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Robust H_∞ filter design for neutral stochastic uncertain systems with time-varying delay ☆

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ABSTRACT

In this paper, the robust H_∞ filtering problem for a class of neutral stochastic systems is discussed. The system under consideration contains parameter uncertainties, Itô-type stochastic disturbances, time-varying delay. The parameter uncertainties are assumed to be time-varying norm-bounded. Using the stochastic Lyapunov stability theory and Itô's differential rule, a full-order filter is designed for all admissible uncertainties and time-varying delay, which is expressed in the form of linear matrix inequality (LMI). The dynamics of the filtering error systems are guaranteed to be robust stochastically mean square asymptotically stable, while achieving a prescribed stochastic robust H_∞ performance level. At the end of this paper, a numerical example is given to demonstrate the usefulness of the proposed method.

1. Introduction

Stochastic modeling has come to play an important role in many branches of science and industry, an area of particular interest has been the automatic control and filter design of stochastic systems, and a great number of results on this subject has been reported in the literatures; see, for example [1–4], and references therein. In 1998, D. Hinrichsen obtained a very important and useful stochastic bounded real lemma for the research of stochastic H_∞ control in the literature [6], and the lemma was applied in the filter design [10]. The classic results of H_2/H_∞ control for deterministic systems which were reported in [13] were also extended to the stochastic systems in the literature [11]. Then, in the following two decades, stochastic robust H_∞ control and H_∞ filter design for the stochastic systems has attracted more and more attention of many experts and has been one of the hottest research areas in the control theory world; see, for example [5,7–12], and the references therein.

Time delay and systems uncertainties are often the two main sources of instability, oscillation and poor performance of control systems, which are encountered in various engineering systems, such as communication, electronics, hydraulic and chemical systems. Therefore, in recent twenty years, considerable attention has been devoted to the studies for stochastic time-delay systems and stochastic uncertain systems, and a great number of results have been reported and various approaches also have been proposed in the literatures; see, for example [14–20], and the references therein.

On the other hand, it is well known that Kalman filtering approach is one of the most popular and most effective ways to deal with the filtering problems [21]. This approach is based on the assumption that the system model is exactly known and

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its disturbances are stationary Gaussian noise with known statistics. However, these assumptions are not exactly satisfied in practical applications [22–24]. Therefore, in the past decade, much research has been discussed depending on the technique of H_∞ filtering, see, for example [25–27], and the references therein.

In this paper, we mainly discuss the robust H_∞ filtering design problem for a class of neutral stochastic uncertain systems with time-varying delay. The rest of the paper is organized as follows: the description of the systems and two lemmas are given in Section 2; in Section 3, the two main results are studied; and a numerical example is given in Section 4 to show the validity of the results and the effectiveness of the proposed approach; Section 5 includes this paper.

Notations. Throughout this paper, for symmetric matrices X and Y, the notation $X \geqslant Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is an identity matrix with appropriate dimensions; the subscript "T" represents the transposition. $E(\cdot)$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. $\mathcal{L}_2[0,\infty)$ is the space of square integrable vector functions over $[0,\infty)$; let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space which relatives to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ algebras $(\mathcal{F}_t)_{t>0} \subset \mathcal{F}$, where Ω is the samples space, $\mathcal{F}$ is σ algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\|\cdot\|_{E_2} = \|E(\cdot)\|_2$, while $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm, R^n and $R^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices respectively.

2. Problem formulation

A class of neutral stochastic uncertain systems with time-varying delay is considered in this paper

$$\begin{aligned}(\Sigma)\quad \mathrm{d}\big[x(t) - D(t)x\big(t-\tau(t)\big)\big] &= \big[A_0(t)x(t) + A_1(t)x\big(t-\tau(t)\big) + G_0 v(t)\big]\,\mathrm{d}t \\ &\quad + \big[H_0(t)x(t) + H_1(t)x\big(t-\tau(t)\big) + G_1 v(t)\big]\,\mathrm{d}w(t),\end{aligned} \tag{2.1}$$

$$\mathrm{d}y(t) = \big[\bar{A}_0(t)x(t) + \bar{A}_1(t)x\big(t-\tau(t)\big) + \bar{G}_0 v(t)\big]\,\mathrm{d}t + \big[\bar{H}_0(t)x(t) + \bar{H}_1(t)x\big(t-\tau(t)\big) + \bar{G}_1 v(t)\big]\,\mathrm{d}w(t), \tag{2.2}$$

$$z(t) = Cx(t), \tag{2.3}$$

$$x(t) = \varphi(t), \quad \forall t \in [-h, 0], \tag{2.4}$$

where $x(t) \in R^n$ is the system state; $v(t) \in R^p$ is the disturbance input of the system which belongs to $\mathcal{L}_2[0,\infty)$, where $\mathcal{L}_2[0,\infty)$ denotes the space of square integrable vector functions over $[0,\infty)$; $y(t) \in R^r$ is the system measured output; $z(t) \in R^q$ is the system control output; $w(t)$ is a zero-mean real scalar Wiener process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Moreover we assume

$$E\big(\mathrm{d}w(t)\big) = 0, \qquad E\big(\mathrm{d}w(t)^2\big) = \mathrm{d}t.$$

In the stochastic systems (Σ), $\tau(t)$ is the system time-varying delay satisfying

$$0 \leqslant \tau(t) \leqslant h < \infty, \qquad \dot{\tau}(t) \leqslant \mu < 1 \tag{2.5}$$

where h, μ are known nonzero constants, $\phi(t)$ is a real-valued initial function on $[-\tau, 0]$ and $A_i(t) = A_i + \Delta A_i(t)$, $\bar{A}_i(t) = \bar{A}_i + \Delta\bar{A}_i(t)$, $H_j(t) = H_j + \Delta H_j(t)$, $\bar{H}_j(t) = \bar{H}_j + \Delta\bar{H}_j(t)$, $i,j = 0,1$, $D(t) = D + \Delta D(t)$, and A_i, C, H_j, $i,j=0,1$, G, $\bar{A}_i$, $\bar{H}_j$, $i,j=0,1$, $\bar{G}$ are known real matrices with appropriate dimension, $\Delta A_i(t)$, $\Delta\bar{A}_i(t)$, $\Delta H_j(t)$, $\Delta\bar{H}_j(t)$, $i,j=0,1$, are unknown matrices with time-varying uncertainties, which satisfy the following conditions

$$\begin{bmatrix} \Delta A_i(t) & \Delta H_j(t) \\ \Delta\bar{A}_i(t) & \Delta\bar{H}_j(t) \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(t) [\, N_i \quad R_j \,], \tag{2.6}$$

$$\Delta D(t) = EF(t)L \tag{2.7}$$

where M_1, M_2, N_i, R_j, E, L, $i,j=0,1$, are known matrices with appropriate dimensions, $F(t): R \to R^{k\times l}$ is an unknown time-varying matrix function, which satisfies

$$F^T(t)F(t) \leqslant I, \quad \forall t > 0. \tag{2.8}$$

The parameter uncertainties are said to be admissible if (2.6)–(2.8) hold.

In this paper, our aim is to design a linear stochastic full-order filter in the following form:

$$(\Sigma_K)\quad \mathrm{d}\xi(t) = A_K\xi(t)\,\mathrm{d}t + B_K\,\mathrm{d}y(t), \tag{2.9}$$

$$\hat{z}(t) = C_K\xi(t) \tag{2.10}$$

where $\xi(t) \in R^n$ is the filter state, $\hat{z}(t) \in R^q$ is the estimation of $z(t)$ in systems (Σ), the matrices A_k, B_k and C_k are the filter matrices with appropriate dimensions, which are to be designed.

Defining $\eta(t) = [\, x^T(t) \quad \xi^T(t) \,]^T$, $e(t) = z(t) - \hat{z}(t)$, then we obtain the filtering error stochastic system as follows:

$$(\Sigma_c)\quad \mathrm{d}\big[\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big] = \big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big) + G_{c0}v(t)\big]\,\mathrm{d}t + \big[H_{c0}(t)\eta(t) + H_{c1}(t)\eta\big(t-\tau(t)\big) + G_{c1}v(t)\big]\,\mathrm{d}w(t), \tag{2.11}$$

$$e(t) = C_c\eta(t) \tag{2.12}$$

where $A_{ci}(t) = A_{ci} + \Delta A_{ci}(t)$, $H_{cj}(t) = H_{cj} + \Delta H_{cj}(t)$, $i, j = 0, 1$, $D_c(t) = D_c + \Delta D_c(t)$, $A_{c0} = \begin{pmatrix} A_0 & 0 \\ B_K\bar{A}_0 & A_K \end{pmatrix}$, $A_{c1} = \begin{pmatrix} A_1 & 0 \\ B_K\bar{A}_1 & 0 \end{pmatrix}$, $G_{c0} = \begin{pmatrix} G_0 \\ B_k\bar{G}_0 \end{pmatrix}$, $H_{c0} = \begin{pmatrix} H_0 & 0 \\ B_K\bar{H}_0 & 0 \end{pmatrix}$, $H_{c1} = \begin{pmatrix} H_1 & 0 \\ B_K\bar{H}_1 & 0 \end{pmatrix}$, $G_{c1} = \begin{pmatrix} G_1 \\ B_k\bar{G}_1 \end{pmatrix}$, $C_c = (C \quad -C_K)$, $D_c = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}$. The uncertainties satisfy the admissible conditions as follows:

$$\Delta D_c(t) = \begin{pmatrix} \Delta D(t) & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} E \\ 0 \end{pmatrix} F(t)\,(L \quad 0), \tag{2.13}$$

$$\Delta A_{c0}(t) = \begin{pmatrix} \Delta A_0(t) & 0 \\ B_K\Delta\bar{A}_0(t) & 0 \end{pmatrix} = \begin{bmatrix} M_1 \\ B_K M_2 \end{bmatrix} F(t)\,[\,N_0 \quad 0\,], \tag{2.14}$$

$$\Delta A_{c1}(t) = \begin{pmatrix} \Delta A_1(t) & 0 \\ B_K\Delta\bar{A}_1(t) & 0 \end{pmatrix} = \begin{bmatrix} M_1 \\ B_K M_2 \end{bmatrix} F(t)\,[\,N_1 \quad 0\,], \tag{2.15}$$

$$\Delta H_{c0}(t) = \begin{pmatrix} \Delta H_0(t) & 0 \\ B_K\Delta\bar{H}_0(t) & 0 \end{pmatrix} = \begin{bmatrix} M_1 \\ B_K M_2 \end{bmatrix} F(t)\,[\,R_0 \quad 0\,], \tag{2.16}$$

$$\Delta H_{c1}(t) = \begin{pmatrix} \Delta H_1(t) & 0 \\ B_K\Delta\bar{H}_1(t) & 0 \end{pmatrix} = \begin{bmatrix} M_1 \\ B_K M_2 \end{bmatrix} F(t)\,[\,R_1 \quad 0\,]. \tag{2.17}$$

Combining (2.13)–(2.17), we obtain

$$\big[\,\Delta A_{c0}(t) \quad \Delta A_{c1}(t) \quad \Delta H_{c0}(t) \quad \Delta H_{c1}(t)\,\big] = \bar{M}F(t)\,[\,\bar{N}_0 \quad \bar{N}_1 \quad \bar{R}_0 \quad \bar{R}_1\,], \tag{2.18}$$

$$\Delta D_c(t) = \bar{E}F(t)\bar{L} \tag{2.19}$$

where $\bar{M} = \begin{pmatrix} M_1 \\ B_k M_2 \end{pmatrix}$, $\bar{N}_0 = (N_0 \quad 0)$, $\bar{N}_1 = (N_1 \quad 0)$, $\bar{R}_0 = (R_0 \quad 0)$, $\bar{R}_1 = (R_1 \quad 0)$, $\bar{E} = \begin{pmatrix} E \\ 0 \end{pmatrix}$, $\bar{L} = (L \quad 0)$.

The stochastic robust H_∞ filter design problems are studied in this paper, which can be formulated as follows: given a prescribed attenuation level $\gamma > 0$, design a linear stochastic filter as the form of (2.9)–(2.10) such that the following two purposes are satisfied:

(P1) The resulting filtering error stochastic system (Σ_c) is stochastically mean square asymptotically stable with the zero disturbance input and for all admissible uncertainties;

(P2) Under zero-initial condition, the following inequality

$$\|e\|_{E_2} < \gamma\|v\|_2 \tag{2.20}$$

holds with the nonzero disturbance input and for all admissible uncertainties.

In the investigation of this paper, we shall mainly use linear matrix inequality (LMI), so we firstly give the following two lemmas without proof.

Lemma 1. *(See [28].) Let D, S and $W > 0$ be real matrices with appropriate dimensions, then for any nonzero vectors x and y with appropriate dimensions, we have*

$$2x^TDSy \leqslant x^TDWD^Tx + y^TS^TW^{-1}Sy.$$

Lemma 2 *(Schur complement). Given three constant matrices S_1, S_2, S_3, where $S_3 = S_3^T < 0$ and $S_1 = S_1^T < 0$, then $S_1 - S_2^TS_3^{-1}S_2 < 0$ if and only if $\begin{pmatrix} S_1 & S_2 \\ S_2 & S_3 \end{pmatrix} < 0$ or $\begin{pmatrix} S_3 & S_2 \\ S_2 & S_1 \end{pmatrix} < 0$.*

3. Main results

In this section, the LMI technology firstly is developed to design a stochastic robust H_∞ filter (Σ_K) such that the neutral stochastic filtering error system (Σ_c) with $v(t) = 0$ is stochastically mean square asymptotically stable.

Theorem 1. *Consider the neutral stochastic uncertain systems* (2.1)–(2.4) *with $v(t) = 0$, there exists a linear stochastic full-order filter* (2.9)–(2.10), *such that the resulting filtering error stochastic systems* (2.11)–(2.12) *are stochastically robust mean square asymp-*

totically stable for all admissible uncertainties (2.13)–(2.19), *if there exist four constants* $\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$, *two matrices* X, Y *with the appropriate dimension and the symmetric positive matrices* P_1, P_2, Q, *such that the following LMI holds*:

$$\Theta = \begin{pmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & 0 & 0 \\ * & * & \Theta_{33} & 0 \\ * & * & * & \Theta_{44} \end{pmatrix} < 0 \tag{3.1}$$

where

$$\Theta_{11} = \begin{pmatrix} \Pi & \bar{A}_0^T Y^T + \varepsilon_0 P_1 M_1 M_2^T Y^T & P_1 A_1 \\ * & X + X^T + \varepsilon_0 Y M_2 M_2^T Y^T & Y \bar{A}_1 \\ * & * & \varepsilon_1 N_1^T N_1 - (1-\mu) Q \end{pmatrix},$$

$$\Pi = Q + P_1 A_0 + A_0^T P_1 + \varepsilon_0 P_1 M_1 M_1^T P_1,$$

$$\Theta_{12} = \begin{pmatrix} A_0^T & \bar{A}_0^T Y^T (P_2^{-1})^T \\ 0 & 0 \\ A_1^T & \bar{A}_1^T Y^T (P_2^{-1})^T \end{pmatrix},$$

$$\Theta_{13} = \begin{pmatrix} H_0^T & \bar{H}_0^T Y^T (P_2^{-1})^T \\ 0 & 0 \\ H_1^T & \bar{H}_1^T Y^T (P_2^{-1})^T \end{pmatrix},$$

$$\Theta_{22} = \Theta_{33} = \begin{pmatrix} \varepsilon_3 M_1 M_1^T - P_1^{-1} & \varepsilon_3 M_1 M_2^T Y^T (P_2^{-1})^T \\ \varepsilon_3 P_2^{-1} Y M_2 M_1^T & \varepsilon_3 Y^T (P_2^{-1})^T M_2 M_2^T Y^T (P_2^{-1})^T - P_2^{-1} \end{pmatrix},$$

$$\Theta_{14} = \begin{pmatrix} 0 & 0 & N_0^T & P_1 M_1 & 0 & N_0^T & R_0^T \\ 0 & 0 & 0 & Y M_2 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & L^T & N_1^T & R_1^T \end{pmatrix},$$

$$\Theta_{44} = diag\big(\varepsilon_2 E E^T - P_1^{-1}, -P_2^{-1}, -\varepsilon_0 I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_3 I\big).$$

Proof. Let $P = diag(P_1, P_2)$, $R = (I \quad 0)$, $A_k = P_2^{-1} X$, $B_k = P_2^{-1} Y$, by the definition of A_{c0}, A_{c1}, H_{c0}, H_{c1}, $\bar{M}$, $\bar{N}_i$, $\bar{R}_j$, $i, j = 0, 1$, $\bar{E}$, $\bar{L}$, D_c, the LMI (3.1) is equivalent to the following LMI:

$$\begin{pmatrix} J_1 & PA_{c1} & A_{c0}^T & H_{c0}^T & 0 & \bar{N}_0^T & P\bar{M} & 0 & \bar{N}_0^T & \bar{R}_0^T \\ * & J_2 & A_{c1}^T & H_{c1}^T & D_c^T & 0 & 0 & \bar{L}^T & \bar{N}_1^T & \bar{R}_1^T \\ * & * & \varepsilon_3 \bar{M}\bar{M}^T - P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varepsilon_3 \bar{M}\bar{M}^T - P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varepsilon_2 \bar{E}\bar{E}^T - P^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_0 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_3 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix} < 0 \tag{3.2}$$

where $J_1 = R^T Q R + PA_{c0} + A_{c0}^T P + \varepsilon_0 P\bar{M}\bar{M}^T P$, $J_2 = \varepsilon_1 \bar{N}_1^T \bar{N}_1 - (1-\mu) R^T Q R$. From the admissible conditions (2.18) and (2.19), we have

$$\begin{pmatrix} 0 & P\Delta A_{c1}(t) & \Delta H_{c0}^T(t) & \Delta A_{c0}^T(t) & 0 \\ * & 0 & \Delta H_{c1}^T(t) & \Delta A_{c1}^T(t) & \Delta D_c^T(t) \\ * & * & -P^{-1} & 0 & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \bar{N}_1^T \\ 0 \\ 0 \\ 0 \end{pmatrix} F^T(t) \begin{pmatrix} P\bar{M} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} P\bar{M} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} F(t) \begin{pmatrix} 0 \\ \bar{N}_1^T \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} 0 \\ \bar{L}^T \\ 0 \\ 0 \\ 0 \end{pmatrix} F^T(t) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{E} \end{pmatrix}^T$$

$$+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{E} \end{pmatrix} F(t) \begin{pmatrix} 0 \\ \bar{L}^T \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} \bar{N}_0^T & \bar{R}_0^T \\ \bar{N}_1^T & \bar{R}_1^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} F^T(t) \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \bar{M} & 0 \\ 0 & \bar{M} \\ 0 & 0 \end{pmatrix}^T$$

$$
\leqslant \varepsilon_1 \begin{pmatrix} 0 \\ \bar{N}_1^T \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \bar{N}_1^T \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \varepsilon_1^{-1} \begin{pmatrix} P\bar{M} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} P\bar{M} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \varepsilon_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{E} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{E} \end{pmatrix}^T
$$

$$
+ \varepsilon_2^{-1} \begin{pmatrix} 0 \\ \bar{L}^T \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \bar{L}^T \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \varepsilon_3 \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \bar{M} & 0 \\ 0 & \bar{M} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \bar{M} & 0 \\ 0 & \bar{M} \\ 0 & 0 \end{pmatrix}^T + \varepsilon_3^{-1} \begin{pmatrix} \bar{N}_0^T & \bar{R}_0^T \\ \bar{N}_1^T & \bar{R}_1^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{N}_0^T & \bar{R}_0^T \\ \bar{N}_1^T & \bar{R}_1^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}^T. \tag{3.3}
$$

By the Schur complement formula, and together with (3.3), the LMI (3.2) implies that

$$
\begin{pmatrix} R^T Q R + PA_{c0} + A_{c0}^T P + \varepsilon_0 P\bar{M}\bar{M}^T P + \varepsilon_0^{-1} \bar{N}_0^T \bar{N}_0 & PA_{c1} & H_{c0}^T & A_{c0}^T & 0 \\ * & -R^T Q R(1-\mu) & H_{c1}^T & A_{c1}^T & D_c^T \\ * & * & -P^{-1} & 0 & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{pmatrix}
$$

$$
+ \begin{pmatrix} 0 & P\Delta A_{c1}(t) & \Delta H_{c0}^T(t) & \Delta A_{c0}^T(t) & 0 \\ * & 0 & \Delta H_{c1}^T(t) & \Delta A_{c1}^T(t) & \Delta D_c^T(t) \\ * & * & -P^{-1} & 0 & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{pmatrix} < 0. \tag{3.4}
$$

Now, set $v(t) = 0$, we obtain the stochastic filtering error system as

$$
\mathrm{d}\big[\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big] = \big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big]\,\mathrm{d}t + \big[H_{c0}(t)\eta(t) + H_{c1}(t)\eta\big(t-\tau(t)\big)\big]\,\mathrm{d}w(t). \tag{3.5}
$$

Define the following Lyapunov function candidate for the systems in (3.3):

$$
V(\eta_t, t) = \big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big)^T P \big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big) + \int_{t-\tau(t)}^{t} \eta^T(s) R^T Q R \eta(s)\,\mathrm{d}s. \tag{3.6}
$$

Applying Itô formula, we have the stochastic differential as

$$
\mathrm{d}V(\eta_t, t) = \mathcal{L}V(\eta_t, t)\,\mathrm{d}t + 2\big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big)^T P\big[H_{c0}(t)\eta(t) + H_{c1}(t)\eta\big(t-\tau(t)\big)\big]\,\mathrm{d}w(t) \tag{3.7}
$$

where

$$
\begin{aligned}
\mathcal{L}V(\eta_t, t) = {} & \big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big)^T P\big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big] \\
& + \big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big]^T P\big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big) \\
& + \big[H_{c0}(t)\eta(t) + H_{c1}(t)\eta\big(t-\tau(t)\big)\big]^T P\big[H_{c0}(t)\eta(t) + H_{c1}(t)\eta\big(t-\tau(t)\big)\big] \\
& + \big[\eta^T(t) R^T Q R \eta(t) - \eta^T\big(t-\tau(t)\big) R^T Q R \eta\big(t-\tau(t)\big)\big(1-\dot{\tau}(t)\big)\big].
\end{aligned} \tag{3.8}
$$

By Lemma 1 and admissible condition (2.18), we have

$$
\begin{aligned}
& \big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big)^T P\big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big] \\
& \quad + \big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big]^T P\big(\eta(t) - D_c(t)\eta\big(t-\tau(t)\big)\big) \\
& = \eta^T(t)\big(PA_{c0}(t) + A_{c0}^T(t)P\big)\eta(t) + \eta^T(t)PA_{c1}(t)\eta\big(t-\tau(t)\big) + \eta^T\big(t-\tau(t)\big)A_{c1}^T(t)P\eta(t) \\
& \quad - \eta^T\big(t-\tau(t)\big)D_c^T(t)P\big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big] \\
& \quad - \big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big]^T PD_c(t)\eta\big(t-\tau(t)\big) \\
& \leqslant \eta^T(t)\big(PA_{c0} + A_{c0}^T P + \varepsilon_0 P\bar{M}\bar{M}^T P + \varepsilon_0^{-1}\bar{N}_0^T\bar{N}_0\big)\eta(t) + \eta^T(t)PA_{c1}(t)\eta\big(t-\tau(t)\big) \\
& \quad + \eta^T\big(t-\tau(t)\big)A_{c1}^T(t)P\eta(t) + \eta^T\big(t-\tau(t)\big)D_c^T(t)PD_c(t)\eta\big(t-\tau(t)\big) \\
& \quad + \big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big]^T P\big[A_{c0}(t)\eta(t) + A_{c1}(t)\eta\big(t-\tau(t)\big)\big].
\end{aligned} \tag{3.9}
$$

Let $\zeta(t) = (\eta(t) \quad \eta(t-\tau(t)))^T$, noticing (3.8) and (3.9), we obtain

$$
\mathcal{L}V(\eta_t, t) \leqslant \zeta^T(t)\Pi(t)\zeta(t) \tag{3.10}
$$

where

$$\Pi(t) = \begin{pmatrix} R^TQR + PA_{c0} + A_{c0}^TP + \varepsilon_0 P\bar{M}\bar{M}^TP \\ \quad + \varepsilon_0^{-1}\bar{N}_0^T\bar{N}_0 + H_{c0}^T(t)PH_{c0}(t) + A_{c0}^T(t)PA_{c0}(t) & PA_{c1}(t) + H_{c0}^T(t)PH_{c1}(t) + A_{c0}^T(t)PA_{c1}(t) \\ A_{c1}^T(t)P + H_{c1}^T(t)PH_{c0}(t) + A_{c1}^T(t)PA_{c0}(t) & D_c^T(t)PD_c(t) - R^TQR(1-\mu) \\ & \quad + H_{c1}^T(t)PH_{c1}(t) + A_{c1}^T(t)PA_{c1}(t) \end{pmatrix}$$

$$= \begin{pmatrix} R^TQR + PA_{c0} + A_{c0}^TP + \varepsilon_0 P\bar{M}\bar{M}^TP + \varepsilon_0^{-1}\bar{N}_0^T\bar{N}_0 & PA_{c1}(t) \\ A_{c1}^T(t)P & D_c^T(t)PD_c(t) - R^TQR(1-\mu) \end{pmatrix}$$

$$+ \begin{pmatrix} H_{c0}^T(t) \\ H_{c1}^T(t) \end{pmatrix} P \begin{pmatrix} H_{c0}^T(t) \\ H_{c1}^T(t) \end{pmatrix}^T + \begin{pmatrix} A_{c0}^T(t) \\ A_{c1}^T(t) \end{pmatrix} P \begin{pmatrix} A_{c0}^T(t) \\ A_{c1}^T(t) \end{pmatrix}^T.$$

Thanks to Schur complement lemma, this is equivalent to the following matrix

$$\begin{pmatrix} R^TQR + PA_{c0} + A_{c0}^TP + \varepsilon_0 P\bar{M}\bar{M}^TP + \varepsilon_0^{-1}\bar{N}_0^T\bar{N}_0 & PA_{c1}(t) & H_{c0}^T(t) & A_{c0}^T(t) & 0 \\ * & -R^TQR(1-\mu) & H_{c1}^T(t) & A_{c1}^T(t) & D_c^T(t) \\ * & * & -P^{-1} & 0 & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{pmatrix}$$

$$= \begin{pmatrix} R^TQR + PA_{c0} + A_{c0}^TP + \varepsilon_0 P\bar{M}\bar{M}^TP + \varepsilon_0^{-1}\bar{N}_0^T\bar{N}_0 & PA_{c1} & H_{c0}^T & A_{c0}^T & 0 \\ * & -R^TQR(1-\mu) & H_{c1}^T & A_{c1}^T & D_c^T \\ * & * & -P^{-1} & 0 & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & P\Delta A_{c1}(t) & \Delta H_{c0}^T(t) & \Delta A_{c0}^T(t) & 0 \\ * & 0 & \Delta H_{c1}^T(t) & \Delta A_{c1}^T(t) & \Delta D_c^T(t) \\ * & * & -P^{-1} & 0 & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -P^{-1} \end{pmatrix}.$$

From the LMI (3.4) we have

$$\mathcal{L}V(\eta_t, t) \leqslant \zeta^T(t)\Pi(t)\zeta(t) < 0 \tag{3.11}$$

which implies the filtering error stochastic systems (2.11)–(2.12) are stochastically robust mean square asymptotically stable for all admissible uncertainties. □

Now, based on Theorem 1, we are able to focus on the analysis of the H_∞ performance of the filtering process in the following theorem.

Theorem 2. *Consider the neutral stochastic uncertain systems* (2.1)–(2.4) *with* $v(t) \neq 0$, *given a scalar* $\gamma > 0$, *then there exists a linear stochastic full-order filter* (2.9)–(2.10), *such that the resulting filtering error stochastic systems* (2.11)–(2.12) *are stochastically robust mean square asymptotically stable and satisfy* $\|e\|_{E_2} < \gamma\|v\|_2$ *for all admissible uncertainties* (2.13)–(2.19) *if there exist four constants* $\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$, *two matrices* X, Y *with appropriate dimension and the symmetric positive matrices* P_1, P_2, Q, *such that the following LMI holds*:

$$\Theta = \begin{pmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & 0 & 0 \\ * & * & \Theta_{33} & 0 \\ * & * & * & \Theta_{44} \end{pmatrix} < 0 \tag{3.12}$$

where

$$\Theta_{11} = \begin{pmatrix} \Pi & \bar{A}_0^TY^T + \varepsilon_0 P_1M_1M_2^TY^T & P_1A_1 & P_1G_0 \\ * & X + X^T + \varepsilon_0 YM_2M_2^TY^T & Y\bar{A}_1 & Y\bar{G}_0 \\ * & * & \varepsilon_1 N_1^TN_1 - (1-\mu)Q & 0 \\ * & * & * & -\gamma^2 I \end{pmatrix},$$

$$\Pi = Q + P_1A_0 + A_0^TP_1 + \varepsilon_0 P_1M_1M_1^TP_1,$$

$$\Theta_{12}=\begin{pmatrix} A_0^T & \bar{A}_0^T Y^T (P_2^{-1})^T \\ 0 & 0 \\ A_1^T & \bar{A}_1^T Y^T (P_2^{-1})^T \\ G_0^T & \bar{G}_0^T Y^T (P_2^{-1})^T \end{pmatrix}, \qquad \Theta_{13}=\begin{pmatrix} H_0^T & \bar{H}_0^T Y^T (P_2^{-1})^T \\ 0 & 0 \\ H_1^T & \bar{H}_1^T Y^T (P_2^{-1})^T \\ G_1^T & \bar{G}_1^T Y^T (P_2^{-1})^T \end{pmatrix},$$

$$\Theta_{22}=\Theta_{33}=\begin{pmatrix} \varepsilon_3 M_1 M_1^T - P_1^{-1} & \varepsilon_3 M_1 M_2^T Y^T (P_2^{-1})^T \\ \varepsilon_3 Y^T (P_2^{-1})^T M_2 M_1^T & \varepsilon_3 P_2^{-1} Y M_2 M_2^T Y^T (P_2^{-1})^T - P_2^{-1} \end{pmatrix},$$

$$\Theta_{14}=\begin{pmatrix} 0 & 0 & N_0^T & P_1 M_1 & 0 & N_0^T & R_0^T & C^T \\ 0 & 0 & 0 & Y M_2 & 0 & 0 & 0 & -C_k^T \\ D & 0 & 0 & 0 & L^T & N_1^T & R_1^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Theta_{44}=diag\big(\varepsilon_2 E E^T - P_1^{-1}, -P_2^{-1}, -\varepsilon_0 I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_3 I, -I\big).$$

In this case, the parameters of a desired filter in the form of (2.9)–(2.10) *are* A_k, B_k, C_k, *then the filter is given by the following form*:

$$\begin{cases} \mathrm{d}\xi(t)=A_K\xi(t)\,\mathrm{d}t+B_K\,\mathrm{d}y(t), \\ \hat{z}(t)=C_K\xi(t) \end{cases} \tag{3.13}$$

where $A_k=P_2^{-1}X$, $B_k=P_2^{-1}Y$.

Proof. Consider the filtering error stochastic system with as follows:

$$(\Sigma_c)\quad \mathrm{d}\big[\eta(t)-D_c(t)\eta\big(t-\tau(t)\big)\big]=\big[A_{c0}(t)\eta(t)+A_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c0}v(t)\big]\,\mathrm{d}t$$
$$+\big[H_{c0}(t)\eta(t)+H_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c1}v(t)\big]\,\mathrm{d}w(t), \tag{3.14}$$

$$e(t)=C_c\eta(t). \tag{3.15}$$

Because the LMI (3.12) implies the LMI (3.1) holds, therefore, by Theorem 1, we have the filtering error stochastic systems (3.14)–(3.15) are stochastically robust mean square asymptotically stable for all admissible uncertainties. Next, we shall show that under the zero-initial condition, the systems (Σ_c) satisfy (P2) for any nonzero $v(t)\in\mathcal{L}[0,\infty)$, to this target, we consider the following Lyapunov function candidate for the systems (3.14)–(3.15)

$$V(\eta_t,t)=\big(\eta(t)-D_c(t)\eta\big(t-\tau(t)\big)\big)^T P\big(\eta(t)-D_c(t)\eta\big(t-\tau(t)\big)\big)+\int_{t-\tau(t)}^{t}\eta^T(s)R^T Q R\eta(s)\,\mathrm{d}s. \tag{3.16}$$

Applying Itô formula, we obtain the stochastic differential as

$$\mathrm{d}V(\eta_t,t)=\mathcal{L}V(\eta_t,t)\,\mathrm{d}t+2\big(\eta(t)-D_c(t)\eta\big(t-\tau(t)\big)\big)^T P\big[H_{c0}(t)\eta(t)+H_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c1}v(t)\big]\,\mathrm{d}w(t) \tag{3.17}$$

where

$$\begin{aligned}\mathcal{L}V(\eta_t,t)&=\big(\eta(t)-D_c(t)\eta\big(t-\tau(t)\big)\big)^T P\big[A_{c0}(t)\eta(t)+A_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c0}v(t)\big]\\&\quad+\big[A_{c0}(t)\eta(t)+A_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c0}v(t)\big]^T P\big(\eta(t)-D_c(t)\eta\big(t-\tau(t)\big)\big)\\&\quad+\big[H_{c0}(t)\eta(t)+H_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c1}v(t)\big]^T P\big[H_{c0}(t)\eta(t)+H_{c1}(t)\eta\big(t-\tau(t)\big)+G_{c1}v(t)\big]\\&\quad+\big[\eta^T(t)R^T Q R\eta(t)-\eta^T\big(t-\tau(t)\big)R^T Q R\eta\big(t-\tau(t)\big)\big(1-\dot{\tau}(t)\big)\big].\end{aligned} \tag{3.18}$$

Let $\zeta(t)=(\eta(t)\quad \eta(t-\tau(t))\quad v(t))^T$; similar to the proof of Theorem 1, we have

$$\mathcal{L}V(\eta_t,t)\leqslant\zeta^T(t)\tilde{\Pi}(t)\zeta(t) \tag{3.19}$$

where

$$\tilde{\Pi}(t)=\begin{pmatrix} R^T Q R+PA_{c0}+A_{c0}^T P+\varepsilon_0 P\bar{M}\bar{M}^T P+\varepsilon_0^{-1}\bar{N}_0^T\bar{N} & PA_{c1}(t) & PG_{c0} \\ A_{c1}^T(t)P & D_c^T(t)PD_c(t)-R^T Q R(1-\mu) & 0 \\ G_{c0}^T P & 0 & 0 \end{pmatrix}$$
$$+\begin{pmatrix} H_{c0}^T(t) \\ H_{c1}^T(t) \\ G_{c1}^T \end{pmatrix} P \begin{pmatrix} H_{c0}^T(t) \\ H_{c1}^T(t) \\ G_{c1}^T \end{pmatrix}^T + \begin{pmatrix} A_{c0}^T(t) \\ A_{c1}^T(t) \\ G_{c0}^T \end{pmatrix} P \begin{pmatrix} A_{c0}^T(t) \\ A_{c1}^T(t) \\ G_{c0}^T \end{pmatrix}^T.$$

By the Schur complement lemma, this is equivalent to the following matrix

$$\begin{pmatrix} R^TQR+PA_{c0}+A_{c0}^TP+\varepsilon_0P\bar{M}\bar{M}^TP+\varepsilon_0^{-1}\bar{N}_0^T\bar{N} & PA_{c1}(t) & PG_{c0} & A_{c0}^T(t) & H_{c0}^T(t) & 0 \\ A_{c1}^T(t)P & -R^TQR(1-\mu) & 0 & A_{c1}^T(t) & H_{c1}^T(t) & D_c^T(t) \\ G_{c0}^TP & 0 & 0 & G_{c0}^T & G_{c1}^T & 0 \\ A_{c0}(t) & A_{c1}(t) & G_{c0} & -P^{-1} & 0 & 0 \\ H_{c0}(t) & H_{c1}(t) & G_{c1} & 0 & -P^{-1} & 0 \\ 0 & D_{c(t)} & 0 & 0 & 0 & -P^{-1} \end{pmatrix}.$$

Hence, considering the stochastic robust H_∞ performance level for the filtering error stochastic systems (3.14)–(3.15), for any $t>0$, we define

$$J(t)=E\left\{\int_0^t\left[z^T(s)z(s)-\gamma^2v^T(s)v(s)\right]\mathrm{d}s\right\}. \tag{3.20}$$

Then, integrating both sides of (3.17) from 0 to t and taking expectations result in

$$E\big(V(\eta_t,t)-V(\eta_0,0)\big)=E\big(V(\eta_t,t)\big)=E\int_0^t\mathcal{L}V(\eta_s,s)\,\mathrm{d}s. \tag{3.21}$$

Immediately we get

$$\begin{aligned} J(t)&=E\left\{\int_0^t\left[z_c^T(s)z_c(s)-\gamma^2v^T(s)v(s)+\mathcal{L}V(\eta_s,s)\right]\mathrm{d}s\right\}-E\big(V(\eta_t,t)\big)\\ &\leqslant E\left\{\int_0^t\left[z_c^T(s)z_c(s)-\gamma^2v^T(s)v(s)+\mathcal{L}V(\eta_s,s)\right]\mathrm{d}s\right\}\\ &\leqslant E\left\{\int_0^t\left[\eta^T(s)\quad\eta^T\big(s-\tau(s)\big)\quad v^T(s)\right]\Upsilon(s)\left[\eta^T(s)\quad\eta^T\big(s-\tau(s)\big)\quad v^T(s)\right]^T\mathrm{d}s\right\} \end{aligned} \tag{3.22}$$

where

$$\Upsilon(t)=\tilde{\Pi}(t)+\begin{pmatrix} C_c^TC_c & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\gamma^2I\end{pmatrix}. \tag{3.23}$$

By using the Schur complement lemma and following the similar line as in the proof of Theorem 1, the LMI (3.12) implies $\Upsilon(t)<0$. This, together with (3.22), we have

$$J(t)<0. \tag{3.24}$$

Therefore, the inequality $\|z\|_{E_2}<\gamma\|v\|_2$ holds. This completes the proof. □

4. Numerical example

In this section, we give an example to show the feasibility of the controller and the usefulness of the proposed techniques in the paper.

Consider the uncertain neutral stochastic systems (2.1)–(2.4), the parameters are given as follows:

$$A_0=\begin{pmatrix}-3 & 0\\ 0.2 & -5\end{pmatrix},\qquad A_1=\begin{pmatrix}-1 & 0.5\\ 0 & -1.5\end{pmatrix},\qquad G_0=\begin{pmatrix}-0.2\\ -0.2\end{pmatrix},\qquad D=\begin{pmatrix}0.2 & 0\\ 1 & 0.5\end{pmatrix},$$

$$H_0=\begin{pmatrix}-2 & 0.5\\ 1 & -0.5\end{pmatrix},\qquad H_1=\begin{pmatrix}-1 & 0.3\\ 0.2 & -1.5\end{pmatrix},\qquad G_1=\begin{pmatrix}-0.2\\ 0.2\end{pmatrix},\qquad \bar{A}_0=\begin{pmatrix}-10 & 0.5\\ 0.2 & -5\end{pmatrix},$$

$$\bar{A}_1=\begin{pmatrix}-1.5 & 0.4\\ 0.5 & -1\end{pmatrix},\qquad \bar{H}_0=\begin{pmatrix}0.5 & 0.5\\ 1 & 0\end{pmatrix},\qquad \bar{G}_0=\begin{pmatrix}-0.1\\ 0.1\end{pmatrix},\qquad \bar{H}_1=\begin{pmatrix}1 & 0.2\\ 0.3 & -1\end{pmatrix},$$

$$\bar{G}_1=\begin{pmatrix}-0.1\\ 0.1\end{pmatrix},\qquad C=\begin{pmatrix}0.2 & 0\\ 0.1 & -0.3\end{pmatrix},\qquad M_1=\begin{pmatrix}-0.2\\ 0.1\end{pmatrix},\qquad M_2=\begin{pmatrix}0.1\\ 0.1\end{pmatrix},\qquad E=\begin{pmatrix}-0.2\\ 0.1\end{pmatrix},$$

$$N_0=(0.1\quad -0.1),\qquad N_1=(-0.1\quad 0.2),\qquad R_0=(-0.2\quad 0.1),$$

$$R_1=(0.1\quad -0.1),\qquad L=(0.2\quad 0.1),\qquad \mu=0.3.$$

In this example, attention is focused on the design of a full-order stochastic filter (Σ_k), such that the stochastic filtering error system is stochastically mean square asymptotically stable for all admissible uncertainties and dissipation level $\gamma = 0.9$, for this objective, we use the Matlab LMI Control Toolbox to solve the LMI (3.12), and obtain the solution as follows:

$$P_1 = \begin{pmatrix} 14.6696 & 0.1740 \\ 0.1740 & 8.3139 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 35.4246 & 0 \\ 0 & 35.4246 \end{pmatrix}, \qquad Q = \begin{pmatrix} 40.0139 & -5.2180 \\ -5.2180 & 43.6390 \end{pmatrix},$$
$$X = \begin{pmatrix} -22.2449 & -6.0106 \\ -3.0757 & -22.2338 \end{pmatrix}, \qquad Y = \begin{pmatrix} -0.0267 & -0.0795 \\ -0.0322 & -0.1081 \end{pmatrix}, \qquad C_k = \begin{pmatrix} 0.0308 & 0.0315 \\ 0.0052 & 0.0011 \end{pmatrix},$$
$$\varepsilon_0 = 33.8610, \qquad \varepsilon_1 = 34.1235, \qquad \varepsilon_2 = 12.7109, \qquad \varepsilon_3 = 1.5400.$$

Therefore, the full-order stochastic filter is given as follows:

$$\begin{cases} \mathrm{d}\xi(t) = A_K \xi(t)\,\mathrm{d}t + B_K\,\mathrm{d}y(t), \\ \hat{z}(t) = C_K \xi(t) \end{cases}$$

where $A_k = P_2^{-1} X = \left(\begin{smallmatrix} -0.6280 & -0.1697 \\ -0.0868 & -0.6276 \end{smallmatrix}\right)$, $B_k = P_2^{-1} Y = \left(\begin{smallmatrix} -0.0008 & -0.0022 \\ -0.0009 & -0.0031 \end{smallmatrix}\right)$.

5. Conclusion

In this paper, the robust H_∞ filtering design problems for a class of neutral stochastic uncertain systems with time-varying norm bounded parameter uncertainties have been studied. The LMI approach and stochastic Lyapunov stability theory have been used to design the full-order stochastic filter, which ensures the resulting stochastic filtering error systems satisfy two purposes (P1) and (P2). At last, a numerical example has been given to show the validity of the result and the effectiveness of the LMI approach.

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