



Micropolar generalized magneto-thermoelasticity with modified Ohm's and Fourier's laws

Magdy A. Ezzat, Emad S. Awad *

Department of Mathematics, Faculty of Education, Alexandria University, Alexandria, Egypt

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ABSTRACT

In this work, we introduce the modified Ohm's law, including the temperature gradient and charge density effects, and the generalized Fourier's law, including the current density effect, to the equations of the linear theory of micropolar generalized magneto-thermoelasticity, allowing the second sound effects. A normal mode analysis is applied to obtain the exact formulas of temperature, displacement, micro-rotation, stresses, couple stresses, electric field, magnetic field and current density. Application is employed to our problem to get the solution in the complete form. The considered variables are presented graphically and discussions are made.

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1. Introduction

The discrepancy between the classical theory of elasticity and the experiments is striking in the dynamical problems characterized by high frequency and small wave length; in such problems the influence of body microstructure becomes significant. So, the theory of micropolar elasticity, which deals with such materials having microstructure, gained a great importance in the last decades and found its applications mainly in the problems of deformation of granular media and multi-molecular systems.

The general theory of linear and non-linear micropolar continuum mechanics was given by Eringen and Şhuhubi [1,2] and Eringen [3,4].

The coupled theory of thermoelasticity was introduced by Biot [5] to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effects on the temperature. The heat equations for both theories are of parabolic type predicting infinite speeds of propagation for heat waves, contrary to physical observations.

Lord and Shulman [6] introduced the generalized theory of thermoelasticity with one relaxation, in which the acceleration of heat flux was taken into account and the Fourier's law of heat conduction was replaced by a generalized version including a constant represents the time-lag needed to establish steady state of heat conduction in an element of volume when a temperature gradient is suddenly imposed on that element, this constant was called relaxation time and Chester (1963) gave a clear physical interpretation for this constant.

The theory of micropolar elasticity was extended to include the thermal effects by Nowacki [7], Eringen [8,9], Tauchert et al. [10], Tauchert [11] and Nowacki and Olszak [12]. The generalized theory of micropolar thermoelasticity, which admits the second sound effect, was established by Boschi and Iesan [13], Sherief et al. [14] introduced another model and proved the uniqueness theorem and solved an axisymmetric thermal shock problem. El-Karamany and Ezzat [15] studied the boundary integral equation formulation for this theory.

* Corresponding author.

E-mail addresses: m_ezzat2000@yahoo.com (M.A. Ezzat), emadawad78@hotmail.com (E.S. Awad).

Nomenclature

λ, μ	Lame's constants	k	thermal conductivity
$\alpha, \beta, \varepsilon, \nu$	micropolar elastic constants	q_i	components of heat flux
j	micro-inertia	τ_0	relaxation time
ρ	density	$\mathbf{D}$	electric displacement vector
u_i	components of displacement	$\mathbf{E}$	induced electric field vector
w_i	components of micro-rotation	$\mathbf{J}$	current density vector
σ_{ij}	components of stresses	$\mathbf{H}$	magnetic intensity vector
μ_{ij}	components of couple stresses	$\mathbf{h}$	induced magnetic field vector
e	dilatation	$\mathbf{H}_0$	initial magnetic field vector
T	absolute temperature	ρ_e	charge density
T_0	reference temperature	ε_0	dielectric constant
θ	$= T - T_0$	μ_0	magnetic permeability
c_e	specific heat at constant strain	σ_0	electric conductivity
α_T	coefficient of linear thermal expansion	δ_{ij}	Kronecker delta function
γ	$= (3\lambda + 2\mu)\alpha_T$	e_{ijk}	permutation symbol

The foundations of magneto-elasticity were presented by Knopoff [16], Chadwick [17] and developed by Kaliski and Petykiewicz [18].

The propagation of electro-magneto-thermoelastic waves in an electrically and thermally conducting solid has been studied by many authors. Paria [19,20] discussed the propagation of plane magneto-thermoelastic waves in an isotropic unbounded medium under the influence of a uniform thermal field and with magnetic field acting transversely to the direction of propagation, Paria used the classical Fourier's law of heat conduction and neglected the electric displacement together with charge density, the coupling between the current density and the heat flow density and the coupling between the temperature gradient and the electric current. Wilson [21] corrected an error in [19,20] and extended Paria's results by introducing a component of the magnetic field parallel to the direction of the propagation. Nayfeh and Nemat-Nasser [22] introduced a more general model of electro-magneto-thermoelasticity which included the relaxation time of heat conduction and the electric displacement current.

Among the authors who considered the generalized theory of magneto-thermoelasticity, Sherief [23] solved a problem for a solid cylinder, Sherief and Ezzat [24] solved a thermal shock half space problem using asymptotic expansion, lately Sherief and Ezzat [25] solved a problem for an infinitely long annular cylinder, while Ezzat [26–28] and Ezzat and Youssef [29] solved some problems for a perfect conducting media.

The linear theory of micropolar generalized magneto-thermoelasticity was studied by Nowacki [30] who studied a two-dimensional problem, Chandrasekhariah [31] investigated the magneto-thermoelastic plane waves in micropolar elasticity. Nath et al. [32] studied magneto-thermoelastic waves in micropolar elastic media.

In this work, a more general model of micropolar generalized electro-magneto-thermoelasticity is used, a two-dimensional problem is formulated and solved using the normal mode analysis, the formulas of temperature, displacement, micro-rotation, stresses, couple stresses, electric field, magnetic field and current density are obtained. Application is employed to our problem to get the solution in the complete form. The considered variables are presented graphically and comparisons and discussions are made.

2. Mathematical model

The variation of electric and magnetic fields inside the medium are given by Maxwell's equations as follows

$$\text{curl} \mathbf{h} = \mathbf{J} + \dot{\mathbf{D}}, \quad (1)$$

$$\text{curl} \mathbf{E} = -\dot{\mathbf{B}}, \quad (2)$$

$$\text{div} \mathbf{B} = 0, \quad \text{div} \mathbf{D} = \rho_e, \quad (3)$$

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad \mathbf{D} = \varepsilon_0 \mathbf{E}, \quad (4)$$

where $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ and $\mathbf{H}_0$ is the applied magnetic field and $\mathbf{h}$ is the perturbation occurred in the total magnetic field by induction, the dot in the above equations denotes to the differentiation with respect to time.

The above equations are supplemented by the modified Ohm's law for media with finite conductivity

$$\mathbf{J} = \sigma_0 [\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}] + \rho_e \dot{\mathbf{u}} - k_0 \text{grad} \theta, \quad (5)$$

where k_0 is the coefficient connecting the temperature gradient and the electric current density.

The equation of balance of linear momentum including the effect of Lorentz force and in the absence of body force can be written in the following form

$$\sigma_{ji,j} + (\mathbf{J} \times \mathbf{B})_i + \rho_e E_i = \rho \ddot{u}_i, \quad (6)$$

where the comma symbol denotes to differentiation with respect to coordinates variables.

The equation of balance of angular momentum without body couples and taking into account the electromagnetic couples can be written as follows

$$e_{ijk}\sigma_{jk} + \mu_{ji,j} + e_{ijk}x_j(\mathbf{J} \times \mathbf{B})_k = \rho jw_i. \quad (7)$$

The equation of energy in the absence of heat source is given by

$$\rho T \dot{\eta} = -q_{i,i}. \quad (8)$$

The entropy η may be written in terms of temperature, in an isotropic media, as follows

$$\rho \eta = \frac{\rho c_e}{T_0} \theta + \gamma e. \quad (9)$$

The generalized Fourier's law including the current density effect is given by

$$q_i + \tau_0 \dot{q}_i = -k\theta_{,i} + \pi_0 J_i, \quad (10)$$

where π_0 is the coefficient connecting the current density with the heat flow density.

In the framework of linear theory, we will neglect all second order quantities and terms of higher orders equations (5)–(8) can be written in the following linearized version

$$\mathbf{J} = \sigma_0[\mathbf{E} + \mu_0 \dot{\mathbf{u}} \times \mathbf{H}_0] - k_0 \text{grad } \theta, \quad (11)$$

$$\sigma_{ji,j} + \mu_0(\mathbf{J} \times \mathbf{H}_0)_i = \rho \ddot{u}_i, \quad (12)$$

$$e_{ijk}\sigma_{jk} + \mu_{ji,j} = \rho jw_i, \quad (13)$$

$$\rho T_0 \dot{\eta} = -q_{i,i}. \quad (14)$$

By eliminating η between (9) and (14) and using (10) we get the equation of heat conduction for the linear theory as follows

$$k \nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho c_e \theta + \gamma T_0 e) + \pi_0 \text{div } \mathbf{J}. \quad (15)$$

Eqs. (1)–(4), (11)–(13) and (15) are supplemented by the following constitutive relations

$$\sigma_{ij} = \lambda u_{k,k} \delta_{ij} + (\mu + \alpha) u_{j,i} + (\mu - \alpha) u_{i,j} - 2\alpha e_{ijk} w_k - \gamma \theta \delta_{ij}, \quad (16)$$

$$\mu_{ij} = \varepsilon w_{k,k} \delta_{ij} + (v + \beta) w_{j,i} + (v - \beta) w_{i,j}, \quad (17)$$

$$e = \text{div } \mathbf{u}. \quad (18)$$

Thus Eqs. (1)–(4), (11)–(13) and (15)–(18) constitute the field equations and constitutive relations of the linear theory of micropolar generalized magneto-thermoelasticity with modified Ohm's law and modified generalized Fourier's law.

3. Formulation of the problem

We consider an isotropic homogeneous micropolar thermoelastic medium undergo an applied constant magnetic field oriented in a fixed direction, say z -axis, $\mathbf{H}_0 = (0, 0, H_0)$ and let the induced magnetic field, as a special case, occur in the same direction of the applied magnetic field, $\mathbf{h} = (0, 0, h)$.

Assuming that all quantities related to the medium is dependent on the Cartesian coordinates x , y and the time t and independent of z . And the deformation of the medium is two-dimensional.

Under the previous considerations we may write the displacement components as $\mathbf{u} = (u, v, 0)$, the micro-rotation $\mathbf{w} = (0, 0, w)$, the electric field is normal to the considered magnetic field $\mathbf{E} = (E_1, E_2, 0)$ and the electric current density is parallel to the electric field $\mathbf{J} = (J_1, J_2, 0)$.

Using the above assumptions and Eqs. (1)–(4) and (11) we can obtain

$$\frac{\partial h}{\partial y} = \sigma_0 \left[E_1 + \mu_0 H_0 \frac{\partial v}{\partial t} \right] + \varepsilon_0 \frac{\partial E_1}{\partial t} - k_0 \frac{\partial \theta}{\partial x}, \quad (19a)$$

$$\frac{\partial h}{\partial x} = -\sigma_0 \left[E_2 - \mu_0 H_0 \frac{\partial u}{\partial t} \right] - \varepsilon_0 \frac{\partial E_2}{\partial t} + k_0 \frac{\partial \theta}{\partial y}, \quad (19b)$$

$$\frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} = \mu_o \frac{\partial h}{\partial t}, \quad (20)$$

$$J_1 = \sigma_o \left[E_1 + \mu_o H_o \frac{\partial v}{\partial t} \right] - k_o \frac{\partial \theta}{\partial x}, \quad (21a)$$

$$J_2 = \sigma_o \left[E_2 - \mu_o H_o \frac{\partial u}{\partial t} \right] - k_o \frac{\partial \theta}{\partial y}, \quad (21b)$$

and using Eqs. (16)–(17) in (12)–(13) we get

$$(\mu + \alpha) \nabla^2 \mathbf{u} + (\lambda + \mu - \alpha) \text{grad}(\text{div} \mathbf{u}) + 2\alpha \text{curl} \mathbf{w} - \gamma \text{grad} \theta + \mu_o (\mathbf{J} \times \mathbf{H}_o) = \rho \ddot{\mathbf{u}}, \quad (22)$$

$$(\nu + \beta) \nabla^2 \mathbf{w} + (\varepsilon + \nu - \beta) \text{grad}(\text{div} \mathbf{w}) + 2\alpha \text{curl} \mathbf{u} - 4\alpha \mathbf{w} = \rho_j \ddot{\mathbf{w}}. \quad (23)$$

Introduce the following non-dimensional variables:

$$\begin{aligned} x^* &= c_o \eta_o x, & y^* &= c_o \eta_o y, & u^* &= c_o \eta_o u, & v^* &= c_o \eta_o v, & t^* &= c_o^2 \eta_o t, & \tau_o^* &= c_o^2 \eta_o \tau_o, \\ \theta^* &= \frac{\gamma \theta}{\rho c_o^2}, & w^* &= \frac{\alpha w}{\mu + \alpha}, & \sigma_{ij}^* &= \frac{\sigma_{ij}}{\mu + \alpha}, & \mu_{ij}^* &= \frac{\alpha \mu_{ij}}{c_o \eta_o (\mu + \alpha) (\nu + \beta)}, \\ h^* &= \frac{\eta_o h}{\sigma_o \mu_o H_o}, & E_i^* &= \frac{\eta_o E_i}{\sigma_o \mu_o^2 H_o c_o}, & J_i^* &= \frac{\eta_o J_i}{\sigma_o^2 \mu_o^2 H_o c_o}, & i &= 1, 2, \end{aligned} \quad (24)$$

where

$$c_o^2 = \frac{\lambda + 2\mu}{\rho}, \quad \eta_o = \frac{\rho c_e}{k}.$$

Now using the non-dimensional variables (24), Eqs. (15)–(17) and (19)–(23) may take the following component form (after dropping the asterisks for convenience)

$$\beta_1^2 \frac{\partial e}{\partial x} + \frac{\partial \varphi}{\partial y} + 2 \frac{\partial w}{\partial y} + \beta_1^2 v_1^2 \varepsilon_2 E_2 - \beta_1^2 \left(\frac{\partial \theta}{\partial x} + k_1 \frac{\partial \theta}{\partial y} \right) = \beta_1^2 \left(\frac{\partial^2 u}{\partial t^2} + v_1 \varepsilon_2 \frac{\partial u}{\partial t} \right), \quad (25)$$

$$\beta_1^2 \frac{\partial e}{\partial y} - \frac{\partial \varphi}{\partial x} - 2 \frac{\partial w}{\partial x} - \beta_1^2 v_1^2 \varepsilon_2 E_1 - \beta_1^2 \left(\frac{\partial \theta}{\partial y} - k_1 \frac{\partial \theta}{\partial x} \right) = \beta_1^2 \left(\frac{\partial^2 v}{\partial t^2} + v_1 \varepsilon_2 \frac{\partial v}{\partial t} \right), \quad (26)$$

$$\nabla^2 w - g_1 \varphi - g_2 w = g_3 \frac{\partial^2 w}{\partial t^2}, \quad (27)$$

$$\nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2} \right) (\theta + \varepsilon_1 e) - \pi_1 \frac{\partial}{\partial t} \left(\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} \right), \quad (28)$$

$$\sigma_{xx} = \beta_1^2 \left(\frac{\partial u}{\partial x} - \theta \right) + \delta_1 \frac{\partial v}{\partial y}, \quad (29)$$

$$\sigma_{yy} = \delta_1 \frac{\partial u}{\partial x} + \beta_1^2 \left(\frac{\partial v}{\partial y} - \theta \right), \quad (30)$$

$$\sigma_{zz} = \delta_1 e - \beta_1^2 \theta, \quad (31)$$

$$\sigma_{xy} = \frac{\partial v}{\partial x} + \delta_2 \frac{\partial u}{\partial y} - 2w, \quad (32)$$

$$\sigma_{yx} = \frac{\partial u}{\partial y} + \delta_2 \frac{\partial v}{\partial x} + 2w, \quad (33)$$

$$\mu_{xz} = \frac{\partial w}{\partial x}, \quad \mu_{zx} = \delta_3 \frac{\partial w}{\partial x}, \quad (34)$$

$$\mu_{yz} = \frac{\partial w}{\partial y}, \quad \mu_{zy} = \delta_3 \frac{\partial w}{\partial y}, \quad (35)$$

$$\frac{\partial h}{\partial y} = v_1 E_1 + \frac{\partial v}{\partial t} + V^2 \frac{\partial E_1}{\partial t} - k_2 \frac{\partial \theta}{\partial x}, \quad (36)$$

$$\frac{\partial h}{\partial x} = -v_1 E_2 + \frac{\partial u}{\partial t} - V^2 \frac{\partial E_2}{\partial t} + k_2 \frac{\partial \theta}{\partial y}, \quad (37)$$

$$\frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} = \frac{\partial h}{\partial t}, \quad (38)$$

$$J_1 = E_1 + \frac{1}{v_1} \frac{\partial v}{\partial t} - k_3 \frac{\partial \theta}{\partial x}, \quad (39)$$

$$J_2 = E_2 - \frac{1}{v_1} \frac{\partial u}{\partial t} - k_3 \frac{\partial \theta}{\partial y}, \quad (40)$$

where

$$\begin{aligned} \beta_1^2 &= \frac{\lambda + 2\mu}{\mu + \alpha}, & v_1 &= \frac{\sigma_0 \mu_0}{\eta_0}, & V^2 &= \frac{c_0^2}{c^2}, & c^2 &= \frac{1}{\mu_0 \varepsilon_0}, & \varepsilon_1 &= \frac{\gamma^2 T_0}{\rho c_0^2 \eta_0 k}, & \varepsilon_2 &= \frac{\mu_0 H_0^2}{\rho c_0^2}, \\ k_1 &= \frac{\mu_0 H_0 k_0}{\gamma}, & k_2 &= \frac{k_1}{v_1 \varepsilon_2}, & k_3 &= \frac{k_2}{v_1}, & g_1 &= \frac{2\alpha^2}{c_0^2 \eta_0^2 (\nu + \beta)(\mu + \alpha)}, & g_2 &= \frac{4\alpha}{c_0^2 \eta_0^2 (\nu + \beta)}, \\ g_3 &= \frac{\rho j c_0^2}{\nu + \beta}, & \pi_1 &= \pi_0^* \varepsilon_2 V^2, & \pi_0^* &= \frac{\pi_0 \sigma_0 \gamma}{k H_0}, & \delta_1 &= \frac{\lambda}{\mu + \alpha}, & \delta_2 &= \frac{\mu - \alpha}{\mu + \alpha}, & \delta_3 &= \frac{\nu - \beta}{\nu + \beta}, \end{aligned}$$

moreover; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is Laplace's operator and

$$\left. \begin{aligned} e &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \\ \varphi &= \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}. \end{aligned} \right\} \quad (41)$$

After simple manipulations for Eqs. (25)–(26) and (36)–(38), one can obtain the following:

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - v_1 \varepsilon_2 \frac{\partial}{\partial t} \right) e = \nabla^2 \theta + v_1^2 \varepsilon_2 \frac{\partial h}{\partial t}, \quad (42)$$

$$\left[\nabla^2 - \beta_1^2 \left(\frac{\partial^2}{\partial t^2} + v_1^2 \varepsilon_2 \frac{\partial}{\partial t} \right) \right] \varphi = \beta_1^2 k_1 \nabla^2 \theta - 2 \nabla^2 w - \beta_1^2 v_1^2 \varepsilon_2 \left(\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} \right), \quad (43)$$

$$\left(\nabla^2 - V^2 \frac{\partial^2}{\partial t^2} + v_1 \frac{\partial}{\partial t} \right) h = \frac{\partial e}{\partial t}, \quad (44)$$

$$\left(v_1 + V^2 \frac{\partial}{\partial t} \right) \left[\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} \right] = \frac{\partial \varphi}{\partial t} + k_2 \nabla^2 \theta. \quad (45)$$

4. Normal mode analysis

We consider here that all considered variables can be expressed at the form of plane wave by expressing it in the following exponential form:

$$[u, v, w, \theta, e, \varphi, \sigma_{ij}, \mu_{ij}, E_i, J_i, h](x, y, t) = [u^*, v^*, w^*, \theta^*, e^*, \varphi^*, \sigma_{ij}^*, \mu_{ij}^*, E_i^*, J_i^*, h^*](x) \exp(\omega t + i a y) \quad (46)$$

where ω is the complex time constant (frequency), a is the wave number in y -axis direction, i is the imaginary unit and $u^*, v^*, w^*, \theta^*, e^*, \varphi^*, \sigma_{ij}^*, \mu_{ij}^*, E_i^*, J_i^*$ and h^* are the amplitudes of the represented plane waves to the considered variables.

Using Eq. (46), Eqs. (27)–(28), (42)–(45) can be reduced to the following equations:

$$(D^2 - (a^2 + \omega^2 + v_1 \varepsilon_2 \omega)) e^* = (D^2 - a^2) \theta^* + v_1^2 \varepsilon_2 h^*, \quad (47)$$

$$\left(D^2 - \left(a^2 + \beta_1^2 \omega^2 + \beta_1^2 v_1 \varepsilon_2 \omega - \frac{\beta_1^2 v_1^2 \varepsilon_2 \omega}{v_1 + V^2 \omega} \right) \right) \varphi^* = \beta_1^2 \left(k_1 - \frac{v_1^2 \varepsilon_2 k_2}{v_1 + V^2 \omega} \right) \theta^* - 2(D^2 - a^2) w^*, \quad (48)$$

$$(D^2 - (a^2 + g_2 + g_3 \omega)) w^* = g_1 \varphi^*, \quad (49)$$

$$\left(D^2 - \left(a^2 + \frac{\omega(1 + \tau_0 \omega)(v_1 + V^2 \omega)}{v_1 + V^2 \omega + \pi_1 \omega k_2} \right) \right) \theta^* = \frac{\varepsilon_1 \omega(1 + \tau_0 \omega)(v_1 + V^2 \omega)}{v_1 + V^2 \omega + \pi_1 \omega k_2} e^* - \frac{\pi_1 \omega^2}{v_1 + V^2 \omega + \pi_1 \omega k_2} \varphi^*, \quad (50)$$

$$(D^2 - (a^2 + V^2 \omega^2 + v_1 \omega)) h^* = \omega e^*, \quad (51)$$

where $D = \frac{d}{dx}$.

Eliminating w^*, e^*, φ^* and h^* between Eqs. (47)–(51) we get the following tenth order differential equation satisfied by θ^* :

$$(D^{10} - \xi_1 D^8 + \xi_2 D^6 - \xi_3 D^4 + \xi_4 D^2 - \xi_5) \theta^*(x) = 0, \quad (52)$$

where

$$\xi_1 = (\alpha_1 + \alpha_3 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_9) - \alpha_4 \alpha_8 - 2g_1,$$

$$\xi_2 = \alpha_1 \xi_{21} - \alpha_2 \omega + \alpha_3 \xi_{23} + \alpha_4 \xi_{24} + \alpha_5 \xi_{25} + \xi_{26} + \alpha_9 \xi_{29},$$

$$\xi_3 = \alpha_1 \xi_{31} + \alpha_2 \xi_{32} + \alpha_3 \xi_{33} + \alpha_4 \xi_{34} + \alpha_5 \xi_{35} + \alpha_6 \xi_{36} + \alpha_7 \xi_{37} - 2a^2 g_1 \alpha_9,$$

$$\xi_4 = \alpha_1 \xi_{41} + \alpha_2 \xi_{42} + \alpha_3 \xi_{43} + \xi_{44}, \quad \xi_5 = \alpha_1 \xi_{51} + \alpha_2 \xi_{52} + \alpha_7 \xi_{57},$$

$$\begin{aligned}
\xi_{21} &= \alpha_3 - \alpha_4\alpha_8 + \alpha_5 + \alpha_6 + \alpha_9, & \xi_{23} &= \alpha_5 + \alpha_6 + \alpha_7 + \alpha_9, \\
\xi_{24} &= -\alpha_8(\alpha_5 + \alpha_9), & \xi_{25} &= \alpha_6 + \alpha_7 + \alpha_9, \\
\xi_{26} &= -a^2(2g_1 + \alpha_4\alpha_8 - \alpha_7), & \xi_{29} &= \alpha_6 + \alpha_7, \\
\xi_{31} &= \alpha_3(\alpha_5 + \alpha_6 + \alpha_9) - \alpha_4\alpha_8(\alpha_5 + \alpha_9 + a^2) + \alpha_5(\alpha_6 + \alpha_9) + \alpha_6(\alpha_9 - 2g_1) - 2g_1(\alpha_9 + a^2), \\
\xi_{32} &= -\omega(\alpha_3 - \alpha_4\alpha_8 + \alpha_5 + \alpha_6 - 2g_1), & \xi_{33} &= \alpha_5(\alpha_6 + \alpha_7 + \alpha_9) + \alpha_6\alpha_9 + \alpha_7(\alpha_9 + a^2), \\
\xi_{34} &= -\alpha_8(\alpha_5(\alpha_9 + a^2) + a^2\alpha_9), & \xi_{35} &= \alpha_6\alpha_9 + \alpha_7(\alpha_9 + a^2), \\
\xi_{36} &= -2g_1(\alpha_9 + a^2), & \xi_{37} &= \alpha_9(a^2 - 2g_1) - 4a^2g_1, \\
\xi_{41} &= \alpha_3(\alpha_5(\alpha_6 + \alpha_9) + \alpha_6\alpha_9) - \alpha_4\alpha_8(\alpha_5(\alpha_9 + a^2) + a^2\alpha_9) + \alpha_5\alpha_6\alpha_9 - 2g_1(\alpha_6(\alpha_9 + a^2) + a^2\alpha_9), \\
\xi_{42} &= -\omega(\alpha_3(\alpha_5 + \alpha_6) - \alpha_4\alpha_8(\alpha_5 + a^2) + \alpha_5\alpha_6 - 2g_1(\alpha_6 + a^2)), \\
\xi_{43} &= \alpha_5(\alpha_6\alpha_9 + \alpha_7(\alpha_9 + a^2)) + a^2\alpha_7\alpha_9, \\
\xi_{44} &= -a^2(\alpha_4\alpha_5\alpha_8\alpha_9 - \alpha_5\alpha_7\alpha_9 + 2g_1(\alpha_6\alpha_9 + \alpha_7(2\alpha_9 + a^2))), \\
\xi_{51} &= \alpha_9(\alpha_3\alpha_5\alpha_6 - a^2(2g_1\alpha_6 + \alpha_4\alpha_5\alpha_8)), \\
\xi_{52} &= a^2\omega(2g_1\alpha_6 + \alpha_4\alpha_5\alpha_8) - \alpha_3\alpha_5\alpha_6\omega, \\
\xi_{57} &= -a^2\alpha_9(2a^2g_1 - \alpha_3\alpha_5),
\end{aligned}$$

and

$$\begin{aligned}
\alpha_1 &= a^2 + \omega^2 + v_1\varepsilon_2\omega, & \alpha_2 &= v_1^2\varepsilon_2, & \alpha_3 &= a^2 + \beta_1^2\omega^2 + \beta_1^2v_1\varepsilon_2\omega - \frac{\beta_1^2v_1^2\varepsilon_2\omega}{v_1 + V^2\omega}, \\
\alpha_4 &= \beta_1^2\left(k_1 - \frac{v_1^2\varepsilon_2k_2}{v_1 + V^2\omega}\right), & \alpha_5 &= a^2 + g_2 + g_3\omega, & \alpha_6 &= a^2 + \frac{\omega(1 + \tau_0\omega)(v_1 + V^2\omega)}{v_1 + V^2\omega + \pi_1\omega k_2}, \\
\alpha_7 &= \frac{\varepsilon_1\omega(1 + \tau_0\omega)(v_1 + V^2\omega)}{v_1 + V^2\omega + \pi_1\omega k_2}, & \alpha_8 &= \frac{\pi_1\omega^2}{v_1 + V^2\omega + \pi_1\omega k_2}, & \alpha_9 &= a^2 + V^2\omega^2 + v_1\omega.
\end{aligned}$$

Again from Eqs. (47)–(51) we find also that e^* , φ^* , h^* and w^* satisfy the same differential equation of θ^* :

$$(D^{10} - \xi_1 D^8 + \xi_2 D^6 - \xi_3 D^4 + \xi_4 D^2 - \xi_5)e^*(x) = 0, \quad (53)$$

$$(D^{10} - \xi_1 D^8 + \xi_2 D^6 - \xi_3 D^4 + \xi_4 D^2 - \xi_5)\varphi^*(x) = 0, \quad (54)$$

$$(D^{10} - \xi_1 D^8 + \xi_2 D^6 - \xi_3 D^4 + \xi_4 D^2 - \xi_5)h^*(x) = 0, \quad (55)$$

$$(D^{10} - \xi_1 D^8 + \xi_2 D^6 - \xi_3 D^4 + \xi_4 D^2 - \xi_5)w^*(x) = 0. \quad (56)$$

In order to get the solution of (52), we firstly factorize it to the following form:

$$\prod_{j=1}^5 (D^2 - m_j^2)\theta^*(x) = 0, \quad (57)$$

where m_j^2 ($j = 1, 2, 3, 4, 5$) are the characteristic roots of the characteristic equation of Eq. (57) which is

$$m^{10} - \xi_1 m^8 + \xi_2 m^6 - \xi_3 m^4 + \xi_4 m^2 - \xi_5 = 0. \quad (58)$$

Thus the solution of (57) which is bounded for the values $x \geq 0$ may be written as

$$\theta^*(x) = \sum_{j=1}^5 A_j(a, \omega)e^{-m_j x}, \quad (59)$$

by the same, the solutions of (53)–(56) which also are bounded for $x \geq 0$ may be written as

$$e^*(x) = \sum_{j=1}^5 A_j^{(1)}(a, \omega)e^{-m_j x}, \quad (60)$$

$$\varphi^*(x) = \sum_{j=1}^5 A_j^{(2)}(a, \omega)e^{-m_j x}, \quad (61)$$

$$h^*(x) = \sum_{j=1}^5 A_j^{(3)}(a, \omega) e^{-m_j x}, \quad (62)$$

$$w^*(x) = \sum_{j=1}^5 A_j^{(4)}(a, \omega) e^{-m_j x}, \quad (63)$$

where $A_j(a, \omega)$, $A_j^{(1)}(a, \omega)$, $A_j^{(2)}(a, \omega)$, $A_j^{(3)}(a, \omega)$, $A_j^{(4)}(a, \omega)$ are some constants depending on a and ω .

The above unknown constants can be reduced to five constants $A_j(a, \omega)$ ($j = 1, 2, 3, 4, 5$) by substituting from (59)–(63) into (47)–(51) we get the following relations:

$$A_j^{(1)}(a, \omega) = \frac{(m_j^2 - a^2)(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} A_j(a, \omega), \quad j = 1, 2, 3, 4, 5, \quad (64)$$

$$A_j^{(2)}(a, \omega) = \frac{\alpha_4(m_j^2 - a^2)(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j(a, \omega), \quad j = 1, 2, 3, 4, 5, \quad (65)$$

$$A_j^{(3)}(a, \omega) = \frac{\omega(m_j^2 - a^2)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} A_j(a, \omega), \quad j = 1, 2, 3, 4, 5, \quad (66)$$

$$A_j^{(4)}(a, \omega) = \frac{g_1 \alpha_4(m_j^2 - a^2)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j(a, \omega), \quad j = 1, 2, 3, 4, 5. \quad (67)$$

Using (64)–(67), Eqs. (60)–(63) become

$$e^*(x) = \sum_{j=1}^5 \frac{(m_j^2 - a^2)(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} A_j(a, \omega) e^{-m_j x}, \quad (68)$$

$$\varphi^*(x) = \sum_{j=1}^5 \frac{\alpha_4(m_j^2 - a^2)(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j(a, \omega) e^{-m_j x}, \quad (69)$$

$$h^*(x) = \sum_{j=1}^5 \frac{\omega(m_j^2 - a^2)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} A_j(a, \omega) e^{-m_j x}, \quad (70)$$

$$w^*(x) = \sum_{j=1}^5 \frac{g_1 \alpha_4(m_j^2 - a^2)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j(a, \omega) e^{-m_j x}. \quad (71)$$

By using Eq. (46) into (41) and eliminating u^* and v^* consequently we can obtain the displacement components as follows

$$u^*(x) = \sum_{j=1}^5 \left\{ \frac{ia\alpha_4(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} - \frac{m_j(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} \right\} A_j e^{-m_j x}, \quad (72)$$

$$v^*(x) = \sum_{j=1}^5 \left\{ \frac{\alpha_4 m_j(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} + \frac{ia(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} \right\} A_j e^{-m_j x}. \quad (73)$$

The electric field components can be obtained by substituting from (46) into (36)–(37) then using Eqs. (59), (70) and (72)–(73) we get

$$E_1^*(x) = \frac{1}{v_1 + V^2 \omega} \sum_{j=1}^5 \left\{ \frac{ia\omega(\alpha_9 - a^2)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} - \frac{\alpha_4 \omega m_j(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} - k_2 m_j \right\} A_j e^{-m_j x}, \quad (74)$$

$$E_2^*(x) = \frac{1}{v_1 + V^2 \omega} \sum_{j=1}^5 \left\{ iak_2 + \frac{\omega m_j(\alpha_9 - a^2)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} + \frac{ia\alpha_4 \omega(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} \right\} A_j e^{-m_j x}. \quad (75)$$

To obtain the components of current density, substituting from (46) into Eqs. (39), (40) and using Eqs. (59), (72)–(75) we get

$$J_1^*(x) = \sum_{j=1}^5 \left\{ \left(k_3 - \frac{k_2}{v_1 + V^2\omega} \right) m_j + \left(\frac{V^2\omega}{v_1(v_1 + V^2\omega)} \right) \left(\frac{\alpha_4\omega m_j(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} \right) \right. \\ \left. + \left(\frac{\alpha_9 - a^2}{v_1 + V^2\omega} + \frac{m_j^2 - \alpha_9}{v_1} \right) \left(\frac{ia\omega}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} \right) \right\} A_j e^{-m_j x}, \quad (76)$$

$$J_2^*(x) = \sum_{j=1}^5 \left\{ ia \left(\frac{k_2}{v_1 + V^2\omega} - k_3 \right) + \left(\frac{V^2\omega}{v_1(v_1 + V^2\omega)} \right) \left(\frac{ia\omega\alpha_4(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} \right) \right. \\ \left. + \left(\frac{\alpha_9 - a^2}{v_1 + V^2\omega} + \frac{m_j^2 - \alpha_9}{v_1} \right) \left(\frac{\omega m_j}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} \right) \right\} A_j e^{-m_j x}. \quad (77)$$

In order to get the components of stresses, substituting from (46) into (29)–(33) and using Eqs. (59), (60), (63) and (72)–(73) we get

$$\sigma_{xx}^*(x) = \sum_{j=1}^5 \left\{ \frac{(\beta_1^2 m_j^2 - a^2 \delta_1)(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} + \frac{ia\alpha_4 m_j(\delta_1 - \beta_1^2)(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} - \beta_1^2 \right\} A_j e^{-m_j x}, \quad (78)$$

$$\sigma_{yy}^*(x) = \sum_{j=1}^5 \left\{ \frac{(\delta_1 m_j^2 - a^2 \beta_1^2)(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} + \frac{ia\alpha_4 m_j(\beta_1^2 - \delta_1)(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} - \beta_1^2 \right\} A_j e^{-m_j x}, \quad (79)$$

$$\sigma_{zz}^*(x) = \sum_{j=1}^5 \left\{ \frac{\delta_1(m_j^2 - a^2)(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} - \beta_1^2 \right\} A_j e^{-m_j x}, \quad (80)$$

$$\sigma_{xy}^*(x) = - \sum_{j=1}^5 \left\{ \frac{\alpha_4[(m_j^2 + a^2\delta_2)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)]}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} + \frac{ia(1 + \delta_2)m_j(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} \right\} A_j e^{-m_j x}, \quad (81)$$

$$\sigma_{yx}^*(x) = - \sum_{j=1}^5 \left\{ \frac{\alpha_4[(\delta_2 m_j^2 + a^2)(m_j^2 - \alpha_5) - 2g_1(m_j^2 - a^2)]}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} + \frac{ia(1 + \delta_2)m_j(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2\omega} \right\} A_j e^{-m_j x}. \quad (82)$$

Moreover, the components of couple stresses can be obtained in a similar manner by substituting from (46) into (34)–(35) and using (63) we get

$$\mu_{xz}^*(x) = \sum_{j=1}^5 \frac{-g_1\alpha_4 m_j(m_j^2 - a^2)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j e^{-m_j x}, \quad (83)$$

$$\mu_{yz}^*(x) = \sum_{j=1}^5 \frac{ia g_1 \alpha_4 (m_j^2 - a^2)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j e^{-m_j x}. \quad (84)$$

Thus Eqs. (60), (68)–(84) constitute amplitudes of plane waves of the considered variables of the model of two-dimensional problems.

5. Application

We consider a micropolar thermoelastic material occupying the semi-space region:

$R = \{(x, y, z): x \geq 0, -\infty < y, z < \infty\}$ and the other semi-space $R^* = \{(x, y, z): x \leq 0, -\infty < y, z < \infty\}$ is a vacuum, let the surface of R is traction free and subjected to decreasing thermal source with time which affects on a narrow band of width $2L$ surrounding z -axis and R^* is kept at room temperature T_0 and the boundary surface between R, R^* is thermally isolated such that the thermal source affects only on R .

In order to apply the appropriate boundary conditions we must firstly obtain the components of induced magnetic and electric field in R^* .

So, let $\mathbf{h}_0 = (0, 0, h_0)$ and $\mathbf{E}_0 = (E_{10}, E_{20}, 0)$ be denoted to the induced magnetic and electric field in R^* respectively, hence Maxwell's equations for region R^* in the non-dimensional form can be simplified to the following:

$$\frac{\partial h_0}{\partial y} = V^2 \frac{\partial E_{10}}{\partial t}, \quad (85)$$

$$\frac{\partial h_0}{\partial x} = -V^2 \frac{\partial E_{20}}{\partial t}, \quad (86)$$

$$\frac{\partial E_{10}}{\partial y} - \frac{\partial E_{20}}{\partial x} = \frac{\partial h_0}{\partial t}. \quad (87)$$

Again, the variables of R^* can be written as a plane wave form as follows

$$[E_{10}, E_{20}, h_0](x, y, t) = [E_{10}^*, E_{20}^*, h_0^*](x) \exp(\omega t + iax). \quad (88)$$

By using Eqs. (85)–(88) we can obtain the considered variables for R^* which are also bounded for $x \leq 0$

$$h_0^*(x) = B(a, \omega) e^{nx}, \quad (89)$$

$$E_{10}^* = \frac{ia}{V^2 \omega} B(a, \omega) e^{nx}, \quad (90)$$

$$E_{20}^* = -\frac{n}{V^2 \omega} B(a, \omega) e^{nx}, \quad (91)$$

where $B(a, \omega)$ is a parameter depending on a and ω , $n = \sqrt{a^2 + V^2 \omega^2}$.

We choose the following boundary conditions for the purpose of determination of the unknown parameters $A_j(a, \omega)$ ($j = 1, 2, 3, 4, 5$) and $B(a, \omega)$.

(4-a) Thermal boundary condition.

$$\text{Let } f(y, t) \text{ be a known function so that } \theta(0, y, t) = f(y, t). \quad (92)$$

(4-b) Mechanical boundary conditions.

Under the assumption that the surface of R is traction free we can get the following conditions

$$\sigma_{xx}(0, y, t) = \sigma_{xy}(0, y, t) = \mu_{xz}(0, y, t) = 0. \quad (93)$$

(4-c) Electromagnetic boundary conditions.

The transverse components of the electric field intensity are continuous across the boundary surface

$$E_2(0, y, t) = E_{20}(0, y, t). \quad (94)$$

The transverse components of the magnetic field intensity are continuous across the boundary surface

$$h(0, y, t) = h_0(0, y, t). \quad (95)$$

By applying Eqs. (46) and (88), Eqs. (92)–(95) will constitute six boundary conditions, in the normal mode form, for our application, with the help of Eqs. (59), (70), (75), (78), (81), (83), (89) and (91) the present boundary conditions may be reduced to

$$\sum_{j=1}^5 A_j = f^*(a, \omega), \quad (96)$$

$$\sum_{j=1}^5 \left\{ \frac{(\beta_1^2 m_j^2 - a^2 \delta_1)(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} + \frac{ia \alpha_4 m_j (\delta_1 - \beta_1^2)(m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} - \beta_1^2 \right\} A_j = 0, \quad (97)$$

$$\sum_{j=1}^5 \left\{ \frac{\alpha_4 [(m_j^2 + a^2 \delta_2)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)]}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} + \frac{ia(1 + \delta_2)m_j(m_j^2 - \alpha_9)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} \right\} A_j = 0, \quad (98)$$

$$\sum_{j=1}^5 \frac{m_j(m_j^2 - a^2)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} A_j = 0, \quad (99)$$

$$\sum_{j=1}^5 \left\{ iak_2 + \frac{\omega m_j (\alpha_9 - a^2)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} + \frac{ia \alpha_4 \omega (m_j^2 - \alpha_5)}{(m_j^2 - \alpha_3)(m_j^2 - \alpha_5) + 2g_1(m_j^2 - a^2)} \right\} A_j + \frac{n(v_1 + V^2 \omega)}{V^2 \omega} B = 0, \quad (100)$$

$$\sum_{j=1}^5 \frac{\omega(m_j^2 - a^2)}{(m_j^2 - \alpha_1)(m_j^2 - \alpha_9) - \alpha_2 \omega} A_j - B = 0. \quad (101)$$

By solving this system of equations for A_j ($j = 1, 2, 3, 4, 5$) and B , we can complete the solution of our problem.

6. Numerical results and discussion

The magnesium crystal [33] is chosen for the purposes of evaluations, the values of physical constants of magnesium, are given at the reference temperature $T_0 = 23^\circ\text{C}$ as follows

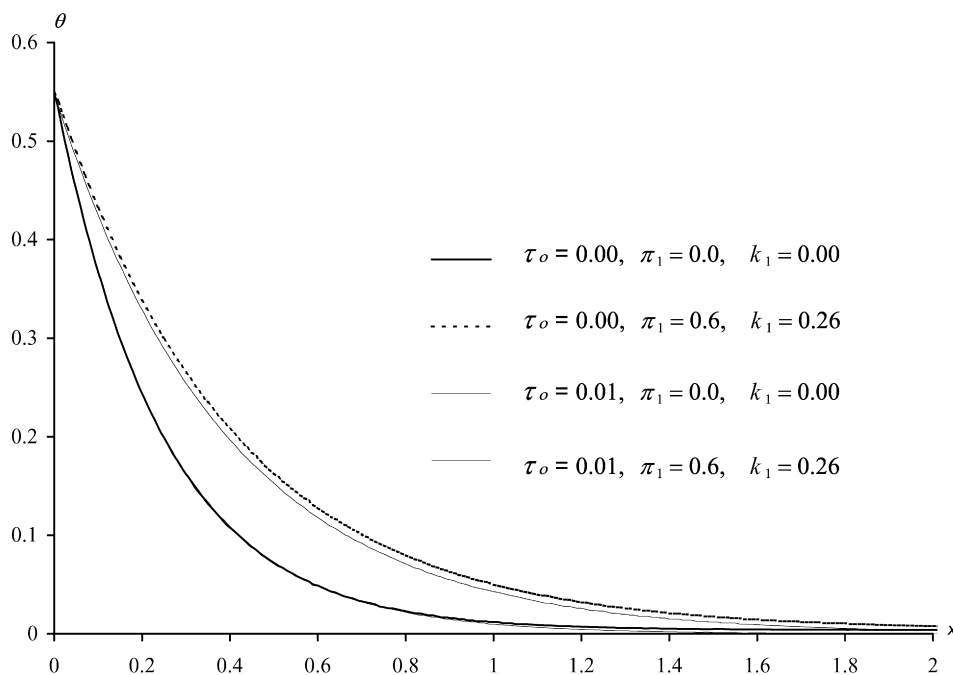


Fig. 1. Variation of temperature with the coefficients of Ohm's and Fourier's laws.

$$\begin{aligned} \lambda &= 9.4 \times 10^{11} \text{ dyne/cm}^2, & \mu &= 4.5 \times 10^{11} \text{ dyne/cm}^2, & \alpha &= 0.5 \times 10^{11} \text{ dyne/cm}^2, \\ \nu + \beta &= 0.779 \times 10^{-4} \text{ dyne/cm}^2, & \rho &= 1.74 \text{ gm/cm}^3, & j &= 0.2 \times 10^{-15} \text{ cm}^2, \\ k &= 0.6 \times 10^{-2} \text{ Cal/cm sec}^\circ\text{C}, & c_e &= 0.23 \text{ Cal/gm}^\circ\text{C}, & \sigma_0 &= 9.36 \times 10^5 \text{ Col}^2/\text{Cal cm sec}, \end{aligned}$$

we consider the following electric constants for our problem

$$\mu_0 = 4\pi \times 10^{-2} \text{ dyne sec}^2/\text{Col}^2, \quad \varepsilon_0 = \frac{10^{-18}}{36\pi} \text{ Col}^2/\text{dyne cm}^2, \quad H_0 = 10^5 \text{ Col/cm sec}.$$

The function $f(y, t)$ applied on the boundary, is taken as follows

$$f(y, t) = \theta_0 H(L - |y|) \exp(-bt),$$

where θ_0 is constant and $H(\cdot)$ is the Heaviside unit step function, putting $f(x, t)$ in normal mode form, we obtain that

$$f^*(a, \omega) = \frac{\theta_0 [\cos(a\ell) - i \sin(a\ell)]}{\exp[(\omega + b)t]}, \quad -L \leq \ell \leq L, \quad t \text{ is a certain value of time.}$$

We have that $\omega = \omega_0 + i\omega_1$ then $e^{\omega t} = e^{\omega_0 t} (\cos \omega_1 t + i \sin \omega_1 t)$, so for small values of time we can take ω is real (i.e. $\omega = \omega_0$), in numerical calculations, the other constants of the problem is taken as follows

$$\omega_0 = 2, \quad a = 2, \quad \tau_0 = 0.01, \quad L = 2, \quad \theta_0 = 1, \quad b = 1, \quad \ell = 0, \quad \pi_1 = 0.6, \quad k_1 = 0.26.$$

The computations are carried out at the non-dimensional time $t = 0.2$, in the plane $y = 0$ and in the range $0 \leq x \leq 2.0$, and we consider the real part of the amplitudes of the field quantities which represented on the vertical axis.

7. Discussion

In all figures, comparisons are made between the theory of micropolar magneto-(coupled and generalized)-thermoelasticity with the presence or absence of modified Ohm's and Fourier's laws, one can notice that the behavior of generalized theory is evident in most figures that the generalized theory predicts finite velocity more than the coupled one.

Fig. 1 describes the variations of the temperatures under four models, it is noticed that the modified Fourier's and Ohm's laws influences is significant, the temperature in the modified model records values higher than these in the old model.

Figs. 2, 3 describe the variations of the normal and transverse displacements, it is noticed that the normal displacement is increased in the modified model while the transverse displacement is decreased.

Fig. 4 describes the variations of the micro-rotation, it is evident that the influence of modified model makes the rotation of the particles decreased.

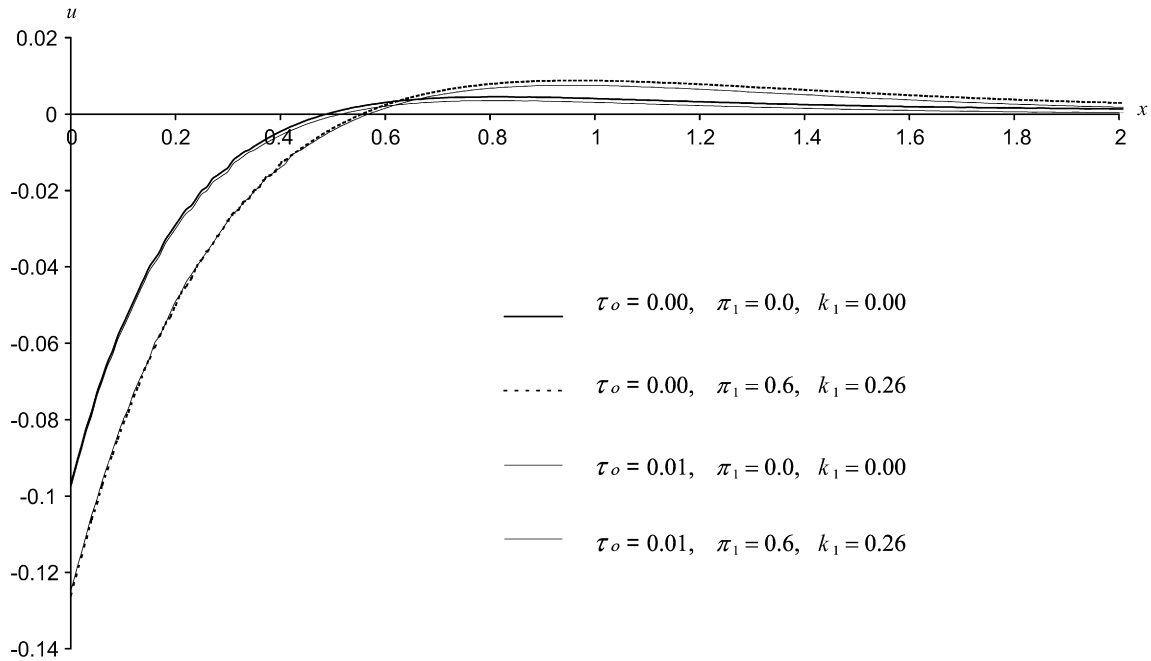


Fig. 2. Variation of normal displacement with the coefficients of Ohm's and Fourier's laws.

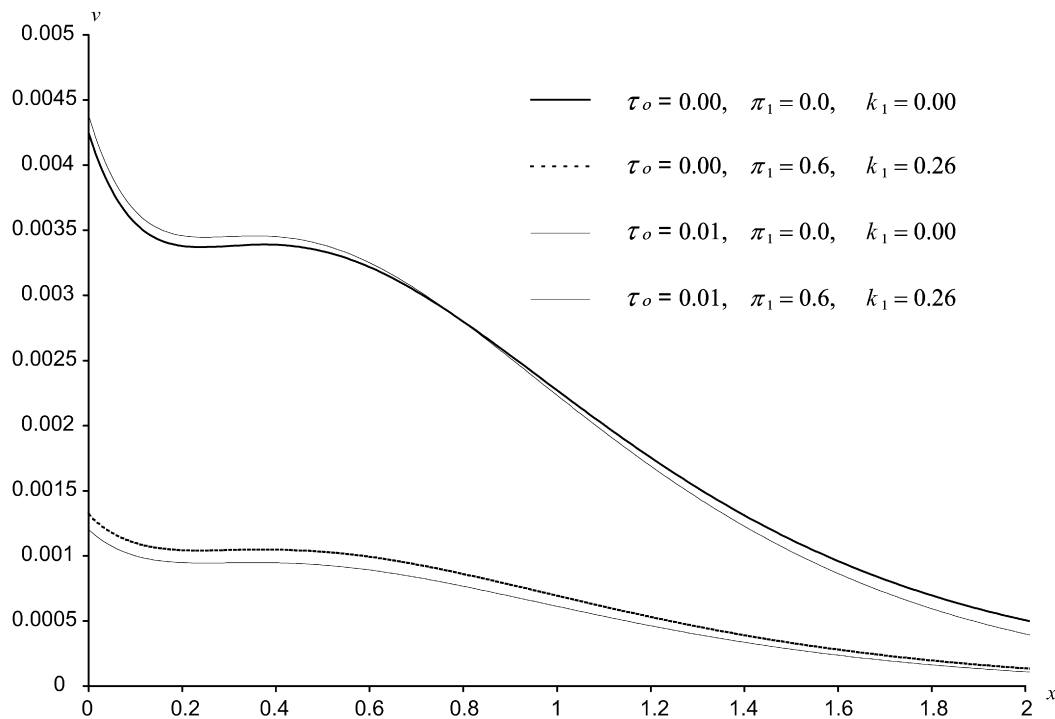


Fig. 3. Variation of transverse displacement with the coefficients of Ohm's and Fourier's laws.

Figs. 5, 6 describe the variations of the normal and one of the shearing stresses, the influence of the modified model affects on the normal stresses by increasing their values and on the shearing stresses by decreasing their values.

Fig. 7 describes the variations of the couple stresses, in accordance with the results appeared in Fig. 4, the values of couple stresses are decreased in the modified model.

Figs. 8, 9 describe the variations of the induced magnetic and electric fields respectively, it is evident that the values of both fields are increased in the modified model.

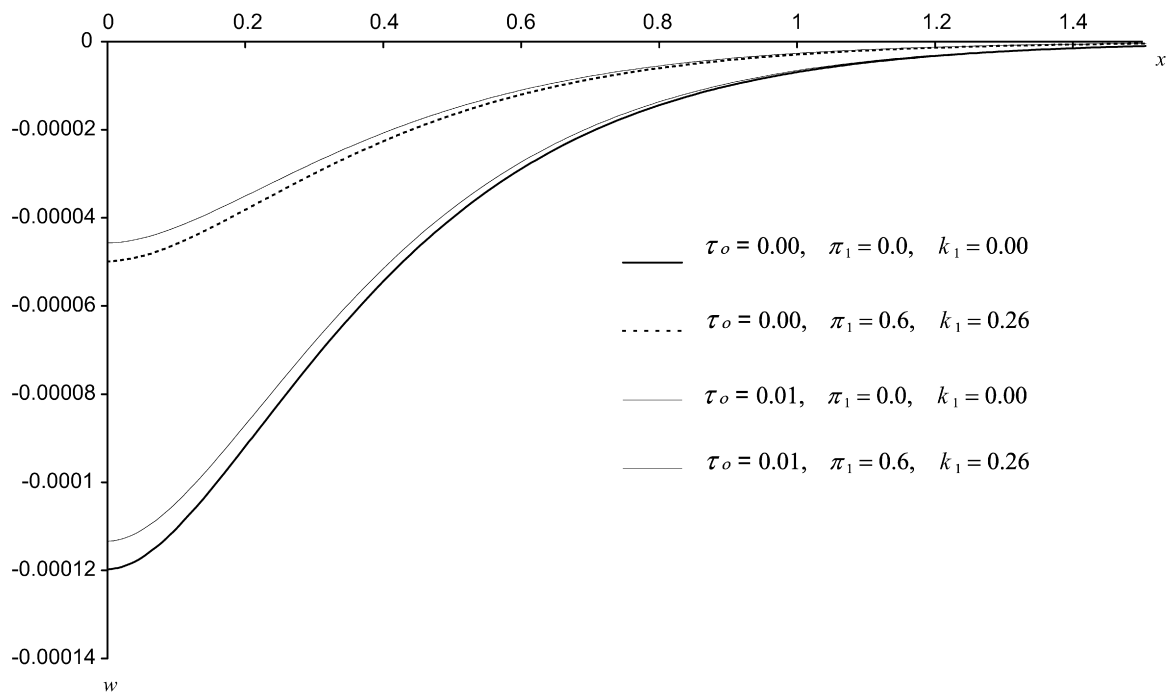


Fig. 4. Variation of micro-rotation with the coefficients of Ohm's and Fourier's laws.

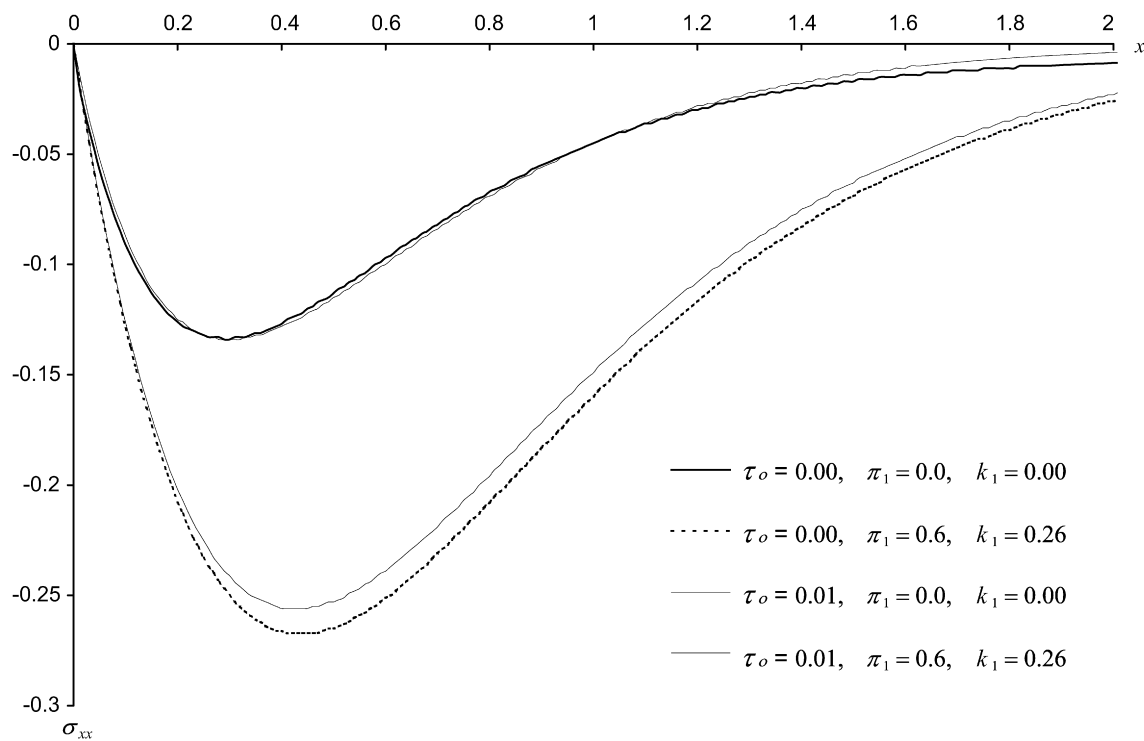


Fig. 5. Variation of normal stresses with the coefficients of Ohm's and Fourier's laws.

8. Conclusion

From the above discussions, one can deduce that the new model of micropolar magneto-generalized thermoelasticity predicts new values for the temperature, displacements, micro-rotation, stresses, couple stresses, electric field and magnetic field which are of similar nature to these in the old model.

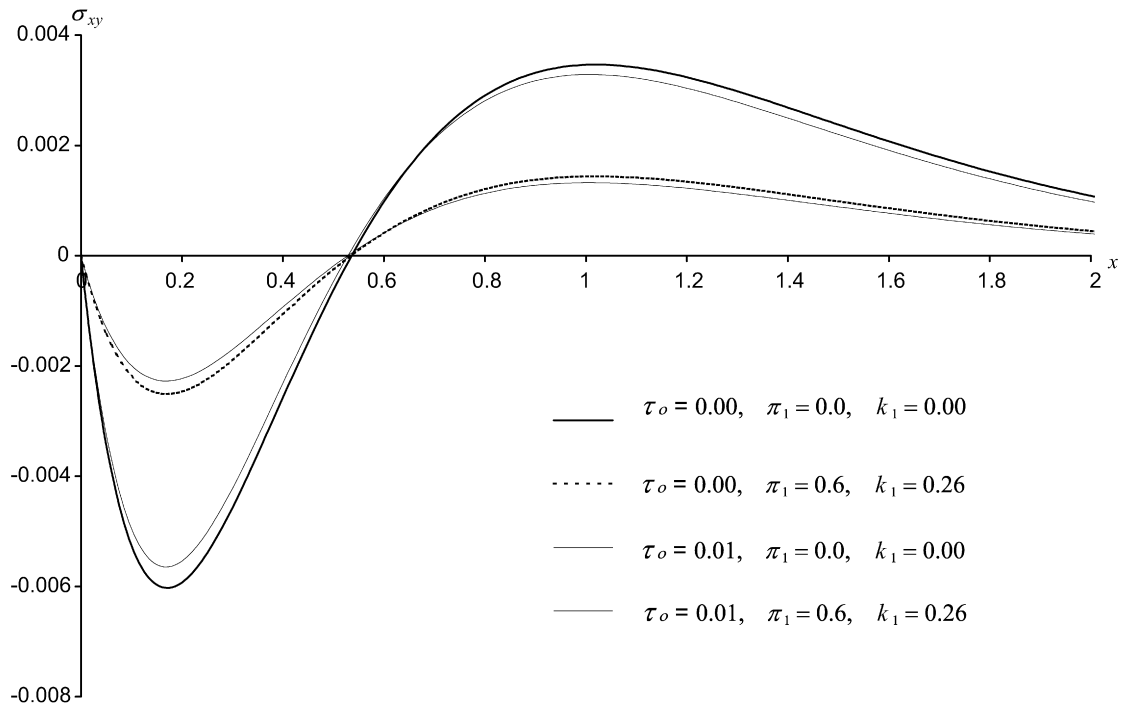


Fig. 6. Variation of shearing stresses with the coefficients of Ohm's and Fourier's laws.

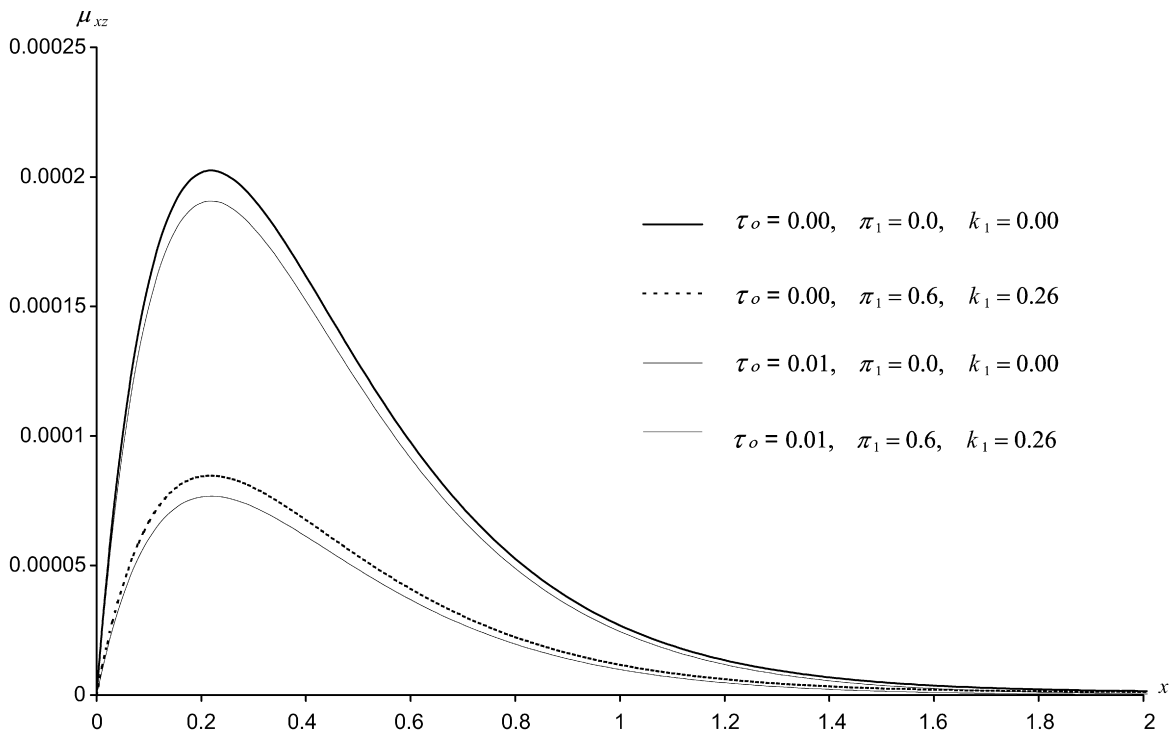


Fig. 7. Variation of couple stresses with the coefficients of Ohm's and Fourier's laws.

The increasing in the values of temperature may be explained as the lost heat generating from the movement of electric current, this heat may be the main reason to make that the deformation of the medium tends to be normal, but the rotation of the particles becomes slower than that in the old model and the components of electric and magnetic fields record values greater than the values recorded in the old model.

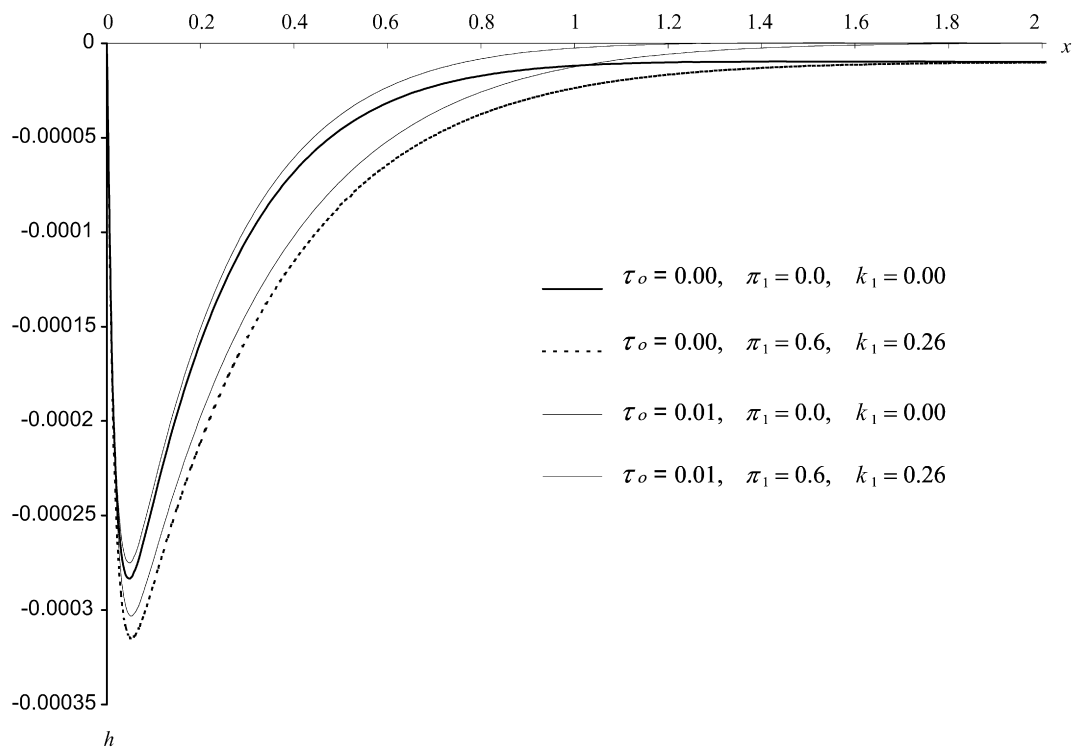


Fig. 8. Variation of magnetic field with the coefficients of Ohm's and Fourier's laws.

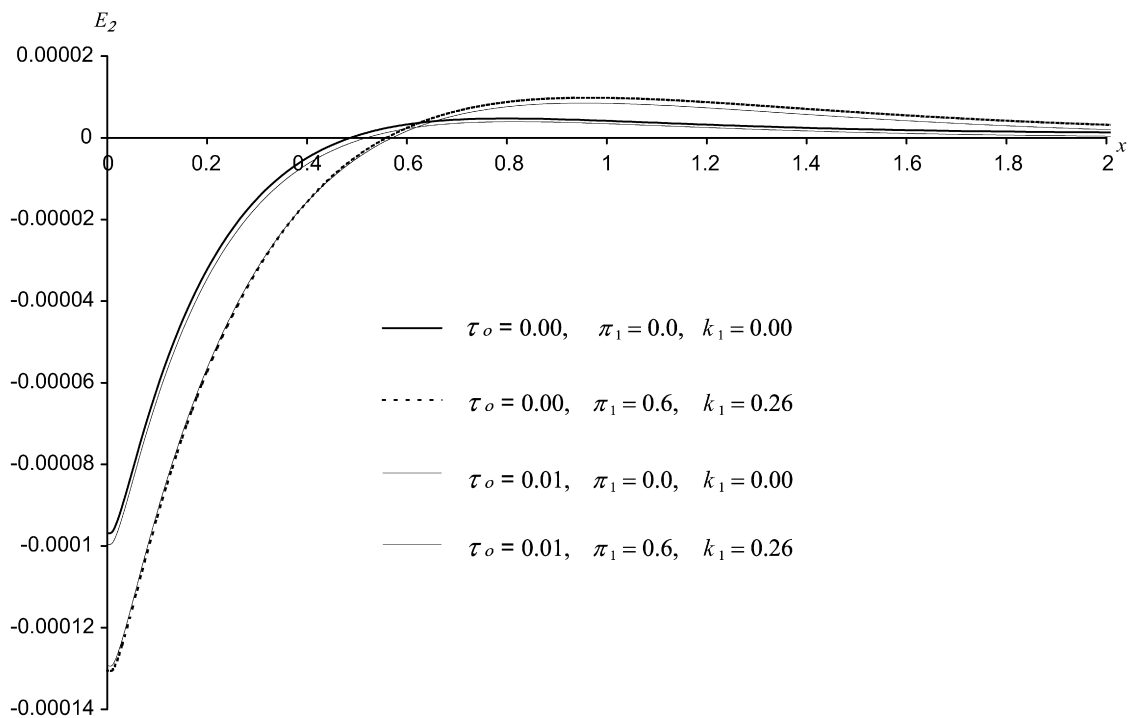


Fig. 9. Variation of transverse electric field with the coefficients of Ohm's and Fourier's laws.

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