



Aggregate preferred correspondence and the existence of a maximin REE



Anuj Bhowmik^{a,1}, Jiling Cao^{a,*}, Nicholas C. Yannelis^b

^a School of Computer and Mathematical Sciences, Auckland University of Technology,
Private Bag 92006, Auckland 1142, New Zealand

^b Department of Economics, Henry B. Tippie College of Business, The University of Iowa,
IA 52242-1994, USA

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ABSTRACT

A general model of a pure exchange differential information economy is studied. In this economic model, the space of states of nature is a complete probability measure space, the space of agents is a finite measure space, and the commodity space is the Euclidean space. Under some appropriate and standard assumptions on agents' characteristics, results on continuity and measurability of the aggregate preferred correspondence similar to that of Aumann [8] are established. These results together with other techniques are then employed to prove the existence of a maximin rational expectations equilibrium (maximin REE) of the economic model.

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1. Introduction

When traders come to a market with different information about the items to be traded, the resulting market prices may reveal to some traders information originally available only to others. The possibility for such inferences rests upon traders having expectations of how equilibrium prices are related to initial information. This endogenous relationship was considered by Radner in his seminal paper [26], where he introduced the concept of a rational expectations equilibrium by imposing on agents the Bayesian (subjective expected utility) decision doctrine. Under the Bayesian decision making, agents maximize their subjective expected utilities conditioned on their own private information and also on the information that the equilibrium prices generate. The resulting equilibrium allocations are measurable with respect to the private

* Corresponding author.

E-mail addresses: anujbhowmik09@gmail.com (A. Bhowmik), jiling.cao@aut.ac.nz (J. Cao), nicholasannelis@gmail.com (N.C. Yannelis).

¹ Current address: Economic Research Unit, Indian Statistical Institute, 203 Barackpore Trunk Road, Kolkata 700108, India.

information of each individual and also with respect to the information the equilibrium prices generate and clear the market for every state of nature. In papers [1,2,26], conditions on the existence of a Bayesian rational expectations equilibrium (REE) were studied and some generic existence results were proved. However, Kreps [22] provided an example that shows that a Bayesian REE may not exist universally. In addition, a Bayesian REE may fail to be fully Pareto optimal and incentive compatible and may not be implementable as a perfect Bayesian equilibrium of an extensive form game, refer to [15] for more details.

It was pointed out in [21] that the market hypothesis fails if the space of states of nature is of a dimension higher than that of the price simplex. Thus, in generic existence theorems of Allen [1,2] and Radner [26], the assumption on the space of states of nature being finite or of sufficiently low dimension relative to the dimension of price simplex is essential. However, it was shown in [20] that if the space of states of nature is of a dimension strictly higher than that of the price simplex, then for a residual set of economies there is a rational expectations equilibrium which is given by a two-to-one and almost discontinuous price function. When the dimensions of both spaces coincide, as mentioned in [3], the existence of an equilibrium fails in finite economies. If the space of agents is a unit interval consisting of imperfectly and perfectly informed agents, under the hypothesis of suitably disperse forecasts, it was shown in [3] that for each state of nature the aggregate excess demand is continuous on the price simplex and satisfies Walras' law. This fact allowed Allen to apply a fixed point theorem to obtain the market clearing price vector for each state of nature and obtain the existence of an ε -rational expectations equilibrium for all $\varepsilon > 0$. The convergence as $\varepsilon \rightarrow 0$ holds for some cases in which open counterexamples to the existence of rational expectations equilibria are known. In the same year, Allen [4] also considered two types of agents (informed and uninformed) and prices carried only incomplete information, when prices conveyed some information from informed agents to uninformed agents. By applying a fixed point theorem, she obtained a new approximate non-revealing rational expectations equilibrium in the sense that the total discrepancy between demand and supply is small. Allen [5] further showed the existence of a rational expectations equilibrium with (strong) ε -market clearing in the sense that the discrepancy between demand and supply is zero for all but one commodity for which the value can be made arbitrarily small.

In a recent paper [13], de Castro et al. introduced a new notion of REE by a careful examination of Kreps' example of the nonexistence of a Bayesian REE. In this formulation, the Bayesian decision making adopted in the papers of [1] and [26] was abandoned and replaced by the maximin expected utility (MEU) (see [14]).² In this new setup, agents maximize their MEU conditioned on their own private information and also on the information the equilibrium prices have generated. Contrary to a Bayesian REE, the resulting maximin REE may not be measurable with respect to the private information of each individual or the information that the equilibrium prices generate.

Although Bayesian REE and maximin REE coincide in some special cases (e.g., fully revealing Bayesian REE and maximin REE), these two concepts are in general not equivalent. Nonetheless, the introduction of the MEU into the general equilibrium modeling enables de Castro et al. to prove that a maximin REE exists universally under the standard continuity and concavity assumptions on the utility functions of agents. Furthermore, they showed that a maximin REE is incentive compatible and efficient. Note that in the economic model considered in [13], it is assumed that there are finitely many states of nature and finitely many agents, and the commodity space is finite-dimensional. Thus, one of open questions is whether their existence theorem can be extended to more general cases. The main motivation of this paper is to tackle this question. We study a general economic model of a pure exchange differential information economy whose space of states of nature is a complete probability measure space, and whose space of agents is a finite measure space. Under appropriate and standard assumptions on agents' characteristics, the existence of a maximin rational expectations equilibrium (maximin REE) in this general economic model is established. To prove our existence result, instead of applying the approaches used in [1,2,26], we modify and extend

² For some recent applications of the MEU approach to the best choice problem, see [10].

Aumann's approach in [8] for a deterministic economic model to our economic model. One of the key ingredients in this approach is the concept of aggregate preferred correspondence.

This paper is organized as follows. In Section 2, mathematical preliminaries and key facts used in this paper are presented. The economic model and assumptions are introduced and explained in Section 3. In Section 4, we modify and extend Aumann's concept of agents' aggregate preferred correspondence to our model. We establish some results on continuity and measurability of the aggregate preferred correspondence. The main results in this section are new and different from those of Aumann in [8] for deterministic economic model, since our model is a differential information economy and we also drop Aumann's assumption on the existence of an assignment that commodity-wise saturates each agent's desire. Moreover, these results are also key techniques for us to prove the existence of a maximin rational expectations equilibrium of the economic model in Section 5. Finally, in Section 6, results and techniques appeared in this paper are compared with relevant results in the literature.

2. Mathematical preliminaries

Let G be a non-empty set. On ℓ -dimensional Euclidean space \mathbb{R}^ℓ , two different but equivalent norms $\|\cdot\|_\infty$ and $\|\cdot\|_1$ are used in this paper, where

$$\|x\|_\infty = \max\{|x_i|: 1 \leq i \leq \ell\} \quad \text{and} \quad \|x\|_1 = \sum_{1 \leq i \leq \ell} |x_i|$$

for each point $x = (x_1, x_2, \dots, x_\ell) \in \mathbb{R}^\ell$. A *correspondence* $F: G \rightrightarrows \mathbb{R}^\ell$ from G to \mathbb{R}^ℓ assigns to each $x \in G$ a subset $F(x)$ of \mathbb{R}^ℓ . Meanwhile, F can also be viewed as a function $F: G \rightarrow 2^{\mathbb{R}^\ell}$, where $2^{\mathbb{R}^\ell}$ denotes the power set of \mathbb{R}^ℓ . Further, F is called *non-empty valued* (resp. *closed-valued*, *compact-valued*, *convex-valued*) if $F(x)$ is a non-empty (resp. closed, compact, convex) subset of \mathbb{R}^ℓ for all $x \in G$. The *graph* of F , denoted by Gr_F , is defined by

$$\text{Gr}_F = \{(x, y) \in G \times \mathbb{R}^\ell: y \in F(x) \text{ and } x \in G\}.$$

For $x \in \mathbb{R}^\ell$ and $A \in 2^{\mathbb{R}^\ell} \setminus \{\emptyset\}$, define $\text{dist}(x, A) = \inf\{d(x, y): y \in A\}$, where d is the Euclidean metric on \mathbb{R}^ℓ . Let $\mathcal{K}_0(\mathbb{R}^\ell)$ be the family of non-empty compact subsets of \mathbb{R}^ℓ . Recall that the *Hausdorff metric* H on $\mathcal{K}_0(\mathbb{R}^\ell)$ is defined such that for any two $A, B \in \mathcal{K}_0(\mathbb{R}^\ell)$,

$$H(A, B) = \max\left\{\sup_{a \in A} \text{dist}(a, B), \sup_{b \in B} \text{dist}(b, A)\right\}.$$

For equivalent definitions of H , refer to [6]. The *Hausdorff metric topology* \mathcal{T}_H on $\mathcal{K}_0(\mathbb{R}^\ell)$ is the topology generated by H . For a closed subset M of \mathbb{R}^ℓ , $\mathcal{K}_0(M)$ and the Hausdorff metric H on $\mathcal{K}_0(M)$ can be defined similarly. If G is a topological space, a non-empty compact-valued correspondence $F: G \rightrightarrows \mathbb{R}^\ell$ is called *Hausdorff continuous* if $F: G \rightarrow (\mathcal{K}_0(\mathbb{R}^\ell), \mathcal{T}_H)$ is continuous. This statement still holds when \mathbb{R}^ℓ is replaced with a closed subset M of \mathbb{R}^ℓ .

Let $\{A_n: n \geq 1\}$ be a sequence of non-empty subsets of \mathbb{R}^ℓ . A point $x \in \mathbb{R}^\ell$ is called a *limit point* of $\{A_n: n \geq 1\}$ if there exist $N \geq 1$ and $x_n \in A_n$ for each $n \geq N$ such that $\{x_n: n \geq N\}$ converges to x . The set of limit points of $\{A_n: n \geq 1\}$ is denoted by $\text{Li } A_n$. Similarly, a point $x \in \mathbb{R}^\ell$ is called a *cluster point* of $\{A_n: n \geq 1\}$ if there exist positive integers $n_1 < n_2 < \dots$ and for each k an $x_k \in A_{n_k}$ such that $\{x_k: k \geq 1\}$ converges to x . The set of cluster points of $\{A_n: n \geq 1\}$ is denoted by $\text{Ls } A_n$. It is clear that $\text{Li } A_n \subseteq \text{Ls } A_n$, and both $\text{Ls } A_n$ and $\text{Li } A_n$ are closed (possibly empty) sets. If $\text{Ls } A_n \subseteq \text{Li } A_n$, then $\text{Li } A_n = \text{Ls } A_n = A$ is called the *limit* of the sequence $\{A_n: n \geq 1\}$. Note that $\text{Ls } A_n = \text{Ls } \bar{A}_n$ and $\text{Li } A_n = \text{Li } \bar{A}_n$. Hence, if A is the limit of $\{A_n: n \geq 1\}$, then A is also the limit of $\{\bar{A}_n: n \geq 1\}$. If A and all A_n 's are closed and contained

in a compact subset $M \subseteq \mathbb{R}^\ell$, then it is well known that $\text{Li } A_n = \text{Ls } A_n = A$ if and only if $\{A_n: n \geq 1\}$ converges to A in the Hausdorff metric topology on $\mathcal{K}_0(M)$.

Let (T, Σ, μ) be a measure space and $\{F_n: n \geq 1\}$, $F: (T, \Sigma, \mu) \rightrightarrows \mathbb{R}^\ell$ be correspondences. Recall that F is said to be *lower measurable* if

$$F^{-1}(V) = \{t \in T: F(t) \cap V \neq \emptyset\} \in \Sigma$$

for every open subset V of \mathbb{R}^ℓ . A *selection* of F is a single-valued function $f: (T, \Sigma, \mu) \rightarrow \mathbb{R}^\ell$ such that $f(t) \in F(t)$ for all $t \in T$. If a selection f of F is measurable (resp. integrable), then it is called a *measurable* (resp. an *integrable*) *selection*. Let \mathcal{S}_F denote the set of integrable selections of F . It is well known that a non-empty closed-valued correspondence $F: (T, \Sigma, \mu) \rightrightarrows \mathbb{R}^\ell$ is lower measurable if and only if there exists a sequence of measurable selections $\{f_n: n \geq 1\}$ of F such that for all $t \in T$, $F(t) = \overline{\{f_n(t): n \geq 1\}}$. If all F'_n s are non-empty closed-valued, lower measurable and at least one of F'_n s is compact-valued, then $\bigcap_{n \geq 1} F_n$ is lower measurable, refer to [19]. If all F'_n s are integrably bounded by the same function, then

$$\text{Ls} \int_T F_n d\mu \subseteq \int_T \text{Ls } F_n d\mu \quad \text{and} \quad \int_T \text{Li } F_n d\mu \subseteq \text{Li} \int_T F_n d\mu.$$

If (S, \mathcal{S}, ν) is another measure space and $f: (T, \Sigma, \mu) \times (S, \mathcal{S}, \nu) \rightarrow \mathbb{R}^\ell$ is jointly measurable, then it is well known that $\int_T f(\cdot, \cdot) d\mu: (S, \mathcal{S}, \nu) \rightarrow \mathbb{R}^\ell$ is measurable. Let $M \subseteq \mathbb{R}^\ell$ be endowed with the relative Euclidean topology, and (Y, ϱ) be a metric space. A function $f: (T, \Sigma, \mu) \times M \rightarrow (Y, \varrho)$ is called *Carathéodory* if $f(\cdot, x)$ is measurable for all $x \in M$, and $f(t, \cdot)$ is continuous for all $t \in T$. It is known that any Carathéodory function is jointly measurable with respect to the Borel structure on M . The *integration* of F over T in the sense of [7] is a subset of \mathbb{R}^ℓ , defined as

$$\int_T F d\mu = \left\{ \int_T f d\mu: f \in \mathcal{S}_F \right\}.$$

If F is non-empty closed-valued and integrably bounded, then $\int_T F d\mu$ is compact, refer to [18, p. 73].

The following special case of the Kuratowski–Ryll–Nardzewski measurable selection theorem will be used in this paper. For more general versions of the measurable selection theorem, see [23] or [19].

Theorem 2.1. *If $F: (T, \Sigma, \mu) \rightarrow \mathbb{R}^\ell$ is a lower measurable correspondence with nonempty closed-valued values, then it has a measurable selection.*

3. Differential information economies

In this paper, a model of a pure exchange economy \mathcal{E} with differential information is considered. The space of states of nature is a complete probability measure space $(\Omega, \mathcal{F}, \nu)$. The space of agents is a measure space (T, Σ, μ) with $0 < \mu(T) < +\infty$. The commodity space is the ℓ -dimensional Euclidean space \mathbb{R}^ℓ , and the positive cone \mathbb{R}_+^ℓ is the *consumption set* for each agent $t \in T$ in every state of nature $\omega \in \Omega$. Each agent $t \in T$ is associated with her/his *characteristics* $(\mathcal{F}_t, U(t, \cdot, \cdot), a(t, \cdot), \mathbb{Q}_t)$, where \mathcal{F}_t is the sub- σ -algebra of \mathcal{F} generated by a partition Π_t of Ω representing the *private information*³ of t , $U(t, \cdot, \cdot): \Omega \times \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ is a *state-dependent utility function* of t and $a(t, \cdot): \Omega \rightarrow \mathbb{R}_+^\ell$ is the *state-dependent initial endowment* of t and \mathbb{Q}_t is a probability measure on Ω giving the *prior belief* of t . The economy extends over two time periods $\tau = 1, 2$. At the ex ante stage ($\tau = 0$), only the above description of the economy is a common knowledge.

³ For a recent general study of information sets, refer to Hervés-Beloso and Monteiro [17].

At the interim stage $\tau = 1$, agent t only knows that the realized state of nature belongs to the event $\mathcal{F}_t(\omega^*)$, where $\mathcal{F}_t(\omega^*)$ is the unique member of Π_t containing the true state of nature ω^* at $\tau = 2$. At the ex post stage ($\tau = 2$), agents execute the trades according to the contract agreed at period $\tau = 1$, and consumption takes place. The coordinate-wise order on \mathbb{R}^ℓ is denoted by \leq and the symbol $x \gg 0$ means that x is an interior point of \mathbb{R}_+^ℓ , and $x > 0$ means that $x \geq 0$ but $x \neq 0$. Let $L_1(\mu, \mathbb{R}_+^\ell)$ be the set of equivalent classes of Lebesgue integrable functions from T to \mathbb{R}_+^ℓ . An *allocation* in \mathcal{E} is a function $f : T \times \Omega \rightarrow \mathbb{R}_+^\ell$ such that $f(\cdot, \omega) \in L_1(\mu, \mathbb{R}_+^\ell)$ for all $\omega \in \Omega$. An allocation f in \mathcal{E} is *feasible* if $\int_T f(\cdot, \omega) d\mu = \int_T a(\cdot, \omega) d\mu$ for all $\omega \in \Omega$.

The following standard assumptions on agents' characteristics shall be used:

- (A₁) The initial endowment function $a : (T, \Sigma, \mu) \times (\Omega, \mathcal{F}, \nu) \rightarrow \mathbb{R}_+^\ell$ is jointly measurable such that $\int_T a(\cdot, \omega) d\mu \gg 0$ for each $\omega \in \Omega$.
- (A₂) $U(\cdot, \cdot, x) : (T, \Sigma, \mu) \times (\Omega, \mathcal{F}, \nu) \rightarrow \mathbb{R}$ is jointly measurable for all $x \in \mathbb{R}_+^\ell$ and $U(t, \omega, \cdot) : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ is continuous for all $(t, \omega) \in T \times \Omega$.
- (A₃) For each $(t, \omega) \in T \times \Omega$, $U(t, \omega, \cdot) : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ is monotone in the sense that if $x, y \in \mathbb{R}_+^\ell$ with $y > 0$, then $U(t, \omega, x + y) > U(t, \omega, x)$.
- (A₄) For each $(t, \omega) \in T \times \Omega$, $U(t, \omega, \cdot) : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ is concave.

Remark 3.1. To prove our main result in Section 5, we need a classical existence result of Hildenbrand in [18], which requires the concavity of utilities. Note that since $\mu(T) < +\infty$, we can decompose T into a disjoint union of two parts, namely, the atomless part T_0 and another part T_1 consisting of at most countably many atoms. In fact, in order to guarantee that the existence result of Hildenbrand in [18] holds, for each $\omega \in \Omega$, we only need the concavity of utility $U(t, \omega, \cdot)$ for $t \in T_1$. But, for the sake of simplicity, for each $\omega \in \Omega$, we assume the concavity of utility $U(t, \omega, \cdot)$ for all $t \in T$.

Let Δ be the simplex of price systems in the deterministic case normalized so that their sum is 1, i.e.,

$$\Delta = \left\{ p \in \mathbb{R}_+^\ell : p \gg 0 \text{ and } \sum_{h=1}^{\ell} p^h = 1 \right\}.$$

Furthermore, we shall equip Δ with the relative Euclidean topology and the Borel structure $\mathcal{B}(\Delta)$ generated by this topology. The *budget correspondence* $B : T \times \Omega \times \Delta \rightrightarrows \mathbb{R}_+^\ell$ is defined by

$$B(t, \omega, p) = \{x \in \mathbb{R}_+^\ell : \langle p, x \rangle \leq \langle p, a(t, \omega) \rangle\}$$

for all $(t, \omega, p) \in T \times \Omega \times \Delta$. Obviously, B is non-empty and closed-valued. For each $\omega \in \Omega$, under (A₁)–(A₄), by Theorem 2 in [18, p. 151], there are $p(\omega) \in \Delta$ and an allocation $f(\cdot, \omega)$ such that $(f(\cdot, \omega), p(\omega))$ is a Walrasian equilibrium of the deterministic economy $\mathcal{E}(\omega)$, given by

$$\mathcal{E}(\omega) = ((T, \Sigma, \mu); \mathbb{R}_+^\ell; (U(t, \omega, \cdot), a(t, \omega)) : t \in T).$$

Define a function $\delta : \Delta \rightarrow \mathbb{R}_+$ by $\delta(p) = \min\{p^h : 1 \leq h \leq \ell\}$, where $p = (p^1, \dots, p^\ell) \in \Delta$. For any $(t, \omega, p) \in T \times \Omega \times \Delta$, let

$$\gamma(t, \omega, p) = \frac{1}{\delta(p)} \sum_{h=1}^{\ell} a^h(t, \omega),$$

and

$$b(t, \omega, p) = (\gamma(t, \omega, p), \dots, \gamma(t, \omega, p)).$$

Define the correspondence $X : T \times \Omega \times \Delta \rightrightarrows \mathbb{R}_+^\ell$ by letting

$$X(t, \omega, p) = \{x \in \mathbb{R}_+^\ell : x \leq b(t, \omega, p)\}$$

for all $(t, \omega, p) \in T \times \Omega \times \Delta$. Note that X is non-empty, compact- and convex-valued such that $B(t, \omega, p) \subseteq X(t, \omega, p)$ for all $(t, \omega, p) \in T \times \Omega \times \Delta$. It can be verified that for every $(t, \omega) \in T \times \Omega$, the correspondence $X(t, \omega, \cdot) : \Delta \rightrightarrows \mathbb{R}_+^\ell$ is Hausdorff continuous. Define correspondences $C, C^X : T \times \Omega \times \Delta \rightrightarrows \mathbb{R}_+^\ell$ by

$$C(t, \omega, p) = \{y \in \mathbb{R}_+^\ell : U(t, \omega, y) \geq U(t, \omega, x) \text{ for all } x \in B(t, \omega, p)\}$$

and

$$C^X(t, \omega, p) = C(t, \omega, p) \cap X(t, \omega, p).$$

By (\mathbf{A}_3) , we can derive that $b(t, \omega, p) \in C(t, \omega, p)$ and thus $b(t, \omega, p) \in C^X(t, \omega, p)$ for all $(t, \omega, p) \in T \times \Omega \times \Delta$. Furthermore, it can be easily checked that

$$B(t, \omega, p) \cap C(t, \omega, p) = B(t, \omega, p) \cap C^X(t, \omega, p)$$

holds for all $(t, \omega, p) \in T \times \Omega \times \Delta$. Note that under (\mathbf{A}_2) , $U(t, \omega, \cdot)$ is continuous on the non-empty compact set $B(t, \omega, p)$. Thus, one has

$$B(t, \omega, p) \cap C(t, \omega, p) \neq \emptyset$$

for all $(t, \omega, p) \in T \times \Omega \times \Delta$.

Remark 3.2. Under (\mathbf{A}_3) , for any $(t, \omega, p) \in T \times \Omega \times \Delta$ and any $x \in C^X(t, \omega, p)$, $\langle p, a(t, \omega) \rangle \leq \langle p, x \rangle$. Indeed, if $\langle p, x_0 \rangle < \langle p, a(t, \omega) \rangle$ for some $x_0 \in C^X(t, \omega, p)$, then we can choose $y \in \mathbb{R}_+^\ell$ such that $y > 0$ and $\langle p, x_0 + y \rangle < \langle p, a(t, \omega) \rangle$. Thus, $x_0 + y \in B(t, \omega, p)$. Since $x_0 \in C^X(t, \omega, p)$, one has $U(t, \omega, x_0) \geq U(t, \omega, x_0 + y)$. However, (\mathbf{A}_3) implies $U(t, \omega, x_0 + y) > U(t, \omega, x_0)$, which is a contradiction.

Following [8], we call $C^X(t, \omega, p)$ the *preferred set* of agent t at the price p and state of nature ω , and call $\int_T C^X(\cdot, \omega, p) d\mu$ the *aggregate preferred set* at the price p and state of nature ω . Moreover, we call

$$\int_T C^X(\cdot, \cdot, \cdot) d\mu : \Omega \times \Delta \rightrightarrows \mathbb{R}_+^\ell$$

the *aggregate preferred correspondence*.

Remark 3.3. Note that our notations are slightly different from those of Aumann in [8]. In fact, the preferred set and the aggregate preferred set defined in [8] are equivalent to our sets $C(t, \omega, p)$ and $\int_T C(\cdot, \omega, p) d\mu$, respectively. But, the preferred set $C^X(t, \omega, p)$ and the aggregate preferred set $\int_T C^X(\cdot, \omega, p) d\mu$ defined in this paper are similar to the sets $D_p(t)$ and $\int_T D_p(\cdot) d\mu$ in [8], respectively. In [8], to define $D_p(t)$, Aumann used the following additional assumption:

(\mathbf{A}_5) There is an allocation⁴ v such that each agent t 's desire is commodity-wise saturated at $v(t)$.

⁴ Note that an allocation is called an assignment in [8].

In this paper, to avoid this assumption, we replace $D_p(t)$ and $\int_T D_p(\cdot) d\mu$ in [8] with $C^X(t, \omega, p)$ and $\int_T C^X(\cdot, \omega, p) d\mu$, respectively.

4. Properties of the aggregate preferred correspondence

In this section, we discuss some descriptive properties of the aggregate preferred correspondence. These properties will be used to derive the existence of a maximin rational expectations equilibrium in our economic model.

Proposition 4.1. *Under (A_1) , for every $(\omega, p) \in \Omega \times \Delta$, $B(\cdot, \omega, p) : (T, \Sigma, \mu) \rightrightarrows \mathbb{R}_+^\ell$ and $X(\cdot, \omega, p) : (T, \Sigma, \mu) \rightrightarrows \mathbb{R}_+^\ell$ are lower measurable.*

Proof. Here, only the proof of lower measurability of $B(\cdot, \omega, p)$ is provided. The other case can be done analogously. Fix $(\omega, p) \in \Omega \times \Delta$. Define a function $h : (T, \Sigma, \mu) \times \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ by letting $h(t, x) = \langle p, x \rangle - \langle p, a(t, \omega) \rangle$ for all $(t, x) \in T \times \mathbb{R}_+^\ell$. Then, $h(\cdot, x)$ is measurable for all $x \in \mathbb{R}_+^\ell$. Note that $B(t, \omega, p) = h(t, \cdot)^{-1}((-\infty, 0])$. Let $V \subseteq \mathbb{R}_+^\ell$ be a non-empty open subset of \mathbb{R}_+^ℓ , and put $V \cap \mathbb{Q}_+^\ell = \{x_k : k \geq 1\}$. It is worth to point out that if $x \in B(t, \omega, p) \cap V$, then $x_k \in B(t, \omega, p)$ for some $k \geq 1$. Since $h(\cdot, x_k)$ is measurable, $\{t \in T : h(t, x_k) \in (-\infty, 0]\} \in \Sigma$ for all $k \geq 1$. Thus,

$$\begin{aligned} B(\cdot, \omega, p)^{-1}(V) &= \bigcup_{k \geq 1} \{t \in T : x_k \in B(t, \omega, p)\} \\ &= \bigcup_{k \geq 1} \{t \in T : h(t, x_k) \in (-\infty, 0]\} \end{aligned}$$

belongs to Σ . It follows that $B(\cdot, \omega, p)$ is lower measurable. \square

Proposition 4.2. *Under (A_1) – (A_3) , $\int_T C^X(\cdot, \cdot, \cdot) d\mu : \Omega \times \Delta \rightrightarrows \mathbb{R}_+^\ell$ is non-empty compact-valued.*

Proof. Fix $(\omega, p) \in \Omega \times \Delta$. By (A_2) , $C^X(t, \omega, p)$ is non-empty closed for all $t \in T$. By the lower measurability of $B(\cdot, \omega, p)$, there exists a sequence $\{f_n : n \geq 1\}$ of measurable functions from (T, Σ, μ) to \mathbb{R}_+^ℓ such that $B(t, \omega, p) = \overline{\{f_n(t) : n \geq 1\}}$ for all $t \in T$. For each $n \geq 1$, define a correspondence $C_n : T \rightrightarrows \mathbb{R}_+^\ell$ by letting

$$C_n(t) = \{x \in \mathbb{R}_+^\ell : U(t, \omega, x) \geq U(t, \omega, f_n(t))\}$$

for all $t \in T$. Obviously, one has $C(t, \omega, p) \subseteq \bigcap_{n \geq 1} C_n(t)$ for all $t \in T$. If $x \in \mathbb{R}_+^\ell \setminus C(t, \omega, p)$ for some $t \in T$, there exists a point $y \in B(t, \omega, p)$ such that $U(t, \omega, y) > U(t, \omega, x)$. By (A_2) , there exists an $n_0 \geq 1$ such that $U(t, \omega, f_{n_0}(t)) > U(t, \omega, x)$. This implies that $x \notin C_{n_0}(t)$, and thus $C(t, \omega, p) = \bigcap_{n \geq 1} C_n(t)$ for all $t \in T$. Fix $n \geq 1$, and define $h : (T, \Sigma, \mu) \times \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ by

$$h(t, x) = U(t, \omega, f_n(t)) - U(t, \omega, x).$$

Clearly, h is Carathéodory. Similar to Proposition 4.1, one can show that C_n is lower measurable. Since $X(\cdot, \omega, p)$ is compact-valued and

$$C^X(\cdot, \omega, p) = \bigcap_{n \geq 1} C_n(\cdot) \cap X(\cdot, \omega, p),$$

then $C^X(\cdot, \omega, p) : (T, \Sigma, \mu) \rightrightarrows \mathbb{R}_+^\ell$ is lower measurable. By Theorem 2.1, $C^X(\cdot, \omega, p)$ has a measurable selection which is also integrable, as $b(\cdot, \omega, p)$ is so. Since $C^X(\cdot, \omega, p)$ is closed-valued and integrably bounded, $\int_T C^X(\cdot, \omega, p) d\mu$ is compact. \square

In [8], the continuity of $\int_T D_p(t)$ in p was proved. As shown in the next theorem, with the help of Proposition 4.2, we are able to establish the continuity of our aggregate preferred correspondence $\int_T C^X(\cdot, \cdot, \cdot) d\mu$ in $p \in \Delta$ with respect to the Hausdorff topology on $\mathcal{K}_0(\mathbb{R}_+^\ell)$.

Theorem 4.3. *Under (\mathbf{A}_1) – (\mathbf{A}_3) , for each $\omega \in \Omega$, $\int_T C^X(\cdot, \omega, \cdot) d\mu : \Delta \rightrightarrows \mathbb{R}_+^\ell$ is Hausdorff continuous.*

Proof. Fix $\omega \in \Omega$. Let $\{p_n : n \geq 1\} \subseteq \Delta$ converge to $p \in \Delta$. Then

$$c = \inf(\{\delta(p_n) : n \geq 1\} \cup \{\delta(p)\}) > 0.$$

For each $(t, \omega) \in T \times \Omega$, let

$$d(t, \omega) = \frac{1}{c} \sum_{h=1}^{\ell} a^h(t, \omega),$$

and

$$\xi(t, \omega) = (d(t, \omega), \dots, d(t, \omega)).$$

Define $M(\omega)$ by

$$M(\omega) = \left\{ x \in \mathbb{R}_+^\ell : x \leq \int_T \xi(\cdot, \omega) d\mu \right\}.$$

Since $X(\cdot, \omega, p_n)$ and $X(\cdot, \omega, p)$ are bounded from above by $\xi(\cdot, \omega)$, then the sets $\int_T C^X(\cdot, \omega, p_n) d\mu$ and $\int_T C^X(\cdot, \omega, p) d\mu$ are contained in the compact subset $M(\omega)$ of \mathbb{R}_+^ℓ . Thus, one only needs to show that $\{\int_T C^X(\cdot, \omega, p_n) d\mu : n \geq 1\}$ converges to $\int_T C^X(\cdot, \omega, p) d\mu$ in the Hausdorff metric topology on $\mathcal{K}_0(M(\omega))$, which is equivalent to

$$\text{Li} \int_T C^X(\cdot, \omega, p_n) d\mu = \text{Ls} \int_T C^X(\cdot, \omega, p_n) d\mu = \int_T C^X(\cdot, \omega, p) d\mu.$$

The above equation can be verified in two steps. First, one verifies

$$\text{Ls} \int_T C^X(\cdot, \omega, p_n) d\mu \subseteq \int_T C^X(\cdot, \omega, p) d\mu.$$

To do this, it is enough to verify that $\text{Ls} C^X(t, \omega, p_n) \subseteq C^X(t, \omega, p)$ for any $t \in T$. Pick $t \in T$ and $x \in \text{Ls} C^X(t, \omega, p_n)$. Then, there exist positive integers $n_1 < n_2 < n_3 < \dots$ and for each k a point $x_k \in C^X(t, \omega, p_{n_k})$ such that $\{x_k : k \geq 1\}$ converges to x . It is obvious that $x \in X(t, \omega, p)$. If $x \notin C^X(t, \omega, p)$, by the continuity of $U(t, \omega, \cdot)$, one can choose some $y \in \mathbb{R}_+^\ell$ such that $\langle p, y \rangle < \langle p, a(t, \omega) \rangle$ and $U(t, \omega, y) > U(t, \omega, x)$. By the Hausdorff continuity of $X(t, \omega, \cdot)$, $\{X(t, \omega, p_{n_k}) : k \geq 1\}$ converges to $X(t, \omega, p)$ in the Hausdorff metric topology. Since $y \in X(t, \omega, p)$, there exists a sequence $\{y_k : k \geq 1\}$ such that $y_k \in X(t, \omega, p_{n_k})$ for all $k \geq 1$ and $\{y_k : k \geq 1\}$ converges to y . It follows that $U(t, \omega, y_k) > U(t, \omega, x_k)$ and $\langle p_{n_k}, y_k \rangle < \langle p_{n_k}, a(t, \omega) \rangle$ for all sufficiently large k , which is a contradiction with $x_k \in C^X(t, \omega, p_{n_k})$ for all $k \geq 1$. Therefore, one must have $x \in C^X(t, \omega, p)$. Secondly, one needs to verify

$$\int_T C^X(\cdot, \omega, p) d\mu \subseteq \text{Li} \int_T C^X(\cdot, \omega, p_n) d\mu.$$

It suffices to verify that $C^X(t, \omega, p) \subseteq \text{Li } C^X(t, \omega, p_n)$ for all $t \in T$. Fix $t \in T$ and $d \in C^X(t, \omega, p)$. If $d = b(t, \omega, p)$, then $b(t, \omega, p_n) \in C^X(t, \omega, p_n)$ and $\{b(t, \omega, p_n) : n \geq 1\}$ converges to d . Assume $d < b(t, \omega, p)$. Select $\delta > 0$ such that

$$d + (0, \dots, \delta, \dots, 0) \leq b(t, \omega, p),$$

and a sequence $\{\delta_i : i \geq 1\}$ in $(0, \delta]$ converging to 0. For each $i \geq 1$, let

$$d^i = d + (0, \dots, \delta_i, \dots, 0),$$

and choose a sequence $\{d_n^i : n \geq 1\}$ such that for each n , $d_n^i \in X(t, \omega, p_n)$ and $\{d_n^i : n \geq 1\}$ converges to d^i . It is claimed that for each $i \geq 1$, $d_n^i \in C^X(t, \omega, p_n)$ for sufficiently large n . Otherwise, there must exist an i_0 and a subsequence $\{d_{n_k}^{i_0} : k \geq 1\}$ of $\{d_n^i : n \geq 1\}$ such that $d_{n_k}^{i_0} \notin C^X(t, \omega, p_{n_k})$. Let $b_k \in B(t, \omega, p_{n_k})$ and $U(t, \omega, b_k) > U(t, \omega, d_{n_k}^{i_0})$ for all $k \geq 1$. Then $\{b_k : k \geq 1\}$ has a subsequence converging to some $b \in B(t, \omega, p)$. (\mathbf{A}_2) and (\mathbf{A}_3) imply

$$U(t, \omega, b) \geq U(t, \omega, d^{i_0}) > U(t, \omega, d),$$

which contradicts with $d \in C^X(t, \omega, p)$. To complete the proof, note that the previous claim implies that for each i , $\{\text{dist}(d^i, C^X(t, \omega, p_n)) : n \geq 1\}$ converges to 0. Since $\{d^i : i \geq 1\}$ converges to d , one concludes that $\{\text{dist}(d, C^X(t, \omega, p_n)) : n \geq 1\}$ converges to 0. This means that $d \in \text{Li } C^X(t, \omega, p_n)$. \square

The next result is crucial for the existence theorem in Section 5. In its proof, the following characterization of lower measurability of correspondences in [6] is used: A correspondence $F : (\Omega, \mathcal{F}, \nu) \rightrightarrows \mathbb{R}_+^\ell$ is lower measurable if and only if for all $y \in \mathbb{R}_+^\ell$, $\text{dist}(y, F(\cdot)) : (\Omega, \mathcal{F}, \nu) \rightarrow \mathbb{R}_+$ is a measurable function.

Theorem 4.4. *Under (\mathbf{A}_1) – (\mathbf{A}_3) , for each $p \in \Delta$, $\int_T C^X(\cdot, \cdot, p) d\mu : (\Omega, \mathcal{F}, \nu) \rightrightarrows \mathbb{R}_+^\ell$ is lower measurable.*

Proof. Fix $p \in \Delta$. Since a and U are $\Sigma \otimes \mathcal{F}$ -measurable and $\Sigma \otimes \mathcal{F} \otimes \mathcal{B}(\mathbb{R}_+^\ell)$ -measurable respectively, by the argument of a result in [27, p. 131], there exist two sequences $\{a_n : n \geq 1\}$ and $\{\psi_n : n \geq 1\}$ of $\Sigma \otimes \mathcal{F}$ -measurable and $\Sigma \otimes \mathcal{F} \otimes \mathcal{B}(\mathbb{R}_+^\ell)$ -measurable functions respectively such that $\{a_n : n \geq 1\}$ uniformly converges to a on $T \times \Omega$ and $\{\psi_n : n \geq 1\}$ uniformly converges to U on $T \times \Omega \times \mathbb{R}_+^\ell$. For each $n \geq 1$, write a_n and ψ_n as

$$a_n = \sum_{i \geq 1} e_i \chi_{T_i^n \times \Omega_i^n} \quad \text{and} \quad \psi_n = \sum_{i \geq 1} v_i \chi_{T_i^n \times \Omega_i^n \times B_i^n},$$

where $e_i \in \mathbb{R}_+^\ell$, $v_i \in \mathbb{R}$, and $\{T_i^n \times \Omega_i^n \times B_i^n : i \geq 1\}$ is a partition of $T \times \Omega \times \mathbb{R}_+^\ell$ for all $n \geq 1$. Choose $N \geq 1$ such that $\|a_n - a\|_\infty < 1$ for all $n \geq N$. By the measurability of $a_n(\cdot, \omega)$, $a_n(\cdot, \omega) \in L_1(\mu, \mathbb{R}_+^\ell)$ for all $\omega \in \Omega$ and all $n \geq 1$ (replacing a_n for all $1 \leq n < N$ by some constant functions, if necessary). Let

$$\gamma_n(t, \omega) = \frac{1}{\delta(p)} \sum_{h=1}^{\ell} a_n^h(t, \omega),$$

and

$$b_n(t, \omega) = (\gamma_n(t, \omega), \dots, \gamma_n(t, \omega)).$$

Define $X_n, B_n, C_n : T \times \Omega \rightrightarrows \mathbb{R}_+^\ell$ such that $X_n(t, \omega) = \{x \in \mathbb{R}_+^\ell : x \leq b_n(t, \omega)\}$,

$$B_n(t, \omega) = \{x \in \mathbb{R}_+^\ell : \langle p, x \rangle \leq \langle p, a_n(t, \omega) \rangle\}$$

and

$$C_n(t, \omega) = \{y \in \mathbb{R}_+^\ell : \psi_n(t, \omega, y) \geq \psi_n(t, \omega, x) \text{ for all } x \in B_n(t, \omega)\}.$$

In addition, define $C_n^X : T \times \Omega \rightrightarrows \mathbb{R}_+^\ell$ such that for all $(t, \omega) \in T \times \Omega$,

$$C_n^X(t, \omega) = (C_n(t, \omega) \cup \{b_n(t, \omega)\}) \cap X_n(t, \omega).$$

For every $n \geq 1$, define the correspondence $H_n : (\Omega, \mathcal{F}, \nu) \rightrightarrows L_1(\mu, \mathbb{R}_+^\ell)$ by letting $H_n(\omega) = \mathcal{S}_{C_n^X(\cdot, \omega)}$. Obviously, $H_n(\omega) \neq \emptyset$ for all $\omega \in \Omega$.

Claim 1. For each $n \geq 1$, H_n is lower measurable. For convenience, let $\Theta : L_1(\mu, \mathbb{R}_+^\ell) \times \Omega \rightarrow \mathbb{R}_+$ be the function such that $\Theta(g, \omega) = \text{dist}(g, H_n(\omega))$ for all $g \in L_1(\mu, \mathbb{R}_+^\ell)$ and $\omega \in \Omega$. To verify the claim, one needs to verify that for all $g \in L_1(\mu, \mathbb{R}_+^\ell)$, $\Theta(g, \cdot)$ is measurable. Since $\Theta(\cdot, \omega) : L_1(\mu, \mathbb{R}_+^\ell) \rightarrow \mathbb{R}_+$ is norm-continuous, it suffices to show that $\Theta(g, \cdot) : (\Omega, \mathcal{F}, \nu) \rightarrow \mathbb{R}_+$ is measurable for every simple function $g = \sum_{j=1}^r x_j \chi_{T_j}$, where $x_j \in \mathbb{R}_+^\ell$. To this end, consider the function $\Gamma : (T, \Sigma, \mu) \times (\Omega, \mathcal{F}, \nu) \rightarrow \mathbb{R}_+$ such that $\Gamma(t, \omega) = \text{dist}(g(t), C_n^X(t, \omega))$ for all $(t, \omega) \in T \times \Omega$. Since Γ is constant on each $(T_i^n \cap T_j) \times \Omega_i^n$, it is jointly measurable. Note that $\Gamma(t, \omega) \leq \|g(t) - b_n(t, \omega)\|$ for all $(t, \omega) \in T \times \Omega$. This implies for all $\omega \in \Omega$, $\Gamma(\cdot, \omega)$ is integrable. Thus, $\Theta(g, \cdot)$ is measurable and Claim 1 is verified if one shows for all $\omega \in \Omega$, $\int_T \Gamma(\cdot, \omega) d\mu = \Theta(g, \omega)$. Assume $\int_T \Gamma(\cdot, \omega_0) d\mu < \Theta(g, \omega_0)$ for some $\omega_0 \in \Omega$. Pick $\varepsilon > 0$ such that

$$\int_T \Gamma(\cdot, \omega_0) d\mu + \varepsilon \mu(T) < \Theta(g, \omega_0).$$

Further, pick $t \in T_i^n \cap T_j$ and $y_{(i,j)} \in C_n^X(t, \omega_0)$ so that

$$\|x_j - y_{(i,j)}\| < \Gamma(t, \omega_0) + \varepsilon.$$

Define $\zeta : T \rightarrow \mathbb{R}_+^\ell$ by $\zeta(t) = y_{(i,j)}$ for all $t \in T_i^n \cap T_j$. Then, $\zeta \in H_n(\omega_0)$ and

$$\|g - \zeta\|_1 < \int_T \Gamma(\cdot, \omega_0) d\mu + \varepsilon \mu(T),$$

which is a contradiction.

Claim 2. The correspondence $\int_T C_n^X(\cdot, \cdot) d\mu : (\Omega, \mathcal{F}, \nu) \rightrightarrows \mathbb{R}_+^\ell$ is lower measurable. Consider the function $\xi : L_1(\mu, \mathbb{R}_+^\ell) \rightarrow \mathbb{R}_+^\ell$ defined by $\xi(f) = \int_T f d\mu$ for all $f \in L_1(\mu, \mathbb{R}_+^\ell)$. Let V be an open subset of \mathbb{R}_+^ℓ . Note that

$$\xi \circ H_n(\omega) = \int_T C_n^X(\cdot, \omega) d\mu$$

for all $\omega \in \Omega$, and

$$(\xi \circ H_n)^{-1}(V) = \{\omega \in \Omega : H_n(\omega) \cap \xi^{-1}(V) \neq \emptyset\}.$$

Since ξ is norm-continuous, by Claim 1, $(\xi \circ H_n)^{-1}(V) \in \mathcal{F}$. This verifies the claim.

Claim 3. For each $\omega \in \Omega$,

$$\text{Li} \int_T C_n^X(\cdot, \omega) d\mu = \text{Ls} \int_T C_n^X(\cdot, \omega) d\mu = \int_T C^X(\cdot, \omega, p) d\mu.$$

To see this, for each $\omega \in \Omega$, put

$$\alpha(\cdot, \omega) = \sup \left\{ b_1(\cdot, \omega), \dots, b_{N-1}(\cdot, \omega), b(\cdot, \omega, p) + \left(\frac{\ell}{\delta(p)}, \dots, \frac{\ell}{\delta(p)} \right) \right\}.$$

Then, $C^X(\cdot, \omega, p)$ and all $C_n^X(\cdot, \omega)$ are bounded from above by $\alpha(\cdot, \omega)$. Now, it suffices to verify that for all $t \in T$,

$$\text{Ls} C_n^X(t, \omega) \subseteq C^X(t, \omega, p) \quad \text{and} \quad C^X(t, \omega, p) \subseteq \text{Li} C_n^X(t, \omega).$$

First, let $x \in \text{Ls} C_n^X(t, \omega)$. If $x = b(t, \omega, p)$, then $\{b_n(t, \omega): n \geq 1\}$ converges to x and $b_n(t, \omega) \in C_n^X(t, \omega)$ for all $n \geq 1$. Otherwise, there exist positive integers $n_1 < n_2 < n_3 < \dots$ and for each k a point $x_k \in C_{n_k}^X(t, \omega)$ such that $\{x_k: k \geq 1\}$ converges to x . Obviously, $x_k \neq b_{n_k}(t, \omega)$ for all sufficiently large k , and $x \in X(t, \omega, p)$. If $x \notin C^X(t, \omega, p)$, there exists some $y \in B(t, \omega, p)$ such that $U(t, \omega, y) > U(t, \omega, x)$. By the continuity of $U(t, \omega, \cdot)$, y can be chosen so that $\langle p, y \rangle < \langle p, a(t, \omega) \rangle$. Since $\{X_{n_k}(t, \omega): k \geq 1\}$ converges to $X(t, \omega, p)$ in the Hausdorff metric topology, there exists a sequence $\{y_k: k \geq 1\}$ such that $y_k \in X_{n_k}(t, \omega)$ for all $k \geq 1$ and $\{y_k: k \geq 1\}$ converges to y . By the inequality

$$|U(t, \omega, x) - \psi_{n_k}(t, \omega, x_k)| < |U(t, \omega, x) - U(t, \omega, x_k)| + |U(t, \omega, x_k) - \psi_{n_k}(t, \omega, x_k)|,$$

the continuity of $U(t, \omega, \cdot)$ and the uniform convergence of $\psi_{n_k}(t, \omega, \cdot)$ to $U(t, \omega, \cdot)$, one concludes that

$$\psi_{n_k}(t, \omega, y_k) > \psi_{n_k}(t, \omega, x_k) \quad \text{and} \quad \langle p, y_k \rangle < \langle p, a_{n_k}(t, \omega) \rangle$$

for sufficiently large k , which contradicts with the fact that $x_k \in C_{n_k}^X(t, \omega)$ for all $k \geq 1$. Hence, $x \in C^X(t, \omega, p)$. Now, let $d \in C^X(t, \omega, p)$. If $d = b(t, \omega, p)$, there is nothing to verify. Thus, $d \in \text{Li} C_n^X(t, \omega)$. Assume $d < b(t, \omega, p)$. Similar to that in the proof of [Theorem 4.3](#), one can show that $d \in \text{Li} C_n^X(t, \omega)$.

To complete the proof, for each $\omega \in \Omega$, put

$$M(\omega) = \left\{ x \in \mathbb{R}_+^\ell: x \leq \int_T \alpha(\cdot, \omega) d\mu \right\}.$$

Clearly, $\overline{\int_T C_n^X(\cdot, \omega) d\mu}$ and $\int_T C^X(\cdot, \omega, p) d\mu$ are contained in the compact set $M(\omega)$. By Claim 3, $\{\int_T C_n^X(\cdot, \omega) d\mu: n \geq 1\}$ converges to $\int_T C^X(\cdot, \omega, p) d\mu$ in $M(\omega)$ in the Hausdorff metric topology. It is well known that a nonempty compact-valued correspondence is lower measurable if and only if it is measurable when viewed as a single-valued function whose range space is the space of nonempty compact sets endowed with the Hausdorff metric topology. By Claim 2, $\int_T C^X(\cdot, \cdot, p) d\mu$ is lower measurable. \square

Corollary 4.5. Under (\mathbf{A}_1) – (\mathbf{A}_3) , $\int_T C^X(\cdot, \cdot, \cdot) d\mu: (\Omega, \mathcal{F}, \nu) \times (\Delta, \mathcal{B}(\Delta)) \rightarrow \mathcal{K}_0(\mathbb{R}_+^\ell)$ is a jointly measurable function, where $\mathcal{K}_0(\mathbb{R}_+^\ell)$ is endowed with the Hausdorff metric topology.

Proof. By [Theorem 4.3](#), for every $\omega \in \Omega$, $\int_T C^X(\cdot, \omega, \cdot) d\mu: \Delta \rightarrow \mathcal{K}_0(\mathbb{R}_+^\ell)$ is continuous. Furthermore, by [Theorem 4.4](#), for every $p \in \Delta$, the correspondence $\int_T C^X(\cdot, \cdot, p) d\mu: (\Omega, \Sigma, \nu) \rightrightarrows \mathbb{R}_+^\ell$ is lower measurable. Hence, for every $p \in \Delta$, $\int_T C^X(\cdot, \cdot, p) d\mu: (\Omega, \Sigma, \nu) \rightarrow \mathcal{K}_0(\mathbb{R}_+^\ell)$ is measurable. This means that $\int_T C^X(\cdot, \cdot, \cdot) d\mu: (\Omega, \mathcal{F}, \nu) \times (\Delta, \mathcal{B}(\Delta)) \rightarrow \mathcal{K}_0(\mathbb{R}_+^\ell)$ is Carathéodory, and therefore is jointly measurable. \square

5. Maximin rational expectations equilibrium

A price system of \mathcal{E} is a measurable function $\pi : (\Omega, \mathcal{F}, \nu) \rightarrow \Delta$. Let $\sigma(\pi)$ be the smallest sub-algebra of \mathcal{F} such that π is measurable and let $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(\pi)$. For each $\omega \in \Omega$, let $\mathcal{G}_t(\omega)$ denote the smallest element of \mathcal{G}_t containing ω . Given $t \in T$, $\omega \in \Omega$ and a price system π , let $B^{REE}(t, \omega, \pi)$ be defined by

$$B^{REE}(t, \omega, \pi) = \{x \in (\mathbb{R}_+^\ell)^\Omega : x(\omega') \in B(t, \omega', \pi(\omega')) \text{ for all } \omega' \in \mathcal{G}_t(\omega)\}.$$

The *maximin utility* of each agent $t \in T$ with respect to \mathcal{G}_t at $x : \Omega \rightarrow \mathbb{R}_+^\ell$ in state $\omega \in \Omega$, denoted by $\underline{U}^{REE}(t, \omega, x)$, is defined by

$$\underline{U}^{REE}(t, \omega, x) = \inf_{\omega' \in \mathcal{G}_t(\omega)} U(t, \omega', x(\omega')).$$

Comparing with $\underline{U}^{REE}(t, \cdot, \cdot)$, the function $U(t, \cdot, \cdot)$ is sometimes called the *ex post* utility of agent t .

Remark 5.1. The maximin utility formation in the sense of REE was introduced by de Castro et al. in [13], where Ω is finite. In this case, for each $t \in T$, Π_t is a partition of Ω consisting of only finitely many elements and $\sigma(\pi)$ is generated by a partition Π_π also consisting of only finitely many elements. Thus, the σ -algebra $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(\pi)$ is generated by the partition $\Pi_t \vee \Pi_\pi$. For each $\omega \in \Omega$, there exists a unique element $\mathcal{G}_t(\omega)$ in $\Pi_t \vee \Pi_\pi$ containing ω . It is clear that $\mathcal{G}_t(\omega)$ is the smallest element of \mathcal{G}_t containing ω . Moreover, since $\mathcal{G}_t(\omega)$ is a finite set, $\underline{U}^{REE}(t, \omega, x)$ is well-defined.

In our case, Ω is fairly general, particularly, can be infinite. The structure of $\sigma(\pi)$ can be complicated. If Ω is infinite, $\sigma(\pi)$ may not be generated by a partition. But, for each $\omega \in \Omega$, there always exists a (unique) smallest element in $\sigma(\pi)$ containing ω . This means that there also exists a (unique) smallest element $\mathcal{G}_t(\omega)$ in \mathcal{G}_t containing ω . Since $\mathcal{G}_t(\omega)$ can be infinite, $\underline{U}^{REE}(t, \omega, x)$ is allowed to take the value $-\infty$ if the above infimum does not exist. This adaptation will not affect the proof of Theorem 5.5 below.

Definition 5.2. (See [13].) Given a feasible allocation f and a price system π , the pair (f, π) is called a *maximin rational expectations equilibrium* (abbreviated as maximin REE) of \mathcal{E} if $f(t, \omega) \in B(t, \omega, \pi(\omega))$ and $f(t, \cdot)$ maximizes $\underline{U}^{REE}(t, \omega, \cdot)$ on $B^{REE}(t, \omega, \pi)$ for all $(t, \omega) \in T \times \Omega$. In this case, f is called a *maximin rational expectations allocation*, and the set of such allocations is denoted by $MREE(\mathcal{E})$.

Let $x : \Omega \rightarrow \mathbb{R}_+^\ell$ be \mathcal{G}_t -measurable. Recall that the *Bayesian expected utility* of agent t with respect to \mathcal{G}_t at x is given by $\mathbb{E}^{\mathbb{Q}_t}[U(t, \cdot, x)|\mathcal{G}_t]$. When Ω is finite, \mathcal{G}_t is generated by the partition $\Pi_t \vee \Pi_\pi$. Then,

$$\mathbb{E}^{\mathbb{Q}_t}[U(t, \cdot, x)|\mathcal{G}_t](\omega) = \sum_{\omega' \in \Pi_t \vee \Pi_\pi(\omega)} U(t, \omega', x(\omega')) \times \frac{\pi(\omega')}{\pi(\Pi_t \vee \Pi_\pi(\omega))},$$

where $\Pi_t \vee \Pi_\pi(\omega)$ is the unique member of $\Pi_t \vee \Pi_\pi$ containing ω .

Definition 5.3. (See [1,26].) Given a feasible allocation f and a price system π , the pair (f, π) is called a *Bayesian rational expectations equilibrium* (abbreviated as Bayesian REE) of \mathcal{E} if

- (i) for each $t \in T$, $f(t, \cdot)$ is \mathcal{G}_t -measurable;
- (ii) for all $(t, \omega) \in T \times \Omega$, $\langle f(t, \omega), \pi(\omega) \rangle \leq \langle a(t, \omega), \pi(\omega) \rangle$;
- (iii) for all $(t, \omega) \in T \times \Omega$,

$$\mathbb{E}^{\mathbb{Q}_t}[U(t, \cdot, f(t, \omega))|\mathcal{G}_t](\omega) = \max_{x \in B^{REE}(t, \omega, \pi(\omega)) \cap L_t^{REE}} \mathbb{E}^{\mathbb{Q}_t}[U(t, \cdot, x)|\mathcal{G}_t](\omega),$$

where L_t^{REE} is defined by

$$L_t^{REE} = \{x \in (\mathbb{R}_+^\ell)^\Omega : x \text{ is } \mathcal{G}_t\text{-measurable}\}.$$

Remark 5.4. Definition 5.2 indicates that at a maximin rational expectations allocation, each individual maximizes his maximin utility conditioned on his private information and the information generated by the equilibrium prices, subject to the budget constraint. Definition 5.3 indicates that at a Bayesian rational expectations allocation, each individual maximizes his Bayesian utility conditioned on his private information and the information generated by the equilibrium prices, subject to the budget constraint. The resulting equilibrium allocations are measurable with respect to the private information of each individual and also with respect to the information the equilibrium prices generate. For detailed discussions on relations and comparison between Bayesian REE and Maximin REE, refer to [13].

It is well known that a Bayesian rational expectations equilibrium may not exist. It only exists in a generic sense and not universally. Moreover, it fails to be fully Pareto optimal and incentive compatible and it is not implementable as a perfect Bayesian equilibrium of an extensive form game, see [15]. In [13], de Castro et al. showed that $MREE(\mathcal{E}) \neq \emptyset$ when Ω and T are finite. Our next theorem extends their result to a more general case.

Theorem 5.5. Under (A_1) – (A_4) , $MREE(\mathcal{E}) \neq \emptyset$.

Proof. Consider the correspondence $Z : (\Omega, \mathcal{F}, \nu) \times (\Delta, \mathcal{B}(\Delta)) \rightrightarrows \mathbb{R}^\ell$ defined by

$$Z(\omega, p) = \int_T C^X(\cdot, \omega, p) d\mu - \int_T a(\cdot, \omega) d\mu.$$

By Proposition 4.2, Z is non-empty compact-valued. In addition, by Theorem 4.3, Z is also Hausdorff continuous in p . By Corollary 4.5 and (A_1) , $Z : (\Omega, \mathcal{F}, \nu) \times (\Delta, \mathcal{B}(\Delta)) \rightarrow \mathcal{K}_0(\mathbb{R}^\ell)$ is jointly measurable. Define another correspondence $F : (\Omega, \mathcal{F}, \nu) \rightrightarrows (\Delta, \mathcal{B}(\Delta))$ by

$$F(\omega) = \{p \in \Delta : Z(\omega, p) \cap \{0\} \neq \emptyset\}.$$

Note that $\mathcal{E}(\omega)$ has a Walrasian equilibrium for each $\omega \in \Omega$. This implies that F is non-empty valued. Furthermore, by Hausdorff continuity of $Z(\omega, \cdot)$, F is also closed-valued. Since $\text{Gr}_F = Z^{-1}(\{0\})$, then $\text{Gr}_F \in \mathcal{F} \otimes \mathcal{B}(\Delta)$. By Theorem 8.1.4 in [6],⁵ F is lower measurable. By Theorem 2.1, there exists a measurable selection $\hat{\pi}$ of F . By the definition of Z , there exists an allocation f such that $f(t, \omega) \in C^X(t, \omega, \hat{\pi}(\omega))$ and $\int_T f(\cdot, \omega) d\mu = \int_T a(\cdot, \omega) d\mu$ for all $(t, \omega) \in T \times \Omega$. Remark 3.2 implies that $\langle \hat{\pi}(\omega), f(t, \omega) \rangle \geq \langle \hat{\pi}(\omega), a(t, \omega) \rangle$ for all $(t, \omega) \in T \times \Omega$. It follows that $\langle \hat{\pi}(\omega), f(t, \omega) \rangle = \langle \hat{\pi}(\omega), a(t, \omega) \rangle$ for μ -a.e. $t \in T$ and all $\omega \in \Omega$. Thus, $f(t, \omega) \in B(t, \omega, \hat{\pi}(\omega))$ for μ -a.e. $t \in T$ and all $\omega \in \Omega$. For every $\omega \in \Omega$, define $T_\omega \subseteq T$ by

$$T_\omega = \{t \in T : f(t, \omega) \in B(t, \omega, \hat{\pi}(\omega)) \cap C(t, \omega, \hat{\pi}(\omega))\}.$$

Then, $\mu(T_\omega) = \mu(T)$ for all $\omega \in \Omega$. Next, for every $\omega \in \Omega$ and every $t \in T \setminus T_\omega$, as $B(t, \omega, \hat{\pi}(\omega)) \cap C(t, \omega, \hat{\pi}(\omega)) \neq \emptyset$, one can pick a point

$$h(t, \omega) \in B(t, \omega, \hat{\pi}(\omega)) \cap C(t, \omega, \hat{\pi}(\omega)),$$

⁵ This is the only place in this paper where the completeness of $(\Omega, \mathcal{F}, \nu)$ is required.

and then define a function $\hat{f} : T \times \Omega \rightarrow \mathbb{R}_+^\ell$ such that

$$\hat{f}(t, \omega) = \begin{cases} f(t, \omega), & \text{if } t \in T_\omega; \\ h(t, \omega), & \text{if } t \in T \setminus T_\omega. \end{cases}$$

It is obvious that $\hat{f}(t, \omega) \in B(t, \omega, \hat{\pi}(\omega)) \cap C(t, \omega, \hat{\pi}(\omega))$ for all $(t, \omega) \in T \times \Omega$. Assume that there are an agent $t_0 \in T$, a state of nature $\omega_{t_0} \in \Omega$ and an element $y(t_0, \cdot) \in B^{REE}(t_0, \omega_{t_0}, \hat{\pi})$ such that

$$\underline{U}^{REE}(t_0, \omega_{t_0}, y(t_0, \cdot)) > \underline{U}^{REE}(t_0, \omega_{t_0}, \hat{f}(t_0, \cdot)).$$

Then, one obtains

$$U(t_0, \omega'_{t_0}, y(t_0, \omega'_{t_0})) > U(t_0, \omega'_{t_0}, \hat{f}(t_0, \omega'_{t_0}))$$

for some $\omega'_{t_0} \in \mathcal{G}_{t_0}(\omega_{t_0})$, which contradicts with $\hat{f}(t_0, \omega'_{t_0}) \in C(t_0, \omega'_{t_0}, \hat{\pi}(\omega'_{t_0}))$. This verifies that $(\hat{f}, \hat{\pi})$ is a maximin rational expectations equilibrium of \mathcal{E} . \square

Remark 5.6. In this remark, we explain how our maximin utility is related to that of Gilboa and Schmeidler in [14], and whether Theorem 5.5 can be extended to the framework of Gilboa and Schmeidler. For any $S \in \mathcal{F}$, let $\mathcal{F}_S = \{F \cap S : F \in \mathcal{F}\}$. The set of beliefs of agent t at state ω is defined by

$$\Delta(\mathcal{G}_t(\omega)) = \{\nu : \mathcal{F}_{\mathcal{G}_t(\omega)} \rightarrow [0, 1] : \nu \text{ is finitely additive and } \nu(\mathcal{G}_t(\omega)) = 1\}.$$

First, we claim that for any \mathcal{F} -measurable function $x : \mathcal{G}_t(\omega) \rightarrow \mathbb{R}_+^\ell$,

$$\inf_{\omega' \in \mathcal{G}_t(\omega)} U(t, \omega', x(\omega')) = \inf_{\nu \in \Delta(\mathcal{G}_t(\omega))} \int_{\mathcal{G}_t(\omega)} U(t, \cdot, x(\cdot)) d\nu.$$

For convenience, we put

$$a = \inf_{\omega' \in \mathcal{G}_t(\omega)} U(t, \omega', x(\omega')) \quad \text{and} \quad b = \inf_{\nu \in \Delta(\mathcal{G}_t(\omega))} \int_{\mathcal{G}_t(\omega)} U(t, \cdot, x(\cdot)) d\nu.$$

Since $a \leq U(t, \omega', x(\omega'))$ for all $\omega' \in \mathcal{G}_t(\omega)$, for any $\nu \in \Delta(\mathcal{G}_t(\omega))$, we have $a \leq \int_{\mathcal{G}_t(\omega)} U(t, \cdot, x(\cdot)) d\nu$. This further implies $a \leq b$. If $a < b$, then we choose some c with $a < c < b$. Define $A = \{\omega' \in \mathcal{G}_t(\omega) : U(t, \omega', x(\omega')) \in (a, c)\}$. From the definition of a , the set A is non-empty. By (A₂), $A \in \mathcal{F}_{\mathcal{G}_t(\omega)}$. Choose $\nu_0 \in \Delta(\mathcal{F}_{\mathcal{G}_t(\omega)})$ such that $\nu_0(A) = 1$ and $\nu_0(\mathcal{G}_t(\omega) \setminus A) = 0$. Thus, $\int_{\mathcal{G}_t(\omega)} U(t, \cdot, x(\cdot)) d\nu_0 < b$. This is a contradiction, which verifies the claim. Therefore, in our model, the maximin utility can be re-formulated as

$$\underline{U}^{REE}(t, \omega, x) = \inf_{\nu \in \Delta(\mathcal{G}_t(\omega))} \int_{\mathcal{G}_t(\omega)} U(t, \cdot, x(\cdot)) d\nu$$

under the \mathcal{F} -measurability assumption of x . Once we realize this, then we can see how Theorem 5.5 can be further extended to the framework of either maximin expected utility in [14] or variational representation of preferences in [24] under the same assumption. Indeed, if we define

$$\underline{U}^{REE}(t, \omega, x) = \inf_{\nu \in K} \int_{\mathcal{G}_t(\omega)} U(t, \cdot, x(\cdot)) d\nu,$$

as the maximin utility function, where K is any non-empty, closed and convex subset of $\Delta(\mathcal{G}_t(\omega))$ as in Gilboa and Schmeidler [14], then Theorem 5.5 still holds. To see this, we only need to verify the last part of the proof of the theorem. Suppose there are an agent $t_0 \in T$, a state of nature $\omega_{t_0} \in \Omega$ and an element $y(t_0, \cdot) \in B^{REE}(t_0, \omega_{t_0}, \hat{\pi})$ such that

$$\underline{U}^{REE}(t_0, \omega_{t_0}, y(t_0, \cdot)) > \underline{U}^{REE}(t_0, \omega_{t_0}, \hat{f}(t_0, \cdot)).$$

Then, we can find a $\nu_0 \in K$ such that

$$\int_{\mathcal{G}_{t_0}(\omega_{t_0})} U(t_0, \cdot, y(t_0, \cdot)) d\nu_0 > \int_{\mathcal{G}_{t_0}(\omega_{t_0})} U(t_0, \cdot, \hat{f}(t_0, \cdot)) d\nu_0.$$

So, there exists a $B \in \mathcal{F}_{\mathcal{G}_{t_0}(\omega_{t_0})}$ with $\nu_0(B) > 0$ such that

$$U(t_0, \omega'_{t_0}, y(t_0, \omega'_{t_0})) > U(t_0, \omega'_{t_0}, \hat{f}(t_0, \omega'_{t_0}))$$

for all $\omega'_{t_0} \in B$, which contradicts with $\hat{f}(t_0, \omega'_{t_0}) \in C(t_0, \omega'_{t_0}, \hat{\pi}(\omega'_{t_0}))$. The other case can be verified similarly.

6. Conclusion

The first application of maximin expected utilities to the general equilibrium theory with differential information appeared in [11], where an existence theorem for a Walrasian equilibrium in an economy was established. However, their MEU formulation is in the ex ante sense, and REE notion was not considered.

It is well known that Walrasian expectations equilibrium may not exist in a differential information economy with Bayesian expected utilities and infinitely many states, see Podczeck et al. [25]. With infinitely many states, Herves-Beloso et al. [16] showed equilibrium existence assuming that the finitely many agents observe a public and a private signal. The publicly observed signal may take infinitely many values but private signals only take finitely many values. That is, the asymmetries among agents' information affect only a finite number of states. The major difference between the model in [16] and ours is that our allocations are not privately measurable, comparing with that in the case of Walrasian expectations equilibrium. Hence, we restrict our attention only on the ex-post utility (that is, we are in a finite dimensional commodity space framework) in Section 4. Later, we use the results obtained in Section 4 to prove the existence theorem in Section 5. Note that the equilibrium prices are \mathcal{F} -measurable and thus, we need the results in Section 4 to establish the existence theorem. But, in the case of Walrasian expectations equilibrium, we deal with privately measurable allocations and if there are infinitely many states, then we are in an infinite dimensional commodity space framework. Thus, our analysis cannot be used to extend the existence of Walrasian expectations equilibrium in [16] to a differential information economy with Bayesian expected utilities and infinitely many states of nature.

Comparing with the existence result on maximin REE in [13], our theorem applies to a more general economic model with an arbitrary finite measure space of agents and an arbitrary complete probability measure space as the space of states of nature, while the later applies only to an economic model which has finitely many agents and finitely many states of nature. The assumptions in our paper are similar to those in [13], except the joint measurability and continuity of utility functions, and the joint measurability of the initial endowment function. The proof techniques in this paper are quite different from those in [13]. Since there are only finitely many agents and states of nature in the model considered in [13], neither measurability nor continuity of utility functions and the initial endowment function plays any role in the proof of the existence of a maximin REE. Instead, the existence of a competitive equilibrium for complete information economies is applied. In contrast, both measurability and continuity of utility functions and the initial

endowment function play key roles in this paper. To establish the existence theorem, techniques in [8] are adopted, measurability and continuity of the aggregate preferred correspondence are investigated. However, for special cases, the techniques can be simplified. For instance, if there are finitely many states of nature, one can still apply the approach employed in [13] and obtains an existence theorem. On the other hand, if there are finitely many agents, then one can show that the demand of each agent is $\mathcal{F} \otimes \mathcal{B}(\Delta)$ -measurable and so is the aggregate demand. Then, an approach similar to that in the proof of Theorem 5.5 can be applied to establish the existence theorem. Further, since the space of states of nature in our model is an abstract probability space, our existence theorem does not depend on the dimension of the space of states of nature.

In [9], Bhowmik and Cao used appropriate notions of properness to characterize Warasian expectations allocations in infinite dimensional commodity spaces. It would be interesting to know if those notions of properness can also be used to provide similar characterizations of maximin REE allocations in infinite dimensional commodity spaces. Recently, a general model of trade ex ante with differential information and uncertain delivery, in which there are finitely many agents and finitely many states of nature, was studied by Correia-da-Silva and Hervés-Beloso in [12], and the existence of equilibrium is established. It would also be interesting to see whether the techniques in this paper can be used to extend the existence of equilibrium in [12] to the framework of infinitely many agents and infinitely many states of nature.

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