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Multiplication operators in Köthe–Bochner spaces ☆

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ABSTRACT

Let X be a Banach space and E an order continuous Banach function space over a finite measure μ. We prove that an operator T in the Köthe–Bochner space $E(X)$ is a multiplication operator (by a function in $L^\infty(\mu)$) if and only if the equality $T(g\langle f, x^*\rangle x) = g\langle T(f), x^*\rangle x$ holds for every $g \in L^\infty(\mu)$, $f \in E(X)$, $x \in X$ and $x^* \in X^*$.

1. Introduction

Throughout the paper E is a Banach function space over a finite measure space (Ω, Σ, μ). It is well known that the functions that define multiplication operators in E are exactly those of $L^\infty(\mu)$ (cf. [12, Theorem 1]). Recall that an orthomorphism in a Banach lattice F is a regular operator $\pi : F \to F$ such that $\pi(f) \perp g = 0$ whenever $f \perp g = 0$, $f, g \in F$. The following characterization of orthomorphisms in Banach function spaces goes back to Zaanen [16, Theorem 8] (cf. [1, Example 2.67]).

Theorem 1.1. *A mapping $T : E \to E$ is an orthomorphism if and only if it is a multiplication operator, that is, there is $g_0 \in L^\infty(\mu)$ such that*

$$T(f) = g_0 f \quad \text{for all } f \in E.$$

On the other hand, it is known that an operator in E is a multiplication operator if and only if it commutes with all multiplication operators in E, cf. [5, Proposition 2.2]. More precisely:

Theorem 1.2. *An operator $T : E \to E$ is a multiplication operator if and only if*

$$T(gf) = gT(f) \quad \text{for all } g \in L^\infty(\mu) \text{ and } f \in E.$$

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In this paper we are interested in multiplication operators in *Köthe–Bochner* spaces $E(X)$, where X is a Banach space. It is easy to see that the characterization given in Theorem 1.2 does not work in the vector-valued case, even for $X = \mathbb{R}^2$. Let us show this with the following example.

Example 1.3. Let $\{e_1, e_2\}$ be the canonical basis of $\mathbb{R}^2$. Each $f \in E(\mathbb{R}^2)$ can be written as $f = f_1e_1 + f_2e_2$, where $f_1(\omega) := \langle f(\omega), e_1\rangle$ and $f_2(\omega) := \langle f(\omega), e_2\rangle$. Consider the operator $T : E(\mathbb{R}^2) \to E(\mathbb{R}^2)$ given by $T(f) := f_2e_1 + f_1e_2$. Notice that for each $g \in L^\infty(\mu)$ and $f \in E(\mathbb{R}^2)$ we have

$$T(gf) = (gf)_2e_1 + (gf)_1e_2 = gf_2e_1 + gf_1e_2 = gT(f).$$

However, for the constant function $f := e_1 \in E(\mathbb{R}^2)$, we have $T(f) = e_2 \neq gf$ for every $g \in L^\infty(\mu)$. So T cannot be a multiplication operator.

Our main goal is to prove the following characterization of multiplication operators in $E(X)$. Here $S(X)$ stands for the subspace of $E(X)$ made up of all simple functions.

Theorem 1.4. *Suppose $S(Y)$ is dense in $E(Y)$ whenever Y is a separable closed subspace of X. Let $T : E(X) \to E(X)$ be an operator. The following statements are equivalent*:

(i) *T is a multiplication operator, that is, there is $g_0 \in L^\infty(\mu)$ such that*

$$T(f) = g_0f \quad \text{for all } f \in E(X).$$

(ii) *The equality*

$$T\big(g\langle f, x^*\rangle x\big) = g\langle T(f), x^*\rangle x$$

holds for every $g \in L^\infty(\mu)$, $f \in E(X)$, $x \in X$ and $x^* \in X^*$.

The assumption on density of simple functions is guaranteed whenever E is order continuous. Thus, our Theorem 1.4 can be applied to the *Lebesgue–Bochner* spaces $L^p(\mu, X)$ for $1 \leqslant p < \infty$. From the technical point of view, the proof of Theorem 1.4 makes use of Markushevich bases. Some related results can be found in [2,13].

At the end of the paper, a similar result for spaces of Pettis integrable functions is also given (Theorem 2.4).

1.1. Terminology

All unexplained terminology can be found in our standard references [7] and [11]. All our linear spaces are real. Given a Banach space Z, the symbol Z^* stands for the topological dual of Z and the duality is denoted by $\langle\cdot,\cdot\rangle$. We write B_Z to denote the closed unit ball of Z. The norm of Z is denoted by $\|\cdot\|_Z$ if needed explicitly. An 'operator' is a linear continuous mapping between Banach spaces. Recall that a Banach space E is called *Banach function space* over a finite measure space (Ω, Σ, μ) if E is a linear subspace of $L^0(\mu)$ such that: (i) if $f \in L^0(\mu)$ and $|f| \leqslant |g|$ μ-a.e. for some $g \in E$, then $f \in E$ and $\|f\|_E \leqslant \|g\|_E$; (ii) the characteristic function χ_A of each $A \in \Sigma$ belongs to E; (iii) the 'identity' defines an operator from E to $L^1(\mu)$. Throughout the paper X is a Banach space. We denote by $E(X)$ the *Köthe–Bochner space* made up of all (equivalence classes of) strongly measurable functions $f : \Omega \to X$ for which the real-valued function $\omega \mapsto \|f(\omega)\|_X$ belongs to E, equipped with the norm

$$\|f\|_{E(X)} := \big\| \|f(\cdot)\|_X \big\|_E.$$

$E(X)$ is a Banach space which coincides with the usual Lebesgue–Bochner space $L^p(\mu, X)$ when $E = L^p(\mu)$, $1 \leqslant p \leqslant \infty$. Given $f \in E(X)$ and $x^* \in X^*$, we write $\langle f, x^*\rangle$ to denote the (equivalence class of the) composition $x^* \circ f$, which belongs to E and satisfies

$$\big\|\langle f, x^*\rangle\big\|_E \leqslant \big\|x^*\big\|_{X^*}\|f\|_{E(X)}.$$

Given $h \in E$ and $x \in X$, we write hx to denote the function of $E(X)$ given by $\omega \mapsto h(\omega)x$. Recall that every $g \in L^\infty(\mu)$ induces a multiplication operator $M_g : E(X) \to E(X)$ by $M_g(f) := gf$. For more information on Köthe–Bochner spaces, we refer the reader to [10].

2. Results

The following lemma is the key to prove Theorem 1.4. Recall first that a *Markushevich basis* (shortly *M-basis*) of X is a family $(x_i, x_i^*)_{i\in I}$, where $x_i \in X$ and $x_i^* \in X^*$, such that: (i) $x_i^*(x_j) = \delta_{i,j}$ (the Kronecker symbol) for every $i, j \in I$, (ii) $X = \overline{\operatorname{span}\{x_i\colon i \in I\}}$ and (iii) $\{x_i^*\colon i \in I\}$ separates the points of X (i.e. for each $x \in X \setminus \{0\}$ there is $i \in I$ such that $x_i^*(x) \neq 0$). It is well known that *every separable Banach space has an M-basis*, cf. [9, Theorem 1.22]. More generally, every weakly compactly generated Banach space has an M-basis, cf. [9, Corollary 5.2]. For complete information on this topic, we refer the reader to [9].

Lemma 2.1. *Suppose X has an M-basis $(x_i, x_i^*)_{i\in I}$. Let $T : E(X) \to E(X)$ be an operator satisfying*

$$T\big(g\langle f, x_j^*\rangle x_i\big) = g\langle T(f), x_j^*\rangle x_i \tag{2.1}$$

for every $g \in L^\infty(\mu)$, $f \in E(X)$ and $i, j \in I$. Then there is $g_0 \in L^\infty(\mu)$ such that $T(f) = g_0 f$ for every $f \in S(X)$. Moreover, if $S(X)$ is dense in $E(X)$, then $T(f) = g_0 f$ for every $f \in E(X)$.

Proof. We first notice that

$$\big\langle T(hx_i), x_j^*\big\rangle = 0, \quad i \neq j,\ h \in E. \tag{2.2}$$

Indeed, (2.1) applied to $f := hx_i \in E(X)$ and $g := 1 \in L^\infty(\mu)$ yields

$$\big\langle T(hx_i), x_j^*\big\rangle x_i = T\big(\langle hx_i, x_j^*\rangle x_i\big) = T(0) = 0,$$

and so $\langle T(hx_i), x_j^*\rangle = 0$ (bear in mind that $x_i \neq 0$).

For each $i \in I$ we define $T_i : E \to E$ by

$$T_i(h) := \big\langle T(hx_i), x_i^*\big\rangle.$$

Observe that T_i is an operator, because

$$\begin{aligned}\big\|T_i(h)\big\|_E &= \big\|\big\langle T(hx_i), x_i^*\big\rangle\big\|_E \\ &\leqslant \big\|x_i^*\big\|_{X^*}\big\|T(hx_i)\big\|_{E(X)} \leqslant \big\|x_i^*\big\|_{X^*}\|T\|\|hx_i\|_{E(X)} \leqslant C_i\|h\|_E\end{aligned}$$

for all $h \in E$, where $C_i := \|x_i^*\|_{X^*}\|T\|\|x_i\|_X$.

We claim that each T_i satisfies

$$T_i(gh) = gT_i(h), \quad g \in L^\infty(\mu),\ h \in E. \tag{2.3}$$

Indeed, (2.1) with $i = j$ applied to $f := hx_i \in E(X)$ yields

$$T(ghx_i) = T\big(g\langle hx_i, x_i^*\rangle x_i\big) = g\big\langle T(hx_i), x_i^*\big\rangle x_i,$$

hence $T_i(gh) = \langle T(ghx_i), x_i^*\rangle = g\langle T(hx_i), x_i^*\rangle = gT_i(h)$.

For each $i \in I$ equality (2.3) allows us to apply Theorem 1.2 to find $g_i \in L^\infty(\mu)$ such that

$$\big\langle T(hx_i), x_i^*\big\rangle = T_i(h) = g_i h, \quad h \in E. \tag{2.4}$$

We claim that $g_i = g_j$ for every $i, j \in I$. Indeed, fix $i \neq j$ and consider the function $f := x_i + x_j \in E(X)$. By (2.4), (2.2) and (2.1) (with $g := 1 \in L^\infty(\mu)$ and $h := 1 \in E$) we have

$$\begin{aligned}g_i x_j &= \big\langle T(x_i), x_i^*\big\rangle x_j \\ &= \big\langle T(x_i), x_i^*\big\rangle x_j + \big\langle T(x_j), x_i^*\big\rangle x_j = \big\langle T(f), x_i^*\big\rangle x_j = T\big(\langle f, x_i^*\rangle x_j\big) = T(x_j)\end{aligned}$$

and, similarly, we also have

$$\begin{aligned}g_j x_j &= \big\langle T(x_j), x_j^*\big\rangle x_j \\ &= \big\langle T(x_i), x_j^*\big\rangle x_j + \big\langle T(x_j), x_j^*\big\rangle x_j = \big\langle T(f), x_j^*\big\rangle x_j = T\big(\langle f, x_j^*\rangle x_j\big) = T(x_j).\end{aligned}$$

Hence $g_i x_j = g_j x_j$ and so $g_i = g_j$ (bear in mind that $x_j \neq 0$).

Therefore, there is $g_0 \in L^\infty(\mu)$ such that

$$\big\langle T(hx_i), x_i^*\big\rangle = g_0 h, \quad h \in E,\ i \in I. \tag{2.5}$$

Fix $A \in \Sigma$ and $x \in X$. Set $h := \chi_A \in E$ and $f := hx \in E(X)$. We will prove that $T(f) = g_0 f$. To this end, fix $\varepsilon > 0$. Since $X = \overline{\operatorname{span}\{x_i\colon i \in I\}}$, we can find $\{i_1, \dots, i_N\} \subset I$ and real numbers $a_{i_1}, \dots, a_{i_N}$ such that

$$\Bigg\|x - \sum_{n=1}^{N} a_{i_n} x_{i_n}\Bigg\|_X \leqslant \varepsilon. \tag{2.6}$$

Set $y := \sum_{n=1}^{N} a_{i_n} x_{i_n} \in X$ and

$$f_0 := hy = \sum_{n=1}^{N} a_{i_n} h x_{i_n} \in E(X), \qquad f_1 := g_0 f_0 = \sum_{n=1}^{N} a_{i_n} g_0 h x_{i_n} \in E(X).$$

For each $i \in I$ equalities (2.2) and (2.5) yield

$$\langle T(f_0), x_i^* \rangle = \sum_{n=1}^{N} a_{i_n} \langle T(h x_{i_n}), x_i^* \rangle = \sum_{n=1}^{N} a_{i_n} \delta_{i_n,i} g_0 h = \langle f_1, x_i^* \rangle,$$

and so $T(f_0) = f_1$ (because $\{x_i^*\colon i \in I\}$ separates the points of X and both $T(f_0)$ and f_1 are strongly measurable), that is, $T(hy) = g_0 hy$. Therefore

$$\|T(f) - g_0 f\|_{E(X)} \leqslant \|T(h(x-y))\|_{E(X)} + \|g_0 h(x-y)\|_{E(X)}.$$

By (2.6) we have

$$\|h(x-y)\|_{E(X)} \leqslant \|h\|_E \|x-y\|_X \leqslant \|h\|_E \varepsilon$$

and so

$$\|T(f) - g_0 f\|_{E(X)} \leqslant C\varepsilon,$$

where $C := (\|T\| + \|g_0\|_{L^\infty(\mu)})\|h\|_E$. As $\varepsilon > 0$ is arbitrary, $T(f) = g_0 f$.

It follows at once that $T(f) = g_0 f$ for every $f \in S(X)$. If in addition $S(X)$ is dense in $E(X)$, the equality $T(f) = g_0 f$ holds for every $f \in E(X)$, because both T and $M_{g_0} : E(X) \to E(X)$, $M_{g_0}(f) := g_0 f$, are continuous. The proof is over. □

We will also need the following lemma.

Lemma 2.2. *Let $T : E(X) \to E(X)$ be an operator satisfying*

$$T(g\langle f, x^* \rangle x) = g\langle T(f), x^* \rangle x \tag{2.7}$$

for every $g \in L^\infty(\mu)$, $f \in E(X)$, $x \in X$ and $x^* \in X^*$. Let $Y \subset X$ be a closed subspace such that $S(Y)$ is dense in $E(Y)$. Then T maps $E(Y)$ into itself.

Proof. Fix $f \in E(Y)$ of the form $f = hy$ for some $y \in Y \setminus \{0\}$ and $h \in E$. By the Hahn–Banach theorem there exist $y^* \in Y^*$ with $\langle y, y^* \rangle = 1$ and $x^* \in X^*$ such that $x^*|_Y = y^*$. By applying (2.7) with $g := 1 \in L^\infty(\mu)$ and $x := y$ we get

$$T(f) = T(\langle f, x^* \rangle y) = \langle T(f), x^* \rangle y \in E(Y).$$

The linearity of T implies that $T(S(Y)) \subset E(Y)$. Since $S(Y)$ is dense in $E(Y)$, T is continuous and $E(Y)$ is closed in $E(X)$, it follows that $T(E(Y)) \subset E(Y)$. □

We can now prove our main result:

Proof of Theorem 1.4. (i) $\Rightarrow$ (ii) is straightforward.

(ii) $\Rightarrow$ (i). Fix $x_0 \in X \setminus \{0\}$ and $x_0^* \in X^*$ with $\langle x_0, x_0^* \rangle = 1$. As in the proof of Lemma 2.1, we can define an operator

$$T_0 : E \to E, \qquad T_0(h) := \langle T(hx_0), x_0^* \rangle,$$

which satisfies

$$T_0(gh) = gT_0(h), \quad g \in L^\infty(\mu),\ h \in E.$$

Thus Theorem 1.2 ensures the existence of $g_0 \in L^\infty(\mu)$ such that

$$\langle T(hx_0), x_0^* \rangle = T_0(h) = g_0 h, \quad h \in E. \tag{2.8}$$

We claim that $T(f) = g_0 f$ for all $f \in E(X)$. Indeed, take any $f \in E(X)$. Since f is strongly measurable, there is $A \in \Sigma$ with $\mu(\Omega \setminus A) = 0$ such that $f(A)$ is separable, cf. [7, Theorem 2, p. 42]. Thus $Y := \overline{\operatorname{span}(f(A) \cup \{x_0\})}$ is a separable closed subspace of X such that $f \in E(Y)$ and $x_0 \in Y$. By Lemma 2.2, we have $T(E(Y)) \subset E(Y)$. Clearly, the restriction $T|_{E(Y)}$ satisfies

$$T|_{E(Y)}(g\langle f_1, y^* \rangle y) = g\langle T|_{E(Y)}(f_1), y^* \rangle y$$

for every $g \in L^\infty(\mu)$, $f_1 \in E(Y)$, $y \in Y$ and $y^* \in Y^*$. Lemma 2.1 applied to $T|_{E(Y)}$ (recall that every separable Banach space has an M-basis) ensures the existence of $g \in L^\infty(\mu)$ such that

$$T(f_1) = gf_1, \quad f_1 \in E(Y).$$

Since both $f_0 := x_0$ and f belong to $E(Y)$, we have $T(f_0) = gf_0 = gx_0$ and $T(f) = gf$. On the other hand, (2.8) applied to $h := 1 \in E$ yields

$$g_0 = \langle T(f_0), x_0^* \rangle = \langle gx_0, x_0^* \rangle = g$$

and so $T(f) = g_0 f$, as claimed. The proof is over. $\square$

It is well known that if E is order continuous then $S(X)$ is dense in $E(X)$, cf. [10, Chapter 3]. Thus, we get the following result which can be applied to the Lebesgue–Bochner spaces $L^p(\mu, X)$ for $1 \leqslant p < \infty$. Notice that, in general, $S(X)$ is not dense in $L^\infty(\mu, X)$, cf. [10, Chapter 3].

Corollary 2.3. *Suppose E is order continuous. Let $T : E(X) \to E(X)$ be an operator. The following statements are equivalent*:

(i) *T is a multiplication operator, that is, there is $g_0 \in L^\infty(\mu)$ such that*

$$T(f) = g_0 f \quad \text{for all } f \in E(X).$$

(ii) *The equality*

$$T\big(g\langle f, x^* \rangle x\big) = g\langle T(f), x^* \rangle x$$

holds for every $g \in L^\infty(\mu)$, $f \in E(X)$, $x \in X$ and $x^* \in X^*$.

We finish the paper by pointing out that some of the previous ideas can also be used when dealing with spaces of *Pettis integrable* functions. Standard references on this topic are [14] and [15]. We write $P(\mu, X)$ to denote the normed space of (equivalence classes of) Pettis integrable functions $f : \Omega \to X$, equipped with the so-called *Pettis norm*

$$\|f\|_{Pe} := \sup_{x^* \in B_{X^*}} \int_\Omega \big|\langle f, x^* \rangle\big| \, d\mu.$$

In this space, two functions $f, g : \Omega \to X$ are identified if and only if, for each $x^* \in X^*$, we have $\langle f, x^* \rangle = \langle g, x^* \rangle$. The subspace of $P(\mu, X)$ made up of all strongly measurable Pettis integrable functions is denoted by $P_s(\mu, X)$. Clearly $S(X) \subset P_s(\mu, X)$. Given $h \in L^1(\mu)$ and $x \in X$, we write hx to denote the function of $P_s(\mu, X)$ given by $\omega \mapsto h(\omega)x$.

Theorem 2.4. *Suppose X has an M-basis $(x_i, x_i^*)_{i \in I}$. Let $Z \subset P(\mu, X)$ be a subspace such that*:

(a) *$P_s(\mu, X) \subset Z$,*
(b) *$gf \in Z$ whenever $g \in L^\infty(\mu)$ and $f \in Z$.*

Let $T : Z \to Z$ be an operator satisfying:

$$T\big(g\langle f, x_j^* \rangle x_i\big) = g\langle T(f), x_j^* \rangle x_i$$

for every $g \in L^\infty(\mu)$, $f \in Z$ and $i, j \in I$. Then there is $g_0 \in L^\infty(\mu)$ such that $T(f) = g_0 f$ for every $f \in S(X)$. Moreover, if $S(X)$ is dense in Z, then $T(f) = g_0 f$ for every $f \in Z$.

Sketch of proof. Just mimic the proof of Lemma 2.1 replacing E by $L^1(\mu)$. Bear in mind that two Pettis integrable functions $f, g : \Omega \to X$ have the same equivalence class in $P(\mu, X)$ if and only if, for each $i \in I$, we have $\langle f, x_i^* \rangle = \langle g, x_i^* \rangle$. Indeed, observe that, given any $A \in \Sigma$, the latter implies that $\langle \int_A f \, d\mu, x_i^* \rangle = \langle \int_A g \, d\mu, x_i^* \rangle$ for every $i \in I$, hence $\int_A f \, d\mu = \int_A g \, d\mu$. $\square$

Several subspaces Z of $P(\mu, X)$ satisfy conditions (a) and (b) of Theorem 2.4, namely: both $P_s(\mu, X)$ and $P(\mu, X)$ (cf. [14, Theorem 4.3]), the subspace of all Birkhoff integrable functions [3,4] and the subspace of all McShane integrable functions [6,8] (when μ is quasi-Radon).

In general, $S(X)$ is not dense in $P(\mu, X)$. Such density condition is guaranteed whenever μ is Radon or X is weakly compactly generated, cf. [14, Section 9] and [15, Chapter 4]. Without additional assumptions, $S(X)$ is always dense in $P_s(\mu, X)$ as well as in the spaces of Birkhoff and McShane integrable functions.

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