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Global solvability and large-time behavior to a three-dimensional chemotaxis-Stokes system modeling coral fertilization ☆

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A B S T R A C T

We consider the chemotaxis-Stokes system

$$\begin{cases} n_t + u\cdot\nabla n = \Delta n - \nabla\cdot(n\chi(c)\nabla c) - mn, & x\in\Omega, \quad t>0,\\ m_t + u\cdot\nabla m = \Delta m - nm, & x\in\Omega, \quad t>0,\\ c_t + u\cdot\nabla c = \Delta c - c + m, & x\in\Omega, \quad t>0,\\ u_t = \Delta u + \nabla P + (n+m)\nabla\phi, & x\in\Omega, \quad t>0,\\ \nabla\cdot u = 0, & x\in\Omega, \quad t>0 \end{cases}$$

under homogenous Neumann boundary conditions in a three-dimensional bounded domain $\Omega\subset\mathbb{R}^3$ with smooth boundary. Here χ is a nondecreasing function on $[0,\infty)$. It is shown that if

$$K_0\chi(K_0) < \frac{\sqrt{2}}{27} \quad \text{with } K_0 = \max\{\|c_0\|_{L^\infty(\Omega)}, \|m_0\|_{L^\infty(\Omega)}\},$$

then the system possesses a globally bounded classical solution.

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1. Introduction

In this paper, we consider the following chemotaxis-fluid system

$$\begin{cases} n_t + u \cdot \nabla n = \Delta n - \chi \nabla \cdot (n\chi(c)\nabla c) - mn, & x \in \Omega, \quad t > 0, \\ m_t + u \cdot \nabla m = \Delta m - nm, & x \in \Omega, \quad t > 0, \\ c_t + u \cdot \nabla c = \Delta c - c + m, & x \in \Omega, \quad t > 0, \\ u_t = \Delta u + \nabla P + (n+m)\nabla \phi, & x \in \Omega, \quad t > 0, \\ \nabla \cdot u = 0, & x \in \Omega, \quad t > 0 \end{cases} \tag{1.1}$$

for the unknown functions $n(x,t)$, $m(x,t)$, $c(x,t)$ and $u(x,t)$ in a bounded domain $\Omega \subset \mathbb{R}^3$ with smooth boundary condition, where ν is the outward normal vector on $\partial\Omega$. To be specific, n, m and c denote the population densities of unfertilized sperms, eggs and the concentration of a chemical released by the latter. u represents the velocity field of the fluid subjecting to an incompressible Navier-Stokes equation with pressure P. The χ measures chemotaxis sensitivity and ϕ is a given potential field. This model was proposed to describe coral fertilization [4,9,10].

Chemotaxis, an oriented movement of cells (or organisms, bacteria) effected by the chemical gradient, plays an important role in describing many biological phenomena. In 1970s, Keller and Segel [8] put forward a mathematical model to describe the aggregation phase of slime mold amoebae

$$\begin{cases} n_t = \Delta n - \nabla \cdot (n\chi(x,n,c)\nabla c), \\ c_t = \Delta c - c + n, \end{cases} \tag{1.2}$$

which has been well investigated in the last decades. The mathematical understanding of (1.2) seems rather complete with respect to some fundamental problems in bounded domains or in whole space, such as global existence and boundedness, finite-time blow-up (see for instance [14,15,17,22]). For more results, one can refer to [3,6,27,28] and the references therein. However, the mathematical study of coral fertilization model seems few.

In [9,10] Kiselev and Ryzhik have studied a two-component system modeling the fertilization process

$$\begin{cases} n_t + u\nabla n = \Delta n - \chi \nabla \cdot (n\nabla c) - \mu n^q, \\ 0 = \Delta c + n, \end{cases} \tag{1.3}$$

in $\mathbb{R}^2$. Here the given vector u in (1.3) models the ambient ocean flow and is independent of n. Compared to (1.1), the densities of sperm and egg are assumed to be identical and the effects of chemical concentration c is ignored in (1.3). In the supercritical case, $q > 2$, it has been proved that the total mass $\int_{\mathbb{R}^2} n(\cdot,t)$ tends to a positive constant $C(\chi, n_0, u)$ as $t \to \infty$, but $C(\chi, n_0, u) \to 0$ as $\chi \to \infty$, with q, n_0 and u fixed. In the $q = 2$ case, a weaker but related effect within finite time intervals is observed. When the second equation of (1.3) is replaced by the equation $c_t + u\nabla c = \Delta c - c + n$, [1] established the well-posedness of regular solution. Moreover, it also showed that the asymptotic behavior of $\int_{\mathbb{R}^2} n(\cdot,t)$ is similar to the supercritical case of (1.3).

In [4], the three-component system modeling coral broadcasting spawning

$$\begin{cases} n_t + u \cdot \nabla n = \Delta n - \nabla \cdot (n\chi(x,n,c)\nabla c) - \mu n^2, \\ c_t + u \cdot \nabla c = \Delta c - c + n, \\ u_t + k(u \cdot \nabla)u = \Delta u + \nabla P + n\nabla \phi, \\ \nabla \cdot u = 0 \end{cases} \tag{1.4}$$

is studied. Compared to (1.1), the convective term $k(u\cdot\nabla)u$ is considered while the interaction of chemotactic movement is neglected. Moreover, if the second equation is replaced by $c_t + u \cdot \nabla c = \Delta c - nc$ and $\mu = 0$, it becomes the system proposed by Tuval et al. [19], which is used to describe the dynamics of swimming bacteria. From the viewpoint of mathematical analysis, the precedents of (1.4) mainly concentrate on the solvability when $\mu = 0$ and the chemotactic sensitivity satisfies the saturation effect $|\chi(x,n,c)| \le C_S(1+n)^{-\alpha}$. In three-dimensional bounded domains, [13] proved the global existence of weak solution under the assumption that $k \in \mathbb{R}$ and with $\alpha > \frac{3}{7}$, [25] proved the existence of a global bounded classical solution under the assumption that $k = 0$ and $\alpha > \frac{1}{3}$, [20] proved that the system possesses a global very weak solution when $k \in \mathbb{R}$ and $\chi(x,n,c)$ is a tensor function with $\alpha > \frac{1}{3}$. For more related works, one can refer to [12,13,16,18,29] and the reference therein.

For the four-component system (1.1), Espejo and Winkler [5] have proved the global existence of classical solution in two-dimensional bounded domain when the convective term $k(u \cdot \nabla)u$ is considered. Recently, in three-dimensional case, [11] shows that the solutions are globally bounded and decay to a spatially homogeneous equilibrium exponentially under the assumption that tensor function χ satisfies $|\chi(x,n,c)| \le (1+n)^{-\alpha}$ with $\alpha > \frac{1}{3}$.

Motivated by the above works, we shall consider the solvability of the system (1.1) in three-dimensional.

To formulate the result, we specify the precise mathematical settings. We shall consider the system (1.1) with the boundary conditions

$$\frac{\partial n}{\partial \nu} = \frac{\partial m}{\partial \nu} = \frac{\partial c}{\partial \nu} = 0, \quad u = 0, \quad x \in \partial\Omega,\ t > 0 \tag{1.5}$$

and the initial conditions

$$n(x,0) = n_0(x), \quad m(x,0) = m_0(x), \quad c(x,0) = c_0(x), \quad u(x,0) = u_0(x), \quad x \in \Omega. \tag{1.6}$$

Throughout the sequel, we assume that

$$\begin{cases} n_0 \in C^0(\overline{\Omega}) \text{ with } n_0 \ge 0 \text{ and } n_0 \not\equiv 0, \\ m_0 \in C^0(\overline{\Omega}) \text{ is nonnegative and } m_0 \not\equiv 0, \\ c_0 \in W^{1,\infty}(\Omega) \text{ is nonnegative and } c_0 \not\equiv 0, \\ u_0 \in D(A^\alpha) \text{ for some } \alpha \in (\frac{3}{4}, 1), \end{cases} \tag{1.7}$$

where A denotes the sectorial operator defined by

$$Au := -\Delta u \text{ for } u \in D(A) := \max\left\{ u \in W^{2,p}(\Omega) : \frac{\partial u}{\partial \nu} = 0 \right\}, \tag{1.8}$$

$\mathcal{P}$ represents the Helmholtz projection from $L^2(\Omega)$ onto $L^2_\sigma(\Omega)$.

We assume that the time-independent function ϕ satisfies

$$\phi \in W^{2,\infty}(\Omega), \tag{1.9}$$

and χ is a positive non-decreasing function on $[0,\infty)$.

Our main results reads as follows.

Theorem 1.1. *Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with smooth boundary. Assume that initial data (n_0, m_0, c_0, u_0) satisfies (1.6)-(1.7). Then if*

$$K_0\chi(K_0) < \frac{\sqrt{2}}{27} \tag{1.10}$$

holds with $K_0 = \max\{\|c_0\|_{L^\infty(\Omega)}, \|m_0\|_{L^\infty(\Omega)}\}$*, the problem (1.1) possesses a globally defined classical solution* (n, c, m, u) *which for each* $p > 3$ *is uniquely determined by the requirements*

$$\begin{cases} n \in C^0(\overline{\Omega} \times [0,\infty)) \cap C^{2,1}(\overline{\Omega} \times (0,\infty)), \\ m \in C^0(\overline{\Omega} \times [0,\infty)) \cap C^{2,1}(\overline{\Omega} \times (0,\infty)), \\ c \in C^0(\overline{\Omega} \times [0,\infty)) \cap C^{2,1}(\overline{\Omega} \times (0,\infty)) \cap L^\infty((0,\infty), W^{1,p}(\Omega)), \\ u \in C^0(\overline{\Omega} \times [0,\infty); \mathbb{R}^3) \cap C^{2,1}(\overline{\Omega} \times (0,\infty); \mathbb{R}^3) \cap L^\infty((0,\infty); D(A^\alpha)), \\ P \in C^{1,0}(\overline{\Omega} \times (0,\infty)), \end{cases}$$

in which n*,* m *and* c *are nonnegative in* $\Omega \times (0,\infty)$ *and solutions are bounded in the sense that one can find a* C_0 *such that*

$$\|n(\cdot,t)\|_{L^\infty(\Omega)} + \|m(\cdot,t)\|_{L^\infty(\Omega)} + \|c(\cdot,t)\|_{W^{1,\infty}(\Omega)} + \|A^\alpha u(\cdot,t)\|_{L^2(\Omega)} \le C, \quad t > 0. \tag{1.11}$$

Remark 1.1. The solution (n, m, c, u) enjoys the following large-time behavior:

$$\|(n - n_\infty)(\cdot,t)\|_{L^\infty(\Omega)} \le K(\delta)e^{-\delta t}, \tag{1.12}$$

$$\|(m - m_\infty)(\cdot,t)\|_{L^\infty(\Omega)} \le K(\delta)e^{-\delta t}, \tag{1.13}$$

$$\|(c - m_\infty)(\cdot,t)\|_{L^\infty(\Omega)} \le K(\delta)e^{-\delta t}, \tag{1.14}$$

$$\|u(\cdot,t)\|_{L^\infty(\Omega)} \le K(\delta)e^{-\delta t}, \tag{1.15}$$

for constants $\delta > 0$ and $K(\delta) > 0$, where $n_\infty = \{⨍_\Omega n_0 - ⨍_\Omega m_0\}_+$ and $m_\infty = \{⨍_\Omega m_0 - ⨍_\Omega n_0\}_+$ (this follows the same arguments as in [11, Section 3]).

Throughout this paper, for simplicity, the integral $\int_\Omega f(x)\mathrm{d}x$ is written as $\int_\Omega f(x)$.

The rest of this paper is organized as follows. In section 2, we show the local existence of the model (1.1) and introduce some important preliminary results. In section 3, several priori estimates are introduced. In section 4, we shall prove Theorem 1.1.

2. Local existence and preliminaries

The first lemma concerns the local solvability of (1.1). The proof is based on a well-established method involving the fixed point theorem and the standard regularity theory of parabolic equations. For more details, please refer to [23].

Lemma 2.1. *Let* $\Omega \subset \mathbb{R}^3$ *be a bounded domain with smooth boundary. Assume that initial data* (n_0, m_0, c_0, u_0) *satisfies (1.6) and (1.7), then there exist a maximal existence time* $T_{max} \in (0,\infty]$ *and functions*

$$\begin{cases} n \in C^0(\overline{\Omega} \times [0,T_{max})) \cap C^{2,1}(\overline{\Omega} \times (0,T_{max})), \\ m \in C^0(\overline{\Omega} \times [0,T_{max})) \cap C^{2,1}(\overline{\Omega} \times (0,T_{max})), \\ c \in C^0(\overline{\Omega} \times [0,T_{max})) \cap C^{2,1}(\overline{\Omega} \times (0,T_{max})) \cap L^\infty((0,T_{max}), W^{1,p}(\Omega)), \\ u \in C^0(\overline{\Omega} \times [0,T_{max}); \mathbb{R}^3) \cap C^{2,1}(\overline{\Omega} \times (0,T_{max}); \mathbb{R}^3), \cap L^\infty((0,T_{max}); D(A^\alpha)) \\ P \in C^{1,0}(\overline{\Omega} \times (0,T_{max})) \end{cases}$$

such that functions (n, m, c, u, P) *solves (1.1) classically in* $\Omega \times (0,T_{max})$*, and* n*,* m *and* c *are nonnegative. Moreover, if* $T_{max} < \infty$*,*

$$\|n(\cdot,t)\|_{L^\infty(\Omega)} + \|m(\cdot,t)\|_{L^\infty(\Omega)} + \|c(\cdot,t)\|_{W^{1,p}(\Omega)} + \|A^\alpha u(\cdot,t)\|_{L^2(\Omega)} \to \infty \tag{2.1}$$

as $t \to T_{max}$, where $\alpha \in (\frac{3}{4}, 1)$ and $p > 3$.

We are going to introduce some elementary properties of the solutions to (1.1).

Lemma 2.2. *Let (n, m, c, u) be the solution of (1.1), then we have*

$$\int_\Omega n(\cdot,t) \le \int_\Omega n_0 \quad \text{for all } t > 0, \tag{2.2}$$

and

$$\|m(\cdot,t)\|_{L^\infty(\Omega)} \le \|m_0\|_{L^\infty(\Omega)} \quad \text{for all } t > 0, \tag{2.3}$$

as well as

$$\|c(\cdot,t)\|_{L^\infty(\Omega)} \le \max\{\|c_0\|_{L^\infty(\Omega)}, \|m_0\|_{L^\infty(\Omega)}\} \quad \text{for all } t > 0. \tag{2.4}$$

Proof. An integral of the first equation in (1.1) on Ω directly shows that $\int_\Omega n(\cdot,t)$ is decreasing in time, which directly results in (2.2). (2.3) and (2.4) follow from the comparison principle. $\square$

Next we recall some inequalities which are useful in the following proof.

Lemma 2.3. *[7] Under the assumption that $m \in \{0, 1\}$, $p \in [1, \infty]$ and $q \in (1, \infty)$, there exists a positive constant C such that*

$$\|u\|_{W^{m,p}(\Omega)} \le C\|(A+I)^\theta u\|_{L^q(\Omega)} \tag{2.5}$$

for any $u \in D((A+I)^\theta)$, where $\theta \in (0,1)$ satisfies

$$m - \frac{n}{p} < 2\theta - \frac{n}{q}. \tag{2.6}$$

In the case that $q \ge p$, the associated diffusion semigroup $\{e^{-t(A+I)}\}$ maps $L^p(\Omega)$ into $D((A+I)^\theta)$, which implies that there exist C, $\gamma > 0$ such that for any $u \in L^p(\Omega)$

$$\|(A+I)^\theta e^{-t(A+I)} u\|_{L^q(\Omega)} \le Ct^{-\theta-\frac{n}{2}(\frac{1}{p}-\frac{1}{q})} e^{-\gamma t}\|u\|_{L^p(\Omega)} \quad \text{for all } u \in L^p(\Omega). \tag{2.7}$$

Moreover, for any $p \in (1, \infty)$ and $\varepsilon > 0$, there is $C(\varepsilon) > 0$ and $\mu > 0$ such that

$$\|(A+I)^\theta e^{-t(A+I)} u\|_{L^q(\Omega)} \le C(\varepsilon)t^{-\theta-\frac{1}{2}-\varepsilon} e^{-\mu t}\|u\|_{L^p(\Omega)} \tag{2.8}$$

for all $\mathbb{R}^n$-valued $u \in L^p(\Omega)$.

The following Gagliardo-Nirenberg interpolation inequality also plays a key role in our proof.

Lemma 2.4. *[7] There exists a constant $C > 0$ such that for all $u \in W^{1,q}(\Omega)$,*

$$\|u\|_{L^p(\Omega)} \le C\|u\|^\eta_{W^{1,q}(\Omega)} \|u\|^{1-\eta}_{L^p(\Omega)}, \tag{2.9}$$

where $p,\ q \geq 1$ *which satisfies* $p(n-q) < nq$, $m \in (0,p)$ *with*

$$\eta = \frac{\frac{n}{m} - \frac{n}{p}}{\frac{n}{m} + 1 - \frac{n}{q}} \in (0,1). \tag{2.10}$$

Finally we recall the following elementary inequality [26].

Lemma 2.5. *Assume that* $y(t) \geq 0$ *satisfies*

$$\begin{cases} y'(t) = -a_1 y^b(t) + a_2 y(t) + a_3, \quad t > 0, \\ y(0) = y_0, \end{cases}$$

where $a_i > 0$ $(i = \{1,2,3\})$ *and* $b > 1$. *Then there exists a constant* $C = C(y_0, a_1, a_2, a_3, b)$ *such that*

$$y(t) \leq C \quad for\ all \quad t > 0. \tag{2.11}$$

3. Priori estimate

This lemma shows that the boundedness of n in $L^p(\Omega)$ implies the boundedness of ∇u in $L^q(\Omega)$.

Lemma 3.1. *Let* (n,m,c,u,P) *be the solutions obtained in Lemma 2.1,* $p \in [1,\infty)$ *and* $r \in [1,\infty]$ *be such that*

$$\begin{cases} r < \frac{3p}{3-p}, & if\ p \leq 3, \\ r \leq \infty, & if\ p > 3. \end{cases}$$

Then for all $K > 0$ *there exists* $C = C(p,r,K)$ *such that*

$$\|Du(\cdot,t)\|_{L^r(\Omega)} \leq C \quad for\ all \quad t \in (0, T_{max}), \tag{3.1}$$

if

$$\|n(\cdot,t)\|_{L^p(\Omega)} \leq K \quad for\ all \quad t \in (0, T_{max}).$$

Proof. Thanks to (2.3), the proof is very similar to [24, Corollary 3.4] and the details are omitted here. □

The next lemma concerns about the bounds of $\int_0^t \int_\Omega |\nabla c|^4 \mathrm{d}x\mathrm{d}t$.

Lemma 3.2. *Assume that (1.5)-(1.7) hold. Then there exists a constant* $C_1 > 0$ *such that*

$$\int_0^t \int_\Omega |\nabla c|^4 \mathrm{d}x\mathrm{d}t \leq C_1 \quad for\ all \quad t \in (0, T_{max}). \tag{3.2}$$

Proof. From [5, Lemma 3.2], we infer that

$$\frac{d}{dt}\int_\Omega |\nabla c|^2 + \int_\Omega |\Delta c|^2 + 2\int_\Omega |\nabla c|^2 \leq C_2\left(\int_\Omega |\nabla u|^2 + 1\right)$$

is valid for all $t \in (0, T_{max})$ with a constant $C_2 > 0$. Thanks to (2.2) and Lemma 3.1, we obtain that there exists a positive constant C_3 such that

$$\int_\Omega |Du|^2 \mathrm{d}x\mathrm{d}t \le C_3$$

for all $t \in (0, T_{max})$. Hence, an integration of the above differential equation asserts that

$$\int_0^t \int_\Omega |\Delta c(\cdot,t)|^2 \mathrm{d}x\mathrm{d}t \le C_4 \quad \text{for all} \quad t \in (0, T_{max})$$

with a constant $C_4 > 0$. With the help of (2.4) and Gagliardo-Nirenberg inequality

$$\|\nabla c(\cdot,t)\|^4_{L^4(\Omega)} \le C_5 \|\Delta c(\cdot,t)\|^2_{L^2(\Omega)} \|c(\cdot,t)\|^2_{L^\infty(\Omega)} + \|c(\cdot,t)\|^4_{L^\infty(\Omega)},$$

we obtain the desired result just by an integration on $(0, t)$. $\square$

Following the idea used in [2,21], a weight function $\phi(c)$ will be adapted to establish the uniform boundedness of $n(x, t)$ in $L^l(\Omega)$ for some $l > 3$.

Lemma 3.3. *Assume that (1.5)-(1.7) and (1.10) hold. Let $(n,\ m,\ c,\ u)$ be a solution of Lemma 2.1, then for some $l > 3$ there exists a positive constant $E_1 > 0$ such that*

$$\|n(\cdot,t)\|_{L^l(\Omega)} \le E_1 \quad \textit{for all} \quad t \in (0, T_{max}). \tag{3.3}$$

Proof. Throughout the proof we denote $K_0 = \max\{\|c_0\|_{L^\infty(\Omega)}, \|m_0\|_{L^\infty(\Omega)}\}$. Given $\gamma = \min\left\{1, \frac{1}{3K_0}\sqrt{\frac{h-1}{2h}}\right\}$, we define

$$\phi(x) = e^{(\gamma x)^2} \quad 0 \le x \le K_0,$$

then

$$1 \le \phi(x) \le e^{(\gamma K_0)^2} \ \text{ for all } 0 \le x \le K_0.$$

Due to

$$\begin{aligned}\int_\Omega n^{l-1}\phi(c) u\cdot\nabla n + \frac{1}{l}\int_\Omega n^l \phi'(c) u \cdot \nabla c &= \int_\Omega n^{l-1}\phi(c) u\cdot\nabla n + \frac{1}{l}\int_\Omega n^l u\cdot\nabla\phi(c) \\ &= \int_\Omega n^{l-1}\phi(c) u\cdot\nabla n - \frac{1}{l}\int_\Omega \phi(c) u\cdot\nabla n^l \\ &= 0,\end{aligned}$$

a direct calculation yields

$$\begin{aligned}
\frac{1}{l}\frac{d}{dt}\int_\Omega n^l\phi(c) =& \int_\Omega n^{l-1}\phi(c)n_t + \frac{1}{l}\int_\Omega n^l\phi'(c)c_t \\
=& \int_\Omega n^{l-1}\phi(c)[\Delta n - u\cdot\nabla n - \nabla\cdot(n\chi(c)\nabla c) - nm] \\
&+ \frac{1}{l}\int_\Omega n^l\phi'(c)[\Delta c - u\cdot\nabla c - c + m] \\
=& -(l-1)\int_\Omega n^{l-2}\phi(c)|\nabla n|^2 - 2\int_\Omega n^{l-1}\phi'(c)\nabla n\cdot\nabla c - \int_\Omega mn^l\phi(c) \\
&+ (l-1)\int_\Omega n^{l-1}\phi(c)\chi(c)\nabla n\cdot\nabla c + \int_\Omega n^l\phi'(c)\chi(c)|\nabla c|^2 - \frac{1}{l}\int_\Omega n^l\phi''(c)|\nabla c|^2 \\
&- \frac{1}{l}\int_\Omega cn^l\phi'(c) + \frac{1}{l}\int_\Omega mn^l\phi'(c) \quad \text{for all } t\in(0,T_{max}).
\end{aligned}$$

According to (2.3) and $\phi'\geq 0$ in $(0,\infty)$, we infer that for all $t\in(0,T_{max})$

$$\begin{aligned}
&\frac{1}{l}\frac{d}{dt}\int_\Omega n^l\phi(c) + (l-1)\int_\Omega n^{l-2}\phi(c)|\nabla n|^2 + \frac{1}{l}\int_\Omega n^l\phi''(c)|\nabla c|^2 \\
&\leq -2\int_\Omega n^{l-1}\phi'(c)\nabla n\cdot\nabla c + (l-1)\int_\Omega n^{l-1}\phi(c)\chi(c)\nabla n\cdot\nabla c + \int_\Omega n^l\phi'(c)\chi(c)|\nabla c|^2 \\
&\quad + \|m(\cdot,t)\|_{L^\infty(\Omega)}\int_\Omega n^l\phi'(c) + \frac{\|m(\cdot,t)\|_{L^\infty(\Omega)}}{l}\int_\Omega n^l\phi'(c).
\end{aligned}\tag{3.4}$$

Using Young's inequality, we have

$$-\int_\Omega n^{l-1}\phi'(c)\nabla n\cdot\nabla c \leq \frac{l-1}{6}\int_\Omega \phi(c)n^{l-2}|\nabla n|^2 + \frac{3}{2(l-1)}\int_\Omega n^l\frac{(\phi'(c))^2}{\phi(c)}|\nabla c|^2$$

and

$$\int_\Omega \phi(c)n^{l-1}\chi(c)\nabla n\cdot\nabla c \leq \frac{1}{6}\int_\Omega \phi(c)n^{l-2}|\nabla n|^2 + \frac{3}{2}\chi(K_0)^2\int_\Omega \phi(c)n^h|\nabla c|^2.$$

Thus

$$\begin{aligned}
&\frac{1}{l}\frac{d}{dt}\int_\Omega \phi(c)n^l + \frac{l-1}{2}\int_\Omega n^{l-2}\phi(c)|\nabla n|^2 + \frac{1}{l}\int_\Omega n^l\phi''(c)|\nabla c|^2 \\
&\leq \frac{3}{l-1}\int_\Omega n^l\frac{(\phi'(c))^2}{\phi(c)}|\nabla c|^2 + \frac{3}{2}(l-1)\chi(K_0)^2\int_\Omega n^l\phi(c)|\nabla c|^2 \\
&\quad + \chi(K_0)\int_\Omega n^l\phi'(c)|\nabla c|^2 + \|m(\cdot,t)\|_{L^\infty(\Omega)}\int_\Omega n^l\phi(c) + \frac{\|m(\cdot,t)\|_{L^\infty(\Omega)}}{l}\int_\Omega n^l\phi'(c).
\end{aligned}\tag{3.5}$$

For any $s\in[0,K_0)$, we set

$$j_1(s) = \frac{1}{l}\phi''(s) = \frac{1}{l}(4\gamma^4 s^2 e^{(\gamma s)^2} + 2\gamma^2 e^{(\gamma s)^2}),$$
$$j_2(s) = \frac{3}{l-1}\frac{(\phi'(s))^2}{\phi(s)} = \frac{3}{l-1}4\gamma^4 s^2 e^{(\gamma s)^2},$$
$$j_3 = \frac{3}{2}(l-1)\|\chi(c)\|^2_{L^\infty(0,K_0)}\phi(s) = \frac{3}{2}(l-1)\chi(K_0)^2 e^{(\gamma s)^2},$$
$$j_4 = \|\chi(c)\|_{L^\infty(0,K_0)}\phi'(s) = 2\chi(K_0)\gamma^2 s e^{(\gamma s)^2}.$$

In order to show that the first three right-hand terms of (3.5) can be dominated by $\frac{1}{l}\int_\Omega \phi''(c)n^l|\nabla c|^2$, it suffices to prove that

$$\frac{j_2(s)}{j_1(s)} \le \frac{1}{3}, \quad \frac{j_3(s)}{j_1(s)} \le \frac{1}{3} \quad \text{and} \quad \frac{j_4(s)}{j_1(s)} \le \frac{1}{3}.$$

Indeed, a direct calculation shows that

$$\frac{j_2(s)}{j_1(s)} \le \frac{\frac{3}{l-1}4\gamma^4 s^2 e^{(\gamma s)^2}}{\frac{1}{h}2\gamma^2 e^{(\gamma s)^2}} \le \frac{6l}{l-1}(\gamma s)^2 \le \frac{6l}{l-1}K_0^2\gamma^2 \le \frac{1}{3}$$

and

$$\frac{j_3}{\frac{1}{3}j_1} \le \frac{\frac{3}{2}(l-1)\chi(K_0)^2 e^{(\gamma s)^2}}{\frac{1}{l}2\gamma^2 e^{(\gamma s)^2}} = \frac{27}{2}l^2 K_0^2 \chi(K_0)^2 \le \frac{1}{3}$$

as well as

$$\frac{j_4(s)}{j_1(s)} \le \chi(K_0)s \le \chi(K_0)K_0 \le \frac{1}{3}$$

are satisfied when $K_0\chi(K_0) \le \frac{\sqrt{2}}{9h}$, which imply that

$$\frac{3}{k-1}\int_\Omega n^k \frac{(\phi'(c))^2}{\phi(c)}|\nabla c|^2 + \frac{3}{2}(k-1)\chi(K_0)^2 \int_\Omega n^k \phi(c)|\nabla c|^2 + \chi(K_0)\int_\Omega n^k \phi'(c)|\nabla c|^2 \le \frac{1}{k}\int_\Omega \phi''(c)n^k|\nabla c|^2.$$

Due to $s\phi'(s) = 2\gamma^2 s\phi(s) \le 2K_0\phi(s) \le \frac{2\sqrt{2}}{9l\chi(K_0)}\phi(s)$, we have

$$\frac{\|m(\cdot,t)\|_{L^\infty(\Omega)}}{l}\int_\Omega n^l\phi'(c) \le \frac{2\sqrt{2}\|m(\cdot,t)\|_{L^\infty(\Omega)}}{9l^2\chi(K_0)}\int_\Omega n^l\phi(c).$$

Then, there is a constant $C_6 > 0$ such that

$$\frac{1}{l}\frac{d}{dt}\int_\Omega n^l\phi(c) + \frac{l-1}{2}\int_\Omega \phi(c)n^{l-2}|\nabla n|^2 \le C_6\int_\Omega n^l\phi(c). \tag{3.6}$$

Applying the Gagliardo-Nirenberg inequality, we obtain that there exist constants C_7, $C_8 > 0$ such that

$$\begin{aligned}\int_\Omega n^l\phi(c) &\le \|n^{\frac{l}{2}}\|^2_{L^2(\Omega)} \le C\|n^{\frac{l}{2}}\|^{2\eta}_{W^{1,2}(\Omega)}\|n^{\frac{h}{2}}\|^{2(1-\eta)}_{L^{\frac{2}{l}}}\\ &\le C_7\left(\|\nabla n^{\frac{l}{2}}\|_{L^2(\Omega)} + \|n^{\frac{l}{2}}\|_{L^2(\Omega)}\right)^{2\eta}\|n\|^{2(1-\eta)}_{L^1(\Omega)}\\ &\le C_8\left(\|\nabla n^{\frac{l}{2}}\|_{L^2(\Omega)} + 1\right)^{2\eta}\end{aligned}$$

with

$$\eta = \frac{3l-3}{3l-1} \in (0,1),$$

which implies that

$$\begin{aligned}\int_\Omega \phi(c)n^{l-2}|\nabla n|^2 \ge \int_\Omega n^{l-2}|\nabla n|^2 &= \frac{4}{l^2}\int_\Omega |\nabla n^{\frac{l}{2}}|^2\\ &\ge \frac{4}{l^2}\left[\frac{1}{C_8^{\frac{1}{\eta}}}\left(\int_\Omega n^l\phi(c)\right)^{\frac{1}{\eta}} - 1\right].\end{aligned} \tag{3.7}$$

Substituting (3.7) into (3.6) gives

$$\frac{1}{l}\frac{d}{dt}\int_\Omega n^l\phi(c) \le -\frac{2(l-1)\phi(K_0)}{l^2C_8^{\frac{1}{\eta}}}\left(\int_\Omega n^l\phi(c)\right)^{\frac{1}{\eta}} + C_9\int_\Omega n^l\phi(c) + \frac{2(l-1)\phi(K_0)}{l^2},$$

then an application of Lemma 2.5 yields that there exists a $E_1 > 0$ such that

$$\|n(\cdot,t)\|_{L^l(\Omega)} \le \left(\int_\Omega n^l\phi(c)\right)^{\frac{1}{l}} \le E_1 \quad \text{for all } t \in (0,T_{max}).$$

Since $K_0\chi(K_0) \le \frac{\sqrt{2}}{9l}$ for some $l > 3$ can be fulfilled by (1.10), the proof is completed. □

4. Proof of Theorem 1.1

In this section we shall prove that the classical solution obtained in the Lemma 2.1 is global. At first, we shall prove the L^2 bounds of $A^\alpha u(\cdot,t)$.

Lemma 4.1. *Under the same assumption in Lemma 3.3. Then there exists a positive constants $E_2 > 0$ such that*

$$\|A^\alpha u(\cdot,t)\|_{L^2(\Omega)} \le E_2 \quad \text{and} \quad \|u(\cdot,t)\|_{L^\infty(\Omega)} \le E_2 \tag{4.1}$$

for all $t \in (0,T_{max})$.

Proof. Applying the Helmholtz projection $\mathcal{P}$ to the fourth equation in (1.1), then its variation-of-constants formula could be written as

$$u(\cdot,t) = e^{-tA}u_0 + \int_0^t e^{-(t-s)A}\mathcal{P}(n+m)\nabla\phi \mathrm{d}s.$$

Thus, by the well-known regularization estimates for the Stokes semigroup, the continuity of the projection $\mathcal{P}$ on $L^r(\Omega;\mathbb{R}^3)$ and choosing $s \in \left(\frac{1}{\frac{1}{2}+\frac{2}{3}(1-\alpha)}, 2\right)$, it yields that there exist constants C_{10}, C_{11}, $C_{12} > 0$ such that

$$\begin{aligned}
\|A^\alpha u(\cdot,t)\|_{L^2(\Omega)} \leq & \|A^\alpha e^{-tA}u_0\|_{L_2(\Omega)} + \int_0^t \|A^\alpha e^{-(t-s)A}\mathcal{P}[(n+m)\nabla\phi]\|_{L^2(\Omega)}\mathrm{d}s \\
\leq & \|A^\alpha e^{-tA}u_0\|_{L_2(\Omega)} \\
& + C_{10}\int_0^t (t-s)^{-\alpha-\frac{3}{2}(\frac{1}{s}-\frac{1}{2})}e^{-\lambda(t-s)}\|\mathcal{P}(n+m)\nabla\phi\|_{L^s(\Omega)}\mathrm{d}s \\
\leq & \|A^\alpha e^{-tA}u_0\|_{L_2(\Omega)} + C_{11}\int_0^t (t-s)^{-\alpha-\frac{3}{2}(\frac{1}{s}-\frac{1}{2})}(\|n\|_{L^s(\Omega)} + \|m\|_{L^s(\Omega)})\mathrm{d}s \\
\leq & \|A^\alpha e^{-tA}u_0\|_{L_2(\Omega)} + C_{12}\int_0^t \sigma^{-\alpha-\frac{3}{2}(\frac{1}{s}-\frac{1}{2})}\mathrm{d}\sigma \leq E_2
\end{aligned}$$

for all $t \in (0, T_{max})$ since the integrability of $\int_0^t \sigma^{-\alpha-\frac{3}{2}(\frac{1}{s}-\frac{1}{2})}\mathrm{d}\sigma$ is asserted by the s chosen above. Furthermore, the L^∞ bounds of $u(\cdot,t)$ is a straightfoward consequence of the embedding $D(A^\alpha) \hookrightarrow L^\infty(\Omega)$ with $\alpha \in (\frac{3}{4}, 1)$, which concludes the proof. $\square$

Lemma 4.2. *Under the same assumptions in Lemma 3.3. Then there exists a positive constant E_3 such that*

$$\|c(\cdot,t)\|_{W^{1,\infty}(\Omega)} \leq E_3 \quad \textit{for all} \quad t \in (0, T_{max}). \tag{4.2}$$

Proof. Let $\tau \in (0, T_{max})$ be given such that $\tau < 1$, and employing ∇ to the both sides of the variation-of-constants formula for c

$$c(\cdot,t) = e^{(t-\tau)\Delta}c(\cdot,\tau) + \int_\tau^t e^{(t-s)\Delta}(-c+m-u\cdot\nabla c)\mathrm{d}s,$$

there are positive constants C_{13}, C_{14}, C_{15} such that

$$\begin{aligned}
\|\nabla c(\cdot,t)\|_{L^4(\Omega)} \leq & \|\nabla e^{(t-\tau)\Delta}c(\cdot,\tau)\|_{L^4(\Omega)} + \int_\tau^t \|\nabla e^{(t-s)\Delta}(-c+m-u\cdot\nabla c)\|_{L^4(\Omega)}\mathrm{d}s \\
\leq & (t-\tau)^{-\frac{1}{2}}\|c(\cdot,\tau)\|_{L^4(\Omega)} + C_{13}\int_0^t (t-s)^{-\frac{1}{2}}(\|c\|_{L^4(\Omega)} + \|m\|_{L^4(\Omega)})\mathrm{d}s \\
& + C_{13}\int_0^t (t-s)^{-\frac{1}{2}}\|u(\cdot,t)\|_{L^\infty(\Omega)}\|\nabla c(\cdot,t)\|_{L^4(\Omega)}\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&\le (t-\tau)^{-\frac{1}{2}}\|c_0\|_{L^4(\Omega)} + C_{14}T_{max}^{\frac{1}{2}} \\
&\quad + C_{14}\left\{\int_0^{T_{max}} s^{-\frac{2}{3}}\mathrm{d}s\right\}^{\frac{3}{4}} \cdot \left\{\int_0^t \|\nabla c(\cdot,s)\|_{L^4(\Omega)}^4 \mathrm{d}s\right\}^{\frac{1}{4}} \\
&\le C_{15} \quad \text{for all} \quad t\in(\tau,T_{max}).
\end{aligned}
$$

Thus, employing results in Lemma 2.3 while picking $\theta \in (\frac{7}{8}, 1)$, it follows that there exist positive constants C_{16} and C_{17} such that

$$
\begin{aligned}
\|c(\cdot,t)\|_{W^{1,\infty}(\Omega)} \le & C_{16}\|(A+1)^{\theta}c(\cdot,t)\|_{L^4(\Omega)} \\
\le & C_{17}t^{-\theta}e^{-\gamma t}\|c_0\|_{L^4(\Omega)} + C_{17}\int_0^t (t-s)^{-\theta}e^{-\gamma(t-s)}\|m(\cdot,t)\|_{L^4(\Omega)}\mathrm{d}s \\
& + C_{17}\int_0^t (t-s)^{-\theta}e^{-\gamma(t-s)}\|u(\cdot,t)\|_{L^\infty(\Omega)}\|c(\cdot,t)\|_{L^4(\Omega)}\mathrm{d}s.
\end{aligned}
$$

Hence, it is obvious to reach the desired result. □

Similarly, we prove L^∞ bounds of $n(\cdot,t)$.

Lemma 4.3. *Under the same assumptions in Lemma 3.3, there is a positive constant E_4 such that*

$$
\|n(\cdot,t)\|_{L^\infty(\Omega)} \le E_4 \quad \textit{for all} \quad t\in(0,T_{max}). \tag{4.3}
$$

Proof. With the help of the variation-constants-formula of $n(\cdot,t)$, we have

$$
\begin{aligned}
n(\cdot,t) =& e^{-(t-\tau)(A+1)}n(\cdot,t) - \int_0^t e^{-(t-s)(A+1)}\nabla\cdot(n\chi(c)\nabla c)\mathrm{d}s \\
& - \int_0^t e^{-(t-s)(A+1)}(u\cdot\nabla n)\mathrm{d}s + \int_0^T e^{-(t-s)(A+1)}(n-nm)\mathrm{d}s \\
=& I_1 + I_2 + I_3 + I_4 \quad \text{for all} \quad t\in(\tau,T_{max}).
\end{aligned}
$$

Then, we estimate L^∞-bound of I_j $(j=1,2,3,4)$. For I_1, it yields that for $\theta_1 \in (\frac{3}{2q}, 1)$

$$
\begin{aligned}
\|I_1\|_{L^\infty(\Omega)} \le \|(A+1)^{\theta_1}I_1\|_{L^q(\Omega)} &\le C_{18}t^{-\theta_1}e^{-\varepsilon t}\|u_0(\cdot,t)\|_{L^q(\Omega)} \\
&\le C_{18}\tau^{-\theta_1}\|u_0\|_{L^\infty(\Omega)} \quad \text{for all} \quad t\in(\tau,T_{max}),
\end{aligned} \tag{4.4}
$$

where $1<q<\infty$ is an arbitrary constant and $\varepsilon>0$.

As to I_2, employing (2.8) of Lemma 2.3 while $m=0$, $q>3$ and $p=\infty$, choosing $\theta_2 \in (\frac{3}{2q}, \frac{1}{2})$ and $\beta_1 \in (0, \frac{1}{2}-\theta_2)$, it follows that there are positive constants C_{19}, C_{20} and C_{21} such that

$$\begin{aligned}
\|I_2\|_{L^\infty(\Omega)} \leq & C_1\|(A+1)^{\theta_2} I_2\|_{L^q(\Omega)} \\
\leq & C_{19}\int_0^t \|(A+1)^{\theta_2} e^{-(t-s)(A+1)}\nabla\cdot(n\chi(c)\nabla c)\|_{L^q(\Omega)}\mathrm{d}s \\
\leq & C_{20}\int_0^t e^{-(t-s)}\|(A+1)^{\theta_2} e^{-(t-s)A}\nabla\cdot(n\chi(c)\nabla c)\|_{L^q(\Omega)}\mathrm{d}s \\
\leq & C_{21}\|\chi(c)\|_{L^\infty(0,K_0)}\int_0^t (t-s)^{-\theta_2-\frac{1}{2}-\beta_1} e^{-(\mu+1)(t-s)}\|n(\cdot,t)\nabla c(\cdot,t)\|_{L^q(\Omega)}\mathrm{d}s
\end{aligned}$$

for all $t \in (0, T_{max})$. Since Lemma 3.3 and 4.2 implies that

$$\|n(\cdot,t)\nabla c(\cdot,t)\|_{L^q(\Omega)} \leq C_{22}(\tau) \quad \text{for all} \quad t \in (\tau, T_{max}),$$

we obtain that for all $t \in (\tau, T_{max})$

$$\begin{aligned}
\|I_2\|_{L^\infty(\Omega)} &\leq C_{22}(\tau)\int_0^t (t-s)^{-\theta_2-\frac{1}{2}-\beta_1} e^{-(\mu+1)(t-s)}\mathrm{d}s \\
&\leq C_{22}(\tau)\int_0^\infty \sigma^{-\theta_2-\frac{1}{2}-\beta_1} e^{-(\mu+1)\sigma}\mathrm{d}\sigma \leq C_{22}(\tau)\Gamma\left(\frac{1}{2}-\theta_2-\beta_1\right),
\end{aligned} \tag{4.5}$$

where $\Gamma(\xi)$ is the Gamma function, which implies the bounds of $\|I_2\|_{L^\infty(\Omega)}$.

Due to $\nabla u = 0$, we rewrite I_3 as follows

$$I_3 = -\int_0^t e^{-(t-s)(A+1)}\nabla(u(\cdot,t)n(\cdot,t))\mathrm{d}s.$$

Since Lemma 3.3 and 4.1 ensure that there is a constant $C_{23} > 0$ such that

$$\|u(\cdot,t)n(\cdot,t)\|_{L^q(\Omega)} \leq \|u(\cdot,t)\|_{L^\infty(\Omega)}\|n(\cdot,t)\|_{L^q(\Omega)} \leq C_{23} \quad \text{for all} \quad t \in (0, T_{max}),$$

we can similarly obtain that there is a $C_{24}(\tau) > 0$ such that

$$\|I_3\|_{L^\infty(\Omega)} \leq C_{24}(\tau) \quad \text{for all} \quad t \in (\tau, T_{max}). \tag{4.6}$$

Finally, by employing Lemma 2.3 while $m = 1$, $q > 3$ and $p > 3$, we obtain that for a given $\theta_3 \in \left(\frac{1}{2}(1-\frac{3}{p}+\frac{3}{q}), 1\right)$ there are positive constants C_{25}, C_{26} such that

$$\begin{aligned}
\|I_4\|_{W^{1,p}(\Omega)} \leq & \|(A+1)^{\theta_3} I_4\|_{L^q(\Omega)} \\
\leq & C_{25}\int_0^t (t-s)^{-\theta_3} e^{-\alpha(t-s)}\|n(\cdot,t) - nm(\cdot,t)\|_{L^q(\Omega)}\mathrm{d}s \\
\leq & C_{25}\int_0^t (t-s)^{-\theta_3} e^{-\alpha(t-s)}(\|n(\cdot,t)\|_{L^q(\Omega)} + \|m(\cdot,t)\|_{L^\infty(\Omega)}\|n(\cdot,t)\|_{L^q(\Omega)})\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&\leq C_{26}\int_0^t (t-s)^{-\theta_3}e^{-\alpha(t-s)}\mathrm{d}s\\
&\leq C_{26}\Gamma(1-\theta_3) \quad \text{for all} \quad t\in(\tau,T_{max}).
\end{aligned}
$$

Due to $q > 3$, an employment of the Sobolev inequality directly shows that

$$\|I_4\|_{L^\infty(\Omega)} \leq C_{27}\|I_4\|_{W^{1,q}(\Omega)} \leq C_{28} \quad \text{for all} \quad t\in(\tau,T_{max}). \tag{4.7}$$

Therefore, it follows from (4.4)-(4.7) that $\|n(\cdot,t)\|_{L^\infty(\Omega)}$ is bounded for all $t\in(\tau,T_{max})$. □

Proof of Theorem 1.1. A combination with Lemma 4.1-4.3 and (2.3) directly yields the global solvability and (1.11). □

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