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Robustness with respect to small delays for exponential stability of non-autonomous systems

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Abstract

Robustness of stability with respect to small delays, e.g., motivated by feedback systems in control theory, is of great theoretical and practical important, but this property does not hold for many systems. In this paper, we introduce the conception of robustness with respect to small time-varying delays for exponential stability of the non-autonomous linear systems. Sufficient conditions are given for the non-autonomous systems to be robust, and examples are provided to illustrate that the conditions are satisfied for a large class of the non-autonomous parabolic systems.

Keywords: Evolution family; Non-autonomous system; Robustness; Small time-varying delay

1. Introduction and problems

For a general feedback control problem, its sensors, processors and actuators all introduce time delays into the feedback loop. It is well known that the finite-dimensional system of the form

$$\frac{dx(t)}{dt} = Ax(t) + Bx(t), \tag{1.1}$$

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where $A, B \in \mathcal{L}(R^n)$, preserves stability when small delays occur in the feedback loop. That is, if system (1.1) is stable, then there is $r_0 > 0$ such that for any $r(\cdot) \in C([0, +\infty), [0, r_0])$ the delay system described by

$$\frac{dx(t)}{dt} = Ax(t) + Bu\bigl(t - r(t)\bigr) \tag{1.2}$$

remains stable (cf. [9,14]). However, this finite-dimensional property of robustness with respect to small delays does not hold for a general class of infinite-dimensional systems described by partial differential equations which are exponentially stabilized by a feedback but are destabilized by arbitrary small time delays in feedback loop. The first examples of this sort appeared in Datko et al. [4] and Huang [10] independently in 1986 (for other examples, see [8,9,11,12,15]). How do small delays in the feedback loop influence the exponential stability achieved by feedback control? This is an important and difficult problem, see Fleming [7]. Huang in [10] gave a sufficient condition for system (1.1) to be robust with respect to small delays, more precisely, he supposed that A is the infinitesimal generator of a C_0-semigroup e^{tA} on Banach space X, e^{tA} is immediately norm continuous and $B \in \mathcal{L}(X)$. For other situation, see, e.g., [9,11] and references therein.

To the best of our knowledge, there are few results available in the literature on robustness with respect to small time-varying delays for exponential stability of the non-autonomous systems. Our main goal here is to extend the results of Huang in [10] to the non-autonomous systems. Consider the non-autonomous system

$$\frac{dx(t)}{dt} = A(t)x(t) + B(t)x(t) \tag{1.3}$$

on a Banach space X, where the linear operators $A(t)$, $t \geqslant 0$, generates an exponentially bounded evolution family $U(t,s)$, $t \geqslant s \geqslant 0$, on X and $B(\cdot) \in C_b([0,\infty), \mathcal{L}(X))$ (i.e., $B(\cdot) \in C([0,\infty), \mathcal{L}(X))$ and $\|B\|_\infty := \sup_{0 \leqslant t < \infty} \|B(t)\| < \infty$), it is well known that there is a unique exponentially bounded evolution family $U_B(t,s)$, $t \geqslant s \geqslant 0$, on X such that

$$U_B(t,s)x = U(t,s)x + \int_s^t U(t,\tau)B(\tau)U_B(\tau,s)x\,d\tau \tag{1.4}$$

for all $t \geqslant s$ and $x \in X$ (cf. [6, Corollary 6.9.18], concerning unexplained concepts and notation in this paper we also refer to this monograph and [13]). Now consider the non-autonomous system with delay

$$\begin{aligned} &\frac{dx(t)}{dt} = A(t)x(t) + B(t)x\bigl(t - r(t)\bigr), \quad t > 0, \\ &x(\theta) = \xi(\theta), \quad \theta \in [-r, 0], \end{aligned} \tag{1.5}$$

where the delays $r(\cdot) \in C([0,\infty); [0,r])$ and the history function $\xi(\cdot) \in C([-r,0]; X)$ which is a Banach space equipped with the sup-norm (i.e., $\|\xi(\cdot)\| := \sup_{\theta \in [-r,0]} \|\xi(\theta)\|$). We can transform (1.5) into

$$\frac{dx(t)}{dt} = \bigl(A(t) + B(t)\bigr)x(t) + B(t)\bigl(x\bigl(t - r(t)\bigr) - x(t)\bigr), \quad t > 0.$$

Similarly, as in the autonomous situation, there exists a unique continuous X-valued function $x(t)$, $t \geqslant -r$, such that

$$\begin{cases} x(t) = U_B(t,0)\xi(0) + \int_0^t U_B(t,\tau)B(\tau)(x(\tau - r(\tau)) - x(\tau))\,d\tau, & t > 0, \\ x(t) = \xi(t), & t \in [-r,0]. \end{cases} \tag{1.6}$$

In this paper we try to find sufficient conditions on $U(t,s)$ and $B(t)$ to ensure that system (1.5) is robust with respect to small (and often inevitable) delays for exponential stability, which is defined below.

Definition. The system (1.5) is called to be robust with respect to small time-varying delays if there exist constants $r_0 > 0$, $M \geqslant 1$, $\omega > 0$ such that for any delays $r(t) \in C([0,\infty);[0,r_0])$, $t \geqslant 0$, and any history functions $\xi(\cdot) \in C([-r_0,0],X)$, the solution $x_t(\theta) := x(t+\theta)$, $t \geqslant 0$, $\theta \in [-r_0,0]$, of the system (1.6) subject to $\xi(\cdot)$ satisfies

$$\left\|x_t(\cdot)\right\| \leqslant Me^{-\omega t}\left\|\xi(\cdot)\right\|, \quad t \geqslant 0.$$

This paper is organized as follows. In Section 2 sufficient conditions on $U(t,s)$ and $B(t)$ are presented to ensure the robust stability with respect to small time-varying delays. Finally, in Section 3, examples are given to illustrate that the conditions in Section 2 are satisfied for a large class of the non-autonomous parabolic systems.

2. Main results

Robustness of stability with respect to small delays, e.g., motivated by feedback systems in control theory, is of great theoretical and practical important, but this property does not hold for many systems. In this section we derive sufficient conditions for system (1.6) to be robust with respect to small delays for exponential stability.

Theorem 1. *Assume that $\{U(t,s)\}_{t \geqslant s \geqslant 0}$ is uniformly norm continuous for (t,s) satisfying $0 < \varepsilon' < s < t - \varepsilon'$, where ε' is an arbitrary small positive number, $B(\cdot) \in C([0,\infty),\mathcal{L}(X))$ and $\|B\|_\infty := \sup_{0 \leqslant t < \infty}\|B(t)\| < \infty$. Then $U_B(t,s)$ is uniformly norm continuous for $0 < \varepsilon' < s < t - \varepsilon'$.*

Proof. For $t_0 > s$, without loss of generality, let $h, \delta \in (0,t_0)$. For (1.4) we have

$$\begin{aligned} &U_B(t_0+h,s)x - U_B(t_0,s)x \\ &\quad = U(t_0+h,s)x - U(t_0,s)x + \int_{t_0}^{t_0+h} U(t_0+h,\tau)B(\tau)U_B(\tau,s)x\,d\tau \\ &\qquad + \int_0^{t_0-\delta} \big(U(t_0+h,\tau) - U(t_0,\tau)\big)B(\tau)U_B(\tau,s)x\,d\tau \end{aligned}$$

$$+\int_{t_0-\delta}^{t_0} \big(U(t_0+h,\tau)-U(t_0,\tau)\big)B(\tau)U_B(\tau,s)x\,d\tau.$$

For any $\varepsilon>0$, let

$$\delta_0 \in \left[0, \min\left\{t_0, \frac{1}{10M^2\|B\|_\infty}\varepsilon\right\}\right]$$

(where $M := \sup_{0\leqslant s\leqslant t\leqslant 2t_0}\{\|U(t,s)\|+\|U_B(t,s)\|\}$). We can deduce that

$$\begin{aligned}
&\left\|\int_{t_0-\delta_0}^{t_0} \big(U(t_0+h,\tau)-U(t_0,\tau)\big)B(\tau)U_B(\tau,s)x\,d\tau\right\| \\
&\quad\leqslant \int_{t_0-\delta_0}^{t_0} \big\|U(t_0+h,\tau)-U(t_0,\tau)\big\|\big\|B(\tau)\big\|\big\|U_B(\tau,s)\big\|\|x\|\,d\tau \\
&\quad\leqslant 2M^2\|B\|_\infty\delta_0\|x\| \leqslant \frac{1}{5}\varepsilon\|x\|.
\end{aligned}\tag{2.1}$$

Similarly, we have

$$\left\|\int_0^{\delta_0}\big(U(t_0+h,\tau)-U(t_0,\tau)\big)B(\tau)U_B(\tau,s)x\,d\tau\right\| \leqslant \frac{1}{5}\varepsilon\|x\|.\tag{2.2}$$

Next, let

$$h_0 \in \left(0, \frac{1}{5M^2\|B\|_\infty}\varepsilon\right].$$

Then for $h\in(0,h_0]$

$$\left\|\int_{t_0}^{t_0+h} U(t_0+h,\tau)B(\tau)U_B(\tau,s)x\,d\tau\right\| \leqslant M^2\|B\|_\infty\|x\|h \leqslant \frac{1}{5}\varepsilon\|x\|.\tag{2.3}$$

Since $U(t,s)$ is norm continuous, there exists $h_1<h_0$ such that for any $h\in[0,h_1]$

$$\big\|U(t_0+h,\tau)-U(t_0,\tau)\big\| \leqslant \frac{1}{5M^2t_0\|B\|_\infty}\varepsilon$$

uniformly for $0<\delta_0<\tau<t_0-\delta_0$. Hence,

$$\begin{aligned}
&\left\|\int_{\delta_0}^{t_0-\delta_0} \big(U(t_0+h,\tau)-U(t_0,\tau)\big)B(\tau)U_B(\tau,s)x\,d\tau\right\| \\
&\quad\leqslant \int_{\delta_0}^{t_0-\delta_0} \big\|U(t_0+h,\tau)-U(t_0,\tau)\big\|\big\|B(\tau)\big\|\big\|U_B(\tau,s)\big\|\|x\|\,d\tau \leqslant \frac{1}{5}\varepsilon\|x\|.
\end{aligned}\tag{2.4}$$

From (2.1)–(2.4), it follows that for any $\varepsilon > 0$ there is $h_1 > 0$ such that

$$\left\| U_B(t_0 + h, \tau) - U_B(t_0, \tau) \right\| \leqslant \varepsilon, \quad h \in [0, h_1],$$

uniformly for $0 < \varepsilon' < \tau < t_0 - \varepsilon'$. The proof has been completed. □

We further prove the robust stability with respect to small delays under the assumptions of Theorem 1.

Theorem 2. *Assume that* $\{U(t,s)\}_{t \geqslant s \geqslant 0}$ *is uniformly norm continuous for* (t,s) *satisfying* $0 < \varepsilon' < s < t - \varepsilon'$, *where* ε' *is an arbitrary small positive number,* $B(\cdot) \in C([0,\infty), \mathcal{L}(X))$ *and* $\|B\|_\infty := \sup_{0 \leqslant t \leqslant \infty} \|B(t)\| < \infty$. *Then system* (1.5) *is robust with respect to small delays for exponential stability.*

Proof. Supposing that $U_B(t,s)$ is exponential stable, i.e., there exist constants $M \geqslant 1$, $\omega > 0$ such that $\|U_B(t,s)\| \leqslant Me^{-\omega(t-s)}$, $t \geqslant s \geqslant 0$. For any $\omega_1 \in (0, \omega)$, let $r_0 > 0$ be such that

$$r_0 M e^{\omega r_0} \left(1 + \|B\|_\infty \left(2 + \frac{1}{\omega - \omega_1} \right) \right) e^{M\|B\|_\infty (1 + e^{\omega r_0}) r_0} < 1.$$

Let $r_1 = \frac{1}{2} r_0$ and $r(t) \in [0, r_1]$. If $x(t)$, $t \geqslant -r$ (for any $r > 0$), is the solution of system (1.6), let $x_t(\theta) := x(t+\theta)$, $-r \leqslant \theta \leqslant 0$. Then

$$x_t(\theta) = \begin{cases} U_B(t+\theta, 0)\xi(0) \\ \quad + \int_0^{t+\theta} U_B(t+\theta, \tau) B(\tau)(x(\tau - r(\tau)) - x(\tau))\, d\tau, & t + \theta > 0, \\ \xi(t+\theta), \quad t+\theta \in [-r, 0]. \end{cases} \tag{2.5}$$

We deduce from (2.5) that for any $t \in [0, r_0]$

$$\begin{aligned} \left\| x_t(\cdot) \right\| &:= \max_{\theta \in [-r_0, 0]} \left\| x(t+\theta) \right\| \leqslant M \left\| \xi(0) \right\| + \int_0^t M \|B\|_\infty \left\| x\big(\tau - r(\tau)\big) - x(\tau) \right\| d\tau \\ &\leqslant M \left\| \xi(\cdot) \right\| + \int_0^t 2M \|B\|_\infty \left\| x_\tau(\cdot) \right\| d\tau. \end{aligned}$$

Hence, for $t \in [0, r_0]$ we have

$$\left\| x_t(\cdot) \right\| \leqslant M \left\| \xi(\cdot) \right\| e^{\int_0^t 2M\|B\|_\infty \, d\tau} \leqslant M_1 \left\| \xi(\cdot) \right\| \tag{2.6}$$

by the Gronwall's inequality (here $M_1 := Me^{2M\|B\|_\infty r_0}$).

On the other hand, we have

$$\begin{aligned} \left\| x_t(\cdot) \right\| &\leqslant M e^{-\omega(t - r_1)} \left\| \xi(\cdot) \right\| \\ &\quad + \int_0^t M e^{-\omega(t - \tau - r_1)} \left\| B(\tau) \right\| \left\| x\big(\tau - r(\tau)\big) - x(\tau) \right\| d\tau \end{aligned} \tag{2.7}$$

for all $t \geqslant 0$ and $r(t) \in [0, r_1]$. From (2.6) we obtain

$$\|x(t - r(t)) - x(t)\| \leqslant 2M_1 \|\xi(\cdot)\| \tag{2.8}$$

for all $t \in [0, r_0]$ and $r(t) \in [0, r_1]$. It follows from Theorem 1 that $U_B(t, s)$ is norm continuous, that is, for $\varepsilon = r_1$ there exists $r_2 = r_2(\varepsilon) \in (0, r_1)$ such that

$$\|U_B(r_1 - h, \tau) - U_B(r_1, \tau)\| < r_1 \tag{2.9}$$

uniformly for $h \in [0, r_2]$ and $0 < \varepsilon' \leqslant \tau < r_1 - \varepsilon'$. Furthermore, notice that $\|U_B(t, s)\| \leqslant Me^{-\omega(t-s)}$, $t \geqslant 0$, so we can deduce for any $\varepsilon > 0$ there exists $\delta > 0$ (independent of t_1 and t_2) such that

$$\|U_B(t_1, \tau) - U_B(t_2, \tau)\| < \varepsilon, \tag{2.10}$$

uniformly for $|t_1 - t_2| < \delta$ and $0 < \varepsilon' < \min\{t_1 - \varepsilon', t_2 - \varepsilon'\}$. We now try to prove that

$$\|x(t - r(t)) - x(t)\| \leqslant M_2 e^{-\omega_1 t} \|\xi(\cdot)\| \tag{2.11}$$

for all $t \geqslant 0$ and $r(t) \in [0, r_2]$. Indeed, for all $t \in [0, r_0]$ we have

$$\|x(t - r(t)) - x(t)\| \leqslant M_2 e^{-\omega_1 t} \|\xi(\cdot)\|$$

where $M_2 = 2M_1 e^{\omega r_0}$ and $\omega_1 \in [0, \omega]$. By induction, we assume that

$$\|x(t - r(t)) - x(t)\| \leqslant M_2 e^{-\omega_1 t} \|\xi(\cdot)\| \tag{2.12}$$

for $t \in [0, nr_0]$ and $r(t) \in [0, r_2]$. Using $\|U_B(t, s)\| \leqslant Me^{-\omega(t-s)}$, (2.5), (2.9), (2.10), and (2.12), for $t \in [nr_0, (n+1)r_0)$ we have

$$\begin{aligned}
\|x(t - r(t)) - x(t)\| &\leqslant \|U_B(t - r(t), t - r_1) - U_B(t, t - r_1)\| \|U_B(t - r_1, 0)\xi(0)\| \\
&\quad + \left(\int_{nr_0 - r_1}^{nr_0} + \int_{nr_0}^{t - r(t)}\right) Me^{-\omega(t - r(t) - \tau)} \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\quad + \left(\int_{nr_0 - r_1}^{nr_0} + \int_{nr_0}^{t}\right) Me^{-\omega(t - \tau)} \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\quad + \int_0^{nr_0 - r_1} \|U_B(t - r(t), t - r_1) - U_B(t, t - r_1)\| \|U_B(t - r_1, \tau)\| \\
&\quad \times \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\leqslant r_1 M e^{-\omega(t - r_1)} \|\xi(\cdot)\| + \int_{nr_0 - r_1}^{nr_0} M(e^{\omega r_1} + 1) e^{-\omega(t - \tau)} \|B(\tau)\| \\
&\quad \times \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\quad + \int_{nr_0 - r_1}^{t} M(e^{\omega r_1} + 1) e^{-\omega(t - \tau)} \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau
\end{aligned}$$

$$
\begin{aligned}
&+ r_1 M \int_0^{nr_0-r_1} e^{-\omega(t-r_1-\tau)} \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\leqslant r_1 M e^{-\omega(t-r_1)} \|\xi(\cdot)\| + \int_{nr_0-r_1}^{nr_0} M(e^{\omega r_1}+1) e^{-\omega(t-\tau)} \|B(\tau)\| M_2 e^{-\omega_1 \tau} \|\xi(\cdot)\| \, d\tau \\
&+ \int_{nr_0}^{t} M(e^{\omega r_1}+1) e^{-\omega(t-\tau)} \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&+ r_1 M \int_0^{nr_0-r_1} e^{-\omega(t-r_1-\tau)} M_2 e^{-\omega_1 \tau} \|B(\tau)\| \|\xi(\cdot)\| \, d\tau.
\end{aligned}
$$

Therefore, we obtain

$$
\begin{aligned}
e^{\omega_1 t} \|x(t - r(t)) - x(t)\| &\leqslant r_1 M e^{\omega r_1} \|\xi(\cdot)\| + M M_2 (e^{\omega r_1}+1) \|B\|_\infty r_1 \|\xi(\cdot)\| \\
&\quad + r M M_2 \|B\|_\infty e^{\omega r_1} \frac{1}{\omega - \omega_1} \|\xi(\cdot)\| \\
&\quad + \int_{nr_0}^{t} M(e^{\omega r_1}+1) \|B\|_\infty e^{\omega_1 \tau} \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\leqslant r_1 M M_2 e^{\omega r_1} \left(1 + \|B\|_\infty \left(2 + \frac{1}{\omega - \omega_1}\right)\right) \|\xi(\cdot)\| \\
&\quad + \int_{nr_0}^{t} M \|B\|_\infty (1 + e^{\omega r_1}) e^{\omega_1 \tau} \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau.
\end{aligned}
$$

Hence, by the Gronwall's inequality we deduce that

$$
\begin{aligned}
e^{\omega_1 t} \|x(t - r(t)) - x(t)\| &\leqslant r_1 M M_2 e^{\omega r_1} \left(1 + \|B\|_\infty \left(2 + \frac{1}{\omega - \omega_1}\right)\right) \\
&\quad \times e^{M\|B\|_\infty (1+e^{\omega r_1})(t - nr_0)} \|\xi(\cdot)\| \\
&\leqslant M_2 \|\xi(\cdot)\|, \qquad (2.13)
\end{aligned}
$$

thus, we obtain (2.11).

From (2.7) and (2.11) we can assert that

$$
\begin{aligned}
\|x_t(\cdot)\| &\leqslant M e^{-\omega(t-r_1)} \|\xi(\cdot)\| + \int_0^t M e^{-\omega(t-\tau-r_1)} \|B(\tau)\| \|x(\tau - r(\tau)) - x(\tau)\| \, d\tau \\
&\leqslant M e^{-\omega(t-r_1)} \|\xi(\cdot)\| + \int_0^t M e^{-\omega(t-\tau-r_1)} \|B(\tau)\| M_2 e^{-\omega_1 \tau} \|\xi(\cdot)\| \, d\tau
\end{aligned}
$$

$$\leqslant Me^{\omega r_1}\left(1 + M_2\|B\|_\infty \frac{1}{\omega - \omega_1}\right)e^{-\omega_1 t}\left\|\xi(\cdot)\right\|.$$

The proof has been completed. □

3. Examples

It is well known that many infinite-dimensional autonomous systems are not robust with respect to small delays (see, e.g., the Introduction). Notice that $U(t,s) = e^{(t-s)A}$ for $t \geqslant s$ is an exponentially bounded evolution family if A is the infinitesimal generator of C_0-semigroup e^{tA}, so there are many non-autonomous systems which are exponentially stabilized by feedback and are destabilized by arbitrary small delays in the feedback loop.

Now we show that the conditions in Theorem 2 are satisfied for a large class of the non-autonomous systems. Consider the abstract Cauchy problem

$$\begin{cases} \frac{d}{dt}x(t) + A(t)x(t) = 0, & t \geqslant s, \\ x(s) = x, \end{cases}$$

where $A(t)$, $t > 0$, satisfies the following assumptions:

(i) The resolvent set $\rho(A(t)) \supset \Sigma = \{\lambda \in C\colon |\arg\lambda| \geqslant \theta_0\}$, $\theta_0 \in (0, \pi/2)$, and $\|(\lambda - A(t))^{-1}\| \leqslant M/(1+\lambda)$, $\lambda \in \Sigma$, $t \geqslant 0$, $M \geqslant 0$;

(ii) $\|A(t)(\lambda - A(t))^{-1}(A(t)^{-1} - A(s)^{-1})\| \leqslant L|t-s|^{\mu}|\lambda|^{-\nu}$, $\lambda \in \Sigma$, $t, s \geqslant 0$, $\mu, \nu \in (0,1]$, $\mu + \nu > 1$, $L > 0$.

The assumption, introduced by Acquistapace and Terreni [1], implies that there is a unique exponential bounded evolution family $U(t,s)$ and the mapping $(t,s) \to U(t,s) \in \mathcal{L}(X)$ is continuous for $t > s$; these results follow from, e.g., [1] and [16]. So, if this non-autonomous parabolic system is exponentially stabilized by the uniform bounded linear feedback operators $B(t)$, $t \geqslant 0$, i.e., the solution of the non-delay Cauchy problem $(d/dt)x(t) = A(t)x(t) + B(t)x(t)$ is uniformly exponential stable, using Theorem 2, then there exists $r_0 > 0$ such that the solution of the delay Cauchy problem $(d/dt)x(t) = A(t)x(t) + B(t)x(t - r(t))$ (for any $r(\cdot) \in C([0, +\infty), [0, r_0])$) is uniformly exponential stable; this means that this non-autonomous system is robust with respect to small time-varying delays.

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