

Two-stream counter-flow heat exchanger equation with time-varying velocities



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ABSTRACT

We study the two-stream counter-flow heat exchanger equation with time-varying fluid velocities. Formulating it into a time-varying boundary control system, a representation formula of the solution is obtained.

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1. Introduction

Heat exchangers are widely used in a large variety of industrial processes and engineering experiments to facilitate the transfer of heat [10,11] between hot and cold fluids or gases. Among which *parallel-flow heat exchangers* and *counter-flow heat exchangers* are most common. Topics related with them have received much attention in recent years, see for instance [1,2,6,7,9,10,14,15,21]. In this paper, we are concerned with the following two-stream counter-flow heat exchanger equation

$$\begin{cases} \frac{\partial z_1}{\partial t}(x, t) = -v_1(t) \frac{\partial z_1}{\partial x}(x, t) + h_1[z_2(x, t) - z_1(x, t)], \\ \frac{\partial z_2}{\partial t}(x, t) = v_2(t) \frac{\partial z_2}{\partial x}(x, t) + h_2[z_1(x, t) - z_2(x, t)], & t > 0, 0 < x < l, \\ z_1(0, t) = u_1(t), \quad z_2(l, t) = u_2(t), & t \geq 0, \\ z_1(x, 0) = z_{10}(x), \quad z_2(x, 0) = z_{20}(x), & 0 < x < l. \end{cases} \quad (1.1)$$

Here $z_1(x, t)$ and $z_2(x, t)$ denote the temperatures of the hot and cold fluids at time t and position x , respectively; $u_1(t)$ and $u_2(t)$ indicate the boundary inputs; $v_1(t)$ and $v_2(t)$ are positive functions reflecting the velocities of the two fluids at time t ; h_1, h_2 are positive thermal constants and l stands for the length of the heat exchanger. See Fig. 1 for the sketch of the heat exchanger. The above system has been studied by Grabowski [6] and its derivation was given in [6, Appendix A]. But, instead of (1.1) itself, what Grabowski actually studied is its variation (see (6) in [6]) corresponding to the *equilibrium state*

$$v_1(t) \equiv v_{1\infty} > 0, \quad v_2(t) \equiv v_{2\infty} > 0, \quad \frac{\partial z_1}{\partial t} = \frac{\partial z_2}{\partial t} = 0.$$

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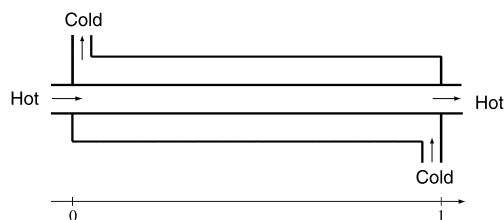


Fig. 1. Sketch of the two-stream counter-flow heat exchanger of unit length. The fluids enter the exchanger from opposite ends.

This leads to a standard autonomous Cauchy system in the state space $Z = L^2(0, l) \times L^2(0, l)$ where the coefficients are no longer time-varying. The control operator is bounded and the unbounded observation operator is defined by the output function

$$y(t) = x_2(0, t),$$

i.e., temperature observation of the cold fluid at $x = 0$. It was shown that the system is *exponentially stable* [6, Lemma 1] and that the observation functional is *infinite-time admissible* [6, Lemma 2]. In addition, using a *circle criterion* coming from the *Leray–Schauder’s fixed point theorem*, the author also investigated the closed-loop system with nonlinear feedback $u(t) = -f[y(t)]$, see [6, Theorem 2] for details.

Whereas many numerical methods have been developed (see e.g. [10, Sect. 3]), some researchers aim at finding solution formulas of heat exchanger equations [1]. This is also the main goal of the present paper. Note that in the case when $v_1(t)$ and $v_2(t)$ are time-independent, following discussion in [9] and formulating (1.1) into a *boundary control system* [4,20], a solution formula can be obtained easily.

The general case gives rise to a time-varying boundary control system and it is more complicated. First of all, instead of *strongly continuous semigroups*, we have to deal with *evolution families*, see for instance [18], [13, Definition 5.5.3] and [5, Definition VI.9.2]. As compared with the semigroup case, in general, we have no explicit expressions for the evolution families which adds many technical difficulties. However, as we will see, the underlying evolution family of (1.1) has an explicit expression and what is more, there is a natural generalization of [4, Theorem 3.3.4] to the time-varying case from which a representation formula of the solution follows.

The rest of this paper is organized as follows. In Section 2, we formulate (1.1) into a time-varying boundary control system. In Section 3, a special type of time-varying boundary control systems including the one associated with (1.1) is studied. A representation formula is given for such systems. Section 4 is devoted to conclusions.

2. A time-varying boundary control system

In this section, we formulate (1.1) into a time-varying boundary control system. First, we introduce the state space

$$Z := L^2(0, l) \times L^2(0, l)$$

and the input space $U := \mathbb{R} \times \mathbb{R}$. Next, we define the operator

$$D := \begin{bmatrix} -\frac{d}{dx} & 0 \\ 0 & \frac{d}{dx} \end{bmatrix} \quad (2.1)$$

with domain

$$\mathcal{D}(D) := \left\{ \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \in H^1(0, l) \times H^1(0, l) : f_1(0) = f_2(l) = 0 \right\}. \quad (2.2)$$

Denote $S_r(t)$ and $S_l(t)$ the right-shift semigroup and left-shift semigroup on $L^2(0, l)$, namely, for each $f \in L^2(0, l)$,

$$(S_r(t)f)(x) = \begin{cases} f(x-t), & x \geq t, \\ 0, & x < t, \end{cases} \quad (S_l(t)f)(x) = \begin{cases} f(x+t), & x+t \leq l, \\ 0, & x+t > l. \end{cases}$$

Then it follows that D is the generator of the C_0 -semigroup

$$S(t) = \begin{bmatrix} S_r(t) & 0 \\ 0 & S_l(t) \end{bmatrix}. \quad (2.3)$$

By the way, it is clear that both $S_r(t)$ and $S_l(t)$ are *nilpotent*, precisely, $S_r(t) = S_l(t) = 0$ for all $t > l$ which implies $S(t)$ is also nilpotent and $S(t) = 0$ for all $t > l$. Further, writing

$$H := \begin{bmatrix} -h_1 & h_1 \\ h_2 & -h_2 \end{bmatrix}, \quad V(t) := \begin{bmatrix} v_1(t) & 0 \\ 0 & v_2(t) \end{bmatrix} \quad (2.4)$$

and

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad z(t) = \begin{bmatrix} z_1(\cdot, t) \\ z_2(\cdot, t) \end{bmatrix}, \quad z_0 = \begin{bmatrix} z_{10} \\ z_{20} \end{bmatrix},$$

(1.1) can be formulated into the following time-varying boundary control system

$$\dot{z}(t) = P(t)z(t), \quad z(t)|_\gamma = u(t), \quad z(0) = z_0. \quad (2.5)$$

Here γ stands for the boundary (two endpoints) of the interval $(0, 1)$ and

$$P(t) := D(t) + H, \quad D(t) := V(t)D. \quad (2.6)$$

This suggests application of the theory of evolution family [13, Definition 5.5.3] (see also [5, Definition VI.9.2]), the generalization of C_0 -semigroup to solve the *nonautonomous* Cauchy problem

$$\dot{x}(t) = A(t)x(t), \quad t \geq s, \quad x(s) = x_0. \quad (2.7)$$

Definition 2.1. Let $U(t, s)$ ($0 \leq s \leq t < \infty$) be a family of bounded operators on Banach space X . It is called an evolution family on $\mathbb{R}_+ := [0, \infty)$ if

- (i) $U(t, t) = I$ for all $t \geq 0$.
- (ii) $U(t, s) = U(t, r)U(r, s)$ for all $0 \leq r \leq s \leq t < \infty$.
- (iii) For each $x \in X$, $(t, s) \mapsto U(t, s)x$ is continuous on $0 \leq s \leq t < \infty$.

We refer to the monographs [3,5,13], the surveys [16,18] and the references therein for more information about evolution families and [20, Chapter 10] for general theory of boundary control systems. The following definition comes from [18, Definition 2.2]:

Definition 2.2. Let $\{A(t)\}_{t \geq 0}$ be a family of generators of C_0 -semigroups and $U(t, s)$ ($0 \leq s \leq t < \infty$) an evolution family on Banach space X . We say that $A(t)$ generates $U(t, s)$ if

- (a) $U(t, s)\mathcal{D}(A(s)) \subset \mathcal{D}(A(t))$ for all $t \geq s \geq 0$.
- (b) For each $x_0 \in \mathcal{D}(A(s))$, the function $t \mapsto U(t, s)x_0$ is continuously differentiable on $[s, \infty)$ and it is the unique solution to (2.7).

Remark 2.3. We note that $A(t)$ generates the evolution family $U(t, s)$ in the above sense if and only if (2.7) is well-posed on $Y_t = \mathcal{D}(A(t))$ in the sense of [5, Definition VI.9.1] (see also [17, Definition 2.1]). In this case, the solution of (2.7) is $U(t, s)x_0$. In addition, it is known that $A(t)$ generates at most one evolution family.

The following definition is taken from [5, Definition VI.9.2]:

Definition 2.4. We call an evolution family $U(t, s)$ solves the Cauchy problem (2.7) on spaces Y_t if there are dense subspaces Y_s of X such that

$$U(t, s)Y_s \subset Y_t \subset \mathcal{D}(A(t)), \quad \forall t \geq s$$

and the function $t \mapsto U(t, s)x_0$ is a solution of (2.7) for each $s \geq 0$ and $x_0 \in Y_s$.

Proposition 2.5. Let A be the generator of a C_0 -semigroup $T(t)$ on Banach space X and $a(t)$ a positive and continuous scalar function on $[0, \infty)$. Then $A(t) = a(t)A$ generates the evolution family

$$U(t, s) = T\left(\int_s^t a(\tau) d\tau\right), \quad 0 \leq s \leq t < \infty. \quad (2.8)$$

Proof. Let $0 \leq s \leq t < \infty$ and $U(t, s)$ be as in (2.8). Since $a(t)$ is positive and continuous, it is trivial to check that $U(t, s)$ is an evolution family and that $U(t, s)\mathcal{D}(A) \subset \mathcal{D}(A)$ for all $t \geq s$. Moreover, for each $x \in \mathcal{D}(A)$, using the chain rule we see easily that $t \mapsto U(t, s)x$ is a solution to

$$\dot{x}(t) = a(t)Ax(t), \quad t \geq s, \quad x(s) = x_0. \quad (2.9)$$

Therefore, the evolution family $U(t, s)$ defined as in (2.8) solves (2.9) on the spaces $Y_t \equiv \mathcal{D}(A)$. Thus, it follows from [8, Proposition 1.4] (see also [12, Proposition 3.10]) that (2.9) is well-posed on $Y_t \equiv \mathcal{D}(A)$. Hence, the solution $t \mapsto U(t, s)x_0$ is also unique and according to Definition 2.2, $U(t, s)$ is the evolution family generated by $A(t) = a(t)A$. \square

Remark 2.6. As an immediate consequence of Proposition 2.5, we know that

$$U_0(t, s) := \begin{bmatrix} S_r(\int_s^t v_1(\tau) d\tau) & 0 \\ 0 & S_l(\int_s^t v_2(\tau) d\tau) \end{bmatrix}, \quad 0 \leq s \leq t < \infty \quad (2.10)$$

is the evolution family generated by $D(t) = V(t)D$ in (2.6) if $v_1(\cdot)$ and $v_2(\cdot)$ are continuous.

3. Solution

First, following the notation in [4, Sect. 3.3] we recall some facts on time-invariant boundary control system

$$\dot{z}(t) = \mathfrak{A}z(t), \quad z(0) = z_0, \quad \mathfrak{B}z(t) = u(t) \quad (3.1)$$

with state space Z and input space U . Here $\mathfrak{A} : \mathcal{D}(\mathfrak{A}) \subset Z \rightarrow Z$, and

$$\mathfrak{B} : \mathcal{D}(\mathfrak{B}) \rightarrow U, \quad \mathcal{D}(\mathfrak{A}) \subset \mathcal{D}(\mathfrak{B}) \subset Z$$

denotes the boundary operator. The standing assumptions in this section are:

- (H₁) the operator $Az = \mathfrak{A}z$ with $\mathcal{D}(A) = \mathcal{D}(\mathfrak{A}) \cap \ker(\mathfrak{B})$ generates a C_0 -semigroup on Z .
 (H₂) there exists $B \in \mathcal{L}(U, Z)$ such that $\text{Ran } B \subset \mathcal{D}(\mathfrak{A})$, $\mathfrak{A}B \in \mathcal{L}(U, Z)$ and $\mathfrak{B}Bu = u$ for all $u \in U$.

Definition 3.1. We call $z(\cdot) \in C^1([0, \infty), Z)$ a solution to (3.1) or (3.2) if it satisfies (3.1) or (3.2) in Z for all $t \geq 0$ or $t \geq s$.

Under the above assumptions, it is well known that (3.1) has a unique solution [4, Theorem 3.3.4]

$$z(t) = Bu(t) + T(t)[z_0 - Bu(0)] + \int_0^t T(t-s)[\mathfrak{A}Bu(s) - B\dot{u}(s)] ds$$

provided $u(\cdot) \in C^2([0, \infty), U)$ and the compatibility condition

$$z_0 - Bu(0) \in \mathcal{D}(A).$$

We shall generalize this result to a special type of time-varying boundary control systems of the form

$$\dot{z}(t) = [a(t)\mathfrak{A} + K]z(t), \quad z(s) = z_0, \quad \mathfrak{B}z(t) = u(t) \quad (3.2)$$

where $K \in \mathcal{L}(Z)$ is a bounded perturbation. This generalization enables us to obtain an expression formula of the solution to (1.1).

Remark 3.2. Throughout this section, $a(t)$ is a scalar function or a little more general, a diagonal matrix function with each component being scalar function, like $V(t)$ in (2.4). The latter case corresponds to the situation when \mathfrak{A} and A are diagonal operator matrices, just like D in (2.1). In this case, $a(t)$ is called positive or continuous if so is each scalar component.

The following result comes from [13, Lemma 5.4.5] which extends the Dyson–Phillips series (see e.g. [5, Corollary III.3.15]) from semigroups to evolution families. Note that in the case when A is a diagonal operator matrix, A generates a contraction semigroup if and only if so does each its component.

Lemma 3.3. Let A be the generator of a contraction semigroup $T(t)$ on Banach space X . Assume that

- (i) $a(\cdot)$ is positive and continuously differentiable on $[0, \infty)$.
 (ii) The operator $K \in \mathcal{L}(X)$.

Then $a(t)A + K$ generates the evolution family

$$U(t, s) = \sum_{k=0}^{\infty} U_k(t, s), \quad 0 \leq s \leq t < \infty. \quad (3.3)$$

Here $U_0(t, s)$ is the evolution family generated by $a(t)A$ and $U_k(t, s)$ is given inductively by

$$U_k(t, s)_X := \int_s^t U_0(t, \tau) K U_{k-1}(\tau, s) x d\tau, \quad k = 1, 2, \dots \quad (3.4)$$

Proof. First, noting that A generates a contraction semigroup, we know each $a(t)A$ generates a contraction semigroup. Consequently, $a(t)A$ is a *stable family* [13, Definition 5.2.1] with stability constants

$$M = 1, \quad \omega = 0.$$

Moreover, from Proposition 2.5 we know that the operator family $a(t)A$ generates an evolution family $U_0(t, s)$ satisfying

$$\|U_0(t, s)\| \leq 1, \quad 0 \leq s \leq t < \infty. \quad (3.5)$$

Next, using [13, Theorem 5.2.3] we deduce that the perturbed family $a(t)A + K$ is also stable with stability constants $M = 1$ and $\mu := \|K\|$. Further, it is clear that

$$\mathcal{D}(a(t)A + K) = \mathcal{D}(a(t)A) = \mathcal{D}(A)$$

for all $t \geq 0$ and $a(\cdot)A$ is strongly continuously differentiable since $a(\cdot)$ is continuously differentiable. Therefore, applying [13, Theorem 5.4.8] (see also the corollary of [19, Theorem 4.4.2]) we deduce that $a(t)A + K$ also generates an evolution family $V(t, s)$ satisfying

$$\|V(t, s)\| \leq e^{\mu(t-s)}, \quad 0 \leq s \leq t < \infty.$$

On the other hand, since (3.5) holds, owing to [13, Lemma 5.4.5], there exists a unique evolution family $U(t, s)$ given by (3.3) and (3.4) such that for each $x_0 \in X$, $U(t, s)x_0$ gives the mild solution to the equation

$$\dot{x}(t) = [a(t)A + K]x(t), \quad x(s) = x_0.$$

Thus, by uniqueness we conclude that $U(t, s) = V(t, s)$ and hence $a(t)A + K$ generates the evolution family $U(t, s)$. This completes the proof. \square

Theorem 3.4. Assume that

- (i) $a(\cdot)$ is positive, continuously differentiable on $[0, \infty)$ and $K \in \mathcal{L}(Z)$.
- (ii) The assumptions (H_1) and (H_2) at the beginning of this section hold and moreover, A generates a contraction semigroup.
- (iii) $u(\cdot) \in C^2([0, \infty), U)$ and $z_0 \in \mathcal{D}(A)$ with $z_0 - Bu(0) \in \mathcal{D}(A)$.

Then the time-varying boundary control system (3.2) has a unique solution

$$\begin{aligned} z(t) &= U(t, s)[z_0 - Bu(0)] + Bu(t) \\ &\quad + \int_s^t U(t, \tau)[a(\tau)\mathfrak{A}Bu(\tau) + KBu(\tau) - B\dot{u}(\tau)]d\tau. \end{aligned} \quad (3.6)$$

Here, $U(t, s)$ is the evolution family given by (3.3).

Proof. First, if (iii) is satisfied, analogous to the proof of [4, Theorem 3.3.3], it is easy to check that the solution of (3.2) and the solution of

$$\begin{cases} \dot{\omega}(t) = [a(t)A + K]\omega(t) - B\dot{u}(t) + [a(t)\mathfrak{A} + K]Bu(t), \\ \omega(s) = \omega_0 \end{cases} \quad (3.7)$$

are related by

$$z(t) = \omega(t) + Bu(t). \quad (3.8)$$

Next, consider the evolution equation

$$\dot{\omega}(t) = [a(t)A + K]\omega(t), \quad \omega(s) = \omega_0. \quad (3.9)$$

From Lemma 3.3 we know that $a(t)A + K$ generates the evolution family (3.3). As an immediate consequence, (3.9) has a unique solution $U(t, s)\omega_0$ for each $\omega_0 \in \mathcal{D}(A)$. Note that it has been shown in the proof of Lemma 3.3 that $a(t)A + K$ is a stable family. Last, denote

$$f(t) := [a(t)\mathfrak{A} + K]Bu(t) - B\dot{u}(t). \quad (3.10)$$

Then the function $f(\cdot)$ is continuously differentiable in view of the assumption $u(\cdot) \in C^2([0, \infty), U)$ and the assumption (H_2) at the beginning of this section. In addition, for each $\omega_0 \in \mathcal{D}(A)$, the function $[a(\cdot)A + K]\omega_0$ is continuously differentiable on $[0, \infty)$ since so is $a(\cdot)$. Thus, it follows immediately from [13, Theorem 5.5.3] (see also [19, Theorem 4.5.3]) that (3.7) has a unique solution which is given by

$$\omega(t) = U(t, s)\omega_0 + \int_s^t U(t, \tau)f(\tau) d\tau. \quad (3.11)$$

Combining (3.10) and (3.11) with (3.8) we complete the proof. \square

The following is the main result of this paper. It says that (1.1) has a unique solution for each pair of inputs and outputs which are smooth enough and satisfy the compatibility condition.

Theorem 3.5. Consider Eq. (1.1) and suppose that

- (i) $v_1(\cdot), v_2(\cdot) \in C^1[0, \infty)$.
- (ii) $u_1(\cdot), u_2(\cdot) \in C^2[0, \infty)$ and $z_{10}, z_{20} \in H^1(0, l)$ with

$$z_{10}(0) = u_1(0), \quad z_{20}(l) = u_2(0).$$

Then (1.1) has a unique solution

$$\begin{aligned} \begin{bmatrix} z_1(t, x) \\ z_2(t, x) \end{bmatrix} &= \begin{bmatrix} (x+1)u_1(t) \\ \frac{x}{l}u_2(t) \end{bmatrix} + U(t, 0) \begin{bmatrix} z_{10}(x) - (1+x)u_1(0) \\ z_{20}(x) - \frac{x}{l}u_2(0) \end{bmatrix} \\ &\quad + \int_0^t U(t, \tau) \begin{bmatrix} \zeta_1(x, \tau) \\ \zeta_2(x, \tau) \end{bmatrix} d\tau. \end{aligned} \quad (3.12)$$

Here, $U(t, s)$ is the evolution family generated by $V(t)D + H$ and

$$\begin{bmatrix} \zeta_1(x, \tau) \\ \zeta_2(x, \tau) \end{bmatrix} := \begin{bmatrix} -v_1(\tau)u_1(\tau) + h_1[\frac{x}{l}u_2(\tau) - (1+x)u_1(\tau)] - (x+1)\dot{u}_1(\tau) \\ \frac{v_2(\tau)}{l}u_2(\tau) + h_2[(1+x)u_1(\tau) - \frac{x}{l}u_2(\tau)] - \frac{x}{l}\dot{u}_2(\tau) \end{bmatrix}.$$

Proof. This is a consequence of Theorem 3.4 and Lemma 3.3. First, since (i) is satisfied, by Lemma 3.3 we know that $V(t)D + H$ generates an evolution family $U(t, s)$ whose expression can be given by (3.3) and (3.4) with $U_0(t, s)$ being as in (2.10). Next, we claim that the assumption (ii) of Theorem 3.4 holds. Actually, the operator D in (2.1) and (2.2) generates a contraction semigroup, therefore, the hypothesis (H_1) at the beginning of this section is satisfied. Take input space $U = \mathbb{R} \times \mathbb{R}$. Let

$$\begin{aligned} \mathfrak{A} &= \begin{bmatrix} -\frac{d}{dx} & 0 \\ 0 & \frac{d}{dx} \end{bmatrix}, \quad \mathcal{D}(\mathfrak{A}) = H^1(0, l) \times H^1(0, l), \\ \mathfrak{B} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} &= \begin{bmatrix} f_1(0) \\ f_2(l) \end{bmatrix}, \quad \forall \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \in \mathcal{D}(\mathfrak{B}) := \mathcal{D}(\mathfrak{A}) \end{aligned}$$

and define $B \in \mathcal{L}(U, Z)$ by

$$B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} := \begin{bmatrix} (x+1)u_1 \\ \frac{x}{l}u_2 \end{bmatrix}, \quad \forall \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in U.$$

Then it is trivial to verify that all the conditions in (H_2) are satisfied. In particular, for each $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \in U$ it holds that

$$\mathfrak{B}Bu = \mathfrak{B} \begin{bmatrix} (x+1)u_1 \\ \frac{x}{l}u_2 \end{bmatrix} = \begin{bmatrix} (x+1)u_1|_{x=0} \\ \frac{x}{l}u_2|_{x=l} \end{bmatrix} = u.$$

Last, letting $s = 0$ in (3.2) and applying Theorem 3.4, simple calculations yield the unique solution (3.12). This ends the proof. \square

4. Conclusions

We have obtained an expression formula of the solution to the two-stream counter-flow heat exchanger equation with time-varying fluid velocities by formulating it into a time-varying boundary control system. Obviously, all the discussions also apply to the two-stream and multiple-stream parallel-flow heat exchanger equations.

In this paper, the velocity functions are assumed to be C^1 and the input functions are assumed to be C^2 . It is meaningful and challenging to relax the smoothness assumptions. The author wonders Lemma 3.3 remains true in the case when $a(\cdot)$ is only continuous. Accordingly, for the velocity functions, it is enough to assume continuity. Note that as indicated in the proof

of Lemma 3.3, we have at least mild solution in this case. We refer to [18, Sect. 2] for some information on time-varying multiplicative perturbation of semigroup generators.

Another aspect deserving further study is to investigate the underlying evolution family $U(t, s)$ qualitatively and quantitatively. It would be interesting to solve (1.1) numerically and then to compare the numerical results with those from theoretical analysis.

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