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# Solutions for a Hydrodynamic Model of Semiconductors[1]

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We study a three-dimensional isothermal hydrodynamic model of semiconductors and prove the existence of local solutions to the initial boundary value problem corresponding to an insulated boundary. © 2000 Academic Press

*Key Words:* semiconductor; hydrodynamic; initial boundary value problem.

## 1. INTRODUCTION

The hydrodynamic model for semiconductor devices is a generalization of the drift diffusion model. It consists of a set of Euler equations with certain source terms and a Poisson equation for electrical potential; see [9]. After appropriate scaling, the three-dimensional system can be written as

$$\begin{aligned} &\rho_t + \nabla \cdot (\rho \mathbf{u}) = 0, \\ &\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} + \frac{1}{\rho}\nabla p - \nabla \phi = -\frac{\mathbf{u}}{\tau}, \\ &\Delta \phi = \rho - I(x), \end{aligned} \tag{1.1}$$

where $\rho > 0$ is the electron density, $\mathbf{u} = (u, v, w)$ the electron velocity, and $\phi$ the electrical potential. The constant $\tau > 0$ is the velocity relaxation time, and $I(x)$ is the given smooth density of positive background ions (doping profile). $p = p(\rho)$ is a given function with $p'(\rho) > 0$.

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The hydrodynamic model of semiconductors has been studied in the literature. A general mathematical survey of the model can be found in [9]. In the case of one space dimension, the existence of steady-state solutions in the transonic case was proved in [4]. For the time-dependent system, the existence of a local smooth solution was proved by using Lagrangian coordinates in [13], and the global existence of weak solutions was established in [14] by the Gudunov scheme and compensated compactness technique, with $p(\rho) = C_p \rho^\gamma$, $1 < \gamma < 5/3$. For $\gamma = 1$, the existence of global solutions for the isothermal hydrodynamic–elliptic system was obtained in [10] by entropy methods. The existence of weak solutions and their relation with the drift diffusion equations was studied in [8]. In the multi-dimensional case, the only result is the existence of a steady-state solution in [2].

In this paper, we will study the non-steady-state solutions of the model in the three-dimensional space and show the existence of local solutions to the initial boundary value problem.

We assume that at the initial time $t = 0$ the distribution of $(\rho, \mathbf{u})$ is known

$$(\rho, \mathbf{u})|_{t=0} = (\rho_0, \mathbf{u}_0)(x), \qquad x \in \Omega, \tag{1.2}$$

where $\Omega$ is a bounded domain in $R^3$ with smooth boundary.

At the boundary $\partial\Omega$, we assume that

$$\mathbf{u} \cdot \nu|_{\partial\Omega} = 0, \tag{1.3}$$

$$\nabla\phi \cdot \nu|_{\partial\Omega} = 0, \tag{1.4}$$

where $\nu$ is the outward unit vector normal to the boundary $\partial\Omega$. Physically, (1.3) means that the walls are streamlines and there is no current flow through $\partial\Omega$, and (1.4) is a Neumann condition for the electrical potential $\phi$ which means that the electrical field is tangential to $\partial\Omega$. That is, the semiconductor device is totally insulated; see [5] for details.

From a mathematical point of view, we can also consider other boundary conditions for the potential $\phi$, such as the Dirichlet boundary condition. It turns out that the problem is much easier for those other standard boundary conditions than for the Neumann condition, because the existence of a solution for the Poisson equation with Neumann condition requires extra compatibility.

We consider the initial boundary value problem

$$\rho_t + (\rho u)_{x_1} + (\rho v)_{x_2} + (\rho w)_{x_3} = 0,$$

$$\rho u_t + \rho u u_{x_1} + \rho v u_{x_2} + \rho w u_{x_3} + p'(\rho)\rho_{x_1} = \rho\left(\phi_{x_1} - \frac{u}{\tau}\right),$$

$$\rho v_t + \rho u v_{x_1} + \rho v v_{x_2} + \rho w v_{x_3} + p'(\rho)\rho_{x_2} = \rho\left(\phi_{x_2} - \frac{v}{\tau}\right),$$

$$\rho w_t + \rho u w_{x_1} + \rho v w_{x_2} + \rho w w_{x_3} + p'(\rho)\rho_{x_3} = \rho\left(\phi_{x_3} - \frac{w}{\tau}\right),$$

$$\Delta\phi = \rho - I(x), \tag{1.5}$$

$$(\rho, u, v, w)|_{t=0} = (\rho_0, u_0, v_0, w_0), \tag{1.6}$$

$$(u, v, w)\cdot\nu|_{\partial\Omega} = 0, \tag{1.7}$$

$$\nabla\phi\cdot\nu|_{\partial\Omega} = 0. \tag{1.8}$$

Introduce the vector $U = (\rho, u, v, w)$ and the matrices

$$A_0(U) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{\rho^2}{p'} & 0 & 0 \\ 0 & 0 & \dfrac{\rho^2}{p'} & 0 \\ 0 & 0 & 0 & \dfrac{\rho^2}{p'} \end{pmatrix}, \quad A_1(U) = \begin{pmatrix} u & \rho & 0 & 0 \\ \rho & \dfrac{\rho^2 u}{p'} & 0 & 0 \\ 0 & 0 & \dfrac{\rho^2 u}{p'} & 0 \\ 0 & 0 & 0 & \dfrac{\rho^2 u}{p'} \end{pmatrix},$$

$$A_2(U) = \begin{pmatrix} v & 0 & \rho & 0 \\ 0 & \dfrac{\rho^2 v}{p'} & 0 & 0 \\ \rho & 0 & \dfrac{\rho^2 v}{p'} & 0 \\ 0 & 0 & 0 & \dfrac{\rho^2 v}{p'} \end{pmatrix}, \tag{1.9}$$

$$A_3(U) = \begin{pmatrix} w & 0 & 0 & \rho \\ 0 & \dfrac{\rho^2 w}{p'} & 0 & 0 \\ 0 & 0 & \dfrac{\rho^2 w}{p'} & 0 \\ \rho & 0 & 0 & \dfrac{\rho^2 w}{p'} \end{pmatrix}.$$

Then the problem (1.5)–(1.8) can be rewritten as

$$\begin{gathered} A_0(U)\,\partial_t U + A_1(U)\,\partial_{x_1} U + A_2(U)\,\partial_{x_2} U + A_3(U)\,\partial_{x_3} U = H_1(U, D\phi), \\ \Delta\phi = H_2(U), \\ U(x,0) = U_0(x), \\ (u,v,w)\cdot\nu|_{\partial\Omega} = 0, \qquad \nabla\phi\cdot\nu|_{\partial\Omega} = 0, \end{gathered} \tag{1.10}$$

where $H_1$ and $H_2$ are sufficient smooth functions of their arguments in the relevant domain. In particular, the first four equations form a hyperbolic symmetric system for $U$.

Because the boundary $\partial\Omega$ is characteristic with respect to the hyperbolic system (1.10) under the boundary condition (1.7), we need to use the partially tangential Sobolev spaces.

Let $D_\sigma$ be a complete set of differential operators which are tangent to the boundary $\partial\Omega$ (see [3]), and let $D$ be the usual differential operators which are in all directions at $\partial\Omega$. For an integer $m \geq 0$, define the space

$$E_m \equiv E_m(\Omega) = \{W \in L^2(\Omega);\ D_\sigma^i D^j W \in L^2,\ i + 2j \leq m\}, \tag{1.11}$$

with norm

$$\|W\|_m^2 = \sum_{i+2j\leq m} \int_\Omega |D_\sigma^i D^j W|^2\, dx, \tag{1.12}$$

and the space

$$E_{m,T} = \{W(t,x);\ \partial_t^s W \in L^\infty([0,T], E_{m-s}),\ 0 \leq s \leq m\}, \tag{1.13}$$

with norm

$$|||\, W \,|||_{m,T} = \sup_{0\leq t\leq T} \left( \sum_{s=0}^{m} \|\partial_t^s W(t)\|_{m-s}^2 \right)^{1/2}. \tag{1.14}$$

For an even number $2m$, spaces $E_{2m}$ have the following embedding property [1]:

$$E_{2m}(\Omega) \subset C^j(\Omega), \qquad m \geq j + 2.$$

In this paper, we prove the existence of a classical solution for the initial boundary value problem (1.10) in the space $E_{m,T}$ with $m \geq 4$ and $T \ll 1$.

The existence of classical solutions always requires compatibility conditions at $(t,x) \in \{0\} \times \partial\Omega$ (refer to [6]).

The initial data $U_0$ are called $k$th-order compatible if there is an approximate solution $(\tilde{U}, \tilde{\phi})$, such that $(\tilde{U}, \tilde{\phi})$ satisfies the initial and boundary conditions (1.6)–(1.8) and such that

$$\begin{aligned} A_0(\tilde{U})\,\partial_t\tilde{U} + A_1(\tilde{U})\,\partial_{x_1}\tilde{U} + A_2(\tilde{U})\,\partial_{x_2}\tilde{U} + A_3(\tilde{U})\,\partial_{x_3}\tilde{U} - H_1\big(\tilde{U}, D\tilde{\phi}\big) \\ = \tilde{F}_1 = O(t^{k-1}), \\ \Delta\tilde{\phi} - H_2(\tilde{U}) = \tilde{F}_2 = O(t^{k-1}). \end{aligned} \tag{1.15}$$

The Neumann condition (1.4) implies that we need to solve the following Neumann problem of the Poisson equation with parameter $t$:

$$\begin{aligned} \Delta\phi &= \rho - I(x), \\ \nabla\phi \cdot \nu|_{\partial\Omega} &= 0. \end{aligned} \tag{1.16}$$

Problem (1.16) is solvable if and only if (see, e.g., [12])

$$\int_\Omega (\,\rho - I(x))\,dx = 0. \tag{1.17}$$

In particular, (1.17) must be satisfied by the initial value $\rho_0$.

Now we are ready to state the main theorem of the paper as follows:

THEOREM 1.1. *Assume that*:

1. $U_0 = (\,\rho_0, u_0, v_0, w_0) \in C^\infty(\overline{\Omega})$.

2. $U_0$ *is compatible up to order* $2m$ *with integer* $m > 3$; *i.e., there exists sufficient smooth functions* $\tilde{U}$, $\tilde{\phi}$ *such that* (1.15) *is satisfied.*

3. *The initial value* $\rho_0$ *satisfies* (1.17); *i.e.*,

$$\int_\Omega \rho_0\,dx = \int_\Omega I(x)\,dx.$$

*Then there exists* $T > 0$ *such that* (1.5)–(1.8) *have a classical solution* $(U, \phi)$, *with*

$$U \in C^1([0,T] \times \Omega), \qquad \phi \in C\big([0,T], C^2(\Omega)\big),$$

*and* $U, D\phi \in E_{2m,T}$.

*Remark* 1.1. $U_0 \in C^\infty(\overline{\Omega})$ is assumed to simplify the discussion (see [7]) and can be obviously relaxed.

*Remark* 1.2. The solution $\phi$ for (1.16) is unique only up to a constant, and problem (1.10) contains only $D\phi$. We will consider only the norms for $D\phi$ in the paper.

The plan of the paper is as follows. In Section 2, we linearize the problem (1.10) and derive the energy estimate. And then, in Section 3, we use the linear iteration to establish the existence of the solution. The appearance of condition (1.17) is a completely new feature in the iteration. We first prove that (1.17) at $t = 0$ implies (1.17) for all $t \in [0, T]$, and then we introduce an error term $e_k$ at every step in the iteration scheme to validate (1.17). Finally, we show that $e_k$ converges to 0.

## 2. LINEARIZED PROBLEM AND *A PRIORI* ESTIMATE

Let $(V, \psi)$ denote the perturbations of $(U, \phi)$ and consider the linearized problem of (1.10) for $(V, \psi)$ at $(U, \phi)$

$$A_0(U)\,\partial_t V + A_1(U)\,\partial_{x_1} V + A_2(U)\,\partial_{x_2} V + A_3(U)\,\partial_{x_3} V = F_1,$$

$$\Delta\psi = F_2, \tag{2.1}$$

$$V|_{t=0} = 0, \tag{2.2}$$

$$(V_2, V_3, V_4)\cdot \nu|_{\partial\Omega} = 0, \tag{2.3}$$

$$\nabla\psi\cdot \nu|_{\partial\Omega} = 0, \tag{2.4}$$

where

$$F_1 \in E_{2m,T}, \qquad F_2 \in E_{2m,T}$$

and

$$\partial_t^j F_1|_{t=0} = 0, \qquad 0 \le j \le 2m - 1. \tag{2.5}$$

It follows from (2.2), (2.1), and (2.5) that the smooth solution $V(t, x)$ satisfies

$$\partial_t^j V|_{t=0} = 0, \qquad 0 \le j \le 2m. \tag{2.6}$$

THEOREM 2.1. *Assume that*

$$\int_\Omega F_2\,dx = 0 \tag{2.7}$$

*for all* $t \geq 0$. *Then there exists a* $\delta > 0$ *such that, for all* $|||U - U_0|||_{2m,T} < \delta$, *the solution* $(V, \psi)$ *of* (2.1)–(2.4) *satisfies the energy estimates*

$$|||V|||^2_{2m,T} \leq C_1 T |||F_1|||^2_{2m,T}, \tag{2.8}$$

$$|||D\psi|||^2_{2m,T} \leq C_2 |||F_2|||^2_{2m,T}, \tag{2.9}$$

*where the constants* $C_1$ *and* $C_2$ *depend only on* $\delta$.

*Proof.* We cover $\overline{\Omega}$ by a finite family of open sets $\mathscr{U}_i$, so that either $\mathscr{U}_i \subset\subset \Omega$ or $\mathscr{U}_i \cap \overline{\Omega}$ is $C^1$ diffeomorphic to $\{x_1 \geq 0\} \cap \{|x| < 1\}$ with $\partial\Omega$ mapping to $\{x_1 = 0\}$.

Choose a finite partition of unity $1 = \sum \varphi_i$ subordinate to this cover and let $(V, \psi) = \sum \varphi_i (V, \psi) = \sum (V_i, \psi_i)$. Changing coordinates in the boundary patches yields functions $(V, \psi) \circ \chi_i$ defined on $\{x_1 \geq 0\} \cap \{|x| < 1\}$, which we continue to call $(V_i, \psi_i)$, where $\chi_i$ are diffeomorphisms.

Thus, by the standard localization technique, it suffices to derive the estimate for each $(V, \psi)$ supported in a small coordinate patch. The estimate for an interior patch $\mathscr{U}_i \subset\subset \Omega$ is straightforward since there is no boundary conditions. We will consider only the boundary patches. A change of independent variables reduces the problem to the case (refer to [11])

$$\Omega = R^3_+, \qquad \operatorname{supp}(V, \psi) \subset \{|x| < 1 \text{ and } x_1 \geq 0\}, \qquad \partial\Omega = \{x;\ x_1 = 0\}.$$

In this case, $D_\sigma$ is generated by $x_1 \partial_{x_1}$, $\partial_{x_2}$, and $\partial_{x_3}$ (refer to [3]). Since $D^i_\sigma$ is a linear combination of $x_1 \partial_{x_1}, x_1^2 \partial^2_{x_1}, \ldots, x_1^i \partial^i_{x_1}$, $\partial_{x_2}, \partial^2_{x_2}, \ldots, \partial^i_{x_2}$, and $\partial_{x_3}, \partial^2_{x_3}, \ldots, \partial^i_{x_3}$, it suffices to prove

$$\sum_{k+r+s+2j-l \leq 2m,\ l \leq j} \|x_1^l \partial^j_{x_1} \partial^k_{x_2} \partial^r_{x_3} \partial^s_t V\|^2_0 \leq CT |||F_1|||^2_{2m,T} \tag{2.10}$$

and

$$\sum_{k+r+s+2j-l \leq 2m,\ l \leq j} \|x_1^l \partial^j_{x_1} \partial^k_{x_2} \partial^r_{x_3} \partial^s_t D\psi\|^2_0 \leq C |||F_2|||^2_{2m,T}. \tag{2.11}$$

Since $V$ and $\psi$ in (2.1) are not coupled, (2.10) and (2.11) can be established separately.

*Proof of* (2.11). Applying $\partial^s_t$ to (2.1) and (2.4), we obtain

$$\Delta \partial^s_t \psi = \partial^s_t F_2,$$

$$\nabla(\partial^s_t \psi) \cdot \nu|_{\partial\Omega} = 0.$$

This is the Neumann problem for the Poisson equation. By assumption (2.7) on $F_2$, there is a solution unique up to a constant such that $\partial_t^s D\phi$ satisfies (see [12])

$$\sum_{s\le 2m} \|\partial_t^s D\psi\|_0^2 \le C \sum_{s\le 2m} \|\partial_t^s F_2\|_0^2 \le C \,|||\, F_2 \,|||\,_{2m,T}^2. \tag{2.12}$$

For the special case $j = 0$, (2.11) is valid just as in (2.12) by applying the operator $\partial_{x_2}^k \partial_{x_3}^r \partial_t^s$ to (2.1) and (2.4). And we have the same estimate as in (2.12)

$$\sum_{k+r+s\le 2m} \|\partial_{x_2}^k \partial_{x_3}^r \partial_t^s D\psi\|_0^2 \le C \,|||\, F_2 \,|||\,_{2m,T}^2. \tag{2.13}$$

The estimate for normal derivatives $\partial_{x_1}^j$ can then be obtained obviously from the differential equation (2.1). This concludes the proof of (2.11).

*Proof of* (2.10). We prove (2.10) by induction on $i = j - l$ and $0 \le i \le m$.

- For $i = 0$, let $V_{jksr} = x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V$. Then, from (2.1), $V_{jksr}$ satisfies

$$A_0 \partial_t V_{jksr} + A_1 \partial_{x_1} V_{jksr} + A_2 \partial_{x_2} V_{jksr} + A_3 \partial_{x_3} V_{jksr} = F_{jksr}, \tag{2.14}$$

where

$$\begin{aligned} F_{jksr} = {} & A_0 x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} F_1\big) \\ & + \Big\{ A_1 \partial_{x_1} V_{jksr} - A_0 x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} A_1 \partial_{x_1} V\big) \Big\} \\ & + \Big\{ A_2 \partial_{x_2} V_{jksr} - A_0 x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} A_2 \partial_{x_2} V\big) \Big\} \\ & + \Big\{ A_3 \partial_{x_3} V_{jksr} - A_0 x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} A_3 \partial_{x_3} V\big) \Big\}. \end{aligned} \tag{2.15}$$

Taking the inner product of (2.14) over $\Omega$ with $V_{jksr}$ and then integrating by parts using the fact that $\operatorname{supp} V \subset \{|x| < 1,\ x_1 \ge 0\}$ yields

$$\begin{aligned} & \partial_t \langle V_{jksr}, A_0 V_{jksr} \rangle - \langle V_{jksr}, A_1 V_{jksr} \rangle|_{x_1 = 0} \\ & \quad = \langle V_{jksr}, \big( (\partial_t A_0) + (\partial_{x_1} A_1) + (\partial_{x_2} A_2) + (\partial_{x_3} A_3) \big) V_{jksr} \rangle \\ & \qquad + 2 \langle F_{jksr}, V_{jksr} \rangle. \end{aligned} \tag{2.16}$$

Since $u = 0$ at $x_1 = 0$ in $A_1$ of (1.9), we have

$$\langle V_{jksr}, A_1 V_{jksr} \rangle|_{x_1 = 0} = 2\rho x_1^{2j} \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V_1 \cdot \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V_2|_{x_1 = 0},$$

which is obviously 0 if $j > 0$. If $j = 0$, since (2.3) is simply $V_2|_{x_1=0} = 0$, $\partial_{x_2}^k \partial_{x_3}^r \partial_t^s V_2|_{x_1=0} = 0$. Therefore, we also have $\langle V_{jksr}, A_1 V_{jksr}\rangle|_{x_1=0} = 0$. Therefore, from (2.16), we have

$$\partial_t \|V_{jksr}(t)\|_0^2 \le C\|V_{jksr}(t)\|_0^2 + \|F_{jksr}(t)\|_0^2. \tag{2.17}$$

Here and in the following, we will use $C$ to denote any constant which is independent of $t$ for $t \le T$.

By Gronwall's inequality and using the fact that $V_{jksr}|_{t=0} = 0$, we obtain

$$\|V_{jksr}(t)\|_0^2 \le C\int_0^t \|F_{jksr}(\tau)\|_0^2 \, d\tau. \tag{2.18}$$

Now we need to estimate $\|F_{jksr}(t)\|_0$. $A_0^{-1}$ is bounded in the expression (2.15) for $\rho_0 \ge \delta_1 > 0$ and $\delta \ll 1$. So it is easy to obtain the estimates

$$\begin{aligned}
&\|x_1^j A_0 \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} F_1\big)\|_0^2 \le C \,|||\, F_1 \,|||\,_{2m,T}^2,\\
&\|A_2 \partial_{x_2} V_{jksr} - A_0 x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} A_2 \partial_{x_2} V\big)\|_0^2\\
&\qquad \le C \sum_{j+k+r+s\le 2m} \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V(t)\|_0^2,\\
&\|A_3 \partial_{x_3} V_{jksr} - A_0 x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s \big(A_0^{-1} A_3 \partial_{x_3} V\big)\|_0^2\\
&\qquad \le C \sum_{j+k+r+s\le 2m} \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V(t)\|_0^2.
\end{aligned}$$

The second term of (2.15) consists of two kinds of terms

$$W_1 \equiv x_1^j \partial_{x_1}^{j_1} \partial_{x_2}^{k_1} \partial_{x_3}^{r_1} \partial_t^{s_1} V, \qquad \text{where } j_1 \le j, \tag{2.19}$$

and

$$W_2 \equiv x_1^j \partial_{x_1}^{j+1} \partial_{x_2}^{k_2} \partial_{x_3}^{r_2} \partial_t^{s_2} V, \qquad \text{where } s_2 + k_2 + r_2 + j \le 2m - 1. \tag{2.20}$$

Obviously, we have

$$\|W_1\|_0^2 \le \sum_{j+k+r+s\le 2m} \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V\|_0. \tag{2.21}$$

To estimate $W_2$, we notice that

$$A_0^{-1}A_1 = \begin{pmatrix} u & \rho & 0 & 0 \\ \dfrac{p'}{\rho} & u & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & u \end{pmatrix}, \qquad A_0^{-1}A_2 = \begin{pmatrix} v & 0 & \rho & 0 \\ 0 & v & 0 & 0 \\ \dfrac{p'}{\rho} & 0 & v & 0 \\ 0 & 0 & 0 & v \end{pmatrix},$$

$$A_0^{-1}A_3 = \begin{pmatrix} w & 0 & 0 & \rho \\ 0 & w & 0 & 0 \\ 0 & 0 & w & 0 \\ \dfrac{p'}{\rho} & 0 & 0 & w \end{pmatrix}.$$

and $u = 0$ on $x_1 = 0$. Therefore, the last two components of vector $W_2$ can be controlled by

$$\sum_{j+k+r+s\le 2m} \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V\|_0.$$

For the first two components of $W_2$, we have

$$\begin{aligned} &\partial_t V_1 + u\partial_{x_1} V_1 + \rho\partial_{x_1} V_2 + \big(\text{terms of } \partial_{x_2} V \text{ and } \partial_{x_3} V\big) = \big(A_0^{-1}F_1\big)_1, \\ &\partial_t V_2 + \frac{p'}{\rho}\partial_{x_1} V_1 + u\partial_{x_1} V_2 + \big(\text{terms of } \partial_{x_2} V \text{ and } \partial_{x_3} V\big) = \big(A_0^{-1}F_1\big)_2. \end{aligned} \tag{2.22}$$

Since $p' > 0$, $\rho > 0$, and $u|_{x_1=0} = 0$, $\partial_{x_1} V_1$ and $\partial_{x_1} V_2$ can be expressed in terms of $\partial_t V$, $\partial_{x_2} V$, $\partial_{x_3} V$, and $F_1$ from (2.22).

Thus, the first two components of $W_2$ can be expressed as linear combinations of

$$x_1^j \partial_{x_1}^j \partial_{x_2}^{k_2+1} \partial_{x_3}^{r_2} \partial_t^{s_2} V,\ x_1^j \partial_{x_1}^j \partial_{x_2}^{k_2} \partial_{x_3}^{r_2+1} \partial_t^{s_2} V,\ x_1^j \partial_{x_1}^j \partial_{x_2}^{k_2} \partial_{x_3}^{r_2} \partial_t^{s_2+1} V,\ x_1^j \partial_{x_1}^j \partial_{x_2}^{k_2} \partial_{x_3}^{r_2} \partial_t^{s_2} F_1,$$

which in turn can be controlled by

$$\|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V\|_0^2,\ \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s F_1\|_0^2.$$

Therefore, the first two components of $W_2$ are controlled by

$$\|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V\|_0,\ \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s F_1\|_0.$$

Summarizing the estimates of all terms of (2.15), we obtain

$$\|F_{jksr}\|_0^2 \le C\left( |||F_1|||_{2m,T}^2 + \sum_{j+k+r+s\le 2m} \|x_1^j \partial_{x_1}^j \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V\|_0^2 \right). \quad (2.23)$$

So, from (2.18) and (2.23), we have

$$\sum_{j+k+r+s\le 2m} \|V_{jksr}(t)\|_0^2$$
$$\le C\int_0^T \left( |||F_1|||_{2m,T}^2 + \sum_{j+k+r+s\le 2m} \|V_{jksr}(\tau)\|_0^2 \right) d\tau.$$

Again, by Gronwall's inequality,

$$\sum_{j+k+r+s\le 2m} \|V_{jksr}(t)\|_0^2 \le C\int_0^T |||F_1|||_{2m,T}^2 \, dz = CT\,|||F_1|||_{2m,T}^2. \quad (2.24)$$

This finishes the proof of (2.10) for $i = j - l = 0$.

• Assuming (2.10) for $0, 1, 2, \ldots, i-1$, we proceed to prove (2.10) for $i$.

Apply the operator $x_1^l \partial_{x_1}^{l+i-1} \partial_{x_2}^k \partial_{x_3}^r \partial_t^s$ to the first two equations of (2.1). Then the first two components of

$$x_1^l \partial_{x_1}^{l+i} \partial_{x_2}^k \partial_{x_3}^r \partial_t^s V \quad (2.25)$$

can be expressed as linear combinations of the following terms:

$$x_1^l \partial_{x_1}^{l+i-1} \partial_{x_2}^{k+1} \partial_{x_3}^r \partial_t^s V,\ x_1^l \partial_{x_1}^{l+i-1} \partial_{x_2}^k \partial_{x_3}^{r+1} \partial_t^s V,$$
$$x_1^l \partial_{x_1}^{l+i-1} \partial_{x_2}^k \partial_{x_3}^r \partial_t^{s+1} V,\ x_1^l \partial_{x_1}^{l+i-1} \partial_{x_2}^k \partial_{x_3}^r \partial_t^s F_1.$$

By the induction assumption, the first three terms can be controlled by $T\,|||F_1|||_{2m,T}$. Since $F = O(t^{2m-1})$, the last term is controlled by $t\,|||x_1^l \partial_{x_1}^{l+i-1} \partial_{x_2}^k \partial_{x_3}^r \partial_t^{s+1} F_1|||_{0,t}$, which in turn is controlled by $T\,|||F_1|||_{2m,T}$.

Applying $x_1^l \partial_{x_1}^{l+i} \partial_{x_2}^k \partial_{x_3}^r \partial_t^s$ to the last two equations of (2.1),

$$\begin{aligned} &\partial_t V_3 + u\partial_{x_1} V_3 + v\partial_{x_2} V_3 + \frac{p'}{\rho}\partial_{x_3} V_1 + w\partial_{x_3} V_3 = \frac{p'}{\rho^2}(F_1)_3, \\ &\partial_t V_4 + u\partial_{x_1} V_4 + v\partial_{x_2} V_4 + \frac{p'}{\rho}\partial_{x_3} V_1 + w\partial_{x_3} V_4 = \frac{p'}{\rho^2}(F_1)_4, \end{aligned} \quad (2.26)$$

we obtain

$$\begin{aligned}(\partial_t + u\partial_{x_1} + v\partial_{x_2} + w\partial_{x_3})x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_3 = G_3,\\ (\partial_t + u\partial_{x_1} + v\partial_{x_2} + w\partial_{x_3})x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_4 = G_4,\end{aligned} \tag{2.27}$$

where $G_3$ and $G_4$ consist of

$$x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s\left(\frac{p'}{\rho^2}F_1\right),\ x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s\left(\frac{p'}{\rho^2}\partial_{x_2}V_1\right),$$

$$\left[x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s, v\partial_{x_2}\right]V_{3,4},\ \left[x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s, w\partial_{x_3}\right]V_{3,4},$$

$$\left[x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s, u\partial_{x_1}\right]V_{3,4}.$$

(2.27) is a system for $x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_3$ and $x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_4$. Since $u = 0$ at $x_1 = 0$, we can derive an energy estimate for its solution by integration by parts without incurring any boundary terms:

$$\|x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_{3,4}(t)\|_0 \le CT\|G_{3,4}\|_{0,T}. \tag{2.28}$$

Now let us estimate the terms in $G_3$ and $G_4$. First of all, we obviously have

$$\left\|x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s\left(\frac{p'}{\rho^2}F_1\right)\right\|_0 \le C|||F_1|||_{2m,T},$$

$$\left\|x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s\left(\frac{p'}{\rho^2}\partial_{x_2}V_1(t)\right)\right\|_0 \le CT|||F_1|||_{2m,T}.$$

$[x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s, v\partial_{x_2}]V_{3,4}$ and $[x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s, w\partial_{x_3}]V_{3,4}$ are either standard lower-order terms of (2.28) or the terms with $x$-derivative one order lower, which are estimated by the induction assumption.

The term $[x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s, u\partial_{x_1}]V_{3,4}$ is a linear combination of the following terms:

$$x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_{3,4},\ x_1^l\partial_{x_1}^{l+i+1}\partial_{x_2}^{k-1}\partial_{x_3}^r\partial_t^s V_{3,4},$$

$$x_1^l\partial_{x_1}^{l+i+1}\partial_{x_2}^k\partial_{x_3}^{r-1}\partial_t^s V_{3,4},\ x_1^l\partial_{x_1}^{l+i+1}\partial_{x_2}^k\partial_{x_3}^r\partial_t^{s-1} V_{3,4}.$$

The first term is a standard lower-order term of (2.27); the other terms can be estimated through another induction on $k + r + s \ge 0$. Consequently, from (2.28) we have

$$\|x_1^l\partial_{x_1}^{l+i}\partial_{x_2}^k\partial_{x_3}^r\partial_t^s V_{3,4}\|_0 \le CT|||F_1|||_{2m,T}. \tag{2.29}$$

This proves (2.10) for $i$ and completes the proof of Theorem 2.1.

## 3. SOLUTION TO THE NON-LINEAR PROBLEM

In this section, we will use the linear iteration to establish the existence of a solution for (1.10) and conclude the proof of the main theorem 1.1. The solution is constructed through the iteration scheme

$$A_0(U^k)\,\partial_t U^{k+1} + \sum_{j=1}^{3} A_j(U^k)\,\partial_{x_j} U^{k+1} = H_1(U^k, D\phi^k),$$

$$\Delta\phi^{k+1} = H_2(U^k) - e_k, \tag{3.1}$$

$$U^{k+1}(0, x_1, x_2, x_3) = U_0(x_1, x_2, x_3), \tag{3.2}$$

$$\left(U_2^{k+1}, U_3^{k+1}, U_4^{k+1}\right)\cdot \nu|_{\partial\Omega} = 0, \tag{3.3}$$

$$\nabla\phi^{k+1}\cdot \nu|_{\partial\Omega} = 0 \tag{3.4}$$

for $k = 0, 1, 2, \ldots,$ with

$$U^0 = \tilde{U}, \qquad \phi^0 = \tilde{\phi}, \tag{3.5}$$

with $(\tilde{U}, \tilde{\phi})$ being the approximate solution.

The term $e_k$ in (3.1) is taken as

$$e_k = \frac{1}{|\Omega|}\int_\Omega \left(U_1^k(x) - I(x)\right) dx, \qquad k = 0, 1, 2, \ldots, \tag{3.6}$$

so that the initial boundary value problem for $\phi^{k+1}$ is well-posed (up to a constant).

For all $U^k$ such that $|||\, U^k - \tilde{U}\,|||_{2m,T} < \delta$, the problem (3.1)–(3.4) can be solved in $[0, T]$ with

$$\partial_t^j (U^{k+1} - \tilde{U})|_{t=0} = 0, \qquad 0 \le j \le 2m, \tag{3.7}$$

$$\partial_t^j \left(D\phi^{k+1} - D\tilde{\phi}\right)|_{t=0} = 0, \qquad 0 \le j \le 2m - 1. \tag{3.8}$$

As usual (see, e.g., [6, 7]), we first show that certain high-order norms of $(U^k, D\phi^k)$ are bounded and then prove that, in a lower-order norm, the map $(U^k, D\phi^k) \mapsto (U^{k+1}, D\phi^{k+1})$ is a contraction.

LEMMA 3.1 (Boundedness in High Norm). *Let* $L = \int_\Omega |I(x)|\,dx + 3\delta$. *There exist* $T_0 = T_0(\delta, L) > 0$ *such that, for* $T \le T_0$,

$$|||\, U^{k+1} - U^0\,|||_{2m,T} \le \delta, \qquad |||\, D\phi^{k+1} - D\phi^0\,|||_{2m,T} \le L \tag{3.9}$$

*for all* $k = 0, 1, 2, \ldots$ .

*Proof.* We prove by induction on $k$. Let $V = U^{k+1} - U^0$, $\psi = \phi^{k+1} - \phi^0$. Then $(V, \psi)$ satisfies

$$\begin{aligned} &A_0(U^k)\,\partial_t V + \sum_{j=1}^{3} A_j(U^k)\,\partial_{x_j} V \\ &\quad = H_1(U^k, D\phi^k) - H_1(U^0, D\phi^0) - \tilde{F}_1 \\ &\qquad + \big(A_0(U^0) - A_0(U^k)\big)\,\partial_t U^0 + \sum_{j=1}^{3} \big(A_j(U^0) - A_j(U^k)\big)\,\partial_{x_j} U^0, \\ &\Delta\psi = H_2(U^k) - H_2(U^0) - \tilde{F}_2 - e_k = (\rho^k - \rho^0) - \tilde{F}_2 - e_k \end{aligned} \tag{3.10}$$

together with the homogeneous initial and boundary conditions. Since $U^{k+1}|_{t=0} = U^0|_{t=0} = U_0$ and consequently $D\phi^k|_{t=0} = D\phi^0|_{t=0}$, the right-hand side of the first equation in (3.10) is 0 at $t = 0$. From (2.8) we have

$$|||\, V \,|||_{2m,T} \le CT, \tag{3.11}$$

where $C$ depends only on $\delta$.

On the other hand, for $T \ll 1$, we obviously have

$$|||\,(\rho^k - \rho^0) - \tilde{F}_2\,|||_{2m,T} \le 2\delta. \tag{3.12}$$

And for $e_k$, we have

$$|||\, e_k \,|||_{2m,T} \le |||\, \rho^k \,|||_{2m,T} + \int_\Omega |I(x)|\,dx \le \delta + \int_\Omega |I(x)|\,dx. \tag{3.13}$$

Taking $T_0 \ll 1$ and combining (3.11)–(3.13), we obtain (3.9).

Lemma 3.1 implies that the iteration (3.1)–(3.4) can proceed infinitely and all the norms $|||\, U^{k+1} \,|||_{2m,T}$ and $|||\, D\phi^{k+1} - D\phi^0 \,|||_{2m,T}$ are uniformly bounded.

LEMMA 3.2 (Contraction in Low Norm). *There exist constants $T_1$, $r_1$, $r_2$, and $\alpha$ with $0 < T_1 \le T_0$, $r_1 + r_2 < 1$, such that, for all $T \le T_1$,*

$$|||\, U^{k+2} - U^{k+1} \,|||_{2,T} \le r_1 |||\, U^{k+1} - U^k \,|||_{2,T} + r_2 |||\, U^k - U^{k-1} \,|||_{2,T}, \tag{3.14}$$

$$|||\, D\phi^{k+1} - D\phi^k \,|||_{2,T} \le \alpha |||\, U^k - U^{k-1} \,|||_{2,T} \tag{3.15}$$

*for all $k = 1, 2, \ldots$.*

*Proof.* Let $V^k = U^{k+1} - U^k$ and $\psi^k = \phi^{k+1} - \phi^k$, $k = 0, 1, 2, \ldots$. Then, from (3.1), $(V^k, \psi^k)$ satisfies

$$\begin{aligned} A_0(U^k)\,\partial_t V^k + \sum_{j=1}^{3} A_j(U^k)\,\partial_{x_j} V^k \\ = H_1(U^k, D\phi^k) - H_1(U^{k-1}, D\phi^{k-1}) \\ - \big(A_0(U^k) - A_0(U^{k-1})\big)\,\partial_t U^k \\ - \sum_{j=1}^{3} \big(A_j(U^k) - A_j(U^{k-1})\big)\,\partial_{x_j} U^k, \\ \Delta\psi^k = H_2(U^k) - H_2(U^{k-1}) - (e_k - e_{k-1}). \end{aligned} \tag{3.16}$$

From the uniform boundedness in Lemma 3.1 and the Banach algebra property of the space $E_{2,T}$, one can easily obtain that, for any $0 < T \le T_0$,

$$\begin{aligned} |||\, H_1(U^k, D\phi^k) - H_1(U^{k-1}, D\phi^{k-1}) \,|||_{2,T} \\ \le B_1\big(|||\, V^{k-1} \,|||_{2,T} + |||\, D\psi^{k-1} \,|||_{2,T}\big), \end{aligned} \tag{3.17}$$

$$\begin{aligned} |||\big(A_0(U^k) - A_0(U^{k-1})\big)\,\partial_t U^k + \sum_{j=1}^{3} \big(A_j(U^k) - A_j(U^{k-1})\big)\,\partial_{x_j} U^k \,|||_{2,T} \\ \le B_2\, |||\, V^{k-1} \,|||_{2,T}. \end{aligned} \tag{3.18}$$

Since $H_2(U^k) - H_2(U^{k-1}) = \rho^k - \rho^{k-1}$ and

$$|||\, e_k - e_{k-1} \,|||_{2,T} \le \int_\Omega |||\, \rho^k - \rho^{k-1} \,|||_{2,T}\, dx,$$

we have

$$|||\, H_2(U^k) - H_2(U^{k-1}) - (e_k - e_{k-1}) \,|||_{2,T} \le B_3\, |||\, V^{k-1} \,|||_{2,T}. \tag{3.19}$$

Applying energy estimates (2.8) and (2.9) to the system (3.16), we obtain

$$|||\, D\phi^{k+1} - D\phi^k \,|||_{2,T} \le C_2 B_3\, |||\, V^{k-1} \,|||_{2,T} \tag{3.20}$$

and

$$\begin{aligned} |||\, U^{k+1} - U^k \,|||_{2,T} &\le C_1 T\big((B_1 + B_2)\, |||\, V^{k-1} \,|||_{2,T} + C_2 B_3\, |||\, D\psi^{k-1} \,|||_{2,T}\big) \\ &= CT\, |||\, V^{k-1} \,|||_{2,T} + CT\, |||\, V^{k-2} \,|||_{2,T}. \end{aligned} \tag{3.21}$$

Choose $T_1$ such that $CT_1 < 1/2$ and let $\alpha = C_2 B_3$, $r_1 = r_2 = CT_1$. This completes the proof of Lemma 3.2.

*Proof of Main Theorem* 1.1. It follows immediately from Lemma 3.2 by taking summation of (3.14) that

$$\sum_{k=1}^{\infty} |||\, U^{k+1} - U^k \,|||_{2,T_1} \le \infty. \tag{3.22}$$

This in turn implies that

$$\sum_{k=1}^{\infty} |||\, D\phi^{k+1} - D\phi^k \,|||_{2,T_1} \le \infty.$$

Therefore, there exists $(U, \phi)$ belonging to $E_{2m,T_1}$ such that

$$\lim_{k\to\infty} \left( |||\, U^k - U \,|||_{2,T_1} + |||\, D\phi^k - D\phi \,|||_{2,T_1} \right) = 0.$$

Since $e_k = \int_\Omega (U_1^k(x) - I(x))\, dx/|\Omega|$, there exists $e \in E_{2m,T_1}$ such that

$$\lim_{k\to\infty} |||\, e_k - e \,|||_{2,T_1} = 0. \tag{3.23}$$

Obviously, $E_{2m,T} \subset H_m$ with $H_m$ being the classic Sobolev space. By the Nirenberg interpolation inequality

$$\|V\|_s \le \|V\|_0^{1-s/m} \|V\|_m^{s/m}, \tag{3.24}$$

where $\|\cdot\|_s$ denotes the classical Sobolev norm of the space $H_s$. Thus, from Lemmas 3.1 and 3.2, we get that, for any $s < m$,

$$\lim_{k\to\infty} \left( \|U^k - U\|_s + \|D\phi^k - D\phi\|_s \right) = 0$$

and

$$\lim_{k\to\infty} \|e_k - e\|_s = 0.$$

For $m > 3$, we have by the standard Sobolev embedding theorem

$$\left(U^k, D\phi^k, e_k\right) \to (U, D\phi, e) \qquad \text{in } C\left([0,T_1], C^1(\Omega)\right). \tag{3.25}$$

Therefore, $(U, D\phi, e)$ satisfies in the classical sense the following

$$A_0(U)\, \partial_t U + A_1(U)\, \partial_{x_1} U + A_2(U)\, \partial_{x_2} U + A_3(U)\, \partial_{x_3} U = H_1(U, D\phi),$$

$$\Delta\phi = \rho - I(x) - e. \tag{3.26}$$

It remains to prove that $e = 0$. Since $U = (U_1, U_2, U_3, U_4) = (\rho, u, v, w)$, we have from the first equation of (3.26)

$$\rho_t + (\rho u)_{x_1} + (\rho v)_{x_2} + (\rho w)_{x_3} = 0. \tag{3.27}$$

Therefore,

$$\int_\Omega \rho_t \, dx + \int_\Omega \nabla \cdot (\rho \mathbf{u}) \, dx = 0.$$

By the divergence theorem, we obtain

$$\int_\Omega \rho_t \, dx + \int_{\partial\Omega} (\rho \mathbf{u}) \cdot \nu \, dS = 0. \tag{3.28}$$

Since $\mathbf{u} \cdot \nu|_{\partial\Omega} = 0$, (3.28) implies

$$\partial_t \int_\Omega \rho \, dx = \int_\Omega \rho_t \, dx = 0.$$

Consequently,

$$\int_\Omega \rho \, dx = \int_\Omega \rho_0 \, dx = \int_\Omega I(x) \, dx. \tag{3.29}$$

Therefore, we have

$$e = \lim_{k\to\infty} e_k = \lim_{k\to\infty} \int_\Omega \left(U_1^k - I(x)\right) dx/|\Omega| = \int_\Omega (\rho_0 - I(x)) \, dx/|\Omega| = 0. \tag{3.30}$$

That is, $U = (\rho, u, v, w)$ and $\phi$ satisfy (1.5)–(1.8) in the classical sense.

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