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A new system of generalized mixed quasi-variational inclusions with (H,η)-monotone operators ☆

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Abstract

In this paper, we introduce and study a new system of generalized mixed quasi-variational inclusions with (H,η)-monotone operators. We prove the convergence of a new iterative algorithm for this system of generalized mixed quasi-variational inclusions. The results in this paper extend and improve some known results in the literature.

Keywords: System of generalized mixed quasi-variational inclusions; (H,η)-monotone operator; Set-valued mapping; Existence; Iterative algorithm

1. Introduction

Variational inclusion problems are among the most interesting and intensively studied classes of mathematical problems and have wide applications in the fields of optimization and control, economics and transportation equilibrium, engineering science. For the past years, many existence results and iterative algorithms for various variational inequality and variational inclusion problems have been studied. For details, please see [1–45] and the references therein.

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Recently, some new and interesting problems, which are called to be system of variational inequality problems were introduced and studied. Pang [27], Cohen and Chaplais [28], Bianchi [29], and Ansari and Yao [15] considered a system of scalar variational inequalities and Pang showed that the traffic equilibrium problem, the spatial equilibrium problem, the Nash equilibrium, and the general equilibrium programming problem can be modeled as a system of variational inequalities. Ansari et al. [30] introduced and studied a system of vector equilibrium problems and a system of vector variational inequalities by a fixed point theorem. Allevi et al. [31] considered a system of generalized vector variational inequalities and established some existence results with relative pseudomonotonicity. Kassay and Kolumbán [16] introduced a system of variational inequalities and proved an existence theorem by the Ky Fan lemma. Kassay et al. [17] studied Minty and Stampacchia variational inequality systems with the help of the Kakutani–Fan–Glicksberg fixed point theorem. Peng and Yang [18,19] introduced a system of quasi-variational inequality problems and proved its existence theorem by maximal element theorems. Verma [20–24] introduced and studied some systems of variational inequalities and developed some iterative algorithms for approximating the solutions of system of variational inequalities in Hilbert spaces. Kim and Kim [25] introduced a new system of generalized nonlinear quasi-variational inequalities and obtained some existence and uniqueness results of solution for this system of generalized nonlinear quasi-variational inequalities in Hilbert spaces. Cho et al. [26] introduced and studied a new system of nonlinear variational inequalities in Hilbert spaces. They proved some existence and uniqueness theorems of solutions for the system of nonlinear variational inequalities.

As generalizations of system of variational inequalities, Agarwal et al. [32] introduced a system of generalized nonlinear mixed quasi-variational inclusions and investigated the sensitivity analysis of solutions for this system of generalized nonlinear mixed quasi-variational inclusions in Hilbert spaces. Kazmi and Bhat [33] introduced a system of nonlinear variational-like inclusions and gave an iterative algorithm for finding its approximate solution. Fang and Huang [34], Verma [35], and Fang et al. [36] introduced and studied a new system of variational inclusions involving H-monotone operators, A-monotone operators, and (H,η)-monotone operators, respectively. Yan et al. [37] introduced and studied a system of set-valued variational inclusions which is more general than the model in [34].

Inspired and motivated by the results in [15–37], the purpose of this paper is to introduce and study a new system of generalized mixed quasi-variational inclusions with (H,η)-monotone operators, which contains the mathematical models in [20–26,34,36,37] as special cases. By use the resolvent technique for the (H,η)-monotone operators, we prove the existence of solutions for this system of generalized mixed quasi-variational inclusions. We also prove the convergence of a new iterative algorithm approximating the solution for this system of generalized mixed quasi-variational inclusions. The result in this paper extends and improves some results in [20–26,34,36,37].

2. Preliminaries

We suppose that $\mathcal{H}$ is a real Hilbert space with norm and inner product denoted by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$, respectively. Let $CB(\mathcal{H})$ denote the families of all nonempty closed bounded subsets of $\mathcal{H}$ and $\tilde{D}(\cdot,\cdot)$ denote the Hausdorff metric on $CB(\mathcal{H})$ defined by

$$\tilde{D}(A,B)=\max\Big\{\sup_{a\in A} d(a,B),\sup_{b\in B} d(A,b)\Big\},\quad \forall A,B\in CB(\mathcal{H}),$$

where $d(a,B)=\inf_{b\in B}\|a-b\|$, $d(A,b)=\inf_{a\in A}\|a-b\|$.

Definition 2.1. [36,38] Let $\eta : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ and $H : \mathcal{H} \to \mathcal{H}$ be two single-valued operators and $M : \mathcal{H} \to 2^{\mathcal{H}}$ be a set-valued operator. M is said to be

(i) η-monotone if

$$\big\langle x - y, \eta(u, v)\big\rangle \geqslant 0, \quad \forall u, v \in \mathcal{H},\ x \in Mu,\ y \in Mv.$$

(ii) (H, η)-monotone if M is η-monotone and $(H + \lambda M)(\mathcal{H}) = \mathcal{H}$ for all $\lambda > 0$.

Remark 2.1. If $\eta(u, v) = u - v$, then the definition of η-monotonicity is that of monotonicity and the definition of (H, η)-monotonicity becomes that of H-monotonicity in [1]. It is easy to know that if $H = I$ (the identity map on $\mathcal{H}$), then the definition of (I, η)-monotone operators is that of maximal η-monotone operators and the definition of I-monotone operators is that of maximal monotone operators. Hence, the class of (H, η)-monotone operators provides a unifying frameworks for classes of maximal monotone operators, maximal η-monotone operators, H-monotone operators (for more details, please see [1,34,36–38]).

Definition 2.2. [1,38] Let $H, g : \mathcal{H} \to \mathcal{H}$, $\eta : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be three single-valued operators. g is said to be

(i) monotone if

$$\langle gu - gv, u - v\rangle \geqslant 0, \quad \forall u, v \in \mathcal{H};$$

(ii) strictly monotone if g is monotone and

$$\langle gu - gv, u - v\rangle = 0 \quad \text{if and only if} \quad u = v;$$

(iii) strongly monotone if there exists a constant $r > 0$ such that

$$\langle gu - gv, u - v\rangle \geqslant r\|u - v\|^2, \quad \forall u, v \in \mathcal{H};$$

(iv) Lipschitz continuous if there exists a constant $s > 0$ such that

$$\big\|g(u) - g(v)\big\| \leqslant s\|u - v\|, \quad \forall u, v \in \mathcal{H};$$

(v) strongly monotone with respect to H if there exists a constant $\gamma > 0$ such that

$$\langle gu - gv, Hu - Hv\rangle \geqslant \gamma\|u - v\|^2, \quad u, v \in \mathcal{H};$$

(vi) η-monotone if

$$\big\langle gu - gv, \eta(u, v)\big\rangle \geqslant 0, \quad \forall u, v \in \mathcal{H};$$

(vii) strictly η-monotone if g is η-monotone and

$$\big\langle gu - gv, \eta(u, v)\big\rangle = 0 \quad \text{if and only if} \quad u = v;$$

(viii) strongly η-monotone if there exists a constant $r > 0$ such that

$$\big\langle gu - gv, \eta(u, v)\big\rangle \geqslant r\|u - v\|^2, \quad \forall u, v \in \mathcal{H}.$$

Remark 2.2. Example 1.1 in [1] shows that the strongly monotonicity with respect to H of g is a generalization of the strongly monotonicity of g.

Definition 2.3. [38] Let $\eta:\mathcal{H}\times\mathcal{H}\to\mathcal{H}$ be a single-valued operator, then for all $u,v\in\mathcal{H}$, $\eta(\cdot,\cdot)$ is said to be

(i) monotone if

$$\langle\eta(u,v),u-v\rangle\geqslant 0;$$

(ii) strongly monotone, if there exists a constant $\delta>0$ such that

$$\langle\eta(u,v),u-v\rangle\geqslant\delta\|u-v\|;$$

(iii) Lipschitz continuous, if there exists a constant $\tau>0$ such that

$$\eta(u,v)\leqslant\tau\|u-v\|.$$

Definition 2.4. [39] Let $M:\mathcal{H}\to CB(\mathcal{H})$ be a set-valued mapping and $N:\mathcal{H}\times\mathcal{H}\to\mathcal{H}$ be a single-valued mapping.

(i) M is said to be $\tilde{D}$-Lipschitz continuous if there exists a constant $\xi>0$ such that

$$\tilde{D}\big(M(u),M(v)\big)\leqslant\xi\|u-v\|,\quad\forall u,v\in\mathcal{H}.$$

(ii) $N(\cdot,\cdot)$ is said to be Lipschitz continuous in the first argument if there exists a constant $\xi>0$ such that

$$\big\|N(u,\cdot)-N(v,\cdot)\big\|\leqslant\xi\|u-v\|,\quad\forall u,v\in\mathcal{H}.$$

(iii) $N(\cdot,\cdot)$ is said to be monotone in the first argument if

$$\langle N(u,\cdot)-N(v,\cdot),u-v\rangle\geqslant 0,\quad\forall u,v\in\mathcal{H}.$$

(iv) $N(\cdot,\cdot)$ is said to be strongly monotone in the first argument if there exists a constant $\alpha>0$ such that

$$\langle N(u,\cdot)-N(v,\cdot),u-v\rangle\geqslant\alpha\|u-v\|^2,\quad\forall u,v\in\mathcal{H}.$$

Definition 2.5. Let $g:\mathcal{H}\to\mathcal{H}$ and $N:\mathcal{H}\times\mathcal{H}\to\mathcal{H}$ be two single-valued mappings.

(i) $N(\cdot,\cdot)$ is said to be monotone with respect to g in the first argument if

$$\langle N(u,\cdot)-N(v,\cdot),gu-gv\rangle\geqslant 0,\quad\forall u,v\in\mathcal{H}.$$

(ii) $N(\cdot,\cdot)$ is said to be strongly monotone with respect to g in the first argument if there exists a constant $\beta>0$ such that

$$\langle N(u,\cdot)-N(v,\cdot),gu-gv\rangle\geqslant\beta\|u-v\|^2,\quad\forall u,v\in\mathcal{H}.$$

In a similar way, we can define the Lipschitz continuity and the strong monotonicity (monotonicity) of $N(\cdot,\cdot)$ with respect to g in the second argument.

Definition 2.6. [36] Let $\eta:\mathcal{H}\times\mathcal{H}\to\mathcal{H}$ be a single-valued operator, $H:\mathcal{H}\to\mathcal{H}$ be a strongly η-monotone operator and $M:\mathcal{H}\to 2^{\mathcal{H}}$ be an (H,η)-monotone operator. Then the resolvent operator $R^{H,\eta}_{M,\lambda}:\mathcal{H}\to\mathcal{H}$ is defined by

$$R^{H,\eta}_{M,\lambda}(x)=(H+\lambda M)^{-1}(x),\quad\forall x\in\mathcal{H}.$$

Lemma 2.1. [36] *Let $\eta:\mathcal{H}\times\mathcal{H}\to\mathcal{H}$ be a single-valued Lipschitz continuous operator with constant τ, $H:\mathcal{H}\to\mathcal{H}$ be a strongly η-monotone operator with constant $\gamma>0$ and $M:\mathcal{H}\to 2^{\mathcal{H}}$ be an (H,η)-monotone operator. Then, the resolvent operator $R_{M,\lambda}^{H,\eta}:\mathcal{H}\to\mathcal{H}$ is Lipschitz continuous with constant τ/γ, i.e.,*

$$\left\| R_{M,\lambda}^{\mathcal{H},\eta}(x)-R_{M,\lambda}^{\mathcal{H},\eta}(y)\right\| \leqslant \frac{\tau}{\gamma}\|x-y\|,\quad \forall x,y\in H.$$

3. A system of generalized mixed quasi-variational inclusions and an iterative algorithm

In this section, we will introduce a new system of generalized mixed quasi-variational inclusions with (H,η)-monotone operators and construct a new iterative algorithm for solving this system of generalized mixed quasi-variational inclusions in Hilbert spaces. In what follows, unless other specified, we always suppose that $\mathcal{H}_1$ and $\mathcal{H}_2$ are two Hilbert spaces, $H_1, g_1: H_1\to H_1$, $H_2, g_2:\mathcal{H}_2\to\mathcal{H}_2$, $\eta_1:\mathcal{H}_1\times\mathcal{H}_1\to\mathcal{H}_1$, $\eta_2:\mathcal{H}_2\times\mathcal{H}_2\to\mathcal{H}_2$, $F,P:\mathcal{H}_1\times\mathcal{H}_2\to\mathcal{H}_1$, $G,Q:\mathcal{H}_1\times\mathcal{H}_2\to\mathcal{H}_2$ are all single-valued mappings and $A,C:\mathcal{H}_1\to CB(\mathcal{H}_1)$, $B,D:\mathcal{H}_2\to CB(\mathcal{H}_2)$ are four set-valued mappings. Let $M:\mathcal{H}_1\to 2^{\mathcal{H}_1}$ be an (H_1,η_1)-monotone operator and $N:\mathcal{H}_2\to 2^{\mathcal{H}_2}$ be an (H_2,η_2)-monotone operator. We consider the following problem of finding (x,y,u,v,w,z) such that $(x,y)\in\mathcal{H}_1\times\mathcal{H}_2$, $u\in A(x)$, $v\in B(y)$, $w\in C(x)$, $z\in D(y)$, and

$$\begin{cases} 0\in F(x,y)+P(u,v)+M(g_1(x)),\\ 0\in G(x,y)+Q(w,z)+N(g_2(y)). \end{cases}\tag{3.1}$$

The problem (3.1) is called a system of generalized mixed quasi-variational inclusions.

Below are some special cases of problem (3.1).

(i) If $g_1\equiv I_1$ (the identity map on $\mathcal{H}_1$), $g_2\equiv I_2$ (the identity map on $\mathcal{H}_2$), $P\equiv 0$ and $Q\equiv 0$, then problem (3.1) reduces to the system of variational inclusions with (H,η)-monotone operators, which is to find $(x,y)\in\mathcal{H}_1\times\mathcal{H}_2$ such that

$$\begin{cases} 0\in F(x,y)+M(x),\\ 0\in G(x,y)+N(y). \end{cases}\tag{3.2}$$

Problem (3.2) was introduced and studied by Fang et al. [36]. From Remark 2.1, it is easy know that problem (3.2) contains the system of variational inclusions with H-monotone operators in [34] as a special case. If M and N are A-monotone and B-monotone, respectively, then problem (3.2) becomes the problem in [35]. It is easy to know that both the system of nonlinear variational-like inequalities and the system of nonlinear variational inequalities in [36] are special cases of problem (3.2).

(ii) If $g_1\equiv I_1$, $g_2\equiv I_2$, $F\equiv 0$ and $G\equiv 0$, then problem (3.1) reduces to the system of set-valued variational inclusions with (H,η)-monotone operators, which is to find (x,y,u,v,w,z) such that $(x,y)\in\mathcal{H}_1\times\mathcal{H}_2$, $u\in A(x)$, $v\in B(y)$, $w\in C(x)$, $z\in D(y)$ and

$$\begin{cases} 0\in P(u,v)+M(x),\\ 0\in Q(w,z)+N(y). \end{cases}\tag{3.3}$$

If $\eta_1(x_1,y_1)=x_1-y_1$ for all $x_1,y_1\in\mathcal{H}_1$, $\eta_2(x_2,y_2)=x_2-y_2$ for all $x_2,y_2\in\mathcal{H}_2$, $A\equiv I_1$ and $D\equiv I_2$, then problem (3.3) becomes the following system of set-valued variational inclusions with H-monotone operators, which is to find (x, y, u, w) such that $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$, $v \in B(y)$, $w \in C(x)$ and

$$\begin{cases} 0 \in P(x, v) + M(x), \\ 0 \in Q(w, y) + N(y). \end{cases} \tag{3.4}$$

Problem (3.4) was introduced and studied by Yan et al. [37]. If $B \equiv I_2$ and $C \equiv I_1$, then problem (3.4) becomes the system of variational inclusions with H-monotone operators considered by Fang and Huang [34]. From [37], we know that the mathematical models of system of variational inequalities in [21–26] are special cases of problem (3.4).

Lemma 3.1. *Let $\eta_1 : \mathcal{H}_1 \times \mathcal{H}_1 \to \mathcal{H}_1$, $\eta_2 : \mathcal{H}_2 \times \mathcal{H}_2 \to \mathcal{H}_2$ be two single-valued operators, $H_1 : \mathcal{H}_1 \to \mathcal{H}_1$ be a strictly η_1-monotone operator and $H_2 : \mathcal{H}_2 \to \mathcal{H}_2$ be a strictly η_2-monotone operator and $M : \mathcal{H}_1 \to 2^{\mathcal{H}_1}$ be an (H_1, η_1)-monotone operator, $N : \mathcal{H}_2 \to 2^{\mathcal{H}_2}$ be an (H_2, η_2)-monotone operator. Then (x, y, u, v, w, z) with $(x, y) \in \mathcal{H}_1 \times \mathcal{H}_2$, $u \in A(x)$, $v \in B(y)$, $w \in C(x)$, $z \in D(y)$ is a solution of the problem* (3.1) *if and only if*

$$g_1(x) = R_{M,\lambda}^{H_1,\eta_1}\big(H_1\big(g_1(x)\big) - \lambda F(x, y) - \lambda P(u, v)\big),$$
$$g_2(y) = R_{N,\rho}^{H_2,\eta_2}\big(H_2\big(g_2(y)\big) - \rho G(x, y) - \rho Q(w, z)\big),$$

where $R_{M,\lambda}^{H_1,\eta_1} = (H_1 + \lambda M)^{-1}$, $R_{N,\rho}^{H_2,\eta_2} = (H_2 + \rho N)^{-1}$, $\lambda > 0$ and $\rho > 0$ are constants.

Proof. The fact directly follows from Definition 2.6. □

For any given $x_0 \in \mathcal{H}_1$, $y_0 \in \mathcal{H}_2$, take $u_0 \in A(x_0)$, $v_0 \in B(y_0)$, $w_0 \in C(x_0)$ and $z_0 \in D(y_0)$. It follows from Lemma 3.1 that there exist $x_1 \in \mathcal{H}_1$ and $y_1 \in \mathcal{H}_2$, such that

$$x_1 = x_0 - g_1(x_0) + R_{M,\lambda}^{H_1,\eta_1}\big(H_1\big(g_1(x_0)\big) - \lambda F(x_0, y_0) - \lambda P(u_0, v_0)\big),$$
$$y_1 = y_0 - g_2(y_0) + R_{N,\rho}^{H_2,\eta_2}\big(H_2\big(g_2(y_0)\big) - \rho G(x_0, y_0) - \rho Q(w_0, z_0)\big).$$

Since $u_0 \in A(x_0)$, $v_0 \in B(y_0)$, $w_0 \in C(x_0)$ and $z_0 \in D(y_0)$, by Nadler's theorem [46], there exist $u_1 \in A(x_1)$, $v_1 \in B(y_1)$, $w_1 \in C(x_1)$ and $z_1 \in D(y_1)$, such that

$$\|u_1 - u_0\| \leqslant (1 + 1)\tilde{D}\big(A(x_1), A(x_0)\big),$$
$$\|v_1 - v_0\| \leqslant (1 + 1)\tilde{D}\big(B(y_1), B(y_0)\big),$$
$$\|w_1 - w_0\| \leqslant (1 + 1)\tilde{D}\big(C(x_1), C(x_0)\big),$$
$$\|z_1 - z_0\| \leqslant (1 + 1)\tilde{D}\big(D(y_1), D(y_0)\big).$$

It follows from Lemma 3.1 that there exist $x_2 \in \mathcal{H}_1$ and $y_2 \in \mathcal{H}_2$, such that

$$x_2 = x_1 - g_1(x_1) + R_{M,\lambda}^{H_1,\eta_1}\big(H_1\big(g_1(x_1)\big) - \lambda F(x_1, y_1) - \lambda P(u_1, v_1)\big),$$
$$y_2 = y_1 - g_2(y_1) + R_{N,\rho}^{H_2,\eta_2}\big(H_2\big(g_2(y_1)\big) - \rho G(x_1, y_1) - \rho Q(w_1, z_1)\big).$$

Again by Nadler's theorem, there exist $u_2 \in A(x_2)$, $v_2 \in B(y_2)$, $w_2 \in C(x_2)$ and $z_2 \in D(y_2)$, such that

$$\|u_2 - u_1\| \leqslant \left(1+\frac{1}{2}\right)\tilde{D}\big(A(x_2), A(x_1)\big),$$
$$\|v_2 - v_1\| \leqslant \left(1+\frac{1}{2}\right)\tilde{D}\big(B(y_2), B(y_1)\big),$$
$$\|w_2 - w_1\| \leqslant \left(1+\frac{1}{2}\right)\tilde{D}\big(C(x_2), C(x_1)\big),$$
$$\|z_2 - z_1\| \leqslant \left(1+\frac{1}{2}\right)\tilde{D}\big(D(y_2), D(y_1)\big).$$

By induction, we can obtain the following iterative algorithm for solving problem (3.1) as follows.

Algorithm 3.1. For any given $x_0 \in \mathcal{H}_1$ and $y_0 \in \mathcal{H}_2$, we can compute the sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ and $\{z_n\}$ by iterative schemes such that

$$x_{n+1} = x_n - g_1(x_n) + R^{H_1,\eta_1}_{M,\lambda}\big(H_1\big(g_1(x_n)\big) - \lambda F(x_n, y_n) - \lambda P(u_n, v_n)\big), \tag{3.5}$$
$$y_{n+1} = y_n - g_2(y_n) + R^{H_2,\eta_2}_{N,\rho}\big(H_2\big(g_2(y_n)\big) - \rho G(x_n, y_n) - \rho Q(w_n, z_n)\big), \tag{3.6}$$
$$u_n \in A(x_n), \qquad \|u_{n+1} - u_n\| \leqslant \left(1+\frac{1}{n+1}\right)\tilde{D}\big(A(x_{n+1}), A(x_n)\big), \tag{3.7}$$
$$v_n \in B(y_n), \qquad \|v_{n+1} - v_n\| \leqslant \left(1+\frac{1}{n+1}\right)\tilde{D}\big(B(y_{n+1}), B(y_n)\big), \tag{3.8}$$
$$w_n \in C(x_n), \qquad \|w_{n+1} - w_n\| \leqslant \left(1+\frac{1}{n+1}\right)\tilde{D}\big(C(x_{n+1}), C(x_n)\big), \tag{3.9}$$
$$z_n \in D(y_n), \qquad \|z_{n+1} - z_n\| \leqslant \left(1+\frac{1}{n+1}\right)\tilde{D}\big(D(y_{n+1}), D(y_n)\big) \tag{3.10}$$

for all $n = 0, 1, 2, \ldots$.

4. Existence of solutions and convergence of an iterative algorithm

In this section, we will prove the existence of solutions for problem (3.1) and the convergence of iterative sequences generated by Algorithm 3.1.

Theorem 4.1. *For $i = 1, 2$, let $\eta_i : \mathcal{H}_i \times \mathcal{H}_i \to \mathcal{H}_i$ be Lipshitz continuous with constant τ_i, $H_i : \mathcal{H}_i \to \mathcal{H}_i$ be strongly η_i-monotone and Lipschitz continuous with constant γ_i and δ_i, respectively, $g_i : \mathcal{H}_i \to \mathcal{H}_i$ be strongly monotone and Lipschitz continuous with constant r_i and s_i, respectively. Let $A, C : \mathcal{H}_1 \to CB(\mathcal{H}_1)$, $B, D : \mathcal{H}_2 \to CB(\mathcal{H}_2)$ be $\tilde{D}$-Lipschitz continuous with constants $l_A > 0$, $l_C > 0$, $l_B > 0$ and $l_D > 0$, respectively. Let $F : \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_1$ be strongly monotone with respect to $\hat{g}_1$ in the first argument with constant $\alpha_1 > 0$, Lipschitz continuous in the first argument with constant $\beta_1 > 0$, and Lipschitz continuous in the second argument with constant $\xi_1 > 0$, respectively, where $\hat{g}_1 : \mathcal{H}_1 \to \mathcal{H}_1$ is defined by $\hat{g}_1(x) = H_1 \circ g_1(x) = H_1(g_1(x))$, $\forall x \in \mathcal{H}_1$. Let $G : \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_2$ be strongly monotone with respect to $\hat{g}_2$ in the second argument with constant $\alpha_2 > 0$, Lipschitz continuous in the second argument with constant $\beta_2 > 0$, and Lipschitz continuous in the first argument with constant $\xi_2 > 0$, respectively, where $\hat{g}_2 : \mathcal{H}_2 \to \mathcal{H}_2$ is defined by $\hat{g}_2(y) = H_2 \circ g_2(y) = H_2(g_2(y))$, $\forall y \in \mathcal{H}_2$. Assume that $P : \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_1$ is Lipschitz continuous in the first and second argument with constants*

$\mu_1 > 0$ and ν_1, respectively, $Q : \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_2$ is Lipschitz continuous in the first and second argument with constants $\mu_2 > 0$ and ν_2, respectively, $M : \mathcal{H}_1 \to 2^{\mathcal{H}_1}$ is an (H_1, η_1)-monotone operator and $N : \mathcal{H}_2 \to 2^{\mathcal{H}_2}$ is an (H_2, η_2)-monotone operator.

If there exist constants $\lambda > 0$ and $\rho > 0$ such that

$$\begin{cases} \sqrt{1+2r_1+s_1^2} + \frac{\tau_1}{\gamma_1}\Big(\sqrt{\delta_1^2 s_1^2 - 2\lambda\alpha_1 + \lambda^2\beta_1^2} + \lambda\mu_1 l_A\Big) + \rho(\xi_2 + \mu_2 l_C)\frac{\tau_2}{\gamma_2} < 1, \\ \sqrt{1+2r_2+s_2^2} + \frac{\tau_2}{\gamma_2}\Big(\sqrt{\delta_2^2 s_2^2 - 2\rho\alpha_2 + \rho^2\beta_2^2} + \rho\nu_2 l_D\Big) + \lambda(\xi_1 + \nu_1 l_B)\frac{\tau_1}{\gamma_1} < 1. \end{cases} \tag{4.1}$$

Then problem (3.1) *admits a solution (x, y, u, v, w, z) and sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$, $\{v_n\}$, $\{w_n\}$, $\{z_n\}$ converge to x, y, u, v, w, z, respectively, where $\{x_n\}$, $\{y_n\}$, $\{u_n\}$, $\{v_n\}$, $\{w_n\}$, $\{z_n\}$ are the sequences generated by Algorithm* 3.1.

Proof. Let $a_n \equiv H_1(g_1(x_n)) - \lambda F(x_n, y_n) - \lambda P(u_n, v_n)$.

By (3.5) and Lemma 2.1, we have

$$\begin{aligned} \|x_{n+1} - x_n\| &= \big\| x_n - g_1(x_n) + R_{M,\lambda}^{H_1,\eta_1}\big(H_1\big(g_1(x_n)\big) - \lambda F(x_n, y_n) - \lambda P(u_n, v_n)\big) \\ &\quad - \big[x_{n-1} - g_1(x_{n-1}) + R_{M,\lambda}^{H_1,\eta_1}\big(H_1\big(g_1(x_{n-1})\big) - \lambda F(x_{n-1}, y_{n-1}) \\ &\quad - \lambda P(u_{n-1}, v_{n-1})\big)\big]\big\| \\ &\leqslant \big\| x_n - x_{n-1} - \big[g_1(x_n) - g_1(x_{n-1})\big]\big\| + \big\| R_{M,\lambda}^{H_1,\eta_1}(a_n) - R_{M,\lambda}^{H_1,\eta_1}(a_{n-1})\big\| \\ &\leqslant \big\| x_n - x_{n-1} - \big[g_1(x_n) - g_1(x_{n-1})\big]\big\| + \frac{\tau_1}{\gamma_1}\|a_n - a_{n-1}\|. \end{aligned} \tag{4.2}$$

Since $g_1 : \mathcal{H}_1 \to \mathcal{H}_1$ is strongly monotone and Lipschitz continuous with constant r_1 and s_1, respectively, we have

$$\begin{aligned} &\big\| x_n - x_{n-1} - \big[g_1(x_n) - g_1(x_{n-1})\big]\big\|^2 \\ &\quad \leqslant \|x_n - x_{n-1}\|^2 - 2\big\langle g_1(x_n) - g_1(x_{n-1}), x_n - x_{n-1}\big\rangle + \big\| g_1(x_n) - g_1(x_{n-1})\big\|^2 \\ &\quad \leqslant \big(1 + 2r_1 + s_1^2\big)\|x_n - x_{n-1}\|^2, \end{aligned} \tag{4.3}$$

$$\begin{aligned} \|a_n - a_{n-1}\| &= \big\| H_1\big(g_1(x_n)\big) - \lambda F(x_n, y_n) - \lambda P(u_n, v_n) \\ &\quad - \big[H_1\big(g_1(x_{n-1})\big) - \lambda F(x_{n-1}, y_{n-1}) - \lambda P(u_{n-1}, v_{n-1})\big]\big\| \\ &\leqslant \big\| H_1\big(g_1(x_n)\big) - H_1\big(g_1(x_{n-1})\big) - \lambda\big[F(x_n, y_n) - F(x_{n-1}, y_n)\big]\big\| \\ &\quad + \lambda\big\| F(x_{n-1}, y_n) - F(x_{n-1}, y_{n-1})\big\| + \lambda\big\| P(u_n, v_n) - P(u_{n-1}, v_{n-1})\big\|. \end{aligned} \tag{4.4}$$

Since $F : \mathcal{H}_1 \times H_2 \to \mathcal{H}_1$ is strongly monotone with respect to $\hat{g}_1 = H_1 \circ g_1$ in the first argument with constant $\alpha_1 > 0$ and Lipschitz continuous in the first argument with constant $\beta_1 > 0$, respectively, we get

$$\begin{aligned} &\big\| H_1\big(g_1(x_n)\big) - H_1\big(g_1(x_{n-1})\big) - \lambda\big[F(x_n, y_n) - F(x_{n-1}, y_n)\big]\big\|^2 \\ &\quad \leqslant \big\| H_1\big(g_1(x_n)\big) - H_1\big(g_1(x_{n-1})\big)\big\|^2 \\ &\qquad - 2\lambda\big\langle F(x_n, y_n) - F(x_{n-1}, y_n), H_1\big(g_1(x_n)\big) - H_1\big(g_1(x_{n-1})\big)\big\rangle \\ &\qquad + \lambda^2\big\| F(x_n, y_n) - F(x_{n-1}, y_n)\big\|^2 \\ &\quad \leqslant \big(\delta_1^2 s_1^2 - 2\lambda\alpha_1 + \lambda^2\beta_1^2\big)\|x_n - x_{n-1}\|^2. \end{aligned} \tag{4.5}$$

Since $F:\mathcal{H}_1\times\mathcal{H}_2\to\mathcal{H}_1$ is Lipschitz continuous in the second arguments with constant $\xi_1>0$, we have

$$\big\|F(x_{n-1},y_n)-F(x_{n-1},y_{n-1})\big\|\leqslant\xi_1\|y_n-y_{n-1}\|. \tag{4.6}$$

It follows the Lipschitz continuity of P, the $\tilde{D}$-Lipschitz continuity of A and B, (3.7) and (3.8) that

$$\begin{aligned}
&\big\|P(u_n,v_n)-P(u_{n-1},v_{n-1})\big\|\\
&\quad\leqslant\big\|P(u_n,v_n)-P(u_{n-1},v_n)\big\|+\big\|P(u_{n-1},v_n)-P(u_{n-1},v_{n-1})\big\|\\
&\quad\leqslant\mu_1\|u_n-u_{n-1}\|+\nu_1\|v_n-v_{n-1}\|\\
&\quad\leqslant\mu_1\left(1+\frac{1}{n}\right)\tilde{D}\big(A(x_n),A(x_{n-1})\big)+\nu_1\left(1+\frac{1}{n}\right)\tilde{D}\big(B(y_n),B(y_{n-1})\big)\\
&\quad\leqslant\mu_1\left(1+\frac{1}{n}\right)l_A\|x_n-x_{n-1}\|+\nu_1 l_B\left(1+\frac{1}{n}\right)\|y_n-y_{n-1}\|.
\end{aligned}\tag{4.7}$$

It follows from (4.2)–(4.7) that

$$\begin{aligned}
\|x_{n+1}-x_n\|\leqslant{}&\Bigg[\sqrt{1+2r_1+s_1^2}\\
&+\frac{\tau_1}{\gamma_1}\left(\sqrt{\delta_1^2s_1^2-2\lambda\alpha_1+\lambda^2\beta_1^2}+\lambda\mu_1 l_A\left(1+\frac{1}{n}\right)\right)\Bigg]\|x_n-x_{n-1}\|\\
&+\lambda\left(\xi_1+\nu_1 l_B\left(1+\frac{1}{n}\right)\right)\frac{\tau_1}{\gamma_1}\|y_n-y_{n-1}\|.
\end{aligned}\tag{4.8}$$

Let $b_n\equiv H_2(g_2(y_n))-\rho G(x_n,y_n)-\rho Q(w_n,z_n)$.
By (3.6) and Lemma 2.1, we have

$$\begin{aligned}
\|y_{n+1}-y_n\|={}&\Big\|y_n-g_2(y_n)+R_{N,\rho}^{H_2,\eta_2}\big(H_2\big(g_2(y_n)\big)-\rho G(x_n,y_n)-\rho Q(w_n,z_n)\big)\\
&-\big[y_{n-1}-g_2(y_{n-1})+R_{N,\rho}^{H_2,\eta_2}\big(H_2\big(g_2(y_{n-1})\big)-\rho G(x_{n-1},y_{n-1})\\
&-\rho Q(w_{n-1},z_{n-1})\big)\big]\Big\|\\
\leqslant{}&\big\|y_n-y_{n-1}-\big[g_2(y_n)-g_2(y_{n-1})\big]\big\|+\big\|R_{N,\rho}^{H_2,\eta_2}(b_n)-R_{N,\rho}^{H_2,\eta_2}(b_{n-1})\big\|\\
\leqslant{}&\big\|y_n-y_{n-1}-\big[g_2(y_n)-g_2(y_{n-1})\big]\big\|+\frac{\tau_2}{\gamma_2}\|b_n-b_{n-1}\|.
\end{aligned}\tag{4.9}$$

Since $g_2:\mathcal{H}_2\to\mathcal{H}_2$ is strongly monotone and Lipschitz continuous with constant r_2 and s_2, respectively, we have

$$\begin{aligned}
&\big\|y_n-y_{n-1}-\big[g_2(y_n)-g_2(y_{n-1})\big]\big\|^2\\
&\quad\leqslant\|y_n-y_{n-1}\|^2-2\big\langle g_2(y_n)-g_2(y_{n-1}),y_n-y_{n-1}\big\rangle+\big\|g_2(y_n)-g_2(y_{n-1})\big\|^2\\
&\quad\leqslant\big(1+2r_2+s_2^2\big)\|y_n-y_{n-1}\|^2,
\end{aligned}\tag{4.10}$$

$$\begin{aligned}
\|b_n-b_{n-1}\|={}&\big\|H_2\big(g_2(y_n)\big)-\rho G(x_n,y_n)-\rho Q(w_n,z_n)\\
&-\big[H_2\big(g_2(y_{n-1})\big)-\rho G(x_{n-1},y_{n-1})-\rho Q(w_{n-1},z_{n-1})\big]\big\|\\
\leqslant{}&\big\|H_2\big(g_2(y_n)\big)-H_2\big(g_2(y_{n-1})\big)-\rho\big[G(x_n,y_n)-G(x_n,y_{n-1})\big]\big\|\\
&+\rho\big\|G(x_n,y_{n-1})-G(x_{n-1},y_{n-1})\big\|+\rho\big\|Q(w_n,z_n)-Q(w_{n-1},z_{n-1})\big\|.
\end{aligned}\tag{4.11}$$

Since $G:\mathcal{H}_1\times H_2\to\mathcal{H}_2$ is strongly monotone with respect to $\hat{g}_2$ in the second argument with constant $\alpha_2>0$ and Lipschitz continuous in the second argument with constant $\beta_2>0$, respectively, we obtain

$$\begin{aligned}
&\bigl\|H_2\bigl(g_2(y_n)\bigr)-H_2\bigl(g_2(y_{n-1})\bigr)-\rho\bigl[G(x_n,y_n)-G(x_n,y_{n-1})\bigr]\bigr\|^2\\
&\quad\leqslant\bigl\|H_2\bigl(g_2(y_n)\bigr)-H_2\bigl(g_2(y_{n-1})\bigr)\bigr\|^2\\
&\qquad-2\rho\bigl\langle G(x_n,y_n)-G(x_n,y_{n-1}),H_2\bigl(g_2(y_n)\bigr)-H_2\bigl(g_2(y_{n-1})\bigr)\bigr\rangle\\
&\qquad+\rho^2\bigl\|G(x_n,y_n)-G(x_n,y_{n-1})\bigr\|^2\\
&\quad\leqslant\bigl(\delta_2^2s_2^2-2\rho\alpha_2+\rho^2\beta_2^2\bigr)\|y_n-y_{n-1}\|^2.
\end{aligned}\tag{4.12}$$

Since $G:\mathcal{H}_1\times\mathcal{H}_2\to\mathcal{H}_2$ is Lipschitz continuous in the first arguments with constant $\xi_2>0$, we have

$$\bigl\|G(x_n,y_{n-1})-G(x_{n-1},y_{n-1})\bigr\|\leqslant\xi_2\|x_n-x_{n-1}\|.\tag{4.13}$$

It follows from the Lipschitz continuity of Q, the $\tilde{D}$-Lipschitz continuity of C and D, (3.9) and (3.10) that

$$\begin{aligned}
&\bigl\|Q(w_n,z_n)-Q(w_{n-1},z_{n-1})\bigr\|\\
&\quad\leqslant\bigl\|Q(w_n,z_n)-Q(w_{n-1},z_n)\bigr\|+\bigl\|Q(w_{n-1},z_n)-Q(w_{n-1},z_{n-1})\bigr\|\\
&\quad\leqslant\mu_2\|w_n-w_{n-1}\|+\nu_2\|z_n-z_{n-1}\|\\
&\quad\leqslant\mu_2\left(1+\frac{1}{n}\right)\tilde{D}\bigl(C(x_n),C(x_{n-1})\bigr)+\nu_2\left(1+\frac{1}{n}\right)\tilde{D}\bigl(D(y_n),D(y_{n-1})\bigr)\\
&\quad\leqslant\mu_2l_C\left(1+\frac{1}{n}\right)\|x_n-x_{n-1}\|+\nu_2l_D\left(1+\frac{1}{n}\right)\|y_n-y_{n-1}\|.
\end{aligned}\tag{4.14}$$

It follows from (4.9)–(4.14) that

$$\begin{aligned}
\|y_{n+1}-y_n\|\leqslant{}&\Biggl[\sqrt{1+2r_2+s_2^2}\\
&+\frac{\tau_2}{\gamma_2}\left(\sqrt{\delta_2^2s_2^2-2\rho\alpha_2+\rho^2\beta_2^2}+\rho\nu_2l_D\left(1+\frac{1}{n}\right)\right)\Biggr]\|y_n-y_{n-1}\|\\
&+\rho\left(\xi_2+\mu_2l_C\left(1+\frac{1}{n}\right)\right)\frac{\tau_2}{\gamma_2}\|x_n-x_{n-1}\|.
\end{aligned}\tag{4.15}$$

By (4.8) and (4.15), we have

$$\begin{aligned}
&\|x_{n+1}-x_n\|+\|y_{n+1}-y_n\|\\
&\quad\leqslant\Biggl[\sqrt{1+2r_1+s_1^2}+\frac{\tau_1}{\gamma_1}\left(\sqrt{\delta_1^2s_1^2-2\lambda\alpha_1+\lambda^2\beta_1^2}+\lambda\mu_1l_A\left(1+\frac{1}{n}\right)\right)\\
&\qquad+\rho\left(\xi_2+\mu_2l_C\left(1+\frac{1}{n}\right)\right)\frac{\tau_2}{\gamma_2}\Biggr]\|x_n-x_{n-1}\|\\
&\qquad+\Biggl[\sqrt{1+2r_2+s_2^2}+\frac{\tau_2}{\gamma_2}\left(\sqrt{\delta_2^2s_2^2-2\rho\alpha_2+\rho^2\beta_2^2}+\rho\nu_2l_D\left(1+\frac{1}{n}\right)\right)\\
&\qquad+\lambda\left(\xi_1+\nu_1l_B\left(1+\frac{1}{n}\right)\right)\frac{\tau_1}{\gamma_1}\Biggr]\|y_n-y_{n-1}\|\\
&\quad\leqslant\theta_n\bigl(\|x_n-x_{n-1}\|+\|y_n-y_{n-1}\|\bigr),
\end{aligned}\tag{4.16}$$

where

$$\theta_n = \max\left\{\sqrt{1+2r_1+s_1^2} + \frac{\tau_1}{\gamma_1}\left(\sqrt{\delta_1^2 s_1^2 - 2\lambda\alpha_1 + \lambda^2\beta_1^2} + \lambda\mu_1 l_A\left(1+\frac{1}{n}\right)\right) + \rho\left(\xi_2 + \mu_2 l_C\left(1+\frac{1}{n}\right)\right)\frac{\tau_2}{\gamma_2},\right.$$
$$\left.\sqrt{1+2r_2+s_2^2} + \frac{\tau_2}{\gamma_2}\left(\sqrt{\delta_2^2 s_2^2 - 2\rho\alpha_2 + \rho^2\beta_2^2} + \rho\nu_2 l_D\left(1+\frac{1}{n}\right)\right) + \lambda\left(\xi_1 + \nu_1 l_B\left(1+\frac{1}{n}\right)\right)\frac{\tau_1}{\gamma_1}\right\}.$$

Let

$$\theta = \max\left\{\sqrt{1+2r_1+s_1^2} + \frac{\tau_1}{\gamma_1}\left(\sqrt{\delta_1^2 s_1^2 - 2\lambda\alpha_1 + \lambda^2\beta_1^2} + \lambda\mu_1 l_A\right) + \rho(\xi_2 + \mu_2 l_C)\frac{\tau_2}{\gamma_2},\right.$$
$$\left.\sqrt{1+2r_2+s_2^2} + \frac{\tau_2}{\gamma_2}\left(\sqrt{\delta_2^2 s_2^2 - 2\rho\alpha_2 + \rho^2\beta_2^2} + \rho\nu_2 l_D\right) + \lambda(\xi_1 + \nu_1 l_B)\frac{\tau_1}{\gamma_1}\right\}.$$

Then $\theta_n \to \theta$ as $n \to \infty$. By (4.1), we know that $0 < \theta < 1$ and so (4.16) implies that x_n and y_n are both Cauchy sequences. Thus, there exist $x \in \mathcal{H}_1$ and $y \in \mathcal{H}_2$ such that $x_n \to x$ and $y_n \to y$ as $n \to \infty$.

Now we prove that $u_n \to u \in A(x)$, $v_n \to v \in B(y)$, $w_n \to w \in C(x)$, $z_n \to z \in D(y)$. In fact, it follows from the Lipschitz continuity of A, B, C, D and (3.7)–(3.10) that

$$\|u_n - u_{n-1}\| \leqslant \left(1+\frac{1}{n}\right) l_A \|x_n - x_{n-1}\|, \tag{4.17}$$

$$\|v_n - v_{n-1}\| \leqslant l_B\left(1+\frac{1}{n}\right) \|y_n - y_{n-1}\|, \tag{4.18}$$

$$\|w_n - w_{n-1}\| \leqslant \left(1+\frac{1}{n}\right) l_C \|x_n - x_{n-1}\|, \tag{4.19}$$

$$\|z_n - z_{n-1}\| \leqslant l_D\left(1+\frac{1}{n}\right) \|y_n - y_{n-1}\|. \tag{4.20}$$

From (4.17)–(4.20), we know that u_n, v_n, w_n, z_n are also Cauchy sequences. Therefore, there exists $u \in \mathcal{H}_1$, $v \in \mathcal{H}_2$, $w \in \mathcal{H}_1$, $z \in \mathcal{H}_2$ such that $u_n \to u$, $v_n \to v$, $w_n \to w$, $z_n \to z$ as $n \to \infty$. Further,

$$d\big(u, A(x)\big) \leqslant \|u - u_n\| + d\big(u_n, A(x)\big) \leqslant \|u - u_n\| + \tilde{D}\big(A(x_n), A(x)\big)$$
$$\leqslant \|u - u_n\| + l_A\|x_n - x\| \to 0.$$

Since $A(x)$ is closed, we have $u \in A(x)$. Similarly, $v \in B(y)$, $w \in C(x)$, $z \in D(y)$. By continuity of $g_1, g_2, H_1, H_2, F, G, P, Q, R_{M,\lambda}^{H_1,\eta_1}, R_{N,\rho}^{H_2,\eta_2}$ and Algorithm 3.1, we know that x, y, u, v, w, z satisfy the following relation:

$$g_1(x) = R_{M,\lambda}^{H_1,\eta_1}\big(H_1\big(g_1(x)\big) - \lambda F(x,y) - \lambda P(u,v)\big),$$
$$g_2(y) = R_{N,\rho}^{H_2,\eta_2}\big(H_2\big(g_2(y)\big) - \rho G(x,y) - \rho Q(w,z)\big).$$

By Lemma 3.1, (x, y, u, v, w, z) is a solution of problem (3.1). This completes the proof. □

Remark 4.1. Theorem 4.1 improves and extends those results in [20–26,34,36,37] in several aspects.

Remark 4.2. By the results in Sections 3 and 4, it is easy to obtain the convergence results of iterative algorithms for the other special cases of problem (2.1) with (H, η)-monotone operators. And we omit them here.

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