^{n} f_i(z)\frac{\partial}{\partial w}\bar{f}_i(w) = \frac{\frac{\partial C(Y(z,w))}{\partial w}}{C(Y(z,w))} + \frac{k}{D(G(z),\overline{G}(w))}\frac{\partial D(G(z),\overline{G}(w))}{\partial w}, \tag{2.4}$$

$$C(Y(z,w)) = \sum_{i=0}^{h} b_i\Gamma(N+i)Y(z,w)^{(N+i)}, Y(z,w) = (1-X(z,w))^{-1}.$$

Here

- when $J = I$, $h = pq+1, b_i = b_{1i}, k = -(p+q+\frac{N}{\mu})$;
- when $J = II$, $h = \frac{p(p+1)}{2}+1, b_i = b_{2i}, k = -(p+1+\frac{N}{\mu})$;
- when $J = III$, $h = \frac{q(q-1)}{2}+1, b_i = b_{3i}, k = -(q-1+\frac{N}{\mu})$;
- when $J = IV$, $h = n+1, b_i = b_{4i}$, $k = -(n+\frac{N}{\mu})$.

In the first three cases,

$$X(z,w) = H(z)\overline{H}(w)^T(det(I-G(z)\overline{G}(w)^T))^{\frac{-1}{\mu}},$$
$$D(G(z),\overline{G}(w)) = det(I-G(z)\cdot\overline{G}(w)^T);$$

in the fourth case,

$$X(z,w) = H(z)\overline{H}(w)^T(1+G(z)G(z)^T\overline{G}(w)\overline{G}(w)^T - 2G(z)\overline{G}(w)^T)^{\frac{-1}{\mu}},$$
$$D(G(z),\overline{G}(w)) = 1+G(z)G(z)^T\overline{G}(w)\overline{G}(w)^T - 2G(z)\overline{G}(w)^T.$$

Since $L(0) = 0$, after simple calculation, we obtained that $D(G(z),\overline{G}(0)) \equiv 1$, $Y(z,0) \equiv 1$ and $C(Y(z,0)) = \sum_{i=0}^{h} b_i\Gamma(N+i)$. Moreover, $\frac{\partial D(G(z),\overline{G}(w))}{\partial w}\,|_{w=0}$ is a polynomial of $G(z)$ in all four cases. Denote $D^\delta = \frac{\partial^\delta}{\partial w^\delta}$. Then we can rewrite (2.4) as follows:

$$F(z)\cdot D^1(\bar{F}(w)) = \phi_1(w, g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z)), \tag{2.5}$$

where $\phi_1(w, g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z))$ is a holomorphic polynomial in L for fixed $w = 0$. Now, differentiating (2.5), we get for any δ the following equation

$$F(z)\cdot D^\delta(\bar{F}(w)) = \phi_\delta(w, g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z)). \tag{2.6}$$

Here for $\delta > 0$ and the fixed $w = 0$, $\phi_\delta(w, g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z))$ is a holomorphic polynomial in $g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z)$.

Now, let $\mathcal{L} := \mathrm{Span}_{\mathbb{C}}\{D^\delta(\bar{F}(w))|_{w=0}\}_{\delta\geq 1}$ be a vector subspace of $\mathbb{C}^n$. Let $\{D^{\delta_j}(\bar{F}(w))|_{w=0}\}_{j=1}^{\tau}$ be a basis for $\mathcal{L}$. Then for a small open disc Δ_0 centered at 0 in $\mathbb{C}$, $\bar{F}(\Delta_0) \subset \mathcal{L}$. Indeed, for any w near 0, we have from the Taylor expansion that

$$\bar{F}(w) = \bar{F}(0) + \sum_{\delta\geq 1}\frac{D^\delta(\bar{F})(0)}{\delta!}w^\delta = \sum_{\delta\geq 1}\frac{D^\delta(\bar{F})(0)}{\delta!}w^\delta \in \mathcal{L}.$$

Now, let ν_j $(j = 1\cdots, n-\tau)$ be a basis of the Euclidean orthogonal complement of $\mathcal{L}$. Then, we have

$$F(z)\cdot\nu_j = 0, \quad \text{for each} \;\; j = 1, \cdots, n-\tau. \tag{2.7}$$

Consider the system consisting of (2.6) at $w = 0$ (with $\delta = \delta_1, \cdots, \delta_\tau$) and (2.7). The coefficient matrix on the left hand side of the system at $w = 0$ with respect to $F(z)$ is

$$\begin{bmatrix} D^{\delta_1}(\bar{F}(w))|_{w=0} \\ \vdots \\ D^{\delta_\tau}(\bar{F}(w))|_{w=0} \\ \nu_1 \\ \vdots \\ \nu_{n-\tau} \end{bmatrix}$$

and is obviously invertible. Note that the right hand side of the system of equations consisting of (2.6) at $w = 0$ (with $\delta = \delta_1, \cdots, \delta_\tau$) is a holomorphic polynomial in $L = (G, H)$. By Cramer's rule, the $f_i(z)$ is a holomorphic polynomials in $(g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z))$.

Step 2. Assume all of the elements $g_1, \cdots, g_{pq}, h_1, \cdots, h_N$ are holomorphic Nash algebraic functions. Recall that a function H is called a holomorphic Nash algebraic function over $V \subset \mathbb{C}^\kappa$ if H is holomorphic over V and there is a non-zero holomorphic polynomial $P(\eta, X)$ in (η, X) such that $P(\eta, H(\eta)) \equiv 0$ for $\eta \in V$. By Step 1, we can write

$$f_i(z) = \hat{f}_i(g_1(z), \cdots, g_{pq}(z), h_1(z), \cdots, h_N(z)), \quad 1 \le i \le n,$$

where $\hat{f}_j(\cdot)$ is a holomorphic polynomial. Then $f_i(z)$ is a holomorphic Nash algebraic function. We consider the following equation

$$\exp(\sum_{i=1}^{n} f_i(z)\bar{f}_i(w)) = K_J(G(z), H(z), \overline{G}(w), \overline{H}(w)), \tag{2.8}$$

which is equivalent to (2.3). By Lemma 2.2 in [6], one can get that $F = (f_1, \cdots, f_n)$ is a constant map. Then Theorem 1.1 is proved.

Step 3. Suppose there exist some elements in $(g_1, \cdots, g_{pq}, h_1, \cdots, h_N)$, which are not Nash algebraic functions. Let $\mathfrak{R}$ be the field of rational functions in z over D. Consider the field extension

$$\mathfrak{F} = \mathfrak{R}(g_1, \cdots, g_{pq}, h_1, \cdots, h_N),$$

namely, the smallest subfield of the field of rational functions over D containing Nash algebraic functions and $g_1, \cdots, g_{pq}, h_1, \cdots, h_N$. In the following, we will write $L = (G, H) = (g_1, \cdots, g_{pq+N})$ for simplication, and let $\mathcal{G} = \{g_1(z), \cdots, g_l(z)\}$ be the maximal algebraic independent subset in $\mathfrak{F}$, thus the transcendence degree of $\mathfrak{F}/\mathfrak{R}(\mathcal{G})$ is 0. Therefore, there exists a small connected open subset U with $0 \in U$ such that for each j with $g_j \notin \mathcal{G}$, we have a holomorphic Nash algebraic function $\hat{g}_j(z, X_1, \cdots, X_l)$ in the neighborhood $\hat{U}$ of $\{(z, g_1(z), \cdots, g_l(z)) | z \in U\}$ in $\mathbb{C} \times \mathbb{C}^l$ such that $g_j(z) = \hat{g}_j(z, g_1(z), \cdots, g_l(z))$ for any $z \in U$.

It is easy to check that for fixed $w = 0$, $\frac{\partial \log K_J(G(z), H(z), \overline{G}(w), \overline{H}(w))}{\partial w}$ is a rational function in $(g_1, \cdots, g_{pq+N})$. In the first step, we have obtained that for each $1 \le i \le n$, $f_i(z)$ is a polynomial in $(g_1, \cdots, g_{pq+N})$. So there exists a holomorphic Nash algebraic function $\hat{f}_i(z, X_1, \cdots, X_l)$ in $\hat{U}$ such that $f_i(z) = \hat{f}_i(z, g_1(z), \cdots, g_l(z))$ for $z \in U$.

Now, we define

$$\Psi(z, X, w) = \sum_{i=1}^{n} \hat{f}_i(z, X)\bar{f}_i(w) - \log K_J(\cdots, X_\gamma, \cdots, \hat{g}_j(z, X), \cdots, \bar{g}_j(w), \cdots)$$

and

$$\Phi_k(z, X, w) = \frac{\partial}{\partial w^k} \Psi(z, X, w)$$

for $(z, X, w) \in \hat{U} \times \mathrm{conj}(U), k = 1, 2, \cdots,$ where $X = (X_1, \cdots, X_l)$.

For any $(z, X) \approx (0, 0)$, $w = 0$, it is easy to check that $\Phi_k(z, X, 0)$ is a holomorphic Nash algebraic function in (z, X) for every k. If $\Phi_k(z, X, 0)$ is nonzero and depends on X for some k, then there exists a holomorphic polynomial $P_k(z, X, t) = A_{d_k}(z, X)t^{d_k} + \cdots + A_{0_k}(z, X)$ of degree d_k in t, with $A_{0_k}(z, X) \not\equiv 0$ such that $P_k(z, X, \Phi_k(z, X, 0)) \equiv 0$. Since $\Psi(z, g_1(z), \cdots, g_l(z), w) \equiv 0$ for $z \in U$, thus $\Phi_k(z, g_1(z), \cdots, g_l(z), 0) \equiv 0$. It follows that $P_k(z, g_1(z), \cdots, g_l(z), \Phi_k(z, g_1(z), \cdots, g_l(z), 0)) = P_k(z, g_1(z), \cdots, g_l(z), 0) = 0$. Therefore, we obtain $A_{0_k}(z, g_1(z), \cdots, g_l(z)) \equiv 0$. This means that $\{g_1(z), \cdots, g_l(z)\}$ are algebraic dependent over $\mathfrak{R}$, which contradicts with the assumption. Therefore, $\psi(z, X, w) = h(z, w)$ which does not depend on X by Taylor expansion, where $h(z, w)$ is a Nash algebraic function in z.

Now, we have obtained the functional identity for any $(z, X, w) \in \hat{U} \times conj(U)$:

$$\sum_{i=1}^{n} \hat{f}_i(z, X)\bar{f}_i(w) = \log K_J(\cdots, X_\gamma, \cdots, \hat{g}_j(z, X), \cdots, \bar{g}_j(w), \cdots) + h(z, w).$$

As in Lemma 2.3 in [6], there exists $(z_0, w_0) \in U \times \mathrm{conj}(U)$ such that

$$\sum_{i=1}^{n} \hat{f}_i(z_0, X)\bar{f}_i(w_0) \not\equiv 0.$$

Therefore, $\sum_{i=1}^{n} \hat{f}_i(z, X)\bar{f}_i(w)$ is a nonconstant holomorphic Nash algebraic function in X. Now we consider the following equation

$$\sum_{i=1}^{n} \hat{f}_i(z_0, X)\bar{f}_i(w_0) = \log K_J(X, \hat{g}_{l+1}(z_0, X), \cdots, \hat{g}_{pq+N}(z_0, X), \overline{g_1}(w_0), \cdots, \overline{g_{pq+N}}(w_0)) + h(z_0, w_0).$$

Then one can get a contradiction with the similar argument in Lemma 2.2 in [6].

Combining the above three steps, we can show that F must be a constant map. The proof of Theorem 1.1 is complete.

3. Generalization

Let $\Omega_j (j = 1, 2, \cdots, m)$ be classic domains and the corresponding Cartan–Hartogs domain be $M_{\Omega_j}(\mu_j)$ with the Bergman metric $\omega_{M_{\Omega_j}(\mu_j)}$. Indeed, we can prove the following slightly more general result by the same argument:

Theorem 3.1. *Let $D \subset \mathbb{C}$ be a connected open subset. Suppose that $F : D \to \mathbb{C}^n$ and $L = (G_1, \cdots, G_m) : D \to M_\Omega = M_{\Omega_1} \times \cdots \times M_{\Omega_m}$ are holomorphic mappings with $L(0) = 0$ such that*

$$F^*\omega_{\mathbb{C}^n} = \sum_{j=1}^{m} \nu_j G_j^* \omega_{M_{\Omega_j}(\mu_j)} \quad on \quad D \tag{3.1}$$

for certain positive constants $\nu_1, \cdots, \nu_m$. Then F must be a constant map. Furthermore, if all $\mu_j's$ are positive, then G is also a constant map.

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